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An Allais Measure of Production Sector Waste due to Quotas

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In this paper we adapt a partial equilibrium approach of Allais and Diewert to measure the efficiency loss in the producing sector due to quotas. The measure of waste is the additional profits available by reallocation subject to constraints that the welfare of persons and firms outside the sector is unaffected. It is relevant to a sector which faces fixed prices for some commodities, but endogenous prices for others Tobacco quotas in the United States are estimated to have caused quota-induced producer sector waste of approximately $95 million per year during 1950–82, or about 3% of the average value of the crop

Key words: deadweight loss, production sector, quotas, waste

Agricultural production quotas are a policy instrument that many countries have adopted as a means of transferring income from consumers to producers. Evaluation of the social waste, or deadweight loss, due to these policies will be an important task as countries adjust to new international trading rules.[1] In addition, the environmental movement has led to potential new uses of quotas to bring resource use and resource contamination closer to socially optimal levels, and this will place further burdens on the adequacy of economic analysis of the welfare effects of quotas. Despite the importance of these issues, the welfare analysis of quotas has received little attention, even though the case of quotas differs from that of price interventions because the analysis must be explicit in quantity space with corresponding evaluations in terms of virtual prices.[2] Furthermore, the current practice in agricultural and resource economics is to measure deadweight loss using Marshallian or Hicksian social surplus triangles, but these concepts imply interpersonal welfare assumptions that many analysts would not wish to make. The contribution of the present study is to extend Diewert's (1983, 1987) concept of production sector waste to the case of quotas to provide an empirically useful alternative deadweight loss measure.

It is useful at this point to clarify the limitations of other approaches to measuring welfare loss due to quotas.[3] We start with the observation that the willingness of a consumer or producer to pay to exchange the current set of external circumstances (prices, for example) for some hypothetical alternative set of circumstances is fundamental to welfare measurement. It is the intuitive notion that underlies consumers' and producers' surplus as introduced by Dupuit and Marshall and later refined by Hicks (1942). If one could evaluate each agent's willingness to pay to exchange the set of effective prices under a quota regime for the hypothetical set of prices that would exist without quotas, then aggregation across individuals would provide a money metric measure of the net welfare effects of the quotas. If those who gain from the change would be willing to pay more for the change than losers would require as compensation for accepting the change, then the excess amount is

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[1] Deadweight loss also referred to as "waste," "excess burden," "social cost," and "distributable surplus," refers to the cost to society of using an incentive-distorting instrument to achieve a transfer, rather than a simple lump-sum transfer of the same amount

[2] The distributional effects of quotas also require special theoretical elaboration relative to the case of tax instruments, since quotas themselves imply both positive and negative transfers among quota market participants, whereas taxes imply transfers from all market participants to the government for subsequent redistribution

[3] Just, Hueth, and Smith (chap 1) provide a good review of conceptual approaches related to willingness to pay, and of the difficulties relating to compensation

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a measure of deadweight loss due to the quotas.

The issue of compensation presents a paradox in this approach to welfare analysis. If the analysis assumes that winners do not actually compensate losers, net aggregate willingness to pay has little if any significance as a welfare measure unless one accepts the assumption that the utility value of a dollar lost by each loser (or the welfare function evaluation of that utility value) is exactly equal to the utility value of a dollar gained by each winner. The Hicks-Kaldor criterion asserts that even in the absence of compensation this measure is a valid guide to social choice, but we assert that the assumption is so strong that it is useful to explore alternative conceptual measures.

Paradoxically, if winners were to compensate losers, then the reference equilibrium usually examined is not the appropriate equilibrium.[4] The analysis must be altered so as to identify a reference equilibrium that is appropriate to the modified income distribution; this requires a general equilibrium approach. Evaluations of deadweight loss in a general equilibrium framework go back to Pareto,[5] but most are based on aggregate net willingness to pay without compensation (such as Boiteux's aggregation of individual consumers' Hicksian variations in a general equilibrium framework, including the contributions by Hotelling, by Harberger, and by many others).

Allais (1943, 1977), however, developed a fully compensated general equilibrium measure of deadweight loss. He proposed to measure waste as the quantity of a good or basket of goods (perhaps money) that could be extracted from an economy by reallocating, subject to the condition that the satisfaction level of every consuming unit remains unchanged.[6] Debreu's coefficient of resource utilization is a related measure of waste, as is Diewert's (1983, 1987) measure of producer sector waste. At an Allais reference equilibrium, compensating transfer payments need not be considered, because the allocation is one that maintains each consuming unit at its original level of satisfaction.

But the Allais approach is still quite demanding in terms of the information required about consumers' utility functions. This is because the analyst must know enough about each consumer indifference surface to be able to identify all alternative consumption bundles that would keep the consumer on that surface. The ambitiousness of this information requirement led Diewert (1983, 1987) to propose a partial equilibrium approach where only reallocations within the producing sector are considered, while using relevant supply and demand conditions to insure that consumers stay at their initial utility levels. This approach requires information only on technology and distortions within the producing sector, and it provides the basis for the approach that we develop in the following analysis.

A Measure of Production Sector Loss Due to Quotas

We begin with Allais's concept of distributable surplus (italics added):

> In a given situation, the maximum distributable surplus of *any good whatever, for a given group of operators* (consumption or production units) disposing of given resources, may be defined as the maximum quantity of this good which can be made available subject to the triple condition (*i*) that all the indexes of preference of the consumption units in the group maintain values which are at least equal to those they had in the situation considered; (*ii*) that the resources used remain at levels which are at most as high as in the initial situation; and (*iii*) that the production this group makes available to the rest of the economy is at least equal to what it supplied in the initial situation (1977, p. 113).

It is clearly within the bounds of this definition to consider money as the reference good (Allais himself later does so), and to consider, as does Diewert, any arbitrary set of production units as the group for which surplus is examined ("sector" hereafter). Condition (*i*) becomes irrelevant because there are no consumers in such a group, but conditions (*ii*) and (*iii*) remain to insure that consumers and others outside the pro-

[4] Referring to figure 1, the usual reference equilibrium is point *f*, where the supply and demand curves are either Marshallian curves or Hicksian curves that identify compensating or equivalent variation, or general equilibrium curves tracing out general equilibrium response to gradual relaxation of the quota intervention (e.g., Thurman). In each of these cases, however, the reference equilibrium *f* is based on the distribution of incomes that result from these equilibrium price adjustments. If the required compensation transfers are then made, every agent who provides or receives a transfer may behave differently, and an equilibrium different than *f* must then be the appropriate reference equilibrium.

[5] Allais (1973, 1977) summarizes the contribution of the early pioneers in this area, while summaries of more recent developments can be found in Auerbach.

[6] Some refer to this as an efficiency measure of deadweight loss, since, in essence, it examines the minimum amount of a basket of goods required to achieve a given welfare objective (namely, the current one).

ducer group are not made worse off by any reference equilibrium considered. Our implementation of the Allais concept is to measure the maximum amount of additional money (profits) that can be extracted from the production sector[7] by a hypothetical reallocation subject to the last two conditions. If the measure is positive, there are more than enough profits at that reference equilibrium to replace the quotas with lump-sum transfers that leave each firm with the same level of profits as under the quotas.

To develop an explicit model of the above concept, it is useful to distinguish between fixed-price commodities and flex-price commodities. The sector is defined to be a price-taker in the markets for fixed-price commodities. For flex-price goods, however, changes in net exports from the sector or reallocations within the sector will affect equilibrium prices. We measure all commodities except fixed inputs as netputs, negative values indicating quantities used and positive values indicating quantities provided. Using superscripts to index firms and subscripts to index commodities, we define $\mathbf{x^f} = (x_1^f, \ldots, x_N^f)'$ as firm f's netput of N fixed-price commodities with price $\mathbf{p} = (p_1, \ldots, p_N)'$; $\mathbf{y^f} = (y_1^f, \ldots, y_M^f)'$ as firm f's netput of M flex-price commodities with price $\mathbf{r} = (r_1, \ldots, r_M)'$; $\mathbf{q^f} = (q_1^f, \ldots, q_I^f)'$ as firm f's netput quota for I quota commodities with price $\mathbf{w} = (w_1, \ldots, w_I)'$; $\mathbf{z^f} = (z_1^f, \ldots, z_J^f)'$ as firm f's quantities of J fixed inputs; and $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Q}$ as summations across the F firms of the corresponding vectors.

Since variables are defined as netputs, the sums $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Q}$ represent net exports of commodities from the sector. Now consider the initial quota-distorted equilibrium in which aggregate sector profit is $\mathbf{p'X^0} + \mathbf{r^{0\prime}Y^0} + \mathbf{w^{0\prime}Q^0}$, where superscripts 0 represent initial values of those prices and quantities that might be affected by any reallocation within the sector. Our measure of production sector efficiency loss is the maximum amount by which this profit could be increased by a hypothetical reallocation within the sector, subject to the constraints that $\mathbf{Y} = \mathbf{Y^0}$, and $\mathbf{Q} = \mathbf{Q^0}$. These two constraints insure that all prices outside the sector are unchanged by the reallocation; in turn, this insures that the welfare of persons outside the production sector remains unchanged, true to the spirit of Allais's more general measure of waste.

[7] In the particular example of tobacco, the production sector will be defined to include both the subsector of farms involved in tobacco production and the tobacco manufacturing sector, with the quotas being imposed on farm-level production

To derive an algebraic expression of this measure of waste, first define a restricted profit function for each firm as

$$(1)\quad \pi^f(\mathbf{p}; \mathbf{y^f}, \mathbf{q^f}, \mathbf{z^f}) \equiv \max_{\mathbf{x}}\left\{\mathbf{p'x^f} : (\mathbf{x^f}, \mathbf{y^f}, \mathbf{q^f}, \mathbf{z^f}) \in T^f\right\} = \mathbf{p'x^{f*}}(\mathbf{p}; \mathbf{y^f}, \mathbf{q^f}, \mathbf{z^f}).$$

A corresponding sector-level restricted profit function can be defined as

$$(2)\quad \Pi(\mathbf{p}; \mathbf{y}, \mathbf{q}, \mathbf{z}) \equiv \textstyle\sum_f \pi^f(\mathbf{p}; \mathbf{y^f}, \mathbf{q^f}, \mathbf{z^f}) = p'\mathbf{X^*}(\mathbf{p}; \mathbf{y}, \mathbf{q}, \mathbf{z})$$

where $\mathbf{y}$, $\mathbf{q}$, and $\mathbf{z}$, respectively, are $(FM) \times 1$, $(FI) \times 1$, and $(FJ) \times 1$ vectors representing the distribution across all firms of flex-price goods, quota goods, and fixed inputs.

We can now define the measure of production sector waste as the maximum extra profit to be generated within the sector by reallocation of both flex-price commodities ($\mathbf{y}$) and quota commodities ($\mathbf{q}$) across firms; i.e.,

$$(3)\quad L(\mathbf{p}; \mathbf{y^0}, \mathbf{q^0}, \mathbf{z}) \equiv \max_{\mathbf{y},\mathbf{q}}\Big\{\Pi(\mathbf{p}; \mathbf{y}, \mathbf{q}, \mathbf{z}) - \Pi(\mathbf{p}; \mathbf{y^0}, \mathbf{q^0}, \mathbf{z}) : \mathbf{Y} = \mathbf{Y^0},\ \mathbf{Q} = \mathbf{Q^0},\ (\mathbf{x^f}, \mathbf{y^f}, \mathbf{q^f}, \mathbf{z^f}) \in T^f\Big\}.$$

Denote the solution to equation (3) as $\mathbf{y^*}$, $\mathbf{q^*}$ (with associated $\mathbf{x^*}$ and aggregate values $\mathbf{Y^*}$, $\mathbf{Q^*}$, and $\mathbf{X^*}$), and denote the solution values of the Lagrangian multipliers as $\mathbf{r^*}$ and $\mathbf{w^*}$, reflecting internal shadow prices for flex-price and quota commodities, respectively. These values represent the hypothetical reference equilibrium against which the quota-distorted equilibrium is compared. The measure of waste can be expressed as

$$\begin{aligned}(4)\quad & L(\mathbf{p}; \mathbf{y^0}, \mathbf{q^0}, \mathbf{z}) \\ & = \Pi(\mathbf{p}; \mathbf{y^*}, \mathbf{q^*}, \mathbf{z}) - \Pi(\mathbf{p}; \mathbf{y^0}, \mathbf{q^0}, \mathbf{z}), \text{ or} \\ & = \mathbf{p'}(\mathbf{X^*} - \mathbf{X^0}), \text{ or} \\ & = \mathbf{p'}(\mathbf{X^*} - \mathbf{X^0}) + \mathbf{r^{0\prime}}(\mathbf{Y^*} - \mathbf{Y^0}) + \mathbf{w^0}(\mathbf{Q^*} - \mathbf{Q^0}), \text{ or} \\ & = \mathbf{p'}(\mathbf{X^*} - \mathbf{X^0}) + \mathbf{r^{*\prime}}(\mathbf{Y^*} - \mathbf{Y^0}) + \mathbf{w^*}(\mathbf{Q^*} - \mathbf{Q^0}).\end{aligned}$$

In other words, since the optimal reallocation within the sector is subject to the constraint of no change in the net sectoral exports $\mathbf{Y}$ and $\mathbf{Q}$, the measure of waste is unaffected by whether or not **Y** and **Q** are included in the measure of profit, nor by whether they are priced at initial external prices or at the internal shadow prices for the hypothetical reallocation.

The reference equilibrium expressed in equations (3) and (4) is of course not directly observable, but it could be computed from full information about the technologies or profit functions and the initial equilibrium, using a suitable method of solving the simultaneous equations system. Second-order approximations of the profit functions would similarly permit approximation of the reference equilibrium. Diewert offers another set of approximations useful when information about flex-price commodities is not available. First, reallocate quota only, subject to firms' independent optimization under the hypothetical assumption that flex-price commodity prices remain at initial levels

$$(5)\quad L^0(\mathbf{p}, \mathbf{r}^0; \mathbf{q}^0, \mathbf{z}) \equiv \max_{\mathbf{q}}\Big\{\Pi(\mathbf{p}, \mathbf{r}^0; \mathbf{q}, \mathbf{z}) - \Pi(\mathbf{p}, \mathbf{r}^0; \mathbf{q}^0, \mathbf{z}): \mathbf{Q} = \mathbf{Q}^0, (\mathbf{x}^f, \mathbf{y}^f, \mathbf{q}^f, \mathbf{z}^f) \in T^f\Big\}.$$

Second, repeat this exercise under the assumption that flex-price commodity prices are fixed at $\mathbf{r}^*$ [the optimal shadow prices from equation (3)], designating this measurement as $L^*(\mathbf{p}, \mathbf{r}^*; \mathbf{q}^0, \mathbf{z})$. Diewert (1987) has shown that these two measures of loss will bracket the loss defined in equation (3), i.e.,

$$(6)\quad L^*(\mathbf{p}, \mathbf{r}^*; \mathbf{q}^0, \mathbf{z}) \le L(\mathbf{p}; \mathbf{y}^0, \mathbf{q}^0, \mathbf{z}) \le L^0(\mathbf{p}, \mathbf{r}^0; \mathbf{q}^0, \mathbf{z}).$$

Thus, using the initial price of **y**, $\mathbf{r}^0$, as a fixed price tends to overstate the Allais-Diewert measure of production sector waste, while using the unobservable internal equilibrium price $\mathbf{r}^*$ tends to understate it.

Interpretation and Contrast with Other Measures

The above measure of loss examines how much more efficiently resources within the producing sector could be allocated, while protecting consumers from any welfare effect. We would like to contrast this measure with other measures; to do so, it is useful to note that the Hicks-Boiteux general equilibrium compensating or equivalent variations and the general equilibrium Allais-Debreu measures can all be expressed as the sum of the change in an appropriately defined aggregate expenditure function and the change in an appropriately defined aggregate profit function.[8] It is clear that, at the very least, our measure differs from those measures in that it consists only of a change in aggregate profits. Beyond that, the change in profit we examine is for a more restricted version of the aggregate profit function than the others; and it is evaluated at a different set of reference prices (thus a different resource allocation as well), so our measure is not simply the "producer component" of any one of these other measures. Furthermore, it is important to recall that our measure is an aggregate dollar measure of firm profits, whereas the Hicks-Boiteux measures (as well as Marshallian consumer surplus), though expressed in terms of dollars, are indexes of consumer utility in which all consumers are weighted equally.[9]

In further contrast to the more general equilibrium measures of Allais and Debreu,[10] our measure constrains the bundle of flex-price goods available to consumers to the original quantities, with only the quantities of fixed-price goods allowed to vary. The Allais-Debreu models consider any reallocation of all goods that keeps consumers on their original indifference surfaces. This permits our measure to be more explicitly partial equilibrium in nature than those of Allais and Debreu.

An examination of two special cases of this loss measure permits further interpretation and comparisons with other measures of deadweight loss. For concreteness, we describe these special cases in terms of quotas in the cigarette-tobacco-producing sector (figure 1). We assume here that quotas are freely transferable among firms. The quota good, tobacco, is demanded by cigarette manufacturing firms within the sector according to derived demand

[8] See Kay and Keen or Fulginiti and Perrin (1993b) for elaboration and demonstration of this point.

[9] The measure described in this paper also differs in two ways from welfare measures based on the Trade Expenditure Function (Anderson and Neary, Vousden; Martin and Alston). First, measures based on the Trade Expenditure Function are general equilibrium Hicks-Boiteux measures of the money-metric equivalent of foregone consumer welfare, including, but not limited to, misallocations in the producing sector. Second, since the Trade Expenditure Function equilibria do not involve compensation, the implicit welfare function specifies equal weights on each consumer's willingness to pay, whereas our measure does not.

[10] A general equilibrium version of L (measuring production and consumption waste) would include expenditure functions describing consumer behavior given the initial utility allocation. See Fulginiti and Perrin (1993b).

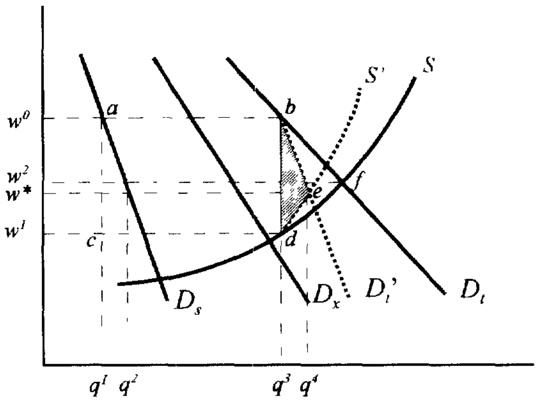

Figure 1. A welfare triangle representing producer sector waste

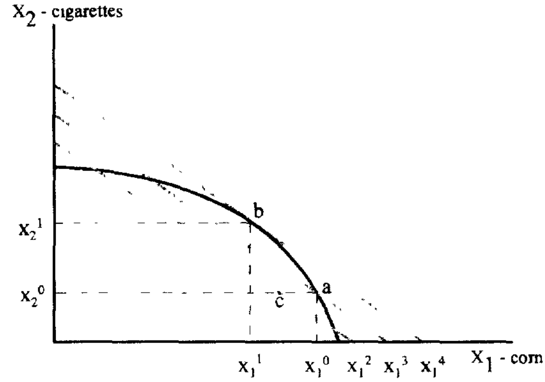

Figure 2. Producer sector waste in production possibilities space

curve D_s, while extra-sectoral demand is D_x, yielding total demand D_t. At the initial distorted equilibrium, quotas to producing firms permit total production of q^3 pounds, while quantity demanded by manufacturing firms (in the absence of user quotas) equals q^1 pounds; the tobacco price is w^0, and net exports from the sector are $Q^0 = q^3 - q^1$. Tobacco rents, area w^0bdw^1, are captured by tobacco-producing firms.

Hypothetical reallocations from the initial distorted equilibrium are constrained by net exports at Q^0, giving rise to the hypothetical total demand curve D_t'. Reallocations are also constrained by holding net exports of other flex-price goods constant at $\mathbf{Y}^0$, which may increase the marginal cost of additional tobacco production yielding the hypothetical supply curve S'. The hypothetical reference reallocation results in an equilibrium with sectoral quota-commodity price w^*, production q^4, sectoral manufacturing demand at q^2, and with net sectoral exports continuing at the initial level. The measure of waste consists of the extra profits equal to the shaded triangle *bde*. This contrasts with the Marshallian surplus triangle *bdf*, which is bounded by the unconstrained market demand and supply curves. At the hypothetical reference equilibrium, net exports from the sector remain at Q^0, and the external price remains at w^0; thus, any rents earned by the sector from restricting outside sales remain at *abdc*. Without loss of generality, the hypothetical reference allocation could include transfers so that each firm is restored to its original level of profits plus rents.

Consider now a second special case of the loss measure, a two-output production sector with quotas applied to a third intermediate good, produced and utilized entirely within the sector. A tobacco-related example would be a sector that exported only cigarettes (x_2) and corn (x_1), with quotas applied to raw tobacco as an intermediate good (figure 2.) With freely tradable quotas, the initial distorted equilibrium might be at point a, with tobacco production quotas effectively limiting cigarette production to x_2^0. If the prices of both cigarettes and corn are fixed to the sector at the ratio indicated by the dashed lines, then the optimal reallocation is at point b, with the loss measured by the distance $x_1^4 - x_1^3$ when expressed in units of corn. If tobacco quotas are not freely tradeable, and therefore not optimally allocated among firms, then the sector may be unable to produce the maximum amount of corn given cigarette production fixed at x_2^0. The initial distorted equilibrium would then occur at some point interior to the production possibilities curve, such as point c; in this case, there would be an additional deadweight loss due to the nontransferability of quotas, equal to the distance $x_1^2x_1^3$.[11]

Second-Order Approximations to the Loss Measure

The expressions for the Allais production loss in equations (3) and (4) do not provide much insight on how the magnitude of the loss depends on the size of the quota. In addition, the allocation $\mathbf{X}^*$ is unobservable, requiring an ap-

[11] In this particular case, the measure coincides with Hicks's (1940) measure of production inefficiency as introduced in his figure 3.2

proximation of the aggregate technology to determine its value. A second-order Taylor series approximation of equation (4) shows the dependence of the measure on the size of the distortion, and it also permits calculation of the producer sector loss from market-observable data. Approximations of L and L^0, at the initial quota allocations $\mathbf{q}^0$, are[12]

$$(7)\quad L(\mathbf{p};\, \mathbf{y}^0,\, \mathbf{q}^0,\, \mathbf{z}) \approx [\mathbf{q}^* - \mathbf{q}^0]'\boldsymbol{\Pi}_{\mathbf{q}} + 1/2[\mathbf{q}^* - \mathbf{q}^0]'\boldsymbol{\Pi}_{\mathbf{qq}}[\mathbf{q}^* - \mathbf{q}^0] + 1/2[\mathbf{q}^* - \mathbf{q}^0]'[\boldsymbol{\Pi}_{\mathbf{qr}}\mathbf{r}_{\mathbf{q}}][\mathbf{q}^* - \mathbf{q}^0]$$

and

$$(8)\quad L^0(\mathbf{p},\, \mathbf{r}^0;\, \mathbf{q}^0,\, \mathbf{z}) \approx [\mathbf{q}^* - \mathbf{q}^0]'\boldsymbol{\Pi}_{\mathbf{q}} + 1/2[\mathbf{q}^* - \mathbf{q}^0]'\boldsymbol{\Pi}_{\mathbf{qq}}[\mathbf{q}^* - \mathbf{q}^0].$$

The first two terms in equation (7) represent the welfare gain in terms of increased producer profit in the markets for fixed-price commodities (holding prices constant), while the last term represents adjustment to this profit due to induced changes in prices of flex-price commodities. This last term illustrates the additional information about firms' flex-price commodity behavior needed in order to evaluate equation (7) as compared to equation (8).

To further illustrate the nature of L^0, consider the situation in which the production sector consists of two distinct sets of firms. In set F_s, all firms are suppliers of tradable-quota commodities with $\mathbf{Q}^s = \Sigma_{F_s}\mathbf{q}^f$, aggregate restricted profits $\Pi^s(\mathbf{p},\, \mathbf{r};\, \mathbf{Q}^s,\, \mathbf{z}^s) = \Sigma_{F_s}\pi^f(\mathbf{p},\, \mathbf{r};\, \mathbf{q}^f,\, \mathbf{z}^f)$, and marginal cost for quota commodities at an initial equilibrium level $\mathbf{w}^s$. In set F_d, all firms are demanders of the quota commodities with $\mathbf{Q}^d = \Sigma_{F_d}\mathbf{q}^f$, aggregate restricted profits $\Pi^d(\mathbf{p},\, \mathbf{r};\, \mathbf{Q}^d,\, \mathbf{z}^d) = \Sigma_{F_d}\pi^f(\mathbf{p},\, \mathbf{r};\, \mathbf{q}^f,\, \mathbf{z}^f)$, and willingess to pay for the quota commodities at the initial equilibrium of $\mathbf{w}^d$. Using $(\mathbf{Q}^{d*} - \mathbf{Q}^{d0}) = -(\mathbf{Q}^{s*} - \mathbf{Q}^{s0})$, equation (8) becomes

$$(9)\quad L^0(\mathbf{p},\, \mathbf{r}^0;\, \mathbf{q}^0,\, \mathbf{z}) \approx 1/2[\mathbf{Q}^{s*} - \mathbf{Q}^{s0}]'[\mathbf{w}^d - \mathbf{w}^s].$$

This is a specific welfare triangle generated by firm behavior when facing exogenous prices $\mathbf{p}$ and $\mathbf{r}^0$, holding aggregate $\mathbf{Q}$ constant. Expression $\mathbf{Q}^{s*}$ is unobservable, but we may approximate it by a first-order Taylor expansion of equation (9) at the distorted equilibrium, yielding an approximation of the triangle as[13]

$$(10)\quad L^0(\mathbf{p},\, \mathbf{r}^0;\, \mathbf{Q}^{s0},\, \mathbf{Q}^{d0},\, \mathbf{z}^s,\, \mathbf{z}^d) \approx -1/2[\mathbf{w}^d - \mathbf{w}^s]'[\boldsymbol{\Pi}^s_{\mathbf{Q}^s\mathbf{Q}^s} + \boldsymbol{\Pi}^d_{\mathbf{Q}^d\mathbf{Q}^d}]^{-1}[\mathbf{w}^d - \mathbf{w}^s].$$

In the case of a single quota commodity, this quantity is the triangle that is shaded in figure 1, where the second derivatives associated with equation (10) are the slopes of S' and D_t'.

Application to U.S. Tobacco

We illustrate the approximate loss measures with application to tobacco quotas in the U.S. The average annual deadweight producer sector loss due to quotas on tobacco production is estimated for the combined tobacco- and cigarette-producing subsectors. The measure consists of the maximum extra profits that could be generated within the sector—an amount over and above that necessary to compensate tobacco-producing firms for quota rents they receive (the transfer which we take to be the objective of the quota policy). This loss measure is particularly useful here because the welfare implications of traditional consumer surplus measures are obscured by externalities in tobacco consumption and by the problems of how to weight U.S. versus foreign consumers.

There are a number of types of tobacco produced in the United States, and most have long been subject to acreage or output quotas with varying degrees of market transferability. In this application we consider the aggregate of all tobaccos for the base period 1950–82. Trade in tobacco products was extensive during this time; about 40% of total tobacco production and 10%–15% of cigarette production were exported, while the share of imported tobacco in cigarettes rose from about 0% to nearly 30%.

Let aggregate profit functions for the tobacco-processing and tobacco-producing subsectors be $\Pi^d(\mathbf{p};\, Q^d,\, \mathbf{z}^d)$ and $\Pi^s(\mathbf{p};\, Q^s,\, \mathbf{z}^s)$, where $\mathbf{p} = (\mathbf{p}_x,\, p_c)$ is the price vector for the relevant netputs, including grain, livestock, foreign tobacco, and cigarettes $[(\mathbf{X},\, X_c)]$; Q^d and Q^s are aggregate quantities of domestic tobacco demanded and supplied; and $\mathbf{z}^d$ and $\mathbf{z}^s$ are vectors of

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[12] To obtain the approximation about the initial value of $\mathbf{q}$, first evaluate L at $\mathbf{r}^*$ so that $L(\mathbf{p},\, \mathbf{r}^*,\, \mathbf{q}^0,\, \mathbf{z}) = \Pi(\mathbf{p},\, \mathbf{r}^*;\, \mathbf{q}^*,\, \mathbf{z}) - \Pi(\mathbf{p},\, \mathbf{r}^*,\, \mathbf{q}^0,\, \mathbf{z})$. Then, recognizing that $\mathbf{r}^*$ is a function of $\mathbf{q}$, use $L_q = \boldsymbol{\Pi}_{\mathbf{q}} + (\boldsymbol{\Pi}_{\mathbf{r}} - \mathbf{y})'\mathbf{r}_{\mathbf{q}}$, where the last term is zero so that $L_{qq} = \boldsymbol{\Pi}_{\mathbf{qq}} + \boldsymbol{\Pi}_{\mathbf{qr}}\mathbf{r}_{\mathbf{q}}$. This permits evaluation of the Taylor expansions of L^*, L^0, or L at $\mathbf{q}$.

[13] At $\mathbf{Q}^*$ we know from equation (5) that $-\boldsymbol{\Pi}^s_{\mathbf{Q}^s} = -\boldsymbol{\Pi}^d_{\mathbf{Q}^d}$. First-order approximation of this equation about the distorted equilibrium yields $\mathbf{w}^s - [\mathbf{Q}^{s*} - \mathbf{Q}^{s0}]\boldsymbol{\Pi}^s_{\mathbf{Q}^s\mathbf{Q}^s} = \mathbf{w}^d - [\mathbf{Q}^{d*} - \mathbf{Q}^{d0}]\boldsymbol{\Pi}^d_{\mathbf{Q}^d\mathbf{Q}^d}$. Substituting $(\mathbf{Q}^{d*} - \mathbf{Q}^{d0}) = -(\mathbf{Q}^{s*} - \mathbf{Q}^{s0})$ and solving for $(\mathbf{Q}^{s*} - \mathbf{Q}^{s0})$, $(\mathbf{Q}^{s*} - \mathbf{Q}^{s0}) = (\mathbf{w}^s - \mathbf{w}^d)'[\boldsymbol{\Pi}^d_{\mathbf{Q}^d\mathbf{Q}^d} + \boldsymbol{\Pi}^s_{\mathbf{Q}^s\mathbf{Q}^s}]^{-1}$

fixed inputs. There are no flex-price commodities because, except for tobacco, there are no other commodities that are both supplied and demanded within this sector, and the sector can be viewed as a price taker relative to the rest of the world economy. We further assume that tobacco quotas apply only to tobacco producers, and that they are optimally distributed among those firms (nontransferability losses are separately measurable, and other estimates are available).[14] This establishes the approximation in equations (9) and (10) as a useful one to measure production sector waste due to the tobacco quotas.

Sumner and Alston estimated a cost function for the U.S. cigarette manufacturing industry defined by $C(\mathbf{p_x}, w; X_c, \mathbf{z})$, where X_c is cigarette output with price p_c, w is the price per pound at which Q^d billion pounds of domestic tobacco are demanded, $\mathbf{p_x}$ is a vector of prices of imported tobacco and other inputs, and $\mathbf{z}$ is a vector of fixed inputs. It can be shown that the second derivative of profits required for equation (10) can be obtained from the estimated cost function C as[15]

$$(11)\quad \Pi^d_{Q^dQ^d}(\mathbf{p}; Q^d, \mathbf{z^d}) = C^{-1}_{w_v w_v}(\mathbf{p_x}, w_v; X^d_c, \mathbf{z^d}).$$

Evaluating the second derivative of the Sumner-Alston cost function at the mean (1950–82) levels of prices and output, and converting from 1972 to 1982 dollars using the CPI index, we evaluate the second derivative of the processor's profit function as

$$(12)\quad \Pi^d_{Q^dQ^d}(\mathbf{p}; Q^d, \mathbf{z^d}) = -1.06$$

in dollars per billion pounds of tobacco.[16] We use the average 1950–82 market price for domestic tobacco, \$1.80/lb in 1982 dollars, as an estimate of w^d.

For the tobacco-producing subsector, a profit function of the kind desired was estimated by Fulginiti and Perrin (1993a) for North Carolina for the period 1950–82. We assume that the results from that study are representative of the entire tobacco-producing subsector of the farm economy. Because that translog profit function must be evaluated at average 1950–82 levels of the variables, we first transform the function to obtain parameters of the profit function, and then scale up these values to the U.S. level by multiplying the estimated slope by the ratio of U.S. to N.C. production.[17] In 1982 dollars, the resulting estimates are

$$(13)\quad \Pi^s_{Q^s}(\mathbf{p}; Q^s, \mathbf{z^s}) = -1.33$$

and

$$\Pi^s_{Q^sQ^s}(\mathbf{p}; Q^s, \mathbf{z^s}) = -0.1035$$

where Q^s is again in billions of pounds.[18] These results imply a wedge of 26% between the average market price and producer price (\$1.80 versus \$1.33), which is consistent with Sumner and Alston's report that quota lease rates averaged 25.6% of market prices during 1977–81. Inserting these estimates into equation (10) yields the estimated production loss of

$$(14)\quad L^0 \approx -0.5(1.80 - 1.33) \cdot (-0.1035 - 1.06)^{-1}(1.80 - 1.33) = 0.095$$

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[14] Production quotas (acreage quotas in earlier years) are assigned to individual farms, and transferability of the quotas among farms, within and between counties, has been subject to limitations of varying severity over the period considered. To the extent that nontransferability results in differing marginal production costs across farms, production inefficiencies are induced beyond those considered in this study. In terms of figure 2, the loss due to nontransferability is $x_i^1 - x_i^2$, while the study here provides only an estimate of the loss from transferable quotas of $x_i^1 - x_i^3$. To capture these additional costs with our methodology we would need an estimate of $\Pi^s(\mathbf{p}, q^1, q^2, \ldots, \mathbf{z^s})$ with as many quota constraints as counties. Alston and Sumner separately estimated the nontransferability portion of losses by noting that quota rental rates in low-rent counties averaged about \$.05/lb less than for the average of all counties, and inferred that nontransferability increased average production costs by this amount (3% of market price, about \$75 million nationally.) Their approach assumed a perfectly elastic supply curve for each county, however. Another estimate of the nontransferability portion of losses is provided by Rucker, Thurman, and Sumner, who estimated marginal cost curves for each county and calculated that nontransferability in North Carolina increased production costs by an average of eight-tenths of 1% of crop value over the period 1977–86. These studies complement the present study in that they provide measures of additional producer sector waste that is due just to the nontransferability of quota.

[15] The relationship of this cost function to a profit function Π^u, where subscript v indicates virtual prices, is

$$\begin{aligned}\Pi^u(\mathbf{p}, w_v, \mathbf{z^d}) &= \max_{\mathbf{X}, X_c, Q}\left\{\mathbf{p_x X} + p_c X_c + w_v Q : (\mathbf{X}, X_c, Q^d, \mathbf{z^d}) \in T\right\}\\ &= \Pi^d(\mathbf{p}, Q^d, \mathbf{z^d}) + w_v Q^d\\ &= -C(\mathbf{p_x}, w : X_c^d, \mathbf{z^d}) + p_c X_c^t\end{aligned}$$

and so $\Pi^u_{w_v w_v} = -C_{w_v w_v}$. Using a result obtained by Fulginiti and Perrin [1993a, equation (20)] and assuming competition in the tobacco industry (Sumner), i.e., $w_v = w$, it can also be shown that

$$\Pi^d_{Q^dQ^d}(\mathbf{p}, Q^d, \mathbf{z^d}) = -\left[\Pi^u_{w_v w_v}(\mathbf{p}, w_v, \mathbf{z^d})\right]^{-1} = C^{-1}_{w_v w_v}(\mathbf{p_x}, w_v, X_c^d, \mathbf{z^d}).$$

[16] This slope implies a domestic demand elasticity of tobacco of −1.64.

[17] This scaling is appropriate if the supply elasticity and price are the same for the United States and for North Carolina.

[18] This slope implies a tobacco supply elasticity of 7.14.

or $95 million in 1982 dollars.

To put this number in perspective, $95 million is about 3% of the average $3.25 billion market value of the tobacco crop during the 1950–82 period. This is a measure of quota-induced deadweight loss due to misallocation of tobacco-cigarette sector resources. This estimate of loss is greater than the $32 million general equilibrium estimate by Fulginiti and Perrin (1993b) that relied on much of the same data. That study assumes U.S. cigarette demand to be identical to U.S. tobacco demand (with a slope of –3.33, compared to –1.06 here), and it treats the U.S. as a closed economy with tobacco and a *numéraire* good as the only commodities.

Other estimates of U.S. tobacco quota deadweight losses have been reported by Johnson, by Johnson and Norton, and by Alston and Sumner. It is of interest to contrast our measure of producer sector waste with their measures of Marshallian social surplus deadweight loss. The deadweight loss triangle of those studies is the one bounded by the worldwide derived demand for U.S. tobacco and the supply of U.S. tobacco. Johnson's deadweight loss calculations for flue-cured tobacco in 1965 use a tobacco supply elasticity of 0.4, a domestic demand elasticity of –0.5, a foreign demand elasticity of –1.5 and price wedge of $0.18. Using the methodology in this paper with these parameter values yields a producer sector deadweight loss of $7.5 million in 1965 dollars, compared to Johnson's calculation of a worldwide Marshallian loss of $21 million. Johnson and Norton estimate world deadweight loss due to this program at $45 million in 1980, using a different set of parameters,[19] but we cannot calculate the implied producer loss because they do not report levels of all variables needed for the calculation. Alston and Sumner examine approximately the same period as our study, but they estimate the tobacco supply elasticity at 5.0, domestic demand elasticity for tobacco at –1.0, foreign demand elasticity at –4.0, and an average price wedge of $0.30 (in 1987 dollars). Using their parameter values and the methodology in this paper, we obtain an average producer sector deadweight loss of $25 million compared to their estimated worldwide Marshallian losses of $73 million in 1987.[20]

Thus, when we use our method with the parameter values of these two other studies, estimated producer sector losses due to tobacco quotas (ignoring any separate losses due to nontransferability) are about a third of the dollar value of the estimated Marshallian deadweight loss. However, these dollar values are not directly comparable, because the Marshallian social surplus can be interpreted only as an index of consumer welfare in which the utility of a dollar for each consumer is equal, whereas the producer sector loss can be directly interpreted as dollars of foregone profit.

Conclusions

We have developed an adaptation of Diewert's production-sector Allais loss as a measure of deadweight loss due to quota restrictions. Deadweight loss is the excess burden suffered by using incentive-distorting instruments rather than a lump-sum transfer to achieve the goal of the quota policy. The advantage of the production sector loss measure, as compared to more general equilibrium notions of loss, is that it does not require information about tastes of individual consumers or welfare weights on various households' marginal utility of money. A disadvantage is that it is only a partial measure of the deadweight loss to society. The Allais-Diewert production sector loss measure examines the additional profits that could be extracted from the production sector, subject to the constraint that the welfare of outsiders is not affected by the internal reallocations considered.

Our particular version of this loss measure is adaptable to a variety of partial equilibrium applications because it permits consideration of commodities whose prices are strictly or partially determined by producer-sector behavior, in addition to quota commodities and fixed-price commodities. Second-order approxima-

[19] They use a tobacco supply elasticity of 1 0, a domestic demand elasticity of –0 2, a foreign demand elasticity of –2 3, and a demand price $0 20 above equilibrium Johnson further estimates a separate acreage restriction loss of $25 million, while Johnson and Norton estimate a separate nontransferability loss of $48 million.

[20] Alston and Sumner estimate separately an additional $75 million deadweight loss due to nontransferability of quotas Also, Alston and Sumner examine the distribution of Marshallian surpluses prior to adding them together to obtain deadweight loss They find that the quota program yields a net Marshallian gain to the U S producing sector of $314 million (total quota rents minus foregone profits) and losses to U.S consumers of $214 million, for a net gain to the U S of $100 million and losses to foreigners of $173 million

tions of this measure of waste, which make use of parameters of very specifically defined profit functions, make possible empirical measurements of the loss.

Given some recent estimates of the necessary profit functions in the case of tobacco production and processing, we are able to estimate that the production sector loss due to tobacco quotas, evaluated at the mean of 1950–82 data, was approximately $95 million per year, or about 3% of the value of the crop. We also applied our method using the estimated parameters of two previous studies of Marshallian deadweight loss due to the U.S. tobacco program, and found that the dollar value of producer sector waste was about a third of the estimates of the Marshallian deadweight loss. However, dollar values of Marshallian deadweight loss are best interpreted as indexes of consumers' well-being (in which the marginal utility of money for all consumers is equal); thus, they are not directly comparable to the dollar values of producer sector waste.

The proposed Allais-Diewert production loss measure provides an estimate of a particular kind of deadweight loss from quotas. Our study thus offers an approach to deadweight loss measurement that, while free of interpersonal welfare comparisons, still allows quantification of the deadweight loss in the use of quotas to transfer income among groups in society. We demonstrate its empirical viability and interpretation in one case, and believe that it can be used to evaluate other changes in quota policies associated with international trade and environmental regulation.

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References

Allais, M. *A la Recherch d'une Discipline Economique*, Vol. 1. Paris: Imprimerie Nationale, 1943.

___. "La Theorie Generale des Surplus et l'Apport Fondamental de Vilfredo Pareto." *Revue d'Economie Politique* 6(1973):1044–97

___. "Theories of General Equilibrium and Maximum Efficiency." *Equilibrium and Disequilibrium Theory*. G Schwodiauer, ed. Hingham MA: D. Reidel, 1977.

Alston, J.M., and D.A. Sumner. "A New Perspective on the Farm Program for U.S. Tobacco." Working Paper No. 88-19, Department of Agricultural Economics, University of California, Davis, 1988.

Anderson, J., and P. Neary. "Trade Reform with Quotas, Partial Rent Retention and Tariffs." *Econometrica* 60(January 1992):57–76.

Auerbach, A. "The Theory of Excess Burden and Optimal Taxation." *Handbook of Public Economics*, Vol. 1. A. Auerbach and M. Feldstein, eds., pp. 61–127. Amsterdam: Elsevier Science Publishers, 1985.

Boiteux, M. "Le Revenue Distruable et les Pertes Economiques." *Econometrica* 19(April 1951):112–33.

Debreu, G. "The Coefficient of Resource Utilization." *Econometrica* 19(July 1951):273–92.

Diewert, W.E. *The Measurement of Economic Benefits of Infrastructure Services*. Lecture Notes in Economics and Mathematical Systems. New York: Springer-Verlag, 1987

___. "The Measurement of Waste Within the Production Sector of an Open Economy." *Scand. J. Econ.* 85(1983):159–79.

Dupuit, J. "De la Mesure de l'utilite des Travaux Publics." *Annales des Ponts et Chaussees* 8, 1944. Translated and reprinted in *AEA Readings in Welfare Economics*. K. Arrow and T. Scitovsky, eds., pp. 255–83. Homewood IL: R.D. Irwin, 1969

Fulginiti L., and R. Perrin "Measures of Waste Due to Quotas." *Amer. J. Agr. Econ.* 75(August 1993b):776–81.

___. "The Theory and Measurement of Producer Response Under Quotas." *Rev. Econ. and Statist.* 75(August 1993a):97–106.

Harberger, A. "The Measurement of Waste." *Amer. Econ Rev* 54(May 1964):58–76.

Hicks, J.R. "Consumers' Surplus and Index Numbers." *Rev Econ. Stud* 9(June 1942):126–37.

___. "The Valuation of the Social Income." *Economica* 7(May 1940):105–24.

Hotelling, H. "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates." *Econometrica* 6(July 1938):242–69.

Johnson, P.R. "The Social Cost of the Tobacco Program." *J. Farm Econ.* 47(May 1965):242–55.

Johnson, P.R., and D.T. Norton, "Social Cost of the Tobacco Program Redux." *Amer J. Agr. Econ.* 65(February 1983):117–19.

Just, R., D. Hueth, and A. Smith. *Applied Welfare Economics and Public Policy*. Englewood Cliffs NJ Prentice-Hall, 1982.

Kay, J., and M. Keen. "Measuring the Inefficiencies of Tax Systems." *J. Public Econ.* 35(April 1988):265–87

Marshall, A. *Principles of Economics*. London: Macmillan, 1920.

Martin, W., and J. Alston. "A Dual Approach to Evaluating Research Benefits in the Presence of

Trade Distortions." *Amer. J. Agr. Econ.* 76(February 1994):26–35.

Pareto, V. "Economie Mathematic." *Encyclopedie des Sciences Mathematiques*. pp. 591–641. Paris: Gauthier-Villars, 1911. Published in English as "Mathematical Economics." *International Economics Papers*, No. 5, pp. 58–102. New York: MacMillan, 1955.

Rucker, R., W. Thurman, and D. Sumner. "Restricting the Market for Quota: An Analysis of Tobacco Production Rights with Corroboration from Congressional Testimony." *J. Polit. Econ.* 103(February 1995):142–75.

Sumner, D.A. "Measurement of Monopoly Behavior: An Application to the Cigarette Industry." *J. Polit. Econ* 89(October 1981):1010–19.

Sumner, D.A., and J.M. Alston. "Substitutability for Farm Commodities: The Demand for U.S. Tobacco in Cigarette Manufacturing." *Amer. J. Agr. Econ.* 69(May 1987):258–65.

Thurman, W. "Applied General Equilibrium Welfare Analysis." *Amer. J. Agr. Econ.* 73(December 1991):1508–16.

Vousden, N. *The Economics of Trade Protection*. Cambridge: Cambridge University Press, 1990.