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NCUWM Poster Abstracts 2013

The Fifteenth Annual Nebraska Conference for Undergraduate Women in Mathematics

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POSTER ABSTRACTS

Posters by Undergraduate Students

Elizabeth Annoni, University of St. Thomas
Nicole Lopez, University of St. Thomas
Classifying open knots by random closures

The recent discovery of knotted behavior in proteins has stimulated discussion about how to classify knots in open arcs. Topologically, all open arcs are equivalent to a straight line segment. However, what people generally call knots, as in shoelaces and headphone cords, are open arcs, and thus are topologically unknotted. Still, there is entanglement in open arcs, and the goal of this project is to classify that entanglement. In particular, we compare a technique of closing via random arcs to an approach where the endpoints are connected in random directions.

Erica Budge, Hope College
Lights Out- Connecting the Dots

"Lights Out Connecting the Dots" Based on a puzzle created by Tiger Electronics, "Lights Out" is a problem in graph theory and linear algebra. A graph begins with vertices that are either on or off, and the edges determine which other vertices are affected when just one vertex is turned on or off. The objective of Lights Out is to turn off all of the lights, in which case a graph has been won. Our work involves generalized Lights Out puzzles in which each light is in one of several states, of which one is designated as "off". For certain families of graphs, we determine which graphs result in puzzles that can always be won regardless of the starting configuration of lights. As part of this process, we develop results that determine how the Lights Out puzzle is affected when two or more graphs are connected together in various ways, or when certain subgraphs are removed.

Magan Carver, University of Montevallo
Elements of Integration

In this poster we will examine the elements of integration. Starting with the Riemann integral, a discussion of its limitations leads naturally to a generalization of the notion of length, and the development of the Lebesgue integral. Some of the fundamental properties of Lebesgue Measure leading to the Lebesgue Integral will be established, along with the Lebesgue Dominated Convergence Theorem.

Mikaela Cashman, Coe College
Domino Patterns

Given a set of rules, the number of dominoes in an m by n board can be minimized taking into account color constraints. Interesting patterns emerge from the exploration of 1, 2 and 3 color boards but proving a minimum covering for a finite board proves difficult. Various methods for approaching these proofs as well as a proof of a covering for an infinite board will be discussed.

Sunita Chepuri, Bowdoin College
Analyzing Large Data Sets with Geometric Linear Algebra

Up to this point, I have learned the theories of geometric linear algebra that I will need in order to do this project, including conditions under which their matrices can be put in particular forms, properties of linear maps, and singular-value decomposition. I have also been learning how to use the linear algebra functions of MATLAB. The first data set with which I will be working is the frequency of each two-letter sequence, or digraph, across languages that use the Latin alphabet. Using singular-value decomposition and a data-visualization technique to find a basis for the row space of this matrix. I will then write the original row vectors at linear combinations of the new basis vectors and use these to create a language family tree. A second data set will be stock prices for different companies over the last 15 years. Similar analysis will allow me to find relationships between stock prices of different companies, which will help predict stock prices.

Brandynne Cho, Saint Mary's College of California
Prime Analysis in Binary

We say that two prime numbers are neighbors when a one-digit change in one primes binary form produces the other. The resulting neighborhoods can be depicted as a graph. The graph of these primes relationships was then analyzed concerning the graphs shape, possible loops in the graph, the strength (or weakness) of the connections between primes, the size of neighborhoods, and how the number of neighbors per prime were distributed throughout the graph. Analysis of the graph prompted further questioning regarding the size of the largest group of neighbors, as well as if there are multiple large neighborhoods.

Elena Crosley, Bowdoin College
Modeling Salmonella dynamics within a finishing pig farm: group structure effects on transmission

Salmonella represents 11% of all foodborne illness in the United States and 28% of all foodborne illness leading to death. One of the largest reservoirs for Salmonella, and a leading cause for the disease in humans, is the pig farming industry. Understanding the disease dynamics on the farm level is a key to developing meaningful strategies to control the disease and mitigate the public health risk. There is a general trend towards larger populations of pigs per pen and larger grower houses. To understand the implications of this trend, we have created a spatially explicit epidemic model of a grower house. The model shows that the number and configurations of the pens in the grower house can have a significant impact on the final disease prevalence and that the physical structure of the grower house cannot be ignored. We also determine that the most helpful strategies for controlling the disease are to decrease the number of pigs per pen and work to reduce the between-pen contact and transmission.

Maricela Cruz, Pomona College
Completing Partial Matrices to Satisfy the Anticommutativity Equation

A solution to the matrix completion problem finds and classifies partial matrix patterns that can be filled in to satisfy interesting properties. For the anticommutativity matrix completion problem we explore what patterns of X we can fill in to satisfy the linear matrix equation $AX = -XA$. In this equation X is a partial matrix with a pattern of specified and unspecified entries and the matrix A is skew symmetric; both X and A are square matrices. We utilize two different approaches to find patterns of specified entries in X that allow it to be completed to anticommute with A. One approach transforms $AX = -XA$ into a linear equation by applying the tensor product. The other provides a means to construct and examine a basis of the solution space to the matrix equation with the eigenvectors of A. Combining these two approaches we show what patterns of X can be completed to satisfy the anticommutativity equation. The tensor product technique and the basis technique apply to many other matrix equations.

Emma Cutler, Bowdoin College
Modeling Earth's Climate: Past and Present

There are many variables that determine the climate of the Earth, but a simple climate model that considers just two or three of these variables can provide insight into the relationships and interactions between these factors. Here, we present a one dimensional Budyko-style climate model that calculates surface temperature and ice volume based on the energy budget of the Earth. We then compare the model output to observations of our current climate and past climate changes. In an effort to explain more of the paleoclimate record, we look at the effects of forcing the model through changes in the amount of solar radiation reaching the surface of the Earth due to orbital variations, and we explore the possibility of including changes in atmospheric carbon dioxide.

Megan DeJager, Utah Valley University
Kristen Smith, Utah Valley University
Mentoring High School Girls

Math Girls Rock! is an outreach program at Utah Valley University that provides mentoring to undergraduate and high school female students. Each year, three undergraduate math and math-ed female students mentor high school girls from the surrounding area, and encourage them to continue their education in mathematics/science beyond secondary school. In this presentation, we will focus on how we have been mentoring high school students at after school meetings. This fall, students have been working on cryptography and probability projects. We will discuss how we used these projects as tools to mentor students and demonstrate to them some applications of math in everyday life.

Andrea Fant, Furman University
NFL Draft

I worked with several professors and three other students on the topic of the NFL draft. We attempted to come up with a program (we used Matlab) that predicts which football players would be most successful in the NFL based on their college statistics and other personal data. We used a combination of several different rating and ranking systems, including methods such as Offensive-Defensive, Colley, and Massey rankings. Using self-developed programs and formulas, we tried to "predict" successful players from earlier years (as early as about 1980). By comparing our predictions to the actual success of these players in the NFL, we were able to assess our program and adjust our formulas.

Anna Flores, Universidad Metropolitana
Effects of population stratification on marker phenotype associations test

Genome Wide Association Studies (GWAS) have reported unprecedented numbers of variants associated to important human traits and diseases. Commonly, the association between markers and phenotypes is assessed using Single Marker Regression (SMR). A commonly used approach to account for PS consists of expanding the SMR with inclusion of Principal Components (PC); this approach accounts for possible additive effects of PCs. However, it does not consider the possibility that the effects of individual variants change across subgroups within a population. Using a collection of wheat lines which are known to exhibit great degree of PS, we evaluated the extent to which the effects of markers may be modulated by ancestry. Our baseline model (M1) ignores the effects of PS, in a second model (M2) PS was accounted for by including additive effects of the first 2 PCs. Finally, we considered a model (M3) which accounts for additive and interaction effects of markers and PCs.

Alexandra Fry, University of Alabama at Birmingham
Tensor Decompositions with Applications

Tensors are multi-dimensional arrays of numerical values. In this poster, tensors and tensor decompositions will be discussed. The order (or rank) of a tensor is the dimensionality of the array needed to represent it. Decompositions of higher-order tensors have numerous applications including the compression of 3-dimensional images. In particular, the singular value decomposition (SVD) is a factorization of a real or complex matrix, which will be presented with some of its useful applications.

Shontrice Garrett, Jackson State University
Rosland Latiker, Jackson State University
Precious Winters, Jackson State University
The Use of Mobile Devices to Encourage Mathematical Discourse

Participation in mathematical dialogue allows students an opportunity to solidify their understanding of mathematical concepts. Student response systems have been shown to encourage student engagement in meaningful classroom discussions. While the cost of these systems is extraordinary, another avenue for utilizing this method of classroom voting has been identified with the use of mobile devices. The goal of this study is to determine how incorporating mobile devices in undergraduate mathematics courses facilitates mathematical discourse at an urban Historically Black College and University (HBCU). The following research questions will be addressed: (a) Are mobile devices accessible and efficient tools for collecting and utilizing student responses? (b) Can mobile devices be used to identify problems/questions that yield inconsistent student responses? (c) Can mobile devices be used as student response systems to identify questions that encourage mathematical discourse?

Amanda Hoisington, University of California, Riverside
Classification of Parseval Frames in $\mathbb{Z}_2^n$ by Transmission Error

It is common for information to be transmitted digitally, thus message vectors can be thought of as binary vectors. In practice, lossy connections produce errors in a transmitted message. In these cases, a spanning set of linearly dependent vectors known as a frame can be used to reduce reconstruction errors. Parseval frames have particularly nice properties that allow them to be used for both deconstruction and reconstruction. Vershynin (2005) proved that for $\mathbb{R}^n$, in order to bound the probability of erroneous reconstruction, the necessary amount of vectors required to survive the transmission is a multiple of $n \log n$. We seek to extend these results to $\mathbb{Z}_2^n$, and in doing so, have constructed a classification of Parseval frames with respect to the probability of faulty reconstruction.

Leah Jean-Louis, Swarthmore College
Completing Partial Matrices to Satisfy the Anticommutativity Equation

A solution to the matrix completion problem finds and classifies partial matrix patterns that can be filled in to satisfy interesting properties. For the anticommutativity matrix completion problem we explore what patterns of X we can fill in to satisfy the linear matrix equation $AX = -XA$. The matrix X is a partial matrix with a pattern of specified and unspecified entries and the matrix A is skew symmetric; both X and A are square matrices. We utilize two different approaches to find patterns of specified entries in X that allow it to be completed to anticommute with A. One approach transforms $AX = -XA$ into a linear equation by applying the tensor product. The other constructs and examines a basis of the solution space to the matrix equation with the eigenvectors of A. We show what patterns of X can be completed to satisfy the equation. Some restrictions on the patterns of X are that X needs to have n specified entries, must be symmetric, and cannot have symmetric locations specified.

Chelsea Jenkins, Piedmont College
When Groups Become Friends

If two positive integers have the same abundancy, then they form a friendly pair. We abstract this notion of friendliness in the context of group theory; specifically, in the category of finitely generated abelian groups. To do so, we define a notion of the abundancy of a finitely generated abelian group by exploiting the Fundamental Theorem to obtain a sequence of integers that uniquely determines such a group up to isomorphism. Subsequently, we look at examples of friendly groups and state conditions under which groups will be friendly.

Melanie King, Mercer University
Aleesha Moran, McKendree University
Sufficient conditions for arc-disjoint Hamiltonian paths and out-branchings in tournaments

We examine conditions sufficient for the existence in a tournament of a Hamiltonian path and out-branching rooted at the same vertex that are arc-disjoint. We show that in a strong tournament the existence is guaranteed and that the-branching can be taken to be path-like or star-like. We show that in a regular tournament on 2n + 1 vertices has a Hamiltonian path and n-1 out-branchings that are mutually arc-disjoint, and we improve several results related to the well-known Kelly conjecture.

Anna Kralovec, Western Oregon University
Secret Sharing
Secret Sharing is a cryptographic scheme in which the secret is broken into several smaller shares, unrecognizable individually. These shares are then distributed to share-holders, and unless a specified number of shareholders cooperate to combine their individual shares, the secret cannot be recreated. In our poster, we investigate an example where secret sharing would be useful, explore the math that makes it secure, and also share applications in today's world.

Rosland Latiker, Jackson State University
see **Shontrice Garrett**

Nicole Lopez, University of St. Thomas
see **Elizabeth Annoni**

Laura Lyman, Reed College
Incongruent Restricted Disjoint Covering Systems with 9-6-3 Constructions

It is impossible for a covering of the integers to be both incongruent and disjoint; however, systems of congruences that cover only finite intervals can satisfy these conditions simultaneously. Such systems are called incongruent restricted disjoint covering systems (IRDCS). We address the question of whether there exists an $n \in \mathbb{N}$ such that there exist IRDCS with 9-6-3 constructions of all lengths greater than or equal to n, which we conjecture in the affirmative for $n = 25$.

Heather Magee, University of Central Oklahoma
Bound Smoothing using Bordered Euclidean Squared Distance Matrices

A distance matrix A which encodes squares of pairwise distances in matrix form is known as a Euclidean Squared Distance Matrix (ESDM). Bordered ESDMs are useful in determining the embedding dimension of points in space. We investigate known methods that use these bordered ESDMs to improve the bounds on unknown distances of four points in three dimensional space (using the tetrangle inequality) and extend these ideas to five points.

Connie Maluwelmeng, Iowa State University
Discrete Morse Theory and Spaces of Trees

The purpose of this research is to find a space that is homotopy equivalent to the three different spaces of rooted trees with ordered vertices: general trees, binary trees, and almost binary trees. Binary trees are defined to have interior vertices with degree 3, leaves with degree 1, and a root with degree 2. We define almost binary trees to have the same traits except that a root can be any degree. We will use Discrete Morse Theory to help identify those spaces homotopy equivalent to each tree space.

Carolyn Mayer, Bowdoin College
Modeling Paleoclimate: Oceanic Impact on the Carbon Cycle

Paleoclimate records retrieved from ice cores and sediment data indicate that the Earths temperature fluctuates periodically. Budyko and others have modeled global temperature changes based on insolation, albedo, heat transport, and atmospheric carbon. Working with a one-dimensional Budyko-style energy balance model, we can understand the contributions of these factors to changing climate. Adding Milankovitch forcing, we see temperature fluctuation in our model with a frequency that is similar to that found in Paleoclimate records. However, the addition of Milankovitch forcing does not produce as large an amplitude of change as seen in the records. Thus a Budyko-style model with Milankovitch forcing alone is not sufficient to model changing temperature on glacial time scales. We are currently investigating the role of the ocean in the global carbon cycle. We hope to add the impact of ocean temperature and alkalinity to our existing model and to observe their impact on changing carbon and temperature.

Aleesha Moran, McKendree University
see **Melanie King**

Lauren Morey, The University of Montana
A new proof of an integral formula for counting perfect matchings in simple graphs

A perfect matching M in a graph G is a collection of pairwise disjoint edges of G with the property that each vertex of G is an end of an edge in M. This talk will examine the question of how many perfect matchings a graph may contain. In [*Combinatorica* **1** (1981), 257–262], Godsil answered this question using an integral counting formula, and he proved it using an induction argument. Emerson and Kayll [*Contributions to Discrete Mathematics* **4** (2009), 89–93] mentioned the possibility of a different, more enlightening proof of Godsil's result, but they provided no details. Here I will present those details. The audience may appreciate the interplay between continuous and discrete mathematics.

Zi Ouyang, University of Dayton
A Solution Algorithm of Fractional Difference Equations

We consider a linear fractional difference equations of arbitrary order,

$$\nabla_0^\nu y(t) + ay(t) = b,$$

where a and b are constants and ν, the order of the difference is positive. ∇_0^ν represents a Riemann-Liouville fractional backward difference operator. A solution algorithm is constructed by using a discrete transform method. Sufficient conditions for convergence of the formal solutions are given. Further discussions include applications of the algorithm to equations of the form

$$\nabla_0^\nu y(t) + ay(t) = f(t).$$

Kristina Pardo, Furman University
College Football Rankings

The purpose of this research is to identify an algorithm based upon team statistics that ranks FBS college football teams more accurately than the BCS rankings system. Ranking methods used included the Massey, Keener, and Colley, as well as variations on these. The rankings were based off of the following statistics: points scored, total yards, passing yards, possession time, and more. For the years 2009-2011, the Massey Method based solely on points scored (Massey Points) had the highest prediction percentage for bowl game outcomes. This method was later modified with a Similar Teams method, which looks through each bowl teams regular season schedule, finds the team most similar to their bowl game opponent. Other ranking systems were developed over the course of the research period, but none were as successful. Continuation of this project will be focused on finding a new method that trumps the modified Massey, or continuing to modify the Massey to improve results.

Jamie Peabody, California State University, Fresno
Cycles and Cycle-related Graphs in $PG(2, q)$

We establish that it is possible to embed k-cycles into $PG(2, q)$, for all $3 \leq k \leq q^2 + q + 1$, as long as a hypothesis on the generators of the finite field $GF(q)$ holds. We have verified this hypothesis to be true for finite fields of prime order $p < 10^{10}$. Furthermore, we expand upon this result to describe the behavior of embeddings of cycle-related graphs, such as wheels and gear graphs. For these families of graphs, we have obtained sharp bounds for embeddability and ways to embed all graphs allowed by these bounds.
Many of our results may be generalized to $AG(2, q)$, and other projective planes.

Samantha Shepler, Winona State University
The Statistics Behind Racial Profiling

Racial profiling is the use of a persons race or ethnicity as the key factor in deciding if a person is a security threat. Many questions have been raised whether the method of racial profiling is ethical, but is it even effective? I am researching the use of racial profiling within airport security through simulations and statistical analyses. The anticipated outcome of this project is to show that strong racial profiling is no more effective than simple uniformly random sampling in catching a potential terrorist.

Kristen Smith, Utah Valley University
see **Megan DeJager**

Molly Stukenholtz, University of Nebraska-Lincoln
Jian Ju (Justine) Yeo, University of Nebraska-Lincoln
A Performance Analysis of the Lincoln Housing Market

In United States history, the collapse of the housing market in 2007 is reputed to be the worst. States like California and Nevada experienced a huge decline in the housing market. The purpose of this project was to compare the local housing market in Lincoln, Nebraska to the nationwide situation. Data from 2006 to 2011 provided by the Lancaster County Assessor was used to analyze the performance of the Lincoln housing market. Thorough analyses using various statistical methods were conducted across time, location, and price range. Results indicate that the Lincoln housing market has been stable over time and even at the time of the national housing market collapse.

Alexa Syryczuk, University of Wisconsin- Eau Claire
Constructing 4-Dimensional Tops

The polar duality transformation takes a polytope with integer lattice points to its polar dual. If the polar dual is also a lattice polytope then we refer to the polytopes as reflexive polytopes. Reflexive polytopes have been classified in 3 dimensions and 4 dimensions, with 4,319 and 473,800,776 classes respectively. A lattice polytope that contains the origin is known as a top, which are related to polytopes. Bouchard and Skarke have classified the 3 dimensional tops corresponding to each class of reflexive 2 dimensional base polytopes.. We use triangulations of 3-dimensional reflexive polytopes to construct new, "exceptional" examples of tops.

Brandy Thibodeaux, University of Louisiana at Lafayette
The Interlace Polynomial of the Wheel Graph

In this paper we study the interlace polynomial of wheel graphs, which are simple graphs in the shape of a wheel. We develop an explicit formula for the interlace polynomial $q(W_n, x)$ for certain n, where W_n is the wheel graph with n external edges. From this explicit formula, we are looking to develop a recursive formula for $q(W_n, x)$. We also give the formulas for the interlace polynomials for some simple tree graphs. Also, we give explicit formulas for the interlace polynomials of the star graph $q(S_n, x)$ where n is the number of edges and the bowtie graph $q(B, x)$. Both star graph and bowtie graph are subgraphs of the wheel graph and aid in computing the interlace of the wheel graph. Our interest in the wheel graph comes from applications of the wheel graph to radio numbering, which is the assignment of labels, traditionally represented by integers, to the vertices of a graph.

Aubrey Thompson, University of Nebraska-Lincoln
Fast Approximation Algorithms for Spectral Clustering

The employee-attrition problem consists of separating the individuals that are likely to quit from those that are not. In this project, we approached this problem using the spectral clustering algorithm. Spectral clustering is a well-known method for separating individuals into groups of similar points according to their characteristics. This method relies on solving an eigenvector problem, which may become computationally expensive for large datasets. To overcome this constraint, we studied several approximations to the spectral clustering algorithm which aim to reduce running time while maintaining the classification structure. We compared these algorithms in terms of running time and accuracy. We also researched improvements to the algorithms; we provide evidence that these improvements are worthwhile. The exact spectral algorithm was used to solve the attrition problem on a large dataset, and the results were compared with the approximations.

Madelyn Twain, Western Oregon University
Lambda Permuations of Power Series on their Circle of Convergence

λ-permutations are bijections of the positive integers which both preserve the convergence of infinite series and allow at least one conditionally divergent series to converge. We apply this concept to the setting of power series and examine the effects of rearrangement on their circle of convergence. In particular, we study the topological properties of the set of divergence of a power series following rearrangement by a λ-permutation.

Kimberly Vilimas, University of Redlands
Mathematical Modeling Interactions between Human and Fish populations: A Bio-Economic Approach

Mathematically modeling interactions between fish and fisherman can be a powerful tool in the formulation of optimal management strategies. In our research, we utilized computer simulations in order to model a system of nonlinear recurrence relations which implement population mathematics as well as economic concepts. We then analyzed the system by means of fixed points, Jacobian stability testing, graphical interpretations, and parameter analysis using the Jury test. The main objectives of this piece are to: (i) recreate and verify the traditional pure open access bio-economic model pioneered by Gordon [1954], Scott [1955] and Schaeffer [1957] and (ii) discuss the implications these results have on the fish industry today, including what needs to be improved and updated.

Marla Williams, Willamette University
Accepted Elasticity in Arithmetic Congruence Monoids

Given $M(a, b)$, a singular local arithmetic congruence monoid (ACM), all elements of $M(a, b)$ other than 1, can be factored into irreducibles, but this factorization is not necessarily unique. Given an element $r \in M(a, b)$, and $\mathcal{L}(r)$ the set of factorization lengths of r in $M(a, b)$, then the elasticity of r is defined to be $\rho(r) = \frac{\max \mathcal{L}(r)}{\min \mathcal{L}(r)}$, and the elasticity of the monoid is $\rho(M(a, b)) = \sup\{\rho(r) | r \in M(a, b)\}$. The elasticity of the monoid is accepted if there is some $r \in M(a, b)$ such that $\rho(r) = \rho(M(a, b))$. We introduce a notation which allows us to characterize new families of ACMs having accepted elasticity.

Precious Winters, Jackson State University
see **Shontrice Garrett**

Jian Ju (Justine) Yeo, University of Nebraska-Lincoln
see **Molly Stukenholtz**