1975

GRAVITATION

GRAVITATION

INTRODUCTION

The members of the solar system - the Sun, the Moon, and the planets - have held a strong fascination for mankind since prehistoric times. The motions of these heavenly bodies were thought to have important specific influences on persons' lives - a belief that is reflected even today in horoscopes and astrological publications. A revolution in man's thinking that occurred about four hundred years ago established the concept of a solar system with planets orbiting about the Sun and moons orbiting about some of the planets. Copernicus, Kepler, Galileo, and Newton were the four scientific leaders chiefly responsible for establishing this new viewpoint. One of its very practical aspects, yet difficult for us earth-bound creatures to grasp, is that the force of gravity gradually diminishes as one recedes from the Earth, in a way beautifully stated by Newton in his universal law of gravitation.

Gravity is a universal force: It acts on every material thing from the smallest nuclear particle to the largest galaxy. It even acts on objects that have zero rest mass, such as photons - the fantastically minute "chunks" in which light comes. One of the most exciting areas of astronomical research today is the "black hole," where the gravitational field may be so immense that not even light can escape!

Newton's law of gravitation is important not only in itself, but also because it serves as a model for the interaction of electric charges, which you will study later. Not only are the force law and the potential-energy function nearly the same, but the concept of a field carries over and becomes even more useful in the calculation of forces between electrically charged particles.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Relate the resultant force acting on a particle to the particle's mass and acceleration (needed for Objectives 1 and 2 of this module)	Newton's Laws Module
*Relate the acceleration of a particle moving in a circular path to its speed and the radius of the path (needed for Objective 2 of this module)	Planar Motion Module
*Use the principle of conservation of total mechanical energy to solve problems of particle motion (needed for Objective 3 of this module)	Conservation of Energy Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Law of gravitation - Use Newton's law of universal gravitation to determine (a) the (vector) gravitational force exerted by one object on another - or the distance or a mass when the force is known; and (b) the gravitational field of an object.

2. Circular orbits - Use the gravitational force law, together with the expression for centripetal acceleration, to find the speed, period, orbital radius, and/or masses of objects moving in circular orbits as a result of gravitational forces.

3. Energy conservation - Determine the potential energy of one object in the gravitational field of another; and use energy conservation to relate changes in this potential energy to changes in kinetic energy and speed of the first object.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Sections 10.8, 10.9, and 10.11 in Chapter 10 of Bueche. Recommended: Read Chapter 14, Sections 14-4 and 14-8 thru 14-10 of Halliday and Resnick (HR)* or Sections 13-5, 13-6, and 13-8 in Chapter 13 of Weidner and Sells (WS).* Optional: Read Sections 10.10, 10.12, and 10.13 of Bueche.

If possible, you should read through a derivation of the result for large, spherically symmetric objects mentioned in General Comment 2 below; and you may wish further discussion of gravitational potential energy, beyond that given in General Comment 4. Both these topics are covered in the Recommended Readings above.

A correction to Figure 10.13: The quartz fiber in a Cavendish balance is actually very fine, and not twisted! In use, the mirror rotates through only a small angle.

Work the problems for Objective 2 starting from the fundamental gravitational and centripetal force expressions. Do not try to remember the equations in Illustrations 10.5 and 10.7.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text[b]	Assigned Problems: Study Guide	Additional Problems (Chap. 10)
1	General Comments 1, 2; Secs. 10.8, 10.9	A	Illus. 10.6	D, E(b), F(a)	J, 12-14, 24
2	General Comment 3; Sec. 10.11	B	Illus. 10.5, 10.7	E(a), F(b), G(c), I(a, c)	K, L, 23, 25, Quest.[c] 1, 11, 13
3	General Comment 4; HR*: Secs. 14-8 thru 14-10; or WS*: Secs. 13-5, 13-6	C a		G(a, b), H, I(b)	M, N, 17

[a]See Example in General Comment 4. [b]Illus. = Illustration(s). [c]Quest = Question(s).

*HR = David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974).

WS = Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Chapter 14 in your text; but Sections 14-3 and 14-5 are optional. Also, you will not be expected to reproduce the derivation in Section 14-4; however, you should read it through because the result, Eq. (14-8), is very important.

At this point, it's quite likely that you haven't yet studied oscillations (as in the module Simple Harmonic Motion). But you need not be dismayed at the two references to simple harmonic motion - they're not critical to your understanding of this module, and you can simply take the stated results about oscillations at face value.

On p. 266 in Section 14-8, the quantity "F(r)" would more appropriately be called "$F_r(r)$." It is really the component of $\vec{F}(r)$ along the outward radial direction (which can be negative and is, in this instance), whereas the notation used makes it look like the magnitude of a force (which cannot be negative).

Work the problems for Objective 2 starting from the fundamental gravitational and centripetal force expressions. Do not try to remember Eq. (14-13) and the subsequent equation.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems (Chap. 14)
1	General Comments 1, 2; Secs. 14-1, 14-2, 14-6	A		D, E(b), F(a)	J, 3, Quest. 1, 26
2	General Comment 3	B	Example*	E(a), F(b), G(c), I(a, c)	K, L, 20, 22-27, Quest.† 5, 8, 12, 21
3	General Comment 4; Secs. 14-8 thru 14-10	C	General Comment 4 Example; Ex.† 4, 5	G(a, b), H, I(b)	M, N, 30-34, 43-45

*Study derivation of Eqs. (14-12) and (14-13) in Section 14-7 (pp. 262, 263).

†Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Chapter 5, Sections 5-4 and 5-5, Chapter 6, Section 6-9, and Chapter 7, Section 7-4. Recommended: Read Sections 14-4 and 14-8 through 14-10 in Chapter 14 of Halliday and Resnick (HR),* or Sections 13-5, 13-6, and 13-8 in Chapter 13 of Weidner and Sells (WS).* Optional: Read Section 6-10 of the text.

If possible, you should read through a derivation of the result for large, spherically symmetric objects mentioned in General Comment 2 below; and you may wish further discussion of gravitational potential energy, beyond that given in General Comment 4 and Section 7-4 in the text. Both these topics are covered in the Recommended Readings above.

Work the problems for Objective 2 starting from the fundamental gravitational and centripetal force expressions. Do not try to remember the equations in Section 6-9.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	General Comments 1, 2; Secs. 5-4, 5-5	A	Sec. 5-4, Ex. 1, 2; Sec. 5-6, Ex. 8	D, E(b), F(a)	J, 5-2, 5-4, 5-8 to 5-12
2	General Comment 3; Sec. 6-9	B	Sec. 6-9, Ex.	E(a), F(b), G(c), I(a, c)	K, L, 6-43 to 6-47, 7-36(a), 7-50(e, f)
3	General Comment 4; Sec. 7-4 after Ex.[a] 5; HR*: Secs. 14-8 thru 14-10; or WS*: Secs. 13-5, 13-6	C b		G(a, b) H, I(b)	M, N, 7-36(b, c)

[a]Ex. = Examples(s). [b]See Example in General Comment 4.

*HR = David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974).

WS = Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Chapter 13 in your text; but Sections 13-4 and 13-7 are optional. Also, you will not be expected to reproduce the derivation in Section 13-8; however, you should read it through because the result, Eq. (13-18) and the implied equation for the force due to a large spherical object, is very important.

Work the problems for Objective 2 starting from the fundamental gravitational and centripetal force expressions. Do not try to remember the equations derived for planetary and satellite motion.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	General Comments 1, 2; Secs. 13-1 thru 13-3	A	Ex.* 13-2	D, E(b), F(a)	J, 13-2, 13-10
2	General Comment 3	B	Ex. 13-1	E(a), F(b), G(c), I(a, c)	K, L, 13-5, 13-8, 13-9, 13-20
3	General Comment 4; Secs. 13-5, 13-6	C †	Ex. 13-4	G(a, b), H, I(b)	M, N, 13-12(b), 13-14 to 13-16, 13-21, 13-26, 13-27

*Ex. = Example(s).

†See Example in General Comment 4.

GENERAL COMMENTS

1. The Gravitational Force Law and the Gravitational Field $\vec{g}$

The LAW OF UNIVERSAL GRAVITATION is easily expressed:

> Every particle (with mass M_2, say) in the universe is attracted toward every other particle (with mass M_1, say) by a force with magnitude
>
> $$F = G(M_1M_2/r^2),$$
>
> where r is the distance between the two particles, and
>
> $$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$$
>
> is an experimentally measured universal constant (see Fig. 1).

(In the figure, M_1 has been placed at the origin for later convenience.) Note that the direction of $\vec{F}$ is exactly along the line joining the two masses. This law can also be expressed vectorially:

> The gravitational force experienced by M_2 due to the presence of M_1 is
>
> $$\vec{F}_{21} = -G(M_1M_2/r^2)\hat{r},$$
>
> where r is again the distance of separation, and $\hat{r}$ is a unit vector pointing in the direction from M_1 toward M_2 (see Fig. 1).

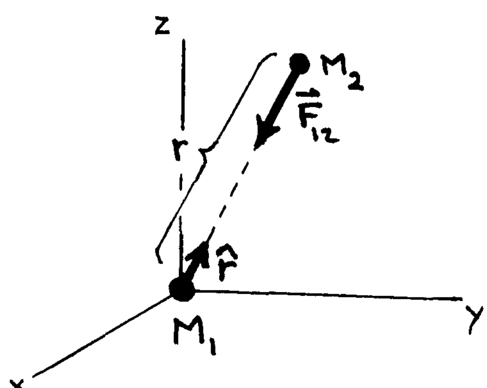


Figure 1

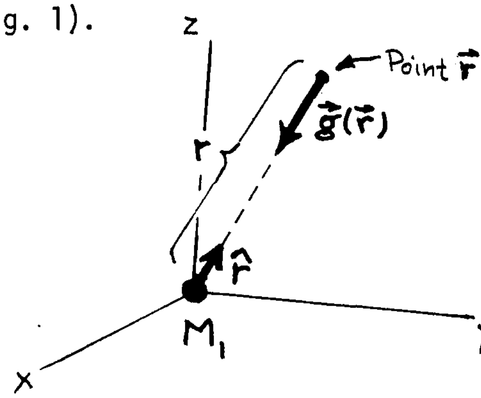


Figure 2

Two masses are required in order to talk about the gravitational force $\vec{F}_{21}$. But notice that if we divide through by M_2, we obtain a quantity

$$\vec{g}(\vec{r}) \equiv \frac{\vec{F}_{21}}{M_2} = -G\left(\frac{M_1}{r^2}\right)\hat{r}$$

that depends only upon the magnitude of M_1 and the <u>point</u> where M_2 is located, relative to M_1. [Since $\vec{r} \equiv r\hat{r}$ is simply the position vector of that point relative to M_1, we have indicated this latter dependence by writing $\vec{g}(\vec{r})$, instead of simply $\vec{g}$.] See Figure 2.

In fact, we don't need M_2 at all in thinking about this quantity $\vec{g}(\vec{r})$, which is called the <u>gravitational field intensity</u> or, more simply, just the <u>gravitational field</u> due to M_1.

This abstraction $\vec{g}$ associated with a single mass M_1 occupies all space surrounding M_1 whether other masses are present or not; and at each point, it can be represented by a vector pointing toward M_1, with the magnitude GM/r^2. Notice that, physically, the gravitational field intensity at a given point is simply the acceleration a very small object would experience <u>if</u> it were placed at that point.

The use of the concept of a force field to describe an interaction at a distance is an exceedingly important technique, and will be developed further in later modules on electric and magnetic interactions. The gravitational field is a central conservative field; central because it acts along the line joining the interacting particles, and conservative (for energy) because it is possible to define a potential energy function of distance. If you have already studied torque and angular momentum, you will recognize that a particle subject only to the centrally directed gravitational force of another body experiences no torque; thus its angular momentum is constant, or "conserved." (For circular orbits, this reduces to the simple result that the speed is constant.) Furthermore, as the particle changes its position (in whatever kind of an orbit), all decreases in kinetic energy are accompanied by equal increases in gravitational potential energy, and vice versa, so that the total energy remains constant. In this module, you will make extensive use of the conservation of energy. Note that because the moving particle is subject to a force, its linear momentum is <u>not</u> constant.

2. <u>"Large" Spherically Symmetric Objects</u>

One of the interesting and useful consequences of the functional form of the law of gravitation (namely, the dependence $1/r^2$) is that the gravitational field of an extended spherically symmetric object of mass M and radius R (see Fig. 3) is exactly the same as the field of a "point" (i.e., very small) object of mass M located at the same place as the center of the sphere. That is, the gravitational field due to 3(a) is exactly the same as the field due to 3(b) for every point $r > R$. (For $r < R$, the fields are very different for these two situations.) Therefore, whenever you encounter a spherically symmetric object, you can simplify the situation by replacing the object with a point object of equal mass - assuming you are interested only in points outside the spherical object, and not, say, in a tunnel through its middle.

Caution: When using the gravitational law in such situations, be sure to use the distance from the center of the gravitational attraction, and not the height above the surface of the Earth or the other body!

Figure 3

(a)

(b)

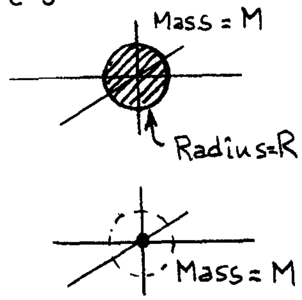
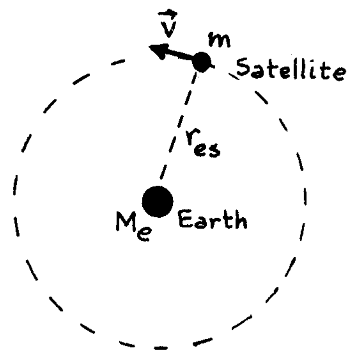


Figure 4



3. Circular Planetary and Satellite Orbits

In reality, the orbits of planets and satellites are never exactly circles, but, rather, more general ellipses. However, the orbits of most planets and of many satellites are near enough to circular that only a very small error results from treating them as circular. This simplifies the calculations greatly, since then you can use what you learned about circular motion in the module Planar Motion. In fact, you found in Problem G of Planar Motion that a circular trajectory such as

$$\vec{r}(t) = 2[\cos(\pi t/4)\hat{i} + \sin(\pi t/4)\hat{j}]\ \text{m}$$

has an acceleration

$$\vec{a}(t) = -\omega^2\vec{r}(t),$$

where $\omega = v/r$. [See, in particular, parts (c) and (d) of that problem.] That is, a particle moving in a circular path of radius r at the constant speed $v = \omega r$ has a centripetal acceleration with magnitude $a_c = \omega^2 r$.

Furthermore, you found in the module Newton's Laws that it takes a force $\vec{F} = m\vec{a}$ to give a particle with mass m the acceleration a. From these last three equations, it follows that the centripetal force

$$F_c = m\omega^2 r = mv^2/r$$

is required to hold a particle in a circular path.

In the case of a satellite moving around Earth as in Figure 4, this centripetal force is provided by the gravitational force of attraction between Earth and the satellite. Since $M_e >> m$, we can ignore the motion of Earth; it acts just like a fixed force center. As you learned in your studies for Objective 1 of this module, the gravitational force acting on the satellite has the magnitude

$$F_g = GM_e m/r_{es}^2.$$

Equating F_c to F_g yields

$$\frac{mv^2}{r_{es}} = G\frac{M_e m}{r_{es}^2} \quad \text{or} \quad v^2 = \frac{GM_e}{r_{es}}.$$

This relation allows us to calculate, say, v in terms of M_e and r_{es}. Once we have found v from a relation such as the above, then it is, of course, easy to find the period T of the circular motion, since vT is just the circumference $2\pi r$ of the circular orbit.

The motion of planets around the Sun is very similar; since the Sun has a mass much greater than that of any planet, it can be treated as a fixed force center, just as the Earth was above.

4. Gravitational Potential Energy

In an earlier unit, you learned that an object of mass m that is raised a distance h in the vicinity of Earth's surface gains potential energy in the amount mgh. Let's show this directly from the law of gravitation, $F = GM_e m/r^2$. The work done lifting m is the integral of the force we must exert through the given distance:

$$W = \int_R^{R+h} \left(\frac{GM_e m}{r^2}\right) dr = GM_e m \int_R^{R+h} \frac{dr}{r^2} = -GM_e m\left[\frac{1}{r}\right]_R^{R+h}$$

$$= -GM_e m\left[\frac{1}{R+h} - \frac{1}{R}\right] = GM_e m \frac{h}{R(R+h)}.$$

When the height h is much less than the radius R of the Earth, this yields the approximate value $W \simeq GM_e mh/R^2 = mgh$.

Note that this is valid only when $h \ll R$.

What if we go all the way from the surface of Earth to infinity? This time we get

$$W(R \text{ to } \infty) = -GM_e m\left[\frac{1}{r}\right]_R^{\infty} = -GM_e m\left[0 - \frac{1}{R}\right] = +\frac{GM_e m}{R}.$$

This is the amount of energy that must be expended to carry a mass m from the surface of Earth to a point infinitely far away. (We are neglecting the presence of the other planets and the Sun.) Since it is convenient and customary to say that an object has zero potential energy at infinity, we see that the gravitational potential energy of an object is a negative quantity (zero only at infinity) that becomes more negative as the object approaches any other massive object. For instance, as an object approaches Earth, it loses more and more potential energy (its potential energy becomes more and more negative), and its kinetic energy becomes correspondingly greater. Recall that space capsules returning from the Moon attain extremely high velocities just before reaching Earth's atmosphere. This is exactly analogous to the example of a car gaining speed as it coasts down a steep hill - potential energy is being transformed into kinetic energy.

If we use the customary symbol $U(r)$ to denote gravitational potential energy, then our result above is just

$$U(r) = -G(M_e m/r),$$

for a point or spherically symmetric mass. A particle in such a gravitational field (with no other forces present) always moves in such a way that the sum of its kinetic and gravitational potential energies is constant:

$$E_i = \text{Total initial (mechanical) energy} = K_i + U_i$$

$$= K_f + U_f = \text{Total final (mechanical) energy} = E_f.$$

This energy-conservation equation is very useful in solving many problems.

Example

Space scientists wish to launch a 100-kg probe to infinity (i.e., far from Earth). How much energy does this require? What initial speed is needed? Ignore the presence of the Sun for this example. ("Initial" means at the time of burnout, which for this problem is only a negligible distance above the surface of Earth.)

Solution

The minimum required energy K_i is that which gets the space probe to "infinity" with zero kinetic energy. That is,

$$K_i + U_i = K_f + U_f = 0 + 0,$$

or

$$K_i = -U_i = \frac{GM_e m}{R_e} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(100)}{6.4 \times 10^6} \text{ J} = 6.3 \times 10^9 \text{ J}.$$

The needed initial speed follows from the relation

$$K_i = (1/2)mv_i^2 = GM_e m/R_e,$$

which yields

$$v_i = \sqrt{2GM_e/R} = 1.12 \times 10^4 \text{ m/s}.$$

ADDITIONAL LEARNING MATERIALS

Auxilliary Reading

Stanley Williams, Kenneth Brownstein, and Robert Gray, Student Study Guide with Programmed Problems to Accompany Fundamentals of Physics and Physics, Parts I and II, by David Halliday and Robert Resnick (Wiley, New York, 1970).

Objective 1: Sections 14-1 and 14-2;

Objective 2: Section 14-8, especially Problems 1 to 7, and 15 to 17;

Objective 3: Sections 14-5 through 14-7, and 14-8, Problems 19 to 28.

Film Loop

Ealing #80-212: Measurement of "G"/The Cavendish Experiment.

Various Texts

Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition: Sections 10.8 through 10.13.

David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing 1974): Chapter 14.

Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition: Sections 5-4, 5-5, 6-9, 6-10, 7-4.

Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1: Chapter 13.

PROBLEM SET WITH SOLUTIONS

Since some of the problems for this module are numerically arduous, you may use the simplified numerical values below (accurate to within a few percent) when working these problems.

$G = (2/3) \times 10^{-10}$ N m^2/kg^2	$\pi^2 = 10$
Mass of Sun = 2.0×10^{30} kg	Radius of Sun = 7.0×10^{8} m
Mass of Earth = 6.0×10^{24} kg	Radius of Earth = 6.4×10^{6} m
Mass of Moon = $(3/4) \times 10^{23}$ kg	Radius of Moon = $(1/6) \times 10^{7}$ m
Mass of Mars = 6.3×10^{23} kg	Radius of Mars = $(1/3) \times 10^{7}$ m
Earth to Moon = $(3/8) \times 10^{9}$ m	Earth to Sun = 1.5×10^{11} m
g on Earth = 10 m/s^2	Saturn to Sun = 1.5×10^{12} m

A(1). A space probe determines that the magnitude of the gravitational field $\vec{g}$ is 1.1 times as large at the surface of Uranus as it is at the surface of Earth. Use the radius of Uranus, $R_u = 2.4 \times 10^7$ m, to determine its mass.

Solution

Since $g = GM/r^2$, we have

$$M_u = gR_u^2/G = \frac{[(1.1)(2.4 \times 10^7)^2]}{[(2/3) \times 10^{-10}]} \text{ kg} = 9.5 \times 10^{25} \text{ kg}.$$

B(2). Communications satellites, such as Telestar, are placed in synchronous orbits around Earth. (A synchronous orbit is an orbit in which the satellite is constantly above the same spot on the surface of Earth.) How far above the surface of Earth must such a satellite be? Be sure to start from the fundamental gravitational and centripetal force equations.

Solution

For a synchronous orbit, the angular velocity of the satellite must be the same as that of Earth, namely, $\omega = 2\pi \times$ (number of revolutions per second) $= (2\pi/24 \times 60 \times 60)$ rad/s.

Since the Earth is much heavier, it is nearly stationary. Therefore, the radius of the satellite's orbit is virtually the same as the distance between the center of the Earth and the satellite; call this distance r. Then the equation $F_{centrip} = F_{grav}$ becomes just

$$mr\omega^2 = GM_e m/r^2$$

or

$$r^3 = GM_e/\omega^2 = [(2/3) \times 10^{-10}](6.0 \times 10^{24})[(12 \times 3600)^2/\pi^2] \text{ m}^3 = 74 \times 10^{21} \text{ m}^3.$$

And thus $r = 4.2 \times 10^7$ m. But the problem asked for the height above the surface of Earth:

$$h = r - R_e = 4.2 \times 10^7 \text{ m} - 0.60 \times 10^7 \text{ m} = 3.6 \times 10^7 \text{ m}.$$

C(3). If an object is fired from the surface of Earth with a great enough speed v_0, it will escape from the gravitational field of Earth and will not return. What initial speed is needed for an object fired vertically to rise to a maximum height $R_e/3$ above the surface of Earth, before it returns? Express your answer in terms of v_0; R_e is the radius of Earth. (You do not need to use any numerical values!)

Solution

At its maximum height the object will not be moving, so that $K_f = 0$. Therefore, $K_i + U_i = K_f + U_f$ becomes

$$\frac{1}{2}(mv_i^2) - \frac{GM_e m}{R_e} = 0 - \frac{GM_e m}{(4R_e/3)}, \qquad v_i^2 = \frac{1}{2}\left(\frac{GM_e}{R_e}\right).$$

Thus

$$v_i = \left(\frac{GM_e}{2R_e}\right)^{\frac{1}{2}} = \left(\frac{[(2/3) \times 10^{-10}](6.0 \times 10^{24})}{2(6.4 \times 10^6)}\right)^{\frac{1}{2}} = 3.8 \times 10^3 \text{ m/s}.$$

Problems

D(1). (a) At what height above the surface of Earth is the gravitational field equal to 5.0 m/s^2? Express your answer in terms of the radius of Earth R_e.

(b) At what point between the Earth and the Sun does an object feel no gravitational force? Express your answer in terms of the masses M_e and M_s, and the Earth-to-Sun distance r_{es}.

E(1,2). Jupiter has a moon with an approximately circular orbit of radius 4.2×10^8 m and a period of 42 h.

(a) What is the magnitude of the gravitational field $\vec{g}$ due to Jupiter at the orbit of this moon?

(b) From (a) and the value of G, find the mass of Jupiter.

F(1,2). Answer the questions below, using only Newton's law of universal gravitation, the centripetal force law, and the following data:

At the surface of the Earth, $g = 9.8$ m/s^2.

The radius of Earth is 6400 km.

The Moon completes one orbit around the Earth every 27.3 d $= 2.40 \times 10^6$ s.

From the Cavendish experiment, $G = 6.7 \times 10^{-11}$ N m^2/kg^2.

(a) What is the mass of Earth?

(b) What is the radius of the Moon's orbit?

G(2,3). Typical satellite orbits back around 1960 were 1.6×10^5 m above the Earth's surface.

(a) What is the potential energy, relative to infinity, of a 1000-kg satellite in such an orbit?

(b) What is its potential energy relative to the Earth's surface?

(c) Find the time that such a satellite requires to complete one orbit. Be sure to start from the fundamental gravitational and centripetal force laws!

H(3). A space traveler in interstellar space is working near her craft when her safety line breaks. At that moment she is 3.00 m away from the center of mass of the craft and drifting away from it at the speed of 1.00 mm/s. If the mass of the craft is 10 000 kg, will she reach a maximum distance and be drawn back, or will she drift away indefinitely?

I(2,3). A 1.00×10^6 kg spaceship making observations in interplanetary space is in a circular orbit about the Sun at a radius of 1.50×10^{11} m (approximately the orbit of Earth).

(a) What is its kinetic energy while in this orbit? [You must start from the gravitational and centripetal force (or acceleration) laws.]

(b) Having completed their observations here, the crew next depart on a voyage to the vicinity of Jupiter's orbit, five times as distant from the Sun (7.5×10^{11} m). There, however, it is not necessary to establish a circular orbit; the ship can arrive there with essentially zero kinetic energy. Purely on the basis of energy conservation, what is the minimum energy that the engines must provide for this voyage?

(c) While in the orbit of part (a), the ship made two complete trips around the Sun. How long, in seconds, was it there?

J(1). Astronauts on the Moon can jump considerably higher than they can on Earth; that is, the acceleration due to gravity is much less. In fact, $g_m = 0.17\ g_e$. The moon is also much smaller: $R_m = 0.27\ R_e$.

(a) Use these data to find the ratio of the masses of the Moon and the Earth, M_m/M_e.

(b) On the straight line between the Earth and the Moon there is a point where a spaceship experiences no gravitational field, because the fields of the Earth and the Moon cancel. How far is this point from the Moon? The Moon is 3.5×10^5 km from the Earth.

K(2). Certain neutron stars are believed to be rotating at about one revolution per second. If such a star has a radius of 30 km, what must be its mass in order that objects on its surface will not be thrown off by the rapid rotation?

L(2). An asteroid revolves about the Sun in a circular orbit once every eight years. Approximately how far is it from the Sun in astronomical units (1 AU is the mean distance from the Sun to Earth)? You must start from the fundamental gravitational and centripetal force (or acceleration) equations.

M(3). What speed is necessary for a 1000-kg spaceship at a distance from the Sun equal to the radius of Saturn's orbit to escape from the Sun's gravitational field?

N(3). A star of mass 2.0×10^{30} kg and another star of mass 4.0×10^{34} kg are initially at rest infinitely far away. They then move directly toward one another under the influence of the gravitational force. Calculate the speed of their impact, which occurs when their centers are separated by 3.0×10^{10} m. The radii of the stars are 1.0×10^{10} m and 2.0×10^{10} m, respectively. Neglect the motion of the more massive star.

Solutions

D(1). (a) $h = r - R_e = 0.41R_e$; (b) $r_s = \sqrt{M_s/M_e}\, r_e$, and thus

$$r_e = \frac{r_{es}}{1 + \sqrt{M_s/M_e}} .$$

E(1,2). (a) $0.73\ \mathrm{m/s^2}$; (b) 1.9×10^{27} kg.

F(1,2). (a) 5.9×10^{24} kg; (b) $gR_e^2/r_m^2 = g'$ (at the Moon's orbit)

$$g' = \frac{v^2}{r_m} = \frac{(2\pi r_m/T)^2}{r_m} = \frac{(2\pi)^2 r_m}{T^2}, \qquad r_m = [gR_e^2T^2/(2\pi)^2]^{1/3} = 3.9 \times 10^8\ \mathrm{m}.$$

G(2,3). (a) -6.1×10^{10} J; (b) 1.6×10^9 J; (c) 88 min.

H(3). Let's hope she has a wrench in her hand that she can throw, since her total energy is

$$E = K + U = (1/2)mv^2 - GMm/r = m(1/2 - 2/9) \times 10^{-6}\ \mathrm{J} > 0!$$

I(2,3). (a) 4.4×10^{14} J; (b) 2.7×10^{14} J; (c) $2\pi \times 10^7$ s.

J(1). (a) $m/M = 0.012$;
(b) 0.35×10^5 km.

K(2). 1.6×10^{25} kg.

L(2). 4.0 AU.

M(3). 1.3×10^4 m/s.

N(3). 1.3×10^7 m/s.

PRACTICE TEST

Use the simplified numerical values below (accurate to within a few percent) in the Practice Test.

$G = (2/3) \times 10^{-10}$ N m^2/kg^2	$\pi^2 = 10$
Mass of Sun = 2.0×10^{30} kg	Radius of Sun = 7.0×10^{8} m
Mass of Earth = 6.0×10^{24} kg	Radius of Earth = 6.4×10^{6} m
Mass of Moon = $(3/4) \times 10^{23}$ kg	Radius of Moon = $(1/6) \times 10^{7}$ m
Mass of Mars = 6.3×10^{23} kg	Radius of Mars = $(1/3) \times 10^{7}$ m
Earth to Moon = $(3/8) \times 10^{9}$ m	Earth to Sun = 1.5×10^{11} m
g on Earth = 10 m/s^2	Saturn to Sun = 1.5×10^{12} m

1. The radius of the planet Jupiter is 11 times that of Earth, and its mass is 310 times as large as that of Earth. Using only these data, find out how the acceleration due to gravity on the surface of Jupiter compares with that on Earth.

2. A 1.0×10^{6} kg spaceship making observations in interplanetary space is in a circular orbit about the Sun at a radius of 1.5×10^{11} m (approximately the orbit of Earth).

   (a) What is its kinetic energy while in this orbit? [You must start from the gravitational and centripetal force (or acceleration) laws.]

   (b) Having completed their observations here, the crew next depart on a voyage to the vicinity of Jupiter's orbit, five times as distant from the Sun (7.5×10^{11} m). There, however, it is not necessary to establish a circular orbit; the ship can arrive there with essentially zero kinetic energy. Purely on the basis of energy conservation, what is the minimum energy that the engines must provide for this voyage?

   (c) While in the orbit of part (a), the ship made two complete trips around the Sun. How long, in seconds, was it there?

Practice Test Answers

1. $g_J = 2.6\, g_e$.

2. (a) 4.4×10^{14} J; (b) 2.7×10^{14} J; (c) $2\pi \times 10^{7}$ s.

GRAVITATION

Mastery Test Form A

Date________________

pass recycle
1 2 3

Name______________________

Tutor________________

Use the simplified numerical values below (accurate to within a few percent) in this Mastery Test.

$G = (2/3) \times 10^{-10}$ N m^2/kg^2	$\pi^2 = 10$
Mass of Sun = 2.0×10^{30} kg	Radius of Sun = 7.0×10^{8} m
Mass of Earth = 6.0×10^{24} kg	Radius of Earth = 6.4×10^{6} m
Mass of Moon = $(3/4) \times 10^{23}$ kg	Radius of Moon = $(1/6) \times 10^{7}$ m
Mass of Mars = 6.3×10^{23} kg	Radius of Mars = $(1/3) \times 10^{7}$ m
Earth to Moon = $(3/8) \times 10^{9}$ m	Earth to Sun = 1.5×10^{11} m
g on Earth = 10 m/s^2	Saturn to Sun = 1.5×10^{12} m

1. On certain Saturdays in the autumn, large numbers of people experience a strong attraction for the football stadium. Do you suppose that this attraction could be gravitational in origin? That is, estimate the gravitational force of attraction exerted by the stadium on an 80-kg football fan one block (133 m) away. Assume a total mass of 1.0×10^7 kg for the stadium (including the fans already assembled there).

2. In 1944, when the first group of astronauts landed on Mars, they discovered the third moon of Mars, which was in a circular orbit 1.0 km above the surface of the planet.

(a) What was the speed of this moon in its orbit? Remember that you are to start from the fundamental centripetal and gravitational force equations!

Since a moon this close to the surface of Mars interfered with their explorations, the astronauts decided to move it into an orbit at a higher altitude. Fortuitously, the angular speed for this new orbit was

$$\omega = \sqrt{42} \times 10^{-6} \text{ rad/s}.$$

(This is especially fortuitous if you use a calculator that does not take cube roots!)

The mass of the moon was 2.0×10^3 kg.

(b) What was the radius of the new orbit?

(c) How much energy did it take to move the moon to its new orbit?

GRAVITATION

Mastery Test   Form B

Date________________

pass   recycle
1   2   3

Name________________

Tutor________________

Use the simplified numerical values below (accurate to within a few percent) in this Mastery Test.

$G = (2/3) \times 10^{-10}$ N m^2/kg^2	$\pi^2 = 10$
Mass of Sun = 2.0×10^{30} kg	Radius of Sun = 7.0×10^{8} m
Mass of Earth = 6.0×10^{24} kg	Radius of Earth = 6.4×10^{6} m
Mass of Moon = $(3/4) \times 10^{23}$ kg	Radius of Moon = $(1/6) \times 10^{7}$ m
Mass of Mars = 6.3×10^{23} kg	Radius of Mars = $(1/3) \times 10^{7}$ m
Earth to Moon = $(3/8) \times 10^{9}$ m	Earth to Sun = 1.5×10^{11} m
g on earth = 10 m/s^2	Saturn to Sun = 1.5×10^{12} m

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1. A meteorite originally at rest in interstellar space falls to the surface of Earth; find the speed with which it hits. For this problem, make the simplifying assumptions (not really justified) that the effect of the Sun, the motion of the Earth, and the retarding force of the atmosphere can be neglected.

2. Because of an accident on a space flight, a 70-kg man is left in deep space, 1.0×10^4 m from the spherical asteroid Juno of mass 1.5×10^{14} kg.

   (a) How fast must he move, propelled by his rocket pack, to achieve a circular orbit around the asteroid at this distance, rather than crashing to its surface? [You must start from the fundamental gravitational and centripetal force (or acceleration) equations.]

   (b) It takes 8 h and 45 min for the rescue ship to arrive. Where should they look for him, relative to the place of the accident?

GRAVITATION

Mastery Test Form C

Date ______________________

pass recycle
1 2 3

Name ______________________ Tutor ______________________

Use the simplified numerical values below (accurate to within a few percent) in this Mastery Test.

$G = (2/3) \times 10^{-10}$ N m^2/kg^2	$\pi^2 = 10$
Mass of Sun = 2.0×10^{30} kg	Radius of Sun = 7.0×10^{8} m
Mass of Earth = 6.0×10^{24} kg	Radius of Earth = 6.4×10^{6} m
Mass of Moon = $(3/4) \times 10^{23}$ kg	Radius of Moon = $(1/6) \times 10^{7}$ m
Mass of Mars = 6.3×10^{23} kg	Radius of Mars = $(1/3) \times 10^{7}$ m
Earth to Moon = $(3/8) \times 10^{9}$ m	Earth to Sun = 1.5×10^{11} m
g on earth = 10 m/s^2	Saturn to Sun = 1.5×10^{12} m

1. When Deathwish Hershey reported for his Space Corps preinduction physical, he brought with him a letter from his psychiatrist to certify that he showed suicidal tendencies in low-g environments. In spite of this, he was assigned to a tour of duty on Mudberry, a perfectly spherical airless asteroid of radius 200 km, where $g = 0.20$ m/s^2. After a month of solitary duty, Deathwish could stand it no longer. Having completed a quick calculation, he fired a bullet parallel to the asteroid's surface at speed v and stood at attention, waiting for it to hit him on the back of the head. Little did he know that his appeal had been successful and that a ship was due to arrive in one and one-half hours to return him to civilian life.

   (a) At what speed v did Deathwish fire the bullet? [You must start from the fundamental gravitational and centripetal force (or acceleration) equations.]

   (b) Did the ship or the fatal bullet arrive first?

2. If Deathwish had fired the bullet vertically instead of horizontally, but with the same speed v as in the preceding problem, how high would it have gone? Or would it have escaped from the asteroid entirely?

MASTERY TEST GRADING KEY - Form A

1. Solution: Hardly, since

$$F_g = \frac{GMm}{r^2} = \frac{[(2/3) \times 10^{-10}](1.0 \times 10^7)(80)}{(133)^2}\ \text{N} = 3.0 \times 10^{-6}\ \text{N}.$$

2. What To Look For: (a) Check that the student really started by equating the centripetal force to the gravitational force. (Equating accelerations is essentially equivalent.) If some other (correct) equation was used, make her/him derive it!

   (c) Make sure that both kinetic and potential energies have been included for both orbits. If the student used one of the relations $E = -K = (1/2)U$ (valid only for such orbits), ask her/him how the problem could be done without assuming this relation. (But don't require the student to go through the arithmetic again.)

   Solutions: (a) $F_{centrip} = F_{grav}$ or $mv^2/R_M = GM_m m/R_M 2$. ($a_{centrip} = a_{grav}$ is also acceptable.) Thus

$$v = \left(\frac{GM_m}{R_m}\right)^{1/2} = \left(\frac{[(2/3) \times 10^{-10}](6.3 \times 10^{23}}{(1/3) \times 10^7}\right)^{1/2}\ \text{ms} = 3.5 \times 10^3\ \text{m/s}.$$

   (b) As above,

$$mr\omega^2 = \frac{GM_m}{r^2} \quad \text{or} \quad r = \left(\frac{GM_m}{\omega^2}\right)^{1/2} = \left(\frac{[(2/3) \times 10^{-10}](6.3 \times 10^{23}}{42 \times 10^{-12}}\right)^{1/2}\ \text{m}$$

$$r = 1.0 \times 10^8\ \text{m}.$$

   (c) $r_i = (1/3) \times 10^7$ m, $r_f = 1.0 \times 10^8$ m, $v_i = 3.5 \times 10^3$ m/s,

   $\omega_f = \sqrt{42} \times 10^{-6}$ rad/s, and $v_f = r_f\omega_f = \sqrt{42} \times 10^2$ m/s;

   thus the energy needed is $\Delta E = K_f + U_f - K_i - U_i = m[\frac{1}{2}v_f^2 - \frac{1}{2}v_1^2 + GM_m(\frac{1}{r_i} - \frac{1}{r_f})]$

$$= (2.0 \times 10^3) \times \{\tfrac{1}{2}(\sqrt{42} \times 10^2)^2 - \tfrac{1}{2}(3.5 \times 10^3)^2 + [(2/3) \times 10^{-10}](6.3 \times 10^{23}) \times (3.0 \times 10^{-7} - 1.0 \times 10^{-8})\}$$

$$= (2.0 \times 10^3)[0.21 - 6.1 + (4.2)(2.9)] \times 10^6$$

   or $\Delta E = 1.3 \times 10^9$ J.

MASTERY TEST GRADING KEY - Form B

1. Solution: $E_f = E_i$ or $K_f + U_f = K_i + U_i = 0 + 0$; thus

$$(1/2)mv_f^2 = K_f = -U_f = +G(M_e/R_e),$$

$$v_f = \left(\frac{2GM_e}{R_e}\right)^{1/2} = \left(\frac{2(2/3 \times 10^{-10})(6.0 \times 10^{24})}{(6.4 \times 10^6)}\right)^{1/2} = 1.1 \times 10^4 \text{ m/s}.$$

2. What To Look For: (a) Check that the student really started by equating the centripetal force to the gravitational force. (Equating accelerations is essentially equivalent.) If some other (correct) equation was used, make her/him derive it.

Solutions: (a) $F_{centrip} = F_{grav}$ or $mv^2/r = GMm/r^2$. Thus,

$$v = \left(\frac{GM}{r}\right)^{1/2} = \left(\frac{[(2/3) \times 10^{-10}](1.5 \times 10^{14})}{1.0 \times 10^4}\right)^{1/2} \text{ m/s} = 1.0 \text{ m/s},$$

the speed he needs to orbit.

(b) $T = \frac{2\pi r}{v} = \frac{2\pi(1.0 \times 10^4)}{(1.0)} = 6.3 \times 10^4$ s.

8 h + 45 min = 8(3600) + 45(60) = 3.15×10^4 s.

They should, therefore, look for him almost exactly on the other side of Juno.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) Check that the student really started by equating the centripetal force to the gravitational force. (Equating accelerations is essentially equivalent.) If some other (correct) equation was used, make her/him derive it!

   Solutions: (a) $F_c = F_g$ (Centripetal force = gravitational force) or

   $$\frac{mv^2}{R} = G\frac{Mm}{R^2} = gm,$$

   $$v = (gR)^{1/2} = [(0.20)(2.0 \times 10^5)]^{1/2} \text{ m/s} = 200 \text{ m/s}.$$

   (b) $T = 2\pi R/v = [2\pi(2.0 \times 10^5)/200] \text{ s} = 2\pi \times 10^3 \text{ s} = 6280 \text{ s}.$

   By comparison, $1\frac{1}{2}$ hour = (1.5)(3600 s) = 5400 s. Cheers!

2. What To Look For: Make sure that both kinetic and potential energies have been included. If the student used one of the relations $E = -K = (1/2)U$ (valid only for such orbits), ask her/him how the problem could be done without assuming this relation. (But don't require the student to go through the arithmetic again.)

   Solution: Use energy conservation: $E_f = E_i$, or $K_f + U_f = K_i + U_i$. Let h be the maximum height of the bullet, the point at which $v_f = 0$; then

   $$0 - \frac{GMm}{R + h} = \frac{1}{2} mv_i^2 - \frac{GMm}{R}.$$

   Since $GM = gR^2$ [see Problem 1(a)], and $v_i = \sqrt{gR}$, this becomes

   $$\frac{mgR^2}{R + h} = -\frac{1}{2}mgR + mgR = \frac{1}{2}mgR.$$

   Thus, $R = (1/2)(R + h)$, and $h = R = 2.0 \times 10^5$ m.

   (If the energy had been so high that the bullet escaped, there would have been no value of h that satisfied this equation.)