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Screw dislocation interacting with interfacial and interface cracks in piezoelectric bimaterials

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Abstract

Interface and interfacial cracks interacting with screw dislocations in piezoelectric bimaterials subjected to antiplane mechanical and in-plane electrical loadings are studied within the framework of linear piezoelectricity theory. Straight dislocations with the Burgers vector normal to the isotropic basal plane near the interface or interfacial crack are considered. The dislocations are characterized by a discontinuous electric potential across the slip plane and are subjected to a line-force and a line-charge at the core. An explicit solution for the screw dislocation in piezoelectric bimaterial with straight interface is found based on the solution of a similar problem for infinite homogenous medium. The obtained relation is independent of the nature of singularity. This fundamental result is used to analyze dislocation interacting with a set of collinear interfacial cracks in piezoelectric bimaterials. Three solutions for the screw dislocation interacting with a semi-infinite crack, finite crack, and edge crack between two bonded dissimilar piezoelectric materials are obtained in closed-form. These solutions can be used as Green's functions for the analyses of interfacial cracks in piezoelectric bimaterials.

Keywords: screw dislocation, interfacial crack, edge crack, piezoelectric bimaterials

1. Introduction

Piezoelectric materials such as ferroelectric ceramics find extensive electromechanical applications in actuators, sensors, and transducers. Piezoelectric ceramics are brittle and liable to cracking at the multitude of scales, from ferroelectric domains to devices [1]. Under high electric fields, cracks and delaminations may initiate from micro-defects such as dislocations, micro-cracks, cavities, inclusions, etc. Reliable predictions of performance and durability of piezoelectric devices require thorough analysis of failure mechanisms and behavior of various defects under coupled electromechanical loading. Since the pioneering works by Parton [2] and Deeg [3], substantial progress has been made on the analysis of electroelastic fields in piezoelectric materials with cracks, cavities, and inclusions [4–14]. The investigations conducted in this area to date are based on linear piezoelectricity theory and focused on determining the electroelastic fields and fracture parameters for cracked piezoelectric materials under various boundary conditions [4–14]. In practice, piezoelectric devices are generally made of piezoelectric layers embedded in a matrix. In such devices, cracks usually initiate and grow near interfaces, in particular near the free-edge of piezoelectric laminates. This is due to both relatively weak interfacial bonding and the mismatch of thermal expansion and Poisson's coefficients of the bonded materials. A thorough study of interfacial fracture in piezoelectric bimaterials is required for the analysis and design of advanced piezoelectric devices. A number of important results on interfacial fracture of piezoelectric bimaterials or laminates loaded with various mechanical and electric loadings applied at crack surfaces were obtained in [1, 15–30]. Stroh formalism was introduced as a convenient mathematical approach to handle anisotropic piezoelectricity by Suo *et al.* [1]. Detailed review of recent developments in fracture mechanics of piezoelectric materials can be found in the review paper by Zhang *et al.* [31].

Dislocation analysis plays an important role in fracture mechanics [32–35]. Dislocation solutions can serve as kernel functions for solving general crack problems, such as crack deflections, kinks, edge cracks, crack/inclusion interactions, etc. In the last decade, dislocation analysis was used for the analysis of cracks in piezoelectric materials. Examples include analysis of forces on dislocations by Pak [36], solutions for dislocation/crack interactions by Qin and Mai [23 and 24], Qin and Zhang [25], Lee *et al.* [37], Kwon and Lee [38], and Chen *et al.* [39], analysis of dislocation/inclusion interaction by Huang and Kuang [40], and others.

To the best of the authors' knowledge, a general solution for a screw dislocation interacting with an interface or collinear interfacial cracks in a piezoelectric bimaterial has not yet been reported. In the present work, we construct a simple continuum model describing the interaction between a single screw dislocation and a straight interface or a set of collinear interfacial cracks in a piezoelectric bimaterial. The straight dislocation with the Burgers vector b is located perpendicular to the isotropic basal plane of a hexagonal crystal exhibiting 6-mm symmetry [4, 36]. In addition to discontinuous electric potential across the slip plane, the dislocation is subjected to a line-force and a line-charge at the core. Complex variable approach and the method developed for the Riemann–Hilbert problems are utilized to solve the governing equations. Three special cases are considered with the screw dislocation interacting, respectively, with semi-infinite, finite, and edge cracks between two bonded dissimilar piezoelectric materials. General potential solutions are obtained explicitly and fracture parameters such as intensity factors (IFs) and energy release rates (ERR) are given in closed-form.

2. Basic equations

Consider two dissimilar piezoelectric half-planes bonded along the x-axis as shown in Figure 1. The piezoelectric materials are supposed to be transversely isotropic with hexagonal symmetry, which have an isotropic basal plane parallel to xy-plane and a poling direction parallel to z-axis.

The piezoelectric boundary value problem is treated as in the case of out-of-plane mechanical displacement and in-plane electric fields such that [4, 36]

$$u_x = u_y = 0, \qquad u_z = u_z(x,y), \tag{1}$$

$$E_x = E_x(x,y), \qquad E_y = E_y(x,y), \qquad E_z = 0. \tag{2}$$

In this case the constitutive relations become

$$\begin{aligned} \sigma_{xz} &= c_{44}\frac{\partial u_z}{\partial x} + e_{15}\frac{\partial \phi}{\partial x}, \qquad & D_x &= e_{15}\frac{\partial u_z}{\partial x} - \varepsilon_{11}\frac{\partial \phi}{\partial x}, \\ \sigma_{yz} &= c_{44}\frac{\partial u_z}{\partial y} + e_{15}\frac{\partial \phi}{\partial y}, \qquad & D_y &= e_{15}\frac{\partial u_z}{\partial y} - \varepsilon_{11}\frac{\partial \phi}{\partial y}, \end{aligned} \tag{3}$$

where σ_{kz} is stress tensor, D_k ($k = x,y$) is electric displacement vector, c_{44} is elastic modulus measured at constant electric field, e_{15} and ε_{11} are piezoelectric and dielectric constants, and ϕ is electric potential.

The electric field is given by

$$E_x = -\frac{\partial \phi}{\partial x}, \qquad E_y = -\frac{\partial \phi}{\partial y}. \tag{4}$$

The governing equations are

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = 0, \qquad \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0. \tag{5}$$

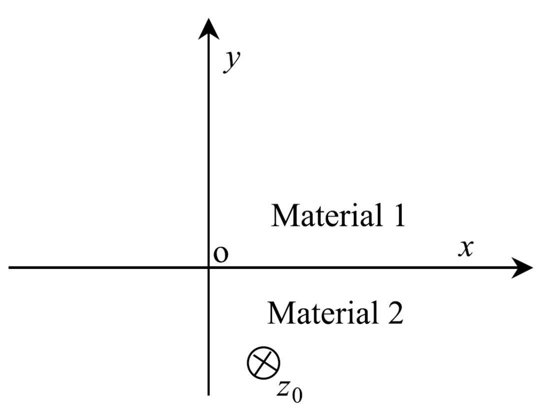

Figure 1. A dislocation interaction with an interface in a piezoelectric bimaterial.

Substituting (3) into (5) yields

$$c_{44}\nabla^2 u_z + e_{15}\nabla^2\phi = 0, \qquad e_{15}\nabla^2 u_z - \varepsilon_{11}\nabla^2\phi = 0. \tag{6}$$

The governing equations (6) can be reduced to the more compact form

$$\nabla^2 u_z = 0, \quad \nabla^2\phi = 0. \tag{7}$$

As a result, the out-of-plane displacement and the electric potential satisfy two decoupled Laplace equations. The solutions can be then expressed in terms of real parts of two analytic functions [4 and 36]. However, in order to facilitate the derivations in this paper, we express the solutions as imaginary parts of two analytic functions $U(z)$ and $\Phi(z)$ as follows

$$u_z = \mathrm{Im}[U(z)], \qquad \phi = \mathrm{Im}[\Phi(z)], \tag{8}$$

where Im denotes the imaginary part of an analytic function.

It is convenient to introduce the complex stress and electric displacement as

$$\begin{aligned} \sigma_{zy} + \mathrm{i}\sigma_{zx} &= c_{44}\left(\frac{\partial u_z}{\partial y} + \mathrm{i}\frac{\partial u_z}{\partial x}\right) + e_{15}\left(\frac{\partial \phi}{\partial y} + \mathrm{i}\frac{\partial \phi}{\partial x}\right), \\ D_y + \mathrm{i}D_x &= e_{15}\left(\frac{\partial u_z}{\partial y} + \mathrm{i}\frac{\partial u_z}{\partial x}\right) - \varepsilon_{11}\left(\frac{\partial \phi}{\partial y} + \mathrm{i}\frac{\partial \phi}{\partial x}\right), \end{aligned} \tag{9}$$

which can be further simplified in the forms of $U(z)$ and $\Phi(z)$ as

$$\sigma_{zy} + \mathrm{i}\sigma_{zx} = c_{44}U'(z) + e_{15}\Phi'(z), \qquad D_y + \mathrm{i}D_x = e_{15}U'(z) - \varepsilon_{11}\Phi'(z). \tag{10}$$

The above show solutions for piezoelectric materials under antiplane mechanical and in-plane electric loading can be represented by two analytic functions satisfying the boundary conditions.

3. Screw dislocation near interface between two bonded dissimilar piezoelectric media

Based on the superposition scheme proposed by Thomson [32] and Suo [33 and 34], the overall solution of this problem can be constructed from the dislocation solution for an infinite homogenous medium and an auxiliary potential (interface image) to satisfy the interface continuity conditions. The out-of-plane displacement and electric potentials $U(z)$ and $\Phi(z)$ of a screw dislocation in an infinite homogenous medium were obtained by Pak [36]. For the purpose of the current paper, we derive this basic solution using the notation (8). Assume that the screw dislocation is located at z_0 ($z_0 = x_0 + \mathrm{i}y_0$). Denote its Burgers vector b, a line-force p, a line-charge q, and an electric potential jump $\Delta\phi$. Assume that the potentials have the following form

$$U(z) = A\ln(z - z_0), \qquad \Phi(z) = B\ln(z - z_0), \tag{11}$$

where $A = A_1 + \mathrm{i}A_2$ and $B = B_1 + \mathrm{i}B_2$ are the singularity quantities to be determined by the dislocation conditions.

Consider a contour C with the outer normal unit vector, n_j, surrounding the dislocation core. The z-component of the Burgers vector is equal to the out-of-plane displacement jump across the slip plane ($x > x_0$, $y = y_0$) as

$$b = \Delta u_z = u_z(x, y_0^-) - u_z(x, y_0^+) = \mathrm{Im}[\Delta U(z)] = \mathrm{Im}\left[\oint_{z_0} U'(z)\,\mathrm{d}z\right] = 2\pi A_1, \tag{12}$$

and the electric potential jump, $\Delta\phi$, across the slip plane is

$$\Delta\phi = \phi(x, y_0^-) - \phi(x, y_0^+) = \mathrm{Im}[\Delta\Phi(z)] = \mathrm{Im}\left[\oint_{z_0} \Phi'(z)\,\mathrm{d}z\right] = 2\pi B_1. \tag{13}$$

From Equations (12) and (13), we have

$$A_1 = \frac{b}{2\pi}, \qquad B_1 = \frac{\Delta\phi}{2\pi}. \tag{14}$$

The integral of z-component traction along the contour C balances the line-force, p, acting at the dislocation core along the z-direction as

$$\begin{aligned} -p &= \oint_{z_0} \sigma_{jz} n_j\,\mathrm{d}l = \int_0^{2\pi} (\sigma_{xz}\cos\theta + \sigma_{yz}\sin\theta) r\,\mathrm{d}\theta = \mathrm{Im}\oint_{z_0} -\mathrm{i}(\sigma_{yz} + \mathrm{i}\sigma_{xz})\,\mathrm{d}z \\ &= \mathrm{Im}\oint_{z_0} \{-\mathrm{i}[c_{44}U'(z) + e_{15}\Phi'(z)]\}\,\mathrm{d}z = 2\pi(c_{44}A_2 + e_{15}B_2), \end{aligned} \tag{15}$$

and the electric charge balance at the dislocation core is expressed as follows

$$\begin{aligned} q &= \oint_{z_0} D_j n_j\,\mathrm{d}l = \int_0^{2\pi} (D_x\cos\theta + D_y\sin\theta) r\,\mathrm{d}\theta = \mathrm{Im}\oint_{z_0} -\mathrm{i}(D_y + \mathrm{i}D_x)\,\mathrm{d}z \\ &= \mathrm{Im}\oint_{z_0} \{-\mathrm{i}[e_{15}U'(z) - \varepsilon_{11}\Phi'(z)]\}\,\mathrm{d}z = 2\pi(e_{15}A_2 - \varepsilon_{11}B_2). \end{aligned} \tag{16}$$

From (15) and (16), we obtain A_2 and B_2 as

$$A_2 = \frac{-p\varepsilon_{11} + qe_{15}}{2\pi(e_{15}^2 + \varepsilon_{11}c_{44})}, \qquad B_2 = -\frac{pe_{15} + qc_{44}}{2\pi(e_{15}^2 + \varepsilon_{11}c_{44})}. \tag{17}$$

Substituting (14) and (17) into (11) yields

$$\begin{aligned} U(z) &= \left[\frac{b}{2\pi} + \mathrm{i}\frac{-p\varepsilon_{11} + qe_{15}}{2\pi(e_{15}^2 + \varepsilon_{11}c_{44})}\right]\ln(z - z_0), \\ \Phi(z) &= \left[\frac{\Delta\phi}{2\pi} - \mathrm{i}\frac{pe_{15} + qc_{44}}{2\pi(e_{15}^2 + \varepsilon_{11}c_{44})}\right]\ln(z - z_0), \end{aligned} \tag{18}$$

which correspond to the expressions obtained by Pak [36].

Now let us consider a screw dislocation interacting with an interface between two bonded dissimilar piezoelectric half-planes. In the following procedure, indices 1 and 2 denote material constants, displacements, electric potentials, stresses, and electric displacements of the upper and lower half-planes, respectively. The x-axis is directed along the interface, as shown in Figure 1. Without loss of generality, we suppose that the screw dislocation is located in the lower half-plane at $z_0 = x_0 + iy_0$ and out-of-plane displacements and electric potentials are expressed in the following form

$$U(z) = \begin{cases} U_1(z), & z \in D_1 \text{ i.e. } (y > 0), \\ U_2(z) + U_0(z), & z \in D_2 \text{ i.e. } (y < 0), \end{cases} \tag{19}$$

and

$$\Phi(z) = \begin{cases} \Phi_1(z), & z \in D_1 \text{ i.e. } (y > 0), \\ \Phi_2(z) + \Phi_0(z), & z \in D_2 \text{ i.e. } (y < 0). \end{cases} \tag{20}$$

Here $U_1(z)$, $U_2(z)$, $\Phi_1(z)$, and $\Phi_2(z)$ are unknown out-of-plane displacement and electric potentials to be determined, and $U_0(z)$ and $\Phi_0(z)$ are the out-of-plane displacement and electric potentials of the screw dislocation in an infinite homogenous piezoelectric material, are expressed by (18) with material constants of the lower half-plane.

In order to simplify the derivation, we introduce five vectors as

$$\mathbf{f}_1(z) = \begin{bmatrix} U_1(z) \\ \Phi_1(z) \end{bmatrix}, \qquad \mathbf{f}_2(z) = \begin{bmatrix} U_2(z) \\ \Phi_2(z) \end{bmatrix}, \qquad \mathbf{f}_0(z) = \begin{bmatrix} U_0(z) \\ \Phi_0(z) \end{bmatrix}, \tag{21}$$

and

$$\mathbf{u} = \begin{bmatrix} u_z \\ \phi \end{bmatrix}, \qquad \mathbf{t} = \begin{bmatrix} \sigma_{yz} \\ D_y \end{bmatrix}. \tag{22}$$

From Equations (8) and (10), and definitions (21) and (22), we have

$$\mathbf{u} = \mathrm{Im}[\mathbf{f}(z)] = -\mathrm{i}/2[\mathbf{f}(z) - \bar{\mathbf{f}}(z)] \tag{23}$$

and

$$\mathbf{t} = L/2[\mathbf{f}'(z) + \bar{\mathbf{f}}'(z)], \tag{24}$$

where

$$L = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix} \tag{25}$$

is the material matrix and ($^-$) denotes the conjugate of an analytic function.

Now let us determine the unknown potentials $U_1(z)$, $U_2(z)$, $\Phi_1(z)$, and $\Phi_2(z)$ by enforcing the out-of-plane displacement, electric potential, mechanical force, and electric displacement continu-

ity across the interface. It should be noted that $U_1(z)$ and $\Phi_1(z)$ are analytic in the upper half-plane, while $U_2(z)$ and $\Phi_2(z)$ are analytic in the lower half-plane. From (19)–(23), the out-of-plane displacement and electric potential continuity across the interface requires that

$$\mathbf{f}_1(\mathrm{x}) - \overline{\mathbf{f}}_1(\mathrm{x}) = \mathbf{f}_2(\mathrm{x}) - \overline{\mathbf{f}}_2(\mathrm{x}) + \mathbf{f}_0(\mathrm{x}) - \overline{\mathbf{f}}_0(\mathrm{x}) \tag{26}$$

Rearranging (26), we obtain

$$\mathbf{f}_1(x) + \overline{\mathbf{f}}_2(x) - \mathbf{f}_0(x) = \mathbf{f}_2(x) + \overline{\mathbf{f}}_1(x) - \overline{\mathbf{f}}_0(x). \tag{27}$$

Equation (27) holds along the whole x-axis. Moreover, the functions at the left-hand side are analytic in the upper half-plane, and those at the right-hand side are analytic in the lower half-plane. With the standard analytic continuity arguments, we have

$$\mathbf{f}_1(z) + \overline{\mathbf{f}}_2(z) - \mathbf{f}_0(z) = 0, \quad z \in D_1 \tag{28}$$

With the same arguments, the stress and electric force continuity across the interface leads to

$$L_1\mathbf{f}'_1(z) - L_2\overline{\mathbf{f}}'_2(z) - L_2\mathbf{f}'_0(z) = 0, \quad z \in D_1 \tag{29}$$

From Equations (28) and (29), we get

$$\begin{aligned} \mathbf{f}_1(z) &= 2(L_1 + L_2)^{-1}L_2\,\mathbf{f}_0(z), \quad z \in D_1 \\ \mathbf{f}_2(z) &= (L_1 + L_2)^{-1}(L_1 - L_2)\overline{\mathbf{f}}_0(z), \quad z \in D_2 \end{aligned} \tag{30}$$

In the above, if letting $L_1 = L_2$, results in (30) cover the solutions (18). Furthermore, a screw dislocation in a half-plane interacting with a traction-free and electrically impermeable surface, $y = 0$, can be constructed as

$$\mathbf{f}(z) = \mathbf{f}_0(z) - \overline{\mathbf{f}}_0(z), \quad z \in D_2 \tag{31}$$

Another meaningful case is a screw dislocation in a half-plane interacting with a rigid and electrically impermeable surface, $y = 0$. The corresponding potentials are

$$U(z) = U_0(z) + \overline{U}_0(z), \qquad \Phi(z) = \Phi_0(z) + \frac{e_{15}}{\varepsilon_{11}}U_0(z) - \overline{\Phi}_0(z), \qquad z \in D_2. \tag{32}$$

4. Screw dislocation near collinear interfacial cracks between two bonded dissimilar piezoelectric media

Suppose a screw dislocation with Burgers vector b, line-force p, line-charge q, and electric potential jump $\Delta\Phi$ is located at $z_0 = x_0 + \mathrm{i}y_0$ in the lower half-plane. The crack surfaces are considered traction-free and electrically impermeable, by analogy with Deeg [3], Pak [4, 9], Sosa and Pak [5], Li *et al.* [6], Sosa [7, 8], Suo *et al.* [1], and Gao *et al.* [13]. According to the superposition scheme by Thomson [32] and Suo [33, 34] depicted in Figure 2, we need to obtain a solution for the interfacial crack problem

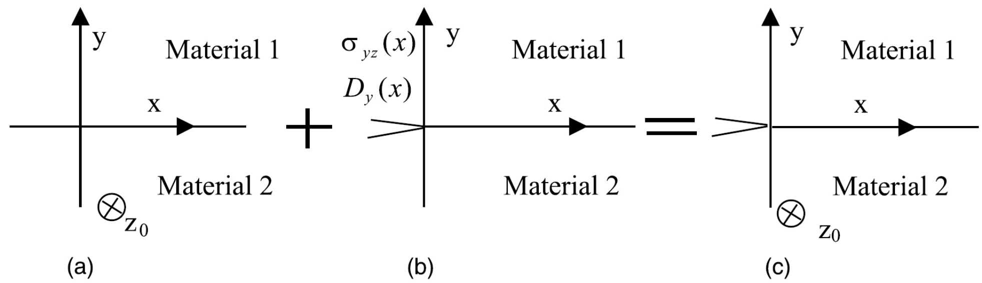

Figure 2. Superposition scheme: (**a**) A screw dislocation with a line-force and a line-charge near interface of two bonded dissimilar piezoelectric media. (**b**) A semi-infinite interfacial crack with traction and electric displacement prescribed on crack surfaces. (**c**) A screw dislocation interaction with a traction-free and electrically impermeable interfacial crack.

with crack surfaces under the action of the negative tractions and electric displacements determined by the potential (19) and (20). In the following we assume the effective potentials in the upper and the lower half-planes are $\mathbf{f}_1(z)$ and $\mathbf{f}_2(z)$, respectively.

Using the notations (21)–(25), the traction and electric displacement continuity along the crack line, $y = 0$, requires

$$L_1[\mathbf{f}'_1(x) + \overline{\mathbf{f}}'_1(x)] = L_2[\mathbf{f}'_2(x) + \overline{\mathbf{f}}'_2(x)]. \tag{33}$$

Rearrangement of (33) gives

$$L_1\mathbf{f}'_1(x) - L_2\overline{\mathbf{f}}'_2(x) = L_2\mathbf{f}'_2(x) - L_1\overline{\mathbf{f}}'_1(x) \tag{34}$$

Equation (34) holds along the whole line, $y = 0$. From standard analytic continuity arguments, it follows that

$$L_1\mathbf{f}'_1(z) = L_2\overline{\mathbf{f}}'_2(z), \qquad z \in D_1 \tag{35}$$

Define the out-of-plane displacement and electric potential jumps across the interface as

$$\mathbf{d}(x) = \begin{bmatrix} u_z(x, 0+) \\ \Phi(x, 0+) \end{bmatrix} - \begin{bmatrix} u_z(x, 0-) \\ \Phi(x, 0-) \end{bmatrix}. \tag{36}$$

With the help of relation (35), the crack surface traction, electric displacement, and the out-of-plane displacement and electric potential jumps can be simplified as

$$\mathbf{t}(x) = L_1/2\mathbf{f}'_1(x) + L_2/2\mathbf{f}'_2(x) \tag{37}$$

and

$$2i\mathbf{d}'(x) = (L_1^{-1} + L_2^{-1})[L_1\mathbf{f}'_1(x) - L_2\mathbf{f}'_2(x)] = H[L_1\mathbf{f}'_1(x) - L_2\mathbf{f}'_2(x)], \tag{38}$$

in which

$$H = (L_1^{-1} + L_2^{-1}) \tag{39}$$

is the bimaterial matrix.

Furthermore, the out-of-plane displacement and electric potential continuity along the bonded portion ($\mathbf{d} = 0$ at $x \notin \Gamma$ and $y = 0$) implies that $L_1\mathbf{f}'_1(z)$ and $L_2\mathbf{f}'_2(z)$ can be analytically extended to the whole plane except on the crack line Γ ($x < 0$ and $y = 0$), and they should satisfy

$$\mathbf{h}(z) = L_1/2\mathbf{f}'_1(z) = L_2/2\mathbf{f}'_2(z), \qquad z \notin \Gamma, \tag{40}$$

where $\mathbf{h}(z)$ is introduced to simplify the further derivation, and Γ denotes the crack line. With the aid of relation (37), the traction and electric displacement conditions at the crack surfaces lead to a system of inhomogeneous Riemann–Hilbert problems as

$$\mathbf{h}^+(x) + \mathbf{h}^-(x) = \mathbf{t}_0(x), \qquad x \in \Gamma, \tag{41}$$

where

$$\mathbf{t}_0(x) = L_1/2[\mathbf{f}'_{10}(x) + \overline{\mathbf{f}}'_{10}(x)] = L_2/2[\mathbf{f}'_{20}(x) + \overline{\mathbf{f}}'_{20}(x)], \tag{42}$$

are the traction and electric displacement along the interface excited by the screw dislocation at z_0.

Assume there are n collinear finite cracks in the intervals (a_j, b_j) and two semi-infinite cracks in the intervals $(-\infty, b_0)$ and $(a_0, +\infty)$, respectively, all located on the interface x-axis. With the standard methods given by Muskhelishvili [41], the general solution of (41) can be expressed as

$$\mathbf{h}(z) = \frac{\chi(z)}{2\pi \mathrm{i}} \int_\Gamma \frac{\mathbf{t}_0(x)\,\mathrm{d}x}{\chi^+(x)(x-z)} + \mathbf{P}(z)\chi(z), \tag{43}$$

with

$$\chi(z) = \prod_{j=0}^{n} (z-a_j)^{-1/2}(z-b_j)^{-1/2}, \tag{44}$$

in which the branch cuts are chosen along the crack lines so that the product for each finite crack has the asymptotic property $1/z$ for large z, and $\mathbf{P}(z)$ is a polynomial which should be chosen to make $\mathbf{h}(z)$ bounded at infinity and the net out-of-plane displacement and electric potential jumps for each of the n finite cracks are zero. This condition leads to n equations

$$\int_{a_j}^{b_j} [\mathbf{h}^+(x) - \mathbf{h}^-(x)]\,\mathrm{d}x = 0, \quad j = 1, 2, \ldots, n. \tag{45}$$

Once $\mathbf{h}(z)$ is obtained, the stresses and IFs can be evaluated. Noting $\mathbf{h}^+(x) = \mathbf{h}^-(x) = \mathbf{h}(x)$ on the bonded portion of the interface, we have the IFs as

$$\mathbf{K} = \sqrt{2\pi} \lim_{x\to a} (x-a)^{1/2}[\mathbf{h}(x) + \bar{\mathbf{h}}(x)], \tag{46}$$

where $\mathbf{K} = \{K_{\mathrm{III}}, K_{\mathrm{D}}\}^{\mathrm{T}}$, and K_{D} is the electric intensity factor.

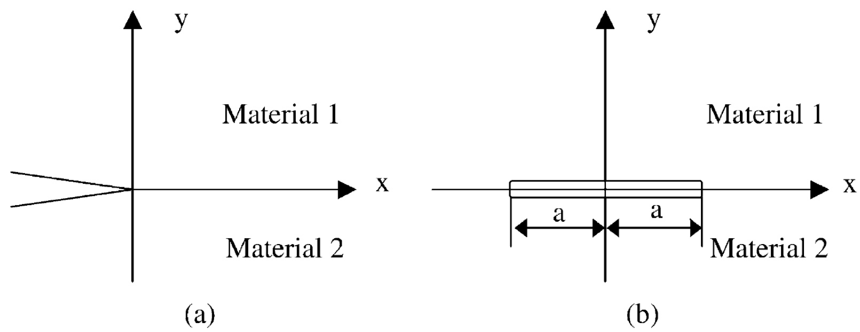

Figure 3. Two typical crack configurations: (**a**) semi-infinite crack and **(b**) finite crack of length $2a$.

The traction and electric displacement at the interface a distance r ahead of the crack tip and the out-of-plane displacement and electric potential jumps a distance r behind of the crack tip are calculated from (37), (38), (40) and (46) as

$$\mathbf{t}(r) = \frac{\mathbf{K}}{\sqrt{2\pi r}}, \qquad \mathbf{d}(r) = \frac{H\mathbf{K}}{\sqrt{2\pi r}}, \tag{47}$$

and the ERR for a unit crack growth along the interface can be evaluated as [1, 34, 42]

$$G = \frac{1}{2\Delta}\int_0^{\Delta} \mathbf{t}^{\mathrm{T}}(\Delta - r)\cdot \mathbf{d}(r)\,\mathrm{d}r = \frac{1}{4}\mathbf{K}^{\mathrm{T}}H\mathbf{K}. \tag{48}$$

Hereafter we consider two typical crack configurations as shown in Figure 3. In the case of a semi-infinite interface crack, we evaluate (43) and (46) as

$$x(z) = z^{-1/2}, \quad \mathbf{P}(z) = 0, \tag{49}$$

and

$$\mathbf{K} = \sqrt{\frac{2}{\pi}}\mathrm{Re}\left[\int_{-\infty}^{0} \frac{\mathbf{t}_0(\xi)}{(-\xi)^{1/2}}\,\mathrm{d}\xi\right]. \tag{50}$$

In the case of a finite interfacial crack of length $2a$, the corresponding results are

$$x(z) = (z^2 - a^2)^{-1/2}, \quad \mathbf{P}(z) = 0, \tag{51}$$

and

$$\mathbf{K} = \frac{1}{\sqrt{\pi a}}\mathrm{Re}\left[\int_{-a}^{a} \left(\frac{a+\xi}{a-\xi}\right)^{1/2} \mathbf{t}_0(\xi)\,\mathrm{d}\xi\right]. \tag{52}$$

In order to evaluate integral (43), we introduce the notation

$$\mathbf{J}_{\Gamma} = \int_{\Gamma} \frac{\mathbf{t}_0(x)\,\mathrm{d}x}{\chi^{+}(x)(x-z)}, \tag{53}$$

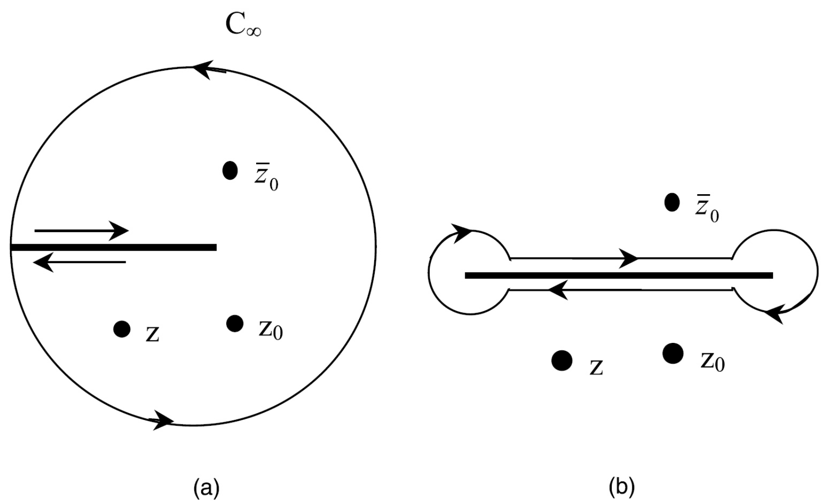

Figure 4. Integration contours: (**a**) semi-infinite crack and (**b**) finite crack of length $2a$.

in which $\mathbf{t}_0(x)$ is determined by (24), (30) and (42) as

$$\mathbf{t}_0(x) = (L_1^{-1} + L_2^{-1})^{-1}\left(\frac{\mathbf{T}}{x - z_0} + \frac{\overline{\mathbf{T}}}{x - \bar{z}_0}\right) \tag{54}$$

and

$$\mathbf{T} = \begin{bmatrix} A_1 + \mathrm{i}A_2 \\ B_1 + \mathrm{i}B_2 \end{bmatrix} = \frac{1}{2\pi}\begin{bmatrix} b \\ \Delta\phi \end{bmatrix} + \frac{L_2^{-1}}{2\pi\mathrm{i}}\begin{bmatrix} p \\ -q \end{bmatrix}. \tag{55}$$

With two contour integrals as shown in Figure 4, we evaluate (53) by the residual theorem.

4.1. A screw dislocation interacting with a semi-infinite interface crack

In this case, as shown in Figure 3 and Figure 4, we have

$$\mathbf{J}_\Gamma = \int_{-\infty}^{0} \frac{\sqrt{x}\,\mathbf{t}_0(x)\,\mathrm{d}x}{(x - z)} \tag{56}$$

and

$$2\mathbf{J}_\Gamma + \int_{C_\infty} \frac{\sqrt{\xi}\,\mathbf{t}_0(\xi)\,\mathrm{d}\xi}{(\xi - z)} = 2\pi\mathrm{i}\sum_{j=1}^{3}\mathrm{Res}\left[\frac{\sqrt{\xi}\,\mathbf{t}_0(\xi)}{(\xi - z)}, z_j\right], \tag{57}$$

in which residues at three isolated poles need evaluating, *i.e.* $z_1 = z$, $z_2 = z_0$, and $z_3 = \bar{z}_0$.

From the above, we have

$$\mathbf{J}_\Gamma = \pi \mathrm{i} H^{-1}\sqrt{z}\left\{\frac{\mathbf{T}}{z-z_0}\left[1-\left(\frac{z_0}{z}\right)^{1/2}\right]+\frac{\overline{\mathbf{T}}}{z-\bar{z}_0}\left[1-\left(\frac{\bar{z}_0}{z}\right)^{1/2}\right]\right\}. \tag{58}$$

Substituting (49) and (58) into (43) yields

$$\mathbf{h}(z) = \frac{H^{-1}}{2}\left\{\frac{\mathbf{T}}{z-z_0}\left[1-\left(\frac{z_0}{z}\right)^{1/2}\right]+\frac{\overline{\mathbf{T}}}{z-\bar{z}_0}\left[1-\left(\frac{\bar{z}_0}{z}\right)^{1/2}\right]\right\}. \tag{59}$$

Combining results ((30), (40) and (59)), we have the potential solutions as

$$\begin{bmatrix} U(z) \\ \Phi(z) \end{bmatrix}' = \begin{cases} 2(L_1+L_2)^{-1}L_2\frac{\mathbf{T}}{z-z_0}+2L_1^{-1}\mathbf{h}(z), & z\in D_1, \\ (L_1+L_2)^{-1}(L_1-L_2)\frac{\overline{\mathbf{T}}}{z-\bar{z}_0}+\frac{\mathbf{T}}{z-z_0}+2L_2^{-1}\mathbf{h}(z), & z\in D_2, \end{cases} \tag{60}$$

where [ ]′ denotes the derivative of an analytic function, L_1 and L_2 are material matrices for the upper and lower half-planes that are determined by (25), and **T** is defined by (55).

Utilizing the definition (46), the IFs can be evaluated as

$$\mathbf{K} = 2\sqrt{2\pi}H^{-1}\,\mathrm{Re}(\mathbf{T}z_0^{-1/2}). \tag{61}$$

4.2. A screw dislocation interacting with a finite interface crack

In this case, as shown in Figure 3 and Figure 4, we need to evaluate the following integral:

$$\mathbf{J}_\Gamma = \int_{-a}^{a}\frac{\sqrt{x^2-a^2}\,\mathbf{t}_0(x)\,\mathrm{d}x}{(x-z)}. \tag{62}$$

With the residual theorem on the exterior domain and the integral contour as shown in Figure 3(b), we have

$$2\mathbf{J}_\Gamma = 2\pi\mathrm{i}\,\mathrm{Res}\left[\frac{\sqrt{\xi^2-a^2}\,\mathbf{t}_0(\xi)}{(\xi-z)},\infty\right]+2\pi\mathrm{i}\sum_{j=1}^{3}\mathrm{Res}\left[\frac{\sqrt{\xi^2-a^2}\,\mathbf{t}_0(\xi)}{(\xi-z)},z_j\right], \tag{63}$$

where the branch cut is chosen from $x = +a$ to $x = -a$, and residues at three isolated poles and infinity need evaluating, *i.e.* $z_1 = z$, $z_2 = z_0$, $z_3 = \bar{z}_0$, and $z_4 = \infty$.

Solving (63) leads to

$$\mathbf{J}_{\Gamma} = -\pi \mathrm{i} H^{-1}\left\{\mathbf{T}\left[1-\frac{(z^2-a^2)^{1/2}}{z-z_0}\left(1-\left(\frac{z_0^2-a^2}{z^2-a^2}\right)\right)^{1/2}\right] + \overline{\mathbf{T}}\left[1-\frac{(z^2-a^2)^{1/2}}{z-\bar{z}_0}\left(1-\left(\frac{\bar{z}_0^2-a^2}{z^2-a^2}\right)\right)^{1/2}\right]\right\}. \tag{64}$$

Substituting (51) and (64) into (43) yields

$$\mathbf{h}(z) = -\frac{H^{-1}}{2}\left\{\mathbf{T}\left[\frac{1}{(z^2-a^2)^{1/2}}-\frac{1}{z-z_0}\left(1-\left(\frac{z_0^2-a^2}{z^2-a^2}\right)^{1/2}\right)\right] + \overline{\mathbf{T}}\left[\frac{1}{(z^2-a^2)^{1/2}}-\frac{1}{z-\bar{z}_0}\left(1-\left(\frac{\bar{z}_0^2-a^2}{z^2-a^2}\right)^{1/2}\right)\right]\right\}. \tag{65}$$

Replacement of $\mathbf{h}(z)$ in (60) with (65) leads to the potentials for a screw dislocation interacting with a finite interfacial crack. As a check, if translating the y-axis to the right crack tip, $x = a$, and letting $a \to \infty$, result (65) returns to (59).

Consequently, substituting (65) into (46) yields the IFs at the right crack tip as

$$\mathbf{K} = 2\sqrt{\frac{\pi}{a}}H^{-1}\,\mathrm{Re}\left\{\mathbf{T}\left[\left(\frac{z_0+a}{z_0-a}\right)^{1/2}-1\right]\right\}. \tag{66}$$

5. Application—Screw dislocation interaction with an interfacial edge crack

Here we utilize the obtained potentials in (60) to find the solution for a screw dislocation interacting with an interfacial edge crack between two bonded quarter-planes of dissimilar piezoelectric materials as shown in Figure 5(a). The free-edge surface and crack surfaces are assumed traction-free and electrically impermeable as in Section 3. Due to the out-of-plane displacement and electric potentials being holomorphic functions as shown in (7), this problem can be solved explicitly by the conformal mapping technique. We introduce the conformal mapping

$$\zeta = (1 + z/a)^2 - 1, \tag{67}$$

which maps two bonded quarter-planes with an interfacial edge crack onto two bonded half-planes with a semi-infinite cut along the negative ξ-axis as shown in Figure 5(b). With the aid of solution (59) and (60), and the mapping (67), we derive the potential solutions as

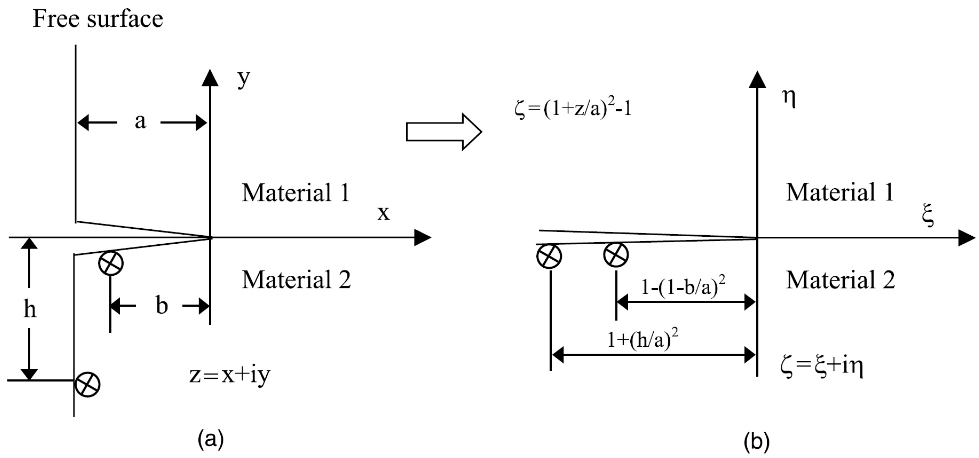

Figure 5. A screw dislocation interaction with an interfacial edge crack in a piezoelectric bimaterial.

$$\begin{bmatrix} U(z) \\ \Phi(z) \end{bmatrix}' = \begin{cases} 4(L_1+L_2)^{-1}L_2\frac{(z+a)\mathbf{T}}{(z+a)^2-(z_0+a)^2} + 2L_1^{-1}\mathbf{h}(z), & z\in D_1, \\ 2(L_1+L_2)^{-1}(L_1-L_2)\frac{(\bar{z}+a)\overline{\mathbf{T}}}{(\bar{z}+a)^2-(\bar{z}_0+a)^2} + \frac{2(z+a)\mathbf{T}}{(z+a)^2-(z_0+a)^2} + 2L_2^{-1}\mathbf{h}(z), & z\in D_2, \end{cases} \tag{68}$$

where

$$\mathbf{h}(z) = H^{-1}\left\{\frac{(z+a)\mathbf{T}}{(z+a)^2-(z_0+a)^2}\left[1-\left(\frac{(z_0+a)^2-a^2}{(z+a)^2-a^2}\right)^{1/2}\right] + \frac{(z+a)\overline{\mathbf{T}}}{(z+a)^2-(\bar{z}_0+a)^2}\left[1-\left(\frac{(\bar{z}_0+a)^2-a^2}{(z+a)^2-a^2}\right)^{1/2}\right]\right\}. \tag{69}$$

Utilizing the definition (46), the IFs can be evaluated as

$$\mathbf{K} = 4\sqrt{\pi a}H^{-1}\,\mathrm{Re}\left\{\frac{\mathbf{T}}{[(z_0+a)^2-a^2]^{1/2}}\right\}, \tag{70}$$

in which $\mathbf{T}$ is the complex quantity of the screw dislocation defined in (55).

As shown in Figure 5(a), we consider two special loading cases with a line-force p and a line-charge q located at $z_0 = -b$ $(0 < b < a)$ and $z_0 = -a - ih$, respectively. Thus from relation (55), $\mathbf{T}$ is simplified as

$$\mathbf{T} = \frac{L_2^{-1}}{2\pi i}\begin{bmatrix} p \\ -q \end{bmatrix}. \tag{71}$$

Using (70) and definition (48), the IFs and ERR for the two cases are evaluated respectively as follows

$$\mathbf{K} = \frac{2\sqrt{a}}{\sqrt{\pi}\sqrt{a^2 - (a-b)^2}} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \tag{72}$$

and

$$G = \frac{a}{\pi[a^2 - (a-b)^2]} \begin{bmatrix} -p \\ q \end{bmatrix}^{\mathrm{T}} L_2^{-1} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \tag{73}$$

for $z_0 = -b$, and

$$\mathbf{K} = \frac{2\sqrt{a}}{\sqrt{\pi}\sqrt{a^2 + h^2}} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \tag{74}$$

and

$$G = \frac{a}{\pi(a^2 + h^2)} \begin{bmatrix} -p \\ q \end{bmatrix}^{\mathrm{T}} L_2^{-1} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \tag{75}$$

for $z_0 = -a - \mathrm{i}h$.

Results (72)–(75)) are in agreement with those given by Li and Fan using the method of dual integral equations [29].

6. Conclusions

General solutions for a screw dislocation interacting with the interface and collinear interfacial cracks between two bonded dissimilar piezoelectric media are obtained. These solutions can be used as fundamental solutions for the analysis of interfacial cracks in piezoelectric bimaterials subjected to arbitrary coupled antiplane mechanical and in-plane electrical loadings. In the current formulation, the governing equations are two decoupled Laplace equations, so a series of interfacial cracks or interfacial edge cracks can be analyzed in closed-form by choosing proper mapping functions. As in linear fracture mechanics, the electroelastic field near the interfacial crack tip loaded by coupled antiplane mechanical and in-plane electric loading exhibits square-root singularity but no oscillations.

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