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Calculus-Based General Physics

Instructional Materials in Physics and Astronomy

1975

EQUILIBRIUM OF RIGID BODIES

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INTRODUCTION

Most of the objects that one sees are in a state of equilibrium, that is, at rest or in a state of uniform motion. Many man-made structures are designed to achieve and sustain a state of equilibrium, and this, in turn, sets requirements to the materials (their sizes and shapes) that can be used. This module will give you some practice in analyzing the forces that result in equilibrium. From this analysis, if you are given the values of an appropriate set of forces you can find the remaining ones. On the other hand, in designing a stable system you can find the requirements of materials and dimensions that will ensure equilibrium.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Apply Newton's second law for the solution of problems involving frictional forces (needed for Objectives 2 and 3 of this module)	Newtons Laws Module
*Locate the center of mass of a uniform rigid body (needed for Objective 1 of this module)	Impulse and Momentum Module
*Define and use the definition of torque (needed for Objectives 1 and 3 of this module)	Rotational Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Equilibrium - Define the following terms and describe the application of each to a physical object or system: first condition of equilibrium (translational); second condition of equilibrium (rotational); center of gravity.

2. Translational equilibrium - Analyze translational equilibrium problems by identifying all forces, making a free-body diagram, and applying the first condition of equilibrium to solve for the unknown parameters. These problems may involve weight acting at the center of gravity, tensions in ropes or wires, compressional forces on rods or hinges, and frictional forces.

3. <u>Rotational equilibrium</u> - Analyze problems involving both the first and second conditions of equilibrium. These problems will involve torques as well as the forces referred to in Objective 2.

GENERAL COMMENTS

The solutions to the problems of this module are rather formal; that is, all problem solutions follow a regular procedure, which if done carefully will almost always produce the desired result. By learning the general procedures and practicing on a few examples you should find no difficulty in solving any problem in this module.

Since your text may not delineate these steps clearly, they are summarized here:

1. Draw an imaginary boundary separating the system under consideration from its surroundings.
2. Draw vectors representing magnitude, direction, and point of application of all external forces to the system. (This is a free-body diagram.)
3. Choose a convenient reference frame, resolve all of the external forces along these axes, and then apply the first condition of equilibrium.
4. Choose a convenient axis, evaluate all of the external torques around it, and apply the second condition of equilibrium.

The resulting simultaneous equations can then be solved for the desired quantities.

TEXT: Frederick J. Bueche, <u>Introduction to Physics for Scientists and Engineers</u>
(McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Study the text Sections 3.1 through 3.4 in Chapter 3 and Section 11.9 in
Chapter 11. Next study the Problem Set in this module, and work text problems,
remembering to use the rules given in General Comments, until you are satisfied
that you have met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text (Chap. 3)	(Chap. 3)
1	Secs. 3.1, 3.2, 3.4, 11.9					
2	Sec. 3.1	A, B	Illus. 3.1 to 3.3	E	3-5, 8, 10	1, 2, 6, 7, 9
3	Sec. 3.2	C, D	Illus. 3.4 to 3.7	F	18-21	22-25

*Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, <u>Fundamentals of Physics</u> (Wiley, New
       York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Study text Sections 12-1 through 12-3 in Chapter 12. Then study the Problem
Set in this module, remembering to use the rules given in General Comments.
Work text problems until you are satisfied that you have met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems (Chap. 12)
		Study Guide	Text*	Study Guide	Text (Chap. 12)	
1	Secs. 12-1, 12-2					
2	Sec. 12-3	A, B	Chap. 5, Ex. 3, 4	E	7, 15, 22, 26	
3	Sec. 12-3	C, D	Chap. 12, Ex. 1, 2	F	6, 10, 12, 20, 29	8, 9, 11, 13, 14, 16-21, 27-29

*Ex. = Example.

TEXT: Francis Weston Sears and Mark W. Zemansky, <u>University Physics</u> (Addison-
Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Study the text Sections 2-1 through 2-7 of Chapter 2 and 3-1 through 3-4 of
Chapter 3. Note that most of Chapter 2 was covered in your study of Newton's
laws. Review it now from the viewpoint of statics.

Study the Problem Set in this module; work Problems E and F, remembering to
use the rules of General Comments. Work text problems until you are satisfied
that you have met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 2-2, 2-6, 3-2, 3-4					
2	Secs. 2-6, 2-7	A, B	Sec. 2-6, Ex. 2 to 5; Sec. 2-7, Ex. 1, 3, 4	E	2-9, 2-10, 2-18, 2-21	2-3 to 2-8, 2-11, 2-12, 2-14, 2-16, 2-19, 2-20, 2-22 to 2-25
3	Secs. 3-2, 3-3	C, D	Sec. 3-2, Ex. 1 to 3	F	3-4, 3-12, 3-17, 3-19	3-3, 3-6 to 3-11, 3-13, 3-15, 3-16, 3-18, 3-20

*Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, <u>Elementary Classical Physics</u>
       (Allyn and Bacon, Boston, 1973), second <u>edition, Vol. 1</u>

<u>SUGGESTED STUDY PROCEDURE</u>

Study the text Sections 12-5 and 12-6 in Chapter 12. Also review Sections 8-1
and 8-2 of Chapter 8 where equilibrium is treated as a special case of Newton's
laws.

Study the Problem Set and work Problems E and F. Work text problems, remembering
to use the rules from General Comments, until you are satisfied that you have
met Objectives 1 to 3.

Try the Practice Test before attempting a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 12-5, 12-6					
2	Sec. 12-6	A, B	Ex. 8-2	E	8-1, 8-2, 8-4, 8-12	
3	Sec. 12-6	C, D	Ex. 12-10	F	12-31, 12-32, 12-34, 12-35	12-29, 12-30, 12-33, 12-37

*Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(2). A block of mass 12.0 kg slides at constant velocity when pushed by a
       force of 50 N applied at 30.0° as shown in Figure 1. Find the coefficient
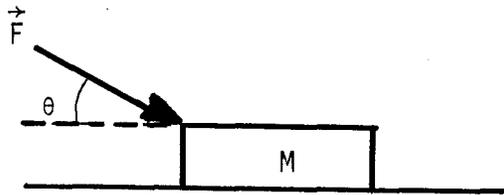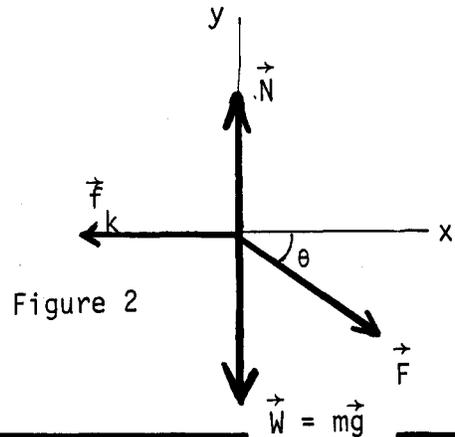   of sliding friction between the block and floor.

Figure 1

Figure 2

Solution

The expression "at constant velocity" suggests that this is a translational
equilibrium problem. First isolate the system, which in this case is just the
block M. Then identify all forces and draw a free-body diagram showing all forces.
See Figure 2: $\vec{W}$ is the weight of the block, $\vec{N}$ is the normal force of the floor
on the block, $\vec{F}$ is the applied force, and $\vec{f}_k$ is the kinetic frictional force
for which we assume $\vec{f}_k = \mu_k\vec{N}$. Since nothing is said in the problem about the
points of application of forces we assume we shall not need to apply the second
condition of equilibrium. Thus, applying the first condition of equilibrium:

$$\Sigma F_y = N - W - F \sin \theta = 0, \qquad \Sigma F_x = F \cos \theta - f_k = 0,$$

and also $f_k = \mu_k N$. Note that in this case the normal force $\vec{N}$ is not simply the
weight. We can combine the last two equations to eliminate the frictional force:

$$F \cos \theta - \mu_k N = 0.$$

This equation can then be combined with the first equation to eliminate the
normal force N and we then solve for μ_k:

$$N = (F \cos \theta)/\mu_k, \quad (F \cos \theta)/\mu_k - W - F \sin \theta = 0, \quad \mu_k = (F \cos \theta)/(W + F \sin \theta).$$

This result is certainly plausible since it is a dimensionless ratio of forces
that is expected, and if we let $\theta = 0$ it reduces to the defining equation for the
coefficient of friction (with W = N).

Substituting in the numbers from the problem we get the answer

$$\mu_k = (50)(\cos 30.0°)/[(12.0)(9.8) + 50 \sin 30.0°] = 0.300.$$

B(2). A weight W is hung from the middle of a tight clothesline 20.0 m long
        (see Fig. 3). The weight causes the center to drop 0.50 m below the
        horizontal (assume the line stretches slightly). Find the tension in
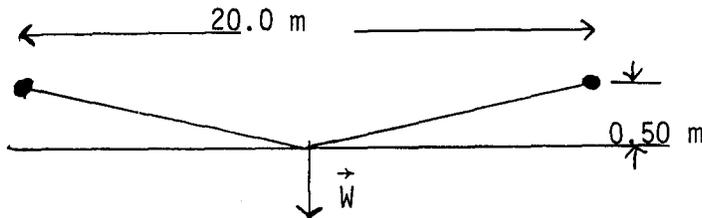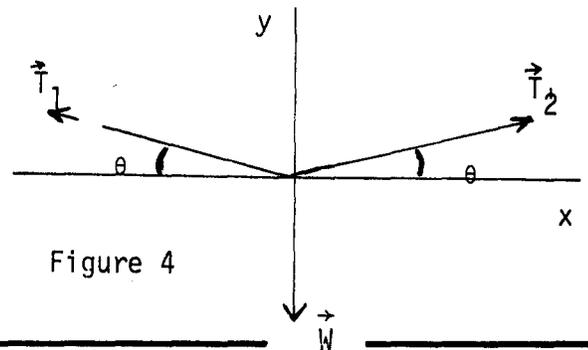   the line in terms of W.

Figure 3

Figure 4

Solution

We assume a first condition of equilibrium and isolate the point of application
of W on the line as the object in equilibrium, resulting in Figure 4.
"Clothesline" implies flexibility, which implies that T must be along the "line,"
and hence the force diagram will have the same angles as the line itself. Thus

$$\tan \theta = 0.50/10.0 = 0.050.$$

$$\Sigma F_x = T_2 \cos \theta - T_1 \cos \theta = 0,$$

hence,

$$T_1 = T_2, \qquad \Sigma F_y = 2T_1 \sin \theta - W = 0, \qquad T_1 = W/(2 \sin \theta) = 10.0W.$$

C(3). A uniform beam 5.0 m long with mass 10.0 kg is hinged at a wall. The
        outer end is supported by a guy wire making an angle of 30.0° with the
        horizontal beam, and an object of mass 20.0 kg is hung on the beam at a
        point 4.5 m from the wall. See Figure 5. Find the tension in the guy
        wire and the vertical and horizontal components of the force on the beam
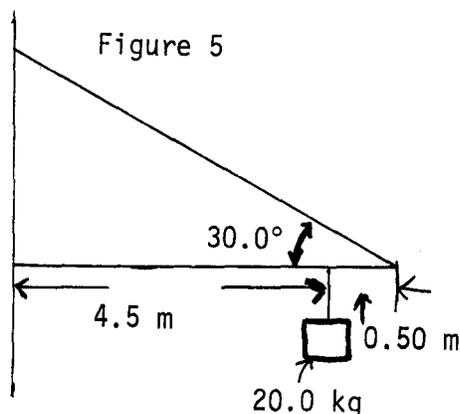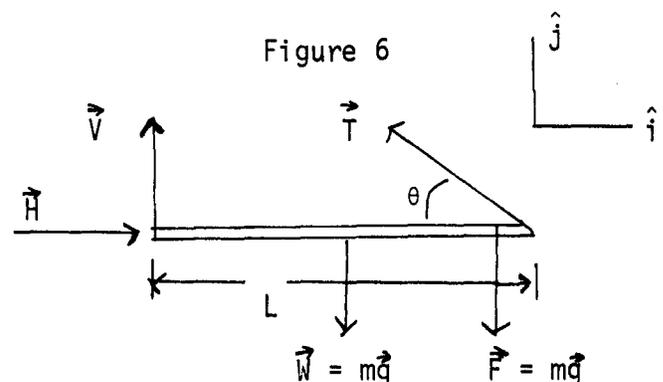   hinge by the wall.

Figure 5

Figure 6

Solution

For the free-body diagram, see Figure 6. The first condition of equilibrium results in the following equations:

$$\Sigma F_y = V + T \sin \theta - W - F = 0, \qquad \Sigma F_x = H - T \cos \theta = 0.$$

The second condition of equilibrium applied to an axis perpendicular to the paper at the hinge gives

$$\Sigma \tau = T(\sin \theta) L - (W)(L/2) - F(0.90)L = 0.$$

Solution of this last equation for T using the values of L, W, F, and θ from the problem gives T = 450 N. The first two equations can then be solved to give

$$H = 390 \text{ N}, \qquad V = 69 \text{ N}.$$

D(3). Physics teachers can be very devious. One such teacher drilled several holes in an otherwise good wooden meterstick and filled the holes with lead. He then gave the meterstick, a knife edge, and a hanging 0.100-kg mass to a student and asked him to find the mass and center of gravity of the modified meterstick. The student found that the stick balanced at 0.58 m with the hanging mass at 0.66 m and also balanced at 0.73 m with the hanging mass at 0.93 m. Find the mass and the center of gravity.
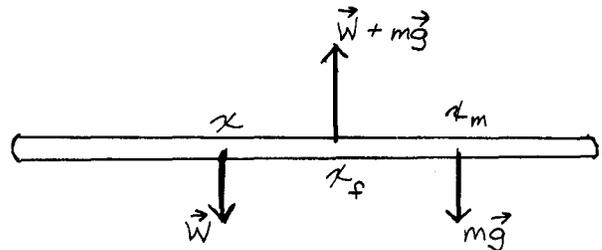

Figure 7

Solution

Assume the meterstick weight W to act at X and write two equations for $\Sigma \tau = 0$ around the 0.0 end. The upward force at the knife edge can be found from the first condition of equilibrium:

$$[W + (0.100)(9.8)](0.58) - WX - (0.100)(9.8)(0.66) = 0,$$

$$[W + (0.100)(9.8)](0.73) - WX - (0.100)(9.8)(0.93) = 0.$$

Simultaneous solution of these equations gives

$$X = 0.48 \text{ m}, \qquad W = 0.78 \text{ N}, \qquad m = 0.080 \text{ kg}.$$

Problems

E(2).   A 2.50-kg mass is hung from the ceiling by a long rope. It is pulled to
the side by a horizontal force of 10.0 N. Find the tension in the long
rope and the angle it will make with the vertical.

F(3).   A uniform ladder 10.0 m long, with a mass of 20.0 kg, rests against a
smooth wall and on a very rough floor, with the base of the ladder
6.0 m from the wall. An 80-kg man has climbed the ladder to 8.0 m
along the ladder from the bottom. Find the horizontal force that the
floor must supply to keep the ladder from slipping.

Solutions

E(2).   Draw a free-body diagram of the forces on the 2.50-kg mass, apply the
first condition of equilibrium to the forces, and solve for the desired quantities:
T = 26.0 N, θ = 22.0°.

F(3).   Draw a free-body diagram showing all forces on the ladder (Fig. 8). Note
that "smooth wall" implies no friction, which implies that F is perpendicular
to the wall. Notice the general similarity of this diagram to the free-body
diagram of Problem C (except for orientation, direction, and magnitude of forces)
and solve in a similar fashion.

$$\Sigma F_y = V - W - W_2 = 0 \text{ (this equation is not necessary)},$$

$$\Sigma F_x = H - F = 0, \qquad \Sigma \tau_B = 10.0F \sin \theta - 5.0W \cos \theta - (8.0)(W_2) \cos \theta = 0$$

The result is H = 540 N.

Figure 8
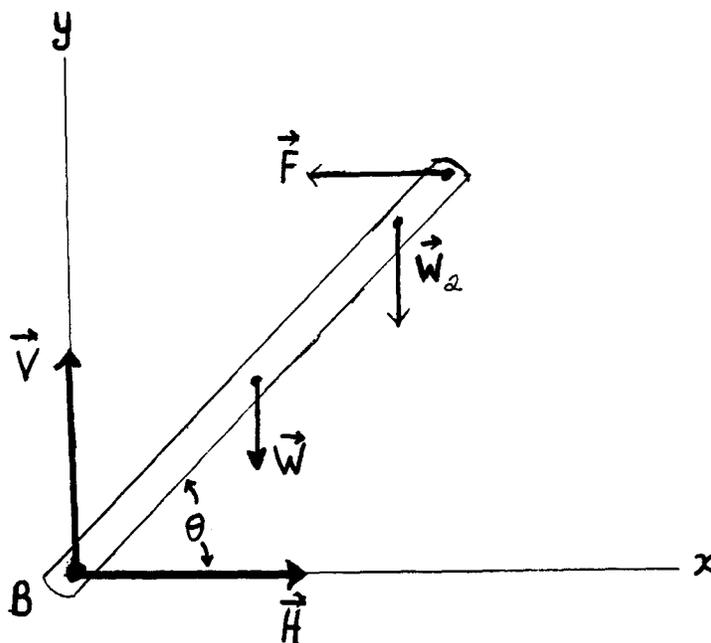
PRACTICE TEST

1. Describe how to find the center of gravity of an irregularly shaped planar object.

2. An object with a weight of 15.0 N is hung from two points as shown in Figure 9. Draw a free-body diagram and find the tension in the two ropes A and B.
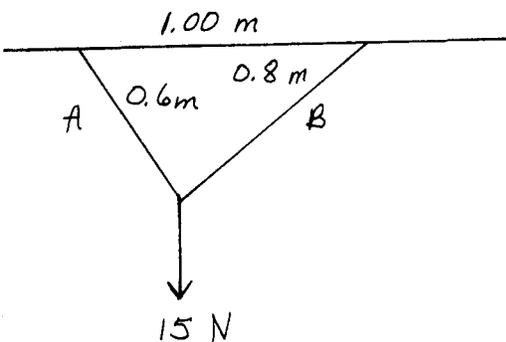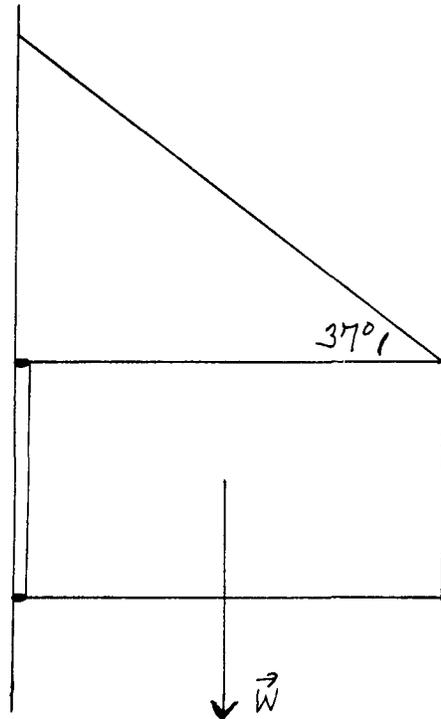
Figure 9

Figure 10

3. A uniform gate of weight W is hung from two hinges as shown in Figure 10, but to relieve the strain on the hinges a guy wire is installed as shown and tightened until there is no horizontal force on the lower hinge. Draw a free-body diagram and find the horizontal force on the top hinge, in terms of W.

Practice Test Answers

1. Assuming a plane object, suspend the object from a point on an edge and mark the vertical line (plumb line); then suspend the object from a different point and again mark the vertical line. The center of gravity is at the intersection of these lines.

2. Choose the point of application of the 15.0-N force as the point in equilibrium and draw a free-body diagram (Fig. 11). Apply the first condition of equilibrium:

$$\Sigma F_x = T_B \cos 37° - T_A \cos 53° = 0, \qquad \Sigma F_y = T_A \sin 53° + T_B \sin 37° - 15 = 0 \cdot$$

Simultaneous solution of these equations gives us

$$T_B = 9.0 \text{ N}, \qquad T_A = 12.0 \text{ N}.$$

3. Draw the free-body diagram as in Figure 12. Then

$$\Sigma F_x = H - T \cos 37° = 0, \qquad \Sigma F_y = V_1 + V_2 + T \sin 37° - W = 0,$$

$$\Sigma \tau = 0 = LT \sin 37° - W(L/2) = 0 \qquad \text{(taking torques about the top hinge),}$$

$$T = \frac{(1/2)W}{\sin 37°} = 0.83 \text{ W}.$$

From the first equation

$$H = T \cos 37° = 0.83W \cos 37° = 0.67W.$$

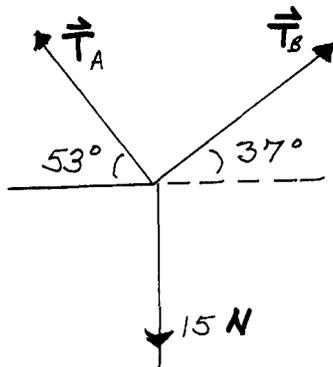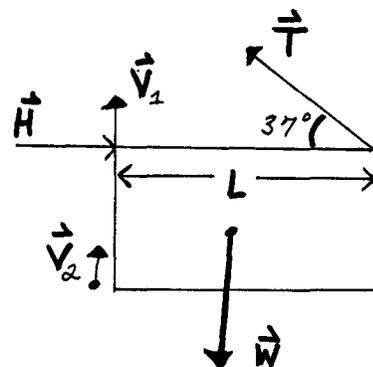Figure 11

Figure 12

EQUILIBRIUM OF RIGID BODIES

Name_____   Tutor_____

1. Describe what is meant by the "center of gravity" of an object.

2. A block of mass 6.0 kg is pulled up an incline at a constant velocity as
    shown in Figure 1. If the coefficient of sliding friction is 0.300, how
    large a force F is required?

3. A weight of 2.00×10^4 N is supported by a beam-and-guy-wire arrangement
    as shown in Figure 2. The beam is uniform and has a mass of 400 kg. Draw
    a free-body diagram and find the vertical and horizontal components of
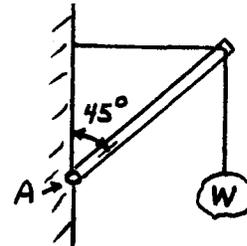   the forces on the hinge at A.
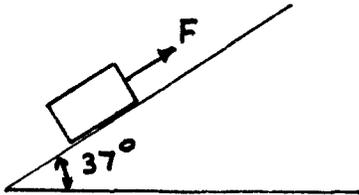
Figure 1

Figure 2

EQUILIBRIUM OF RIGID BODIES

Name_____ Tutor_____

1. State the first condition of equilibrium (translational) and describe how it differs from the second condition of equilibrium (rotational).

2. Two weights of 17.0 N each are suspended by cords 2.00 m long and attached to the ceiling at points 4.0 m apart. The weights are then connected with a cord that is 2.00 m long as shown in Figure 1. Draw a free-body diagram and find the tension in the connecting cord.

3. A playground seesaw, made from a plank 4.0 m long with a uniformly distributed mass of 50 kg, is to be used by two children with masses 60 kg and 40 kg. If the children are to sit at the ends of the plank, where should the fulcrum be placed for a balance?
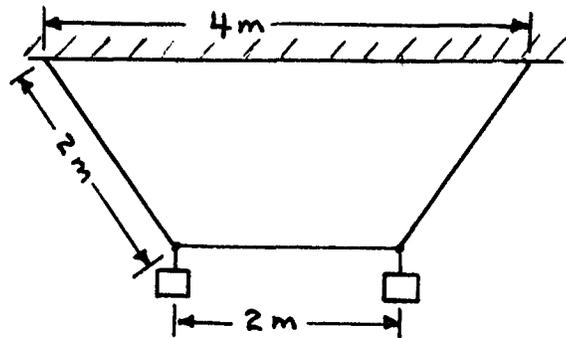

Figure 1

EQUILIBRIUM OF RIGID BODIES

Name_____ Tutor_____

1. State the second condition of equilibrium (rotational), and describe how it
    differs from the first condition of equilibrium (translational).

2. A rope of tensile strength 4500 N is tied to two sturdy posts 40 m apart.
    The rope is broken by applying a transverse force at the center. If the
    rope broke when the center was displaced by 1.00 m, how large was the
    transverse force?

3. A nonuniform rod of length L is attached to a wall by a hinge A as shown
    in Figure 1. The lower end rests on a frictionless floor. The center of
    gravity is (1/3)L from the lower end. Draw a free-body diagram, and find
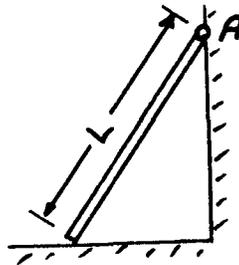   the force on the hinge.

Figure 1

1. Describe what is meant by the "center of gravity" of an object.

2. The coefficient of static friction between the surface and Block A, of weight 80 N ., is 0.40. Find the maximum weight of Block B which still keeps the system in equilibrium. The angle α is known. $\alpha = 37°$

3.
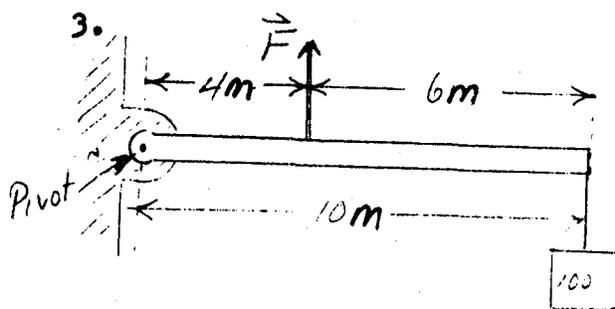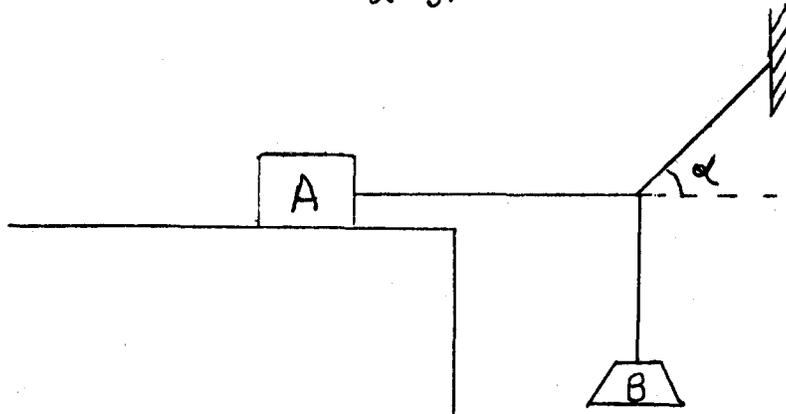
A 100N weight hangs at the end of a 10 m rod which is free to rotate about the other end. Calculate the magnitude of the vertical force required to prevent rotation when the rod is horizontal.

Courtesy of University of Missouri-Rolla
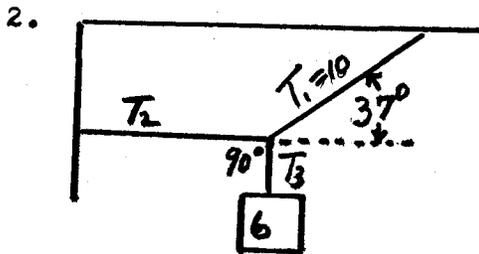
1. State the second condition of equilibrium and describe how it differs from the first condition of equilibrium.

2.

A six Newton weight is supported as shown. Draw the free body vector diagram for the <u>knot</u>.

Calculate T_2 for the above system

3. A horizontal bar 5 m long and weighing 30 N is supported at its two ends. If the force supporting it at one end is 12 N , find the distance of the center of gravity of the bar from that end.

Courtesy of University of Missouri-Rolla
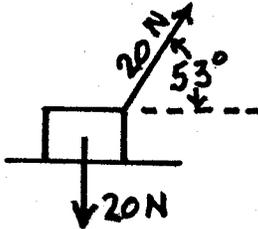
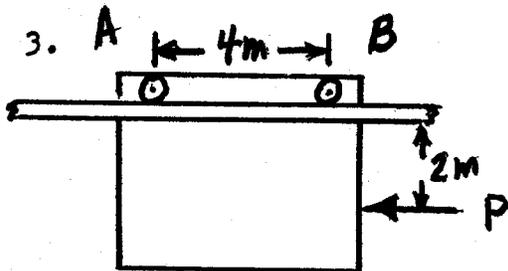1. Describe what is meant by the "center of gravity" of an object.

2.

A 20 N block is being pulled by a 10 N force at an angle of 53° above the horizontal. The block is moving at a uniform velocity. Draw a free body vector diagram showing all forces ON THE BLOCK. (don't forget friction) $\mu = \frac{1}{2}$

Calculate the normal force on the plane.

3.

A garage door is mounted on an overhead rail as shown. The wheels A and B have rusted so that they do not roll, but slide along the track. The coefficient of sliding friction is 0.5. The distance between the wheels is 4 m ; each is 1 m from the edge of the door. The door is symmetrical and weighs 160 N. It is pushed to the left at constant velocity by a horizontal force P, 2 m below the wheels. Find the vertical component of force exerted on each wheel by the track.

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. (a) Be sure the student has made the concise statement asked for in the question. (b) Ask the student for the meaning of one or two key words in his answer.	1. Center of gravity - That point about which the resultant of the gravitational torques on all the particles composing the rigid body is zero.
2. (a) Check for a free-body diagram showing only directions and magnitudes of all <u>external</u> forces (not wrong, but explain to student). (b) Check for a correct statement of $\Sigma F_x = 0$ and $\Sigma F_y = 0$. (c) Be sure that he has obtained the correct answer, including magnitude, direction, and units.	2. 49 N up the incline.
3. (a) Check for a free-body diagram showing only directions, magnitudes, and points of application of all external forces. (b) Check for a correct statement of $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma \tau = 0$. (Be sure an explicit choice of axis has been made for calculation of torques.) (c) Be sure student has obtained the requested answer, including magnitude, direction, and units.	3. $\vec{V} = 2.40 \times 10^4$ N up, $\vec{H} = 2.20 \times 10^4$ N to right.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. (a) Be sure that the student has made the concise statement asked for in the question. (b) Ask the student for the meaning of one or two key words in his answer.	1. First condition of equilibrium (translational) - The vector sum of the forces acting on the object equals zero, i.e., the object has a constant velocity.
2. (a) Check for a free-body diagram showing only directions and magnitudes of all external forces. (b) Check for a correct statement of $F_x = 0$ and $F_y = 0$. (c) Be sure that student has obtained the correct answer including magnitude, direction and units.	2. 9.8 N along horizontal cord.
3. (a) Check for a free-body diagram showing only directions and magnitude, and points of application of all external forces (if missing, not wrong). (b) Check for a correct statement of $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma \tau = 0$. (Be sure an explicit choice of axis has been made for calculation of torques.) (c) Be sure that student has obtained the requested answer including magnitude, direction, and units.	3. 1.70 m from the 60-kg child.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. (a) Be sure that the student has made the concise statement asked for in the question. (b) Ask the student for the meaning of one or two key words in his answer.	1. Second condition of equilibrium (rotational) - The sum of the torques at a chosen pivot point equals zero, i.e., the object will not rotate.
2. (a) Check for a free-body diagram showing only directions and magnitudes of external forces (not wrong if missing). (b) Check for a correct statement of $\Sigma F_x = 0$ and $\Sigma F_y = 0$. (c) Be sure that student has obtained the requested answer, including magnitude and units.	2. 400 N.
3. (a) Check for a free-body diagram showing only directions, magnitudes, and points of application of all external forces. (b) Check for a correct statement of $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma \tau = 0$. (Be sure an explicit choice of axis has been made for calculation of torques.) (c) Be sure that student has obtained the requested answer including magnitude, and direction.	3. $\vec{F} = W/3$ upwards with no horizontal component.