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Mathematics and Evolution

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Introduction

What is game theory and how does it apply to Evolutionary Biology? Game theory was originally developed to help portray how individuals interact with each other and it was mainly used in the economics and political science fields. This area of mathematical study came about after World War II and was developed by John von Neumann and Oskar Morgenstern. It wasn't until the 1970's that George Price and John Maynard Smith began to apply game theory to evolution in trying to predict the behavior of animals in a species. There are two main evolutionary trends we will study using game theory: 1) what happens to a species when some animals display aggressive behavior, but back down instead of fighting, and 2) how can animals cooperate together to live in harmony while still able to attain their own personal goals.

John Nash, while working for the RAND Corporation, introduced what is now called the *Nash-Equilibrium*. The Nash-Equilibrium is reached in a non-zero sum game when no one person can increase their outcome or "fitness" by deciding to change their strategy unless the other person also changes strategy. Smith later advanced the idea of the Nash-Equilibrium and applied it to evolution to establish the *evolutionarily stable strategy* (ESS). An ESS is a type of Nash-Equilibrium in which the ideas are applied only to populations and provisions are included for stability. The definition of an ESS is a strategy played by all members of an identical population, where no individual has any advantage over another, and mutant (or alternative) strategies can be reduced or surpassed.

In this paper, I will illustrate exactly how games are used to study the two questions mentioned earlier. Before I discuss game theory as it applies to evolutionary biology, let me first give you an example of what is meant by a game.

A zero-sum game is a game that is played where the gain or loss of one player is equal to the gain or loss of the other player. The sum of the gain of one player and the loss of the other player is zero. For example, sports competitions are zero-sum games; one team wins and the other team loses. Another example is Rochambeau or rock-paper-scissors. The two players must each show their choice at the same time and one person wins while the other person loses.

To begin, we consider a scenario in which Jim and Ruth play a zero-sum game called "Confuse the Teachers". Jim and Ruth each have four possible moves. When Jim wins, Ruth pays him money and when Ruth wins, Jim pays her money. A matrix of possible payoffs for each outcome of this particular version of the game is shown below.

		Ruth			
		A	**B**	**C**	**D**
	A	12	-1	1	0
Jim	**B**	5	1	7	-20
	C	3	2	4	3
	D	-10	0	0	16

In this matrix, the positive numbers represent the money that Ruth pays Jim and the negative numbers represent the money that Jim pays Ruth. For example, if Jim and Ruth both selected move A, Ruth would have to pay Jim $12.

There are many different strategies to consider when playing a zero-sum game. First, we need to think about what it means for one strategy to dominate another strategy. A strategy is *dominant* if each outcome in that strategy is as good as or better than each outcome in another strategy <u>and</u> at least one outcome is definitely better. Note in the example above that from

Ruth's perspective the strategy of selecting B dominates the selection of C. Each outcome in B is as good or better than those in C and in fact, if Ruth were to play B and Jim were to play A, she would get paid $1 which is definitely better than any outcome in C. The *Dominance Principle* states that a rational person should never play a dominated strategy, so as long as Ruth is rational, she would never play C. In light of these considerations, what are the options in this game?

Assuming that Jim and Ruth both want to maximize their own payoff or outcome, they should each play a strategy that would give them their highest possible outcome. For this game, if Jim wants to maximize his outcome, he could play D in hopes of Ruth also playing D. However, Jim's chance for loss if Ruth plays A is also great. If Ruth wanted to maximize her outcome, she could play D in hopes that Jim would play B, but her chance for loss if Jim plays D is high. If we look at each of the strategies rationally, Jim should always play C because he always wins money with C. Ruth on the other hand should always play B because she loses very little, and has the opportunity to win if Jim happens to play A. Although the higher numbers of 16 (for Jim) and -20 (for Ruth) are tempting, if they were to make those choices, the chance of losing big is high.

This leads into the discussion of equilibrium in a zero-sum game. An equilibrium outcome for this game is an outcome where if both players are selecting certain choices, then neither player has an incentive or desire to switch to a different choice. In analyzing this game for Jim and Ruth, their equilibrium outcome is (C, B) in the order of (Jim, Ruth). This means that if Ruth were set on playing B, Jim's best option is C and he wouldn't win anymore by switching to another choice. For Ruth, if Jim were set on playing C, to minimize her losses, she should play B. An equilibrium outcome does not necessarily maximize anyone's payoff, it may instead minimize a player's losses. This scenario simply serves as an example of what a game might entail. It should be noted that while games used in predicting animal behavior are similar, they are not necessarily zero-sum games.

Game Theory and Evolution

Game theory is an area of study where mathematics is applied to social and environmental situations in which an attempt is made to describe behavior in planned situations where an individual's triumph in making choices depends on the choices of others. John Maynard-Smith was a founding member of the University of Sussex and the Dean from 1965-1985. He became a professor emeritus at the University of Sussex. Smith is well known for his work in applying game theory to evolution and his pivotal idea of the evolutionarily stable strategy (ESS). Smith was the first to use Hawk-Dove terms as a model of conflict in a two-person game. This model consists of two players engaged in a game in which neither player wants to concede to the other, but a conflict is the worst possible outcome for both players. The Hawk-Dove model is primarily applied to game theory and evolutionary biology.

Consider the scenario where there is a population of birds. Some of these birds display aggressive, or "hawk-like" behavior (and thus will be referred to as Hawks), while other birds in the population exhibit passive or "dove-like" behavior (and hence are referred to as Doves). The competition within this species is over resources because all birds want to be as "fit" as possible. If a Hawk is competing for a resource, it will always fight the other bird. If a Dove competes for a resource with another Dove it will display (show aggressive behavior), but not fight and run away if things start to look bad. In this encounter, one Dove will outlast the other in displaying and thus gain the resource while the other Dove will give up and run away. If a Dove meets a Hawk, the Dove will simply run away leaving the prey item for the Hawk. If a Hawk competes for a prey item with another Hawk, one will win and one will lose (and probably be badly injured during the conflict). Each Hawk has a 50% chance of winning against another Hawk and in a Hawk-Dove conflict, the Hawk will always win. In a Dove-Dove conflict, since one will win and the other will lose, each Dove has a 50% chance of winning.

To analyze which type of bird has a better chance of survival, “fitness” points can be assigned to each of these conflicts. The winner of any conflict will win 50 “fitness” points for gaining the prey item. In a Hawk-Hawk conflict, the Hawk that loses will most likely be hurt so he will lose 100 “fitness” points. In a Dove-Hawk conflict, the Dove will run away and neither gain nor lose any points and the Hawk will receive the 50 “fitness” points for gaining the prey. In a Dove-Dove conflict, both will display behavior, but one will eventually give up and run away so the other Dove will gain the 50 “fitness” points for the prey. Each Dove will lose 10 “fitness” points for the energy it takes to display.

We now compute the payoffs for each conflict. Since the Hawk-Hawk and Dove-Dove conflicts are based on probabilities (50% chance of winning or losing) those payoffs are actually average payoffs.

Hawk-Hawk conflict – 50-50 chance	½ (50) + ½ (-100) = -25 points (per Hawk)
Hawk-Dove conflict – Hawk always wins	50 points (Hawk) 0 points (Dove)
Dove-Dove conflict – each displays 1 wins	½ (50-10) + ½ (-10) = 15 points (per Dove)

This can be shown in a payoff matrix:

	Hawk	Dove
Hawk	(-25,-25)	(50,0)
Dove	(0,50)	(15,15)

Using this payoff matrix, we can think about different mixes of populations for the birds. If the population had a lot of Hawks, there would be a lot of conflict and many Hawks would be losing fitness points because of injury. This population could then see a surge in the number of Doves s as the Hawks would diminish in fitness in a predominantly Hawk population while the Doves would not lose many fitness points since they won't fight. If the population was dominated by Doves, there wouldn't be any fighting, however such a population could see a dramatic increase in the number of Hawks as they could easily gain access to resources since they will always win a Hawk-Dove conflict. Neither of these situations constitutes an ESS for the bird population. The question then becomes, are there some numbers of Hawks and Doves where neither type of behavior provides any advantage over the other? Recall that an ESS is a type of Nash-Equilibrium in which the ideas are applied only to populations and provisions are included for stability. The definition of an ESS is a strategy played by all members of an identical population, where no individual has any advantage over another, and mutant (or alternative) strategies can be reduced or surpassed.

In order for both the Hawks and Doves to survive in the same population, I need to find an ESS where neither has an advantage over the other. If the population in this situation had ¼ Hawks and ¾ Doves, I can figure payoffs for each.

Hawk
¼ (-25) + ¾(50) = -6.25 + 37.50 = 31.25

Dove
¼ (0) + ¾ (15) = 0 + 11.25 = 11.25

For this case, it is obvious that a population consisting of ¼ Hawks and ¾ Doves gives the Hawks an advantage so it is not the ESS. To find the ESS for this Hawk-Dove population, we need to consider two things: the payoffs for the Hawks and the Doves, and when the payoffs are equal. Thus to determine the ESS we establish the following:

Finding the ESS:
Let x = fraction of population that are Hawks
Let y = fraction of population that are Doves

Then the constraints can be written as

Payoff Hawks = Payoff Doves	and	Hawks + Doves = 1
-25x + 50y = 0x + 15y	and	x + y = 1

To find the ESS, I need to solve these two equations. Since x + y = 1, solving for x gives x = 1 – y. Substituting this into the other equation gives:

$$-25(1-y) + 50y = 0(1 - y) + 15y$$
$$-25 + 25y + 50y = 15y$$
$$-25 + 75y = 15y$$
$$60y = 25$$
$$y = 25/60 = 5/12$$

Thus y = 5/12 and x = 1- 5/12 = 7/12. Using this split for the Hawk-Dove bird population we refigure their payoffs:

Hawk: 7/12(-25) + 5/12(50) = -14.58 + 20.83 = 6.25

Dove: 7/12(0) + 5/12(15) = 0 + 6.25 = 6.25

Since both the Hawks and the Doves have the same average payout, or gain of "fitness" points, with this split of the population, the ESS for this Hawk-Dove scenario is to have a population of 7/12 Hawks and 5/12 Doves. This means that if the population of birds has 7/12 Hawks and 5/12 Doves, neither has an advantage over the other and both types of birds will survive and thrive by earning enough "fitness" points through obtaining prey or other resources. Evolution will drive a population towards the ESS through natural selection. This is true because the presence of an ESS causes mutant strategies to be overpowered. Without an ESS, mutants could have better fitness than the average member of the population and thus cause significant changes .

What happens if the value of gaining a prey item were to change? Let's look at a case where gaining prey will earn a bird 150 "fitness" points instead of just 50. I will re-work this analysis to see what happens to the ESS.

Hawk-Hawk conflict – 50-50 chance ½ (150) + ½ (-100) = 25 points (per Hawk)

Hawk-Dove conflict – Hawk always wins 150 points (Hawk)
0 points (Dove)

Dove-Dove conflict – each displays, 1 wins ½ (150-10) + ½ (-10) = 65 points (per Dove)

Again, this can be shown in a payoff matrix:

	Hawk	**Dove**
Hawk	(25,25)	(150,0)
Dove	(0,150)	(65,65)

Just by looking at this payoff matrix, I notice immediately that there is never going to be a loss of fitness points, even during a Hawk-Hawk conflict. This seems to me to give hawk-like behavior an extreme advantage because they will always be gaining in fitness and the Doves, while they won't lose any fitness points either, will only gain in fitness when faced with another Dove. Is there an ESS for this situation? Let's find out:

<u>Finding the ESS:</u>

Let x = fraction of population that are Hawks
Let y = fraction of population that are Doves

Then to find the ESS, I need to solve the following two equations:

Payoff Hawks = Payoff Doves	and	Hawks + Doves = 1
$25x + 150y = 0x + 65y$	and	$x + y = 1$

Since $x + y = 1$, solving for x gives $x = 1 – y$. Substituting this into the other equation gives:

$$25(1-y) + 150y = 0(1 – y) + 65y$$
$$25 - 25y + 150y = 65y$$
$$25 + 125y = 65y$$
$$60y = -25$$
$$y = -25/60 = -5/12$$

Since $y = -5/12$ it follows that $x = 1+ 5/12 = 17/12$. Since it is not possible to have the split of the population consist of a negative fraction and one that is larger than 1, there is no ESS for this situation. Although I already had doubts before I actually tried to find the ESS, showing this calculation proves to me that in order for a population to have an ESS, it is important to carefully select the value of gains and losses. In the first simulation, the gain was less than the loss and in the second case the gain was greater than the loss. In a conflict, the injury to the Hawk should be great because they are risking a lot in the fight. That is why it is important that when analyzing a situation such as this, the value of the resources needs to be less than the loss due to injury. In a situation where the gain is more than the loss, there is no penalty for fighting, therefore why would any bird choose to be a Dove? This would almost force the entire population to adopt the dominant Hawk strategy, which would result in many fights. Hawks would always receive the highest or best fitness.

Up to this point, I have talked about the birds being either a Hawk or a Dove, but in reality, the birds usually have characteristics of both types and apply them at different times. The ESS for this population of birds, 7/12 Hawks and 5/12 Doves, really translates to the percentage of time that each individual bird would display each type of behavior. The ESS correlates to the combination of Hawks and Doves where switching from one to the other would not provide any advantage. According to evolution, birds will advance to use the Hawk strategy 7/12 (58%) of the time and the Dove strategy 5/12 (42%) of the time.

The Prisoner's Dilemma

Cooperation among individuals has long been a mystery. What would make some people selfish all the time and other people cooperate or compromise in hopes of gaining in the long run? The Prisoner's Dilemma problem is a well known example of a non-zero sum game that deals with two people having to choose to either cooperate or not cooperate with each other. The original problem is as follows:

> *"Two suspects are arrested by the police. The police have insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal: if one testifies for the prosecution against the other and the other remains silent, the betrayer goes free and the silent accomplice receives the full 10-year sentence. If both remain silent, both prisoners are sentenced to only six months in jail for a minor charge. If each betrays the other, each receives a five-year sentence. Each must make the choice of whether to betray the other or to remain silent. Each one is assured that the other would not know about the betrayal before the end of the investigation and that there will be no revenge for the betrayal. What should each do?"*

In this case, each prisoner needs to consider what happens if they choose to cooperate (stay silent) with their friend or if they choose to not cooperate (confess) with their friend. This can be shown in a payoff matrix where the numbers represent the payoff to each person. The negative numbers represent time they spend in prison, and they are negative because prison is considered bad so it is represented by a loss.

		Prisoner 1	
		Stay silent	**Confess**
Prisoner 2	**Stay silent**	(-0.5, -0.5)	(-10,0)
	confess	(0,-10)	(-5,-5)

Looking at this payoff matrix, there is no strategy that is dominated by another so either prisoner could play either option, therefore no strategy can be ruled out. In a case such as the Prisoner's Dilemma, there are many things to be considered. Will these two people ever have any dealings again? How much does each trust the other? How strong is the friendship between the two prisoners? Has either or both spent time in prison before?

All of these questions are things to think about when trying to predict the behavior of each individual. If they aren't good friends and aren't likely to have any dealings again, they are more likely to try to confess first and get a better deal for themselves. If either or both has spent time in prison before, they may be more likely to stay quiet in hopes that the other does as well and receive very little time in prison. All of these choices come down to how much they trust each other.

Although no strategy can be eliminated because it is dominated by another strategy, there is an equilibrium in this situation. The Nash-equilibrium applies to this situation. This game is a non-zero sum game since the gain of one player does not equal the loss of the other player. The equilibrium in this case is for both prisoners to confess. This surprised me when I first computed the equilibrium, so I carefully examined each possible situation.

- If both Prisoner 1 and Prisoner 2 choose to stay silent:
  - Prisoner 2 would have incentive to switch to confessing to receive no prison time. **Not an equilibrium**
  - Prisoner 1 would have incentive to switch to confessing to receive no prison time. **Not an equilibrium**

- If Prisoner 1 chooses to stay silent and Prisoner 2 chooses to confess:
  - Prisoner 1 would have incentive to switch to confessing also. **Not an equilibrium**
  - Prisoner 2 has no incentive to switch to a different choice, but that doesn't matter since Prisoner 1 has incentive to switch. **Not an equilibrium**

- If Prisoner 1 chooses to confess and Prisoner 2 chooses to stay silent:
  - Prisoner 1 would have no incentive to switch to a different choice.
  - Prisoner 2 would have incentive to switch to confessing also. **Not an equilibrium.**

- If both Prisoner 1 and Prisoner 2 choose to confess:
  - Prisoner 1 has no incentive to switch to a different option.
  - Prisoner 2 has no incentive to switch to a different option. **Equilibrium found**

It may seem crazy to some people, but the equilibrium for this situation is for both prisoners to confess and receive five years each in prison. This does however make sense when you consider the questions I asked earlier. The question of trust plays a huge role in this situation. If one takes the chance and chooses to stay silent, they could either get very little time in prison, or get all the time in prison. For both prisoners, the best option is for both to confess and split the time between them because with that choice neither has an advantage over the other.

What happens when this situation occurs between two people more than once? Robert Aumann, a Nobel Prize winner, found that rational players repeating this game over and over maintain the cooperative outcome. Again, thinking logically, this makes sense. If there is only one time that this situation will occur between two people, most individuals are more likely to look out for themselves first and not really worry too much about the other person. Choosing to stay silent could lead to a big loss for one player if this is only happening for one game. When individuals have many interactions together, they have a memory of what happened the last time and that may affect what the choice that is made this time. To test this theory of evolutionary cooperation, I played this iterated game with two different people and the average payouts of each were surprisingly different.

To begin with, I explained the Prisoner's Dilemma to each person that I played with. Instead of calling the choices "rat out" and "don't rat out", I made two cards and "N" card for not cooperate and a "C" card for cooperate (this means cooperate or not cooperate with each other, not the police). We each had a set of these cards and we would make our choice and lay it face down on the table. When both cards were on the table we would reveal our choices and I then recorded the outcome and computed our payoffs. The payoffs were computed using a matrix similar to that in the original Prisoner's Dilemma problem with the ½ 's being changed to 1's to make it easier to compute average payout in the end. The payoff matrix used for these iterated games is below.

		Kacy	
		Cooperate	**Not Cooperate**
Chris & Taryn	**Cooperate**	(-1, -1)	(-10,0)
	Not Cooperate	(0,-10)	(-5,-5)

Robert Axelrod, a professor at the University of Michigan, is well known for his work on Evolutionary Cooperation. Axelrod wrote a book entitled *The Evolution of Cooperation* (1984), in which he gives details about a tournament he organized where he invited many colleagues to develop computer programs that would play a strategy for an iterated Prisoner's Dilemma game over and over. He found that the best strategy developed was also the most simple and it was called "Tit for Tat", and was developed by Anatol Rapaport. The strategy was simple in that the computer would cooperate the first iteration and then after that, simply do what the opponent did on the previous iteration.

I first played with my husband and we played 100 games of this iterated Prisoner's Dilemma. I am not sure if I didn't explain the situation well enough or not, but I was slightly disappointed to see that the cooperation strategy was not upheld in this iterated Prisoner's Dilemma game. I had previously decided that I was going to use the "Tit for Tat" strategy mentioned by Axelrod. In the beginning, I forgot that "Tit for Tat" has the first iteration as cooperate so the first three iterations were not in that strategy, but I then began to use it the rest of the time. My husband said he didn't really have a strategy. At times he said he was simply making the same choice for three iterations then switching for three iterations. He then said he considered each iteration a new situation and that his opponent was someone he didn't know so he was trying to do what was best for himself. Although there were a few times where we were able to both cooperate, the rest were split between one cooperation and one non-cooperation and both non-cooperating (see Appendix A). I was really surprised when I computed our average payoffs and we had exactly the same, -4.16 or 4.16 years in prison. Even more surprising to me was that it was very close to the Nash Equilibrium of both of us confessing and receiving 5 years each.

I played the iterated Prisoner's Dilemma again with a friend and had completely different results. I stuck to the "Tit for Tat" strategy completely this time, and again we played 100 iterations of the Prisoner's Dilemma. Taryn, my opponent also started out with cooperation which meant that I would cooperate on the next turn. This pattern continued for the entire 100 iterations. We did uphold the cooperation strategy and I must say that by the end we were both laughing because nothing had changed for either of us. Computing our average payouts was very easy since there were 100 iterations where we each received -1, 1 year in prison, so our average payout was 1 year in prison (see Appendix B).

Although in both iterated Prisoner's Dilemma games that I played, the payoffs were the same, that is not always the case. While playing with my husband, we played (N,C) the same number of times that we played (C,N) which is why we ended up having the same average payoff in the end. With Taryn, since we both cooperated the entire time, we ended with the same average payoff. Using the iterated Prisoner's Dilemma genuinely illustrates the evolution of cooperation within a species. Going back to the original Prisoner's Dilemma problem, if both players only had contact or interactions with each other once, then they would definitely do what is in their best interest. With the iterated Prisoner's Dilemma, we are able to see what can happen with multiple interactions.

I began to think about applying this situation to actual interactions between individuals. I can see how this can apply to negotiations between countries that may be in an arms race. Neither country can be certain what the other is doing so their immediate reaction may be to build up their weapons supply, but it is imperative to remember that this may not be the only negotiation with this other country and if one betrays now, the consequences during the next interaction could be drastic.

What about a real-life example of the Prisoner's Dilemma where one party is innocent and one is guilty? The innocent person is not likely to confess to something they didn't do. The guilty person, on the other hand, may only looking out for himself and confess and testify against the innocent person to receive no jail time

I also think of honor boxes at places such as golf courses. Early in the golf season, there may not be anyone available to actually work so an honor box is placed outside for people to pay their fees. It may be tempting for people to not pay (not cooperate), but if no one pays, then the honor system won't work and will be taken away. That may mean that the golf course is closed until people are available to be there which would ruin it for everyone.

The Prisoner's Dilemma is a good example of what can happen when individuals are confronted with situations where they have to make a choice and that choice may hurt someone else. The iterated Prisoner's Dilemma really showed me that it does matter how many "games" or interactions people may have before a trust or cooperation begins to emerge. It may seem tempting to make the selfish choice, but the consequences of the next encounter could cost one or both dearly.

Although cooperation between individuals and society is interesting, it is important to remember that this theory can illustrate how cooperation evolves within certain species. Thinking of a pack of wolves, a single wolf is only able to kill rather small prey on their own. If the entire pack were to work together, they would be able to kill larger prey and have food available for all. Cooperation between individuals is an important part of what makes life within a species, and society, work. Individuals within the population that are able to cooperate with each other are rewarded through the acquiring of resources. Non cooperation however, has dire consequences that may result in the loss of fitness within a species. Using game theory and iterated games to help predict cooperative behavior has many implications in the social sciences and even in government.

References

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Appendix A

Game #	Me	Chris	My Score	Chris Score	Game #	Me	Chris	My Score	Chris Score
1	N	C	0	-10	51	C	C	-1	-1
2	N	N	-5	-5	52	C	N	-10	0
3	C	C	-1	-1	53	N	C	0	-10
4	C	N	-10	0	54	C	C	-1	-1
5	N	N	-5	-5	55	C	C	-1	-1
6	N	C	0	-10	56	C	N	-10	0
7	C	N	-10	0	57	N	N	-5	-5
8	N	C	0	-10	58	N	C	0	-10
9	C	N	-10	0	59	C	C	-1	-1
10	N	C	0	-10	60	C	N	-10	0
11	C	C	-1	-1	61	N	C	0	-10
12	C	N	-10	0	62	C	N	-10	0
13	N	N	-5	-5	63	N	C	0	-10
14	N	C	0	-10	64	C	C	-1	-1
15	C	C	-1	-1	65	C	C	-1	-1
16	C	N	-10	0	66	C	C	-1	-1
17	N	N	-5	-5	67	C	N	-10	0
18	N	C	0	-10	68	N	N	-5	-5
19	C	N	-10	0	69	N	N	-5	-5
20	N	C	0	-10	70	N	N	-5	-5
21	C	N	-10	0	71	N	C	0	-10
22	N	N	-5	-5	72	C	C	-1	-1
23	N	C	0	-10	73	C	C	-1	-1
24	C	C	-1	-1	74	C	N	-10	0
25	C	C	-1	-1	75	N	N	-5	-5
26	C	N	-10	0	76	N	N	-5	-5
27	N	C	0	-10	77	N	C	0	-10
28	C	N	-10	0	78	C	C	-1	-1
29	N	C	0	-10	79	C	C	-1	-1
30	C	N	-10	0	80	C	N	-10	0
31	N	C	0	-10	81	N	N	-5	-5
32	C	N	-10	0	82	N	N	-5	-5
33	N	C	0	-10	83	N	N	-5	-5
34	C	C	-1	-1	84	N	C	0	-10
35	C	N	-10	0	85	C	C	-1	-1
36	N	N	-5	-5	86	C	N	-10	0
37	N	C	0	-10	87	N	C	0	-10
38	C	N	-10	0	88	C	C	-1	-1
39	N	C	0	-10	89	C	N	-10	0
40	C	N	-10	0	90	N	N	-5	-5
41	N	N	-5	-5	91	N	C	0	-10
42	N	C	0	-10	92	C	N	-10	0
43	C	N	-10	0	93	N	N	-5	-5
44	N	C	0	-10	94	N	C	0	-10

45	C	C	-1	-1	95	C	N	-10	0
46	C	N	-10	0	96	N	C	0	-10
47	N	C	0	-10	97	C	C	-1	-1
48	C	N	-10	0	98	C	N	-10	0
49	N	N	-5	-5	99	N	C	0	-10
50	N	C	0	-10	100	C	N	-10	0
			-217	-227				-199	-189

Kacy - Average Payout = -4.16 **Chris - Average Payout = -4.16**

Appendix B

Game #	Me	Taryn	My Score	Taryn Score	Game #	Me	Taryn	My Score	Taryn Score
1	C	C	-1	-1	51	C	C	-1	-1
2	C	C	-1	-1	52	C	C	-1	-1
3	C	C	-1	-1	53	C	C	-1	-1
4	C	C	-1	-1	54	C	C	-1	-1
5	C	C	-1	-1	55	C	C	-1	-1
6	C	C	-1	-1	56	C	C	-1	-1
7	C	C	-1	-1	57	C	C	-1	-1
8	C	C	-1	-1	58	C	C	-1	-1
9	C	C	-1	-1	59	C	C	-1	-1
10	C	C	-1	-1	60	C	C	-1	-1
11	C	C	-1	-1	61	C	C	-1	-1
12	C	C	-1	-1	62	C	C	-1	-1
13	C	C	-1	-1	63	C	C	-1	-1
14	C	C	-1	-1	64	C	C	-1	-1
15	C	C	-1	-1	65	C	C	-1	-1
16	C	C	-1	-1	66	C	C	-1	-1
17	C	C	-1	-1	67	C	C	-1	-1
18	C	C	-1	-1	68	C	C	-1	-1
19	C	C	-1	-1	69	C	C	-1	-1
20	C	C	-1	-1	70	C	C	-1	-1
21	C	C	-1	-1	71	C	C	-1	-1
22	C	C	-1	-1	72	C	C	-1	-1
23	C	C	-1	-1	73	C	C	-1	-1
24	C	C	-1	-1	74	C	C	-1	-1
25	C	C	-1	-1	75	C	C	-1	-1
26	C	C	-1	-1	76	C	C	-1	-1
27	C	C	-1	-1	77	C	C	-1	-1
28	C	C	-1	-1	78	C	C	-1	-1
29	C	C	-1	-1	79	C	C	-1	-1
30	C	C	-1	-1	80	C	C	-1	-1
31	C	C	-1	-1	81	C	C	-1	-1
32	C	C	-1	-1	82	C	C	-1	-1
33	C	C	-1	-1	83	C	C	-1	-1
34	C	C	-1	-1	84	C	C	-1	-1
35	C	C	-1	-1	85	C	C	-1	-1
36	C	C	-1	-1	86	C	C	-1	-1
37	C	C	-1	-1	87	C	C	-1	-1
38	C	C	-1	-1	88	C	C	-1	-1
39	C	C	-1	-1	89	C	C	-1	-1
40	C	C	-1	-1	90	C	C	-1	-1
41	C	C	-1	-1	91	C	C	-1	-1
42	C	C	-1	-1	92	C	C	-1	-1
43	C	C	-1	-1	93	C	C	-1	-1
44	C	C	-1	-1	94	C	C	-1	-1

45	C	C	-1	-1	95	C	C	-1	-1
46	C	C	-1	-1	96	C	C	-1	-1
47	C	C	-1	-1	97	C	C	-1	-1
48	C	C	-1	-1	98	C	C	-1	-1
49	C	C	-1	-1	99	C	C	-1	-1
50	C	C	-1	-1	100	C	C	-1	-1
			-50	-50				-50	-50

Kacy - Average Payout = **-1** **Taryn - Average Payout =** **-1**