1975

IMPULSE AND MOMENTUM

IMPULSE AND MOMENTUM

INTRODUCTION

You have already learned that you stub your toe harder trying to kick larger masses. Now imagine another unpleasant activity: catching a bowling ball. This gets harder to do as the ball is dropped from higher places. The difficulty depends both on the ball's mass and its velocity just before you apply the stopping force. This force can be applied in different ways. Any winner of an egg-throwing contest will tell you the way to stop an object with the least force is to spread the stopping process out over a maximum time.

This module will develop the above "folk physics" into a system of concepts and equations; and even a new law that is believed to be more fundamental than the laws from which it will be derived. This wonderful anomaly will not be further explored in this module, but it does indicate some of the philosophical richness and curiosity that continues to be part of this science. The concepts are "center of mass" and "linear momentum"; the law is called "conservation of linear momentum."

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Describe the motion of a body moving in a plane (needed for Objectives 1 to 3 of this module)	Planar Motion Module
*Solve problems requiring the application of Newton's second and third laws (needed for Objectives 3 and 4 of this module)	Newton's Laws Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Center of mass - Write the formulas for the center of mass (c.m.) of a system and explain all the terms. Write the formulas for the linear momentum of a system. Explain all the terms.

2. Linear momentum - Given the masses, positions, and velocities of all particles in a system, find the position and velocity of the center of mass, and the total (vector) linear momentum.

3. Impulse - Given a force versus time graph or function for a system, calculate the change of the system's linear momentum.

4. Linear-momentum conservation - Recognize conditions for which the linear momentum of a system is conserved.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 9, Section 9.1 first. This explains the center of mass. Then read Chapter 7, Section 7.1, which introduces linear momentum and impulse and the relation between them.

Now read Section 9.2, which partially shows how to calculate the velocity of a system's center of mass; You solve Eq. (9.3) for $v_{x(c.m.)}$ and similar but unspecified equations for $v_{y(c.m.)}$ and $v_{z(c.m.)}$. The velocity components are then combined into a single velocity: $\vec{v}_{c.m.} = v_{x(c.m.)}\hat{i} + v_{y(c.m.)}\hat{j} + v_{z(c.m.)}\hat{k}$. This section also relates the external forces acting on a system to the motion of the system's center of mass.

Although Section 7.1 describes a single particle, Sections 9.1 and 9.2 have shown how to reduce a system of masses into a somewhat equivalent single particle. This particle has the total mass of the system and is located at the center of mass of the system. It is equivalent only in that it has the same linear momentum as the system. If you do not need to know details of the internal structure of the system, you can apply the ideas of Section 7.1 to systems of masses by treating them as a single particle located at the center of mass. For example, the earth is regarded as a particle by the scientists who calculate very high-altitude satellite orbits, but such an earth would be without meaning to most geologists.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text* (Illus.)	Assigned Problems: Study Guide	Assigned Problems: Text* (Illus.)	Additional Problems
1	Secs. 9.1, 7.1	A		A		
2	Secs. 9.1, 9.2	A, B	9.1	A, B	9.1	Chap. 9, Probs. 1, 3, 4
3	Secs. 7.1, 7.2	C	7.1, 7.2	C	7.1, 7.2	Chap. 7, Quest.* 10, 12
4	Sec. 7.4	D	7.3, 7.4, 9.1	D	7.3, 7.4, 9.1	Chap. 7, Quest.* 5, 7

*Illus. = Illustration(s). Quest. = Question(s).

Continue to read Sections 7.2, 7.3, and 7.4. Keep in mind that now when the text specifies a particle it can also be interpreted as referring to the c.m. of a system of masses.

Read the General Comments; and read and understand how to solve the problem set. If you need additional help work some of the Additional Problems.

Try the Practice Test.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read all of Chapter 8. Understand and know Eq. (8-3b). It contains all the previous equations in this chapter. Equation (8-4b) is Eq. (8-3b) in calculus form. The next important equation is (8-8). It also sums up the arguments of Section 8-2. Section 8-3 starts out with a definition you must memorize: the linear momentum of a particle, Eq. (8-9). Equation (8-10) will be used in Sections 8-4 and 8-5. It is a statement of Newton's second law. Section 8-4 shows how to calculate the total linear momentum of a system of particles: Eq. (8-12). Equation (8-13) can be used to find $\vec{v}_{c.m.}$. Equation (8-10) appears again in a more restricted form as Eq. (8-15): the internal forces have been eliminated. You should know why. Section 8-5 begins with the important equations describing the conservation of linear momentum, but paradoxically does not give them numbers.

Read and understand Examples 1 to 6 in this chapter. Then read Section 9-2 in Chapter 9. Read the General Comments; and read and understand how to solve the Problem Set. If you need additional help, work some of the Additional Problems.

Try the practice test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 8-1	A			
2	Secs. 8-1, 8-2, 8-4	A, B	Chap. 8, Ex.* 1, 2, 3	A, B	Chap. 8, Quest. 1 to 4, 10; Probs. 1 to 13, 21, 22
3	Sec. 9-2	C	Chap. 9, Ex. 1 to 4	C	Chap. 9, Quest. 3, 4, 8; Probs. 1 to 13
4	Sec. 8-5	D	Chap. 8, Ex. 4, 5, 6	D	Chap. 8, Quest. 5, 8, 9; Probs. 23 to 39

*Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

University Physics does not include center of mass, and therefore other texts must be suggested for this important topic. Read the indicated sections in one of the three texts listed below.

Author and Text	Topic	Section
Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition	center of mass motion of c.m.	9.1 9.2
David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)	center of mass motion of c.m.	8-1 8-2, 8-4
Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1	motion of c.m. center of mass	6-1, 8-4 6-2

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems†
1	Use one of the readings recommended above. Sec. 8-1	A	Ex.* 1, 2 (Sec. 8-2)	A	
2	Use one of the readings recommended above	A, B		A, B	B: Chap. 9, 1, 3, 4 (noncalculus), 5-8 (calculus) HR: Chap. 8, Quest.* 1-3, 10; Probs. 1-13, Ex. 1-3 WS: 6-1, 6-3 to 6-8 (calculus)
3	Sec. 8-1	C	Ex. 1 (Sec. 8-1)	C	8-3, 8-4 (calculus)
4	Sec. 8-2	D		D	8-5

*Ex. = Example(s). Quest. = Question(s).
†B = Bueche. HR = Halliday and Resnick. WS = Weidner and Sells.

The above readings develop the idea of replacing a system of masses by an imaginary but somewhat equivalent mass particle. This particle has the same total mass as the system and is located at a place called the center of mass (c.m.) of the system. In Chapter 8, Sections 8-1 and 8-2, University Physics develops the ideas of momentum and impulse for particles, and these concepts can be applied to all systems of masses since you now know how to reduce systems to equivalent particles.

Read Chapter 8. Examples 1 and 2 in Section 8-2 show that the momentum of a system of particles is the sum (vector) of the momenta of the particles. Read the General Comments; and read and understand how to solve the Problem Set. If you need additional help, work some of the Additional Problems.

Try the Practice Test.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 5, Section 5-5. This section defines the linear momentum of a particle: Eq. (5-2). It furthermore shows by a worked example that in a particular collision the total linear momentum of the two colliding particles does not change. The total linear momentum of a system of particles is defined in Eq. (5-5). Work through Examples 5-1 to 5-3. You should draw figures showing the linear-momentum vectors of the systems before and after the collision.

Now read Chapter 6, Sections 6-1 and 6-2. These sections show you how to calculate the velocity and the location of a system's center of mass (c.m.). They also give you a method of reducing any collection of masses to a single somewhat equivalent particle located at the c.m. Now you have the tools to apply any rules for particles to collections of masses. The remainder of this module will show you how to use these tools.

Read Section 7-4 in Chapter 7. Here you are shown the relationship between the linear momentum of a system and the forces acting on the system: Eq. (7-6). Example 7-3 should be understood.

Read Chapter 8, Section 8-4. This is an important section because it tells you the general requirements for a system in order that its linear momentum be conserved. Example 8-8 is a good review of the ideas presented in this module. There are summaries at the ends of the chapters that gather the ideas into a few lines.

Read the General Comments; and read and understand how to solve the Problem Set. If you need additional help, work some of the Additional Problems.

Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	Sec. 5-5, Chap. 6, Sec. 8-4	A		A		
2	Sec. 5-5, Chap. 6, Sec. 8-4	A, B		A		5-3, 6-1 to 6-8, 6-5 (calculus)
3	Sec. 7-4	C		C		7-2 to 7-9, 7-11, 7-12
4	Sec. 8-4	D		D		

GENERAL COMMENTS

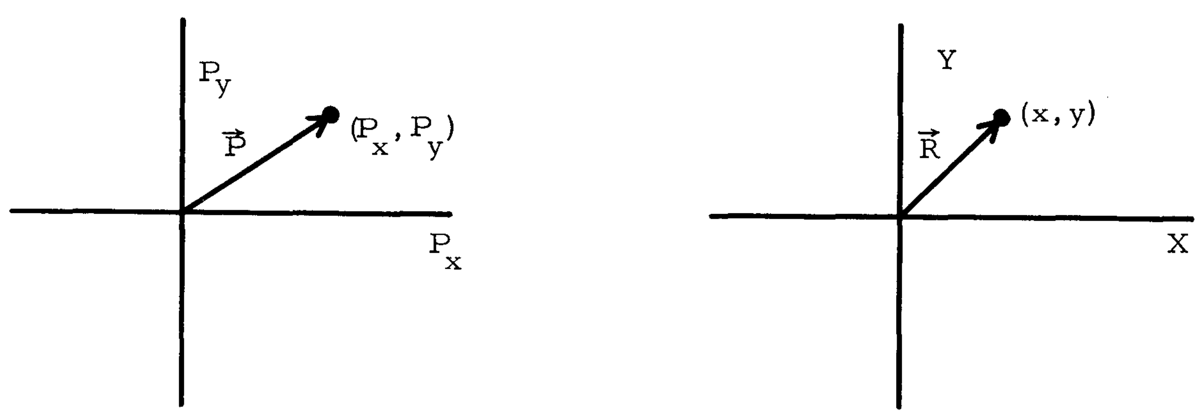
1. Center of Mass

The center of mass (c.m.) of a system is an intrinsic property of that system. Although the formulas for the position of the c.m. give it as the distance from the origin of a coordinate frame, the position of the c.m. does not move with respect to the system if the origin is moved. The distances from the origin to the c.m. will change, of course. But this is only because the origin or the system as a whole has moved.

This property allows you to pick any point as the origin in a problem where you have to find a c.m. A clever choice may simplify your calculation. Look for a symmetry axis in your system and place the origin somewhere on it. Or you can place the origin on one of the system's particles.

2. Linear Momentum

Linear-momentum vectors are added like any other vectors, but they exist in linear-momentum space (see Figure 1). Two-dimensional examples of a linear-momentum space and an ordinary coordinate space are shown below. Points in ordinary space show position, and vectors in this space show displacements. Points and vectors in linear-momentum space represent linear momenta. You have no information about the position of anything in linear-momentum space. Sometimes when solving problems you may draw displacement and linear-momentum vectors in the same figure and not realize that they are superimposing ordinary and linear-momentum spaces. Watch out! This can lead to mistakes such as attempts to add displacement and linear-momentum vectors.



Momentum Space Coordinate Space

Figure 1

3. Coordinate System

You will learn that the total linear momentum of a system is the product of the total mass of the system and the velocity of the system's c.m. If the system has zero resultant external forces acting on it, its c.m. is not accelerated. Thus the c.m. moves at constant velocity, and a coordinate system whose origin is placed at the c.m. and moves with it will be an inertial coordinate system. Furthermore, the total linear momentum of the system relative to that origin is zero because the velocity of the c.m. relative to the origin (at the c.m.) is zero. Here is another case where a good choice of coordinate-system origin may sometimes simplify a problem. By placing the origin at the system's c.m. (providing the c.m. is not accelerating) you can use the fact that the total linear momentum of the system is zero.

This placement of the coordinate system's origin establishes what is called the center-of-mass coordinate system or center-of-mass reference frame.

PROBLEM SET WITH SOLUTIONS

A(1). (a) A system of several particles is shown in Figure 2. You are told the mass of the particles and their position coordinates relative to the coordinate axis. Explain how to find the center of mass of the system.

(b) At the instant shown in Figure 2 the particles are in motion. m_1 is moving upward (+z direction) with speed v_1. m_2 is moving to the left (-y direction) with speed v_2. m_3 is moving toward m_1 with speed v_3. Explain how to find the total linear momentum of the system.

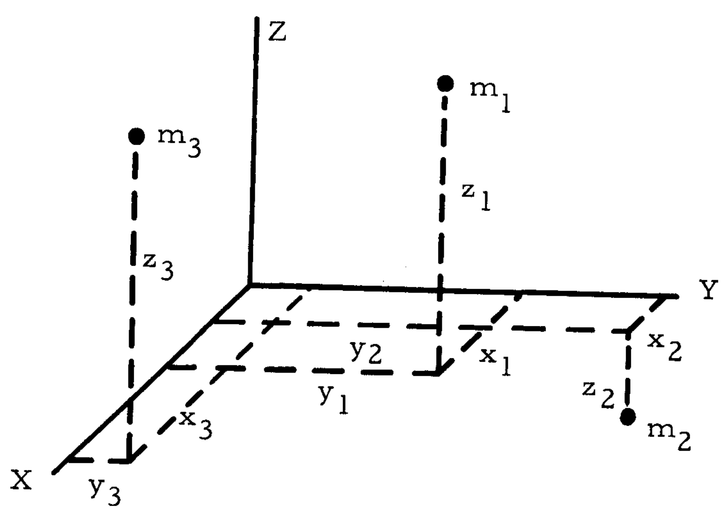


Figure 2

<u>Solution</u>

(a) By inspection of the figure you can see that the c.m. will be at a place that has x, y, and z coordinates. Your texts give formulas for each of these coordinates:

$$x_{c.m.} = \sum_{i=1}^{N} x_i m_i / \sum_{i=1}^{N} m_i = \frac{1}{M} \sum_{i=1}^{N} m_i x_i = X,$$

$$y_{c.m.} = \sum_{i=1}^{N} y_i m_i / \sum_{i=1}^{N} m_i = \frac{1}{M} \sum_{i=1}^{N} m_i y_i = Y,$$

$$z_{c.m.} = \sum_{i=1}^{N} z_i m_i / \sum_{i=1}^{N} m_i = \frac{1}{M} \sum_{i=1}^{N} m_i z_i = Z.$$

Since this problem uses particles and not extended bodies you can use the c.m. formulas for particles. If the masses were not particles you would have to use the calculus formulas for c.m. coordinates.

The c.m. of the system (see Figure 3) is located at the point having coordinates ($x_{c.m.}$, $y_{c.m.}$, $z_{c.m.}$). You can also express the position of the c.m. by specifying the vector $\vec{r}_{c.m.}$. Use unit vectors and write

$$\vec{r}_{c.m.} = x_{c.m.}\hat{i} + y_{c.m.}\hat{j} + z_{c.m.}\hat{k}.$$

The problem could also have been given to you in terms of the masses and their position (or displacement) vectors. You would then have the additional first step of resolving the position vectors into their components, i.e., finding the x_1, y_1, z_1, x_2,....

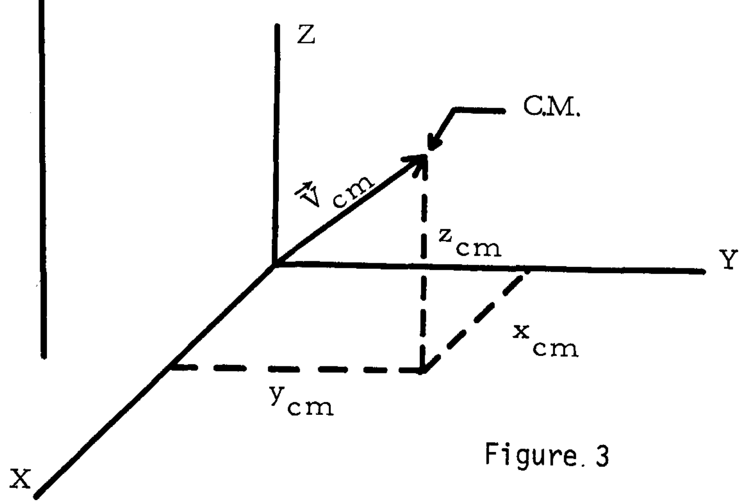


Figure 3

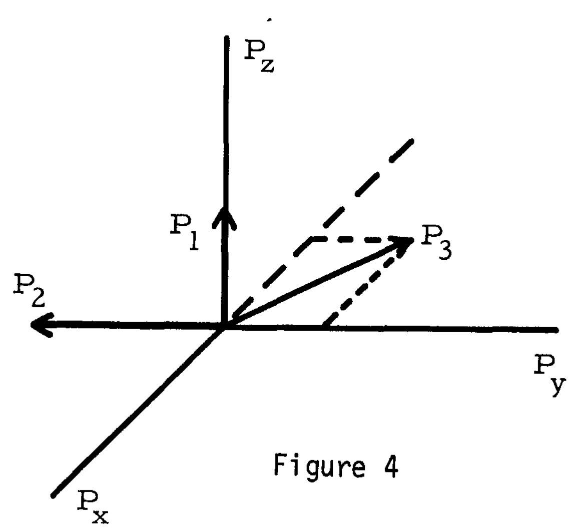


Figure 4

(b) The total linear momentum of the system is the sum (vector) of the momenta of its parts. In this problem you must calculate the linear momentum of each particle and then add them to obtain the total linear momentum. The momentum of each particle is $\vec{p} = m\vec{v}$, where m is the mass and v is its velocity. The momenta vectors would look as shown in Figure 4, and the total linear momentum is

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = M\vec{V}_x = m_1\vec{v}_{1x} + m_2\vec{v}_{2x} + m_3\vec{v}_{3x}.$$

Since the three momentum vectors of the particles are not collinear nor even coplanar, the best way to add them is by their components. Resolve the velocity vectors into their x, y, and z components and add them:

$$V_x = v_{1x} + v_{2x} + v_{3x},$$

$$V_y = v_{1y} + v_{2y} + v_{3y},$$

$$V_z = v_{1z} + v_{2z} + v_{3z}.$$

Then

$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k},$$

and finally

$$\vec{P} = M\vec{V}.$$

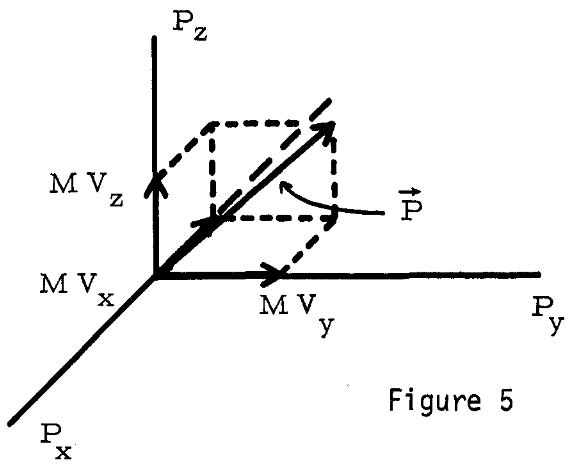


Figure 5

The total linear momentum and its components would look as shown in Figure 5. Although it's not asked for in this problem, you should recognize that <u>$\vec{V}$ is the velocity of the center of mass of the system</u>.

B(2). Three particles are moving radially outward from the coordinate origin, at angles of 120° to one another in the xy plane. Their masses are $m_1 = 1.00$ kg, $m_2 = 2.00$ kg, and $m_3 = 0.80$ kg, and their speeds are $v_1 = 6.0$ m/s, $v_2 = 2.00$ m/s, and $v_3 = 10.0$ m/s, respectively.

(a) Sketch the system in a coordinate frame.

(b) Which particle has the momentum of greatest magnitude?

(c) What is the total momentum of the three-particle system?

(d) Find the velocity of the c.m.

<u>Solution</u>

(a) Draw an xy coordinate system and show the particles and their motions on it (Figure 6). There are many ways to place the particles on the coordinate system. In Figure 6 one of the particle's trajectories was placed along the y axis. This will save effort later if it becomes necessary to resolve the velocities or momenta into components in the x and y directions.

Also draw the momentum vectors for the three particles (Figure 7). The Magnitudes may be wrong in this figure because the linear momenta have not yet been calculated; but this gives you an idea of the momenta and their directions. Now start answering the questions.

(b) Linear momentum is $\vec{p} = m\vec{v}$, and its magnitude is p = mv. For each particle:

$p_1 = m_1v_1 = (1.00 \text{ kg})(6.0 \text{ m/s}) = 6.0 \text{ kg m/s}$,

$p_2 = m_2v_2 = (2.00 \text{ kg})(2.00 \text{ m/s}) = 4.0 \text{ kg m/s}$,

$p_3 = m_3v_3 = (0.80 \text{ kg})(10.0 \text{ m/s}) = 8.0 \text{ kg m/s}$.

Particle 3 has the largest linear-momentum magnitude. We can now redraw Figure 7 to scale as Figure 8.

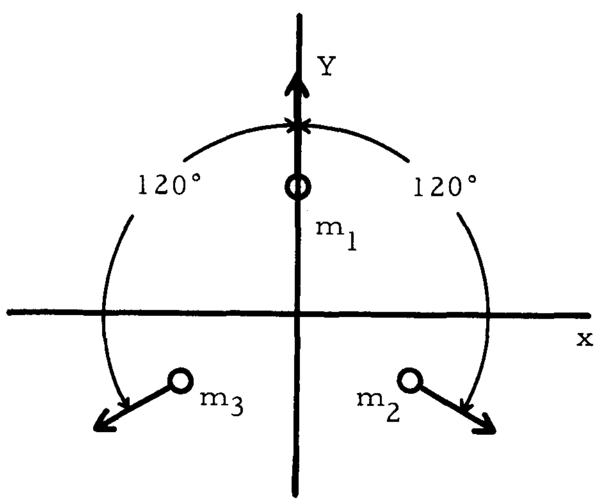


Figure 6

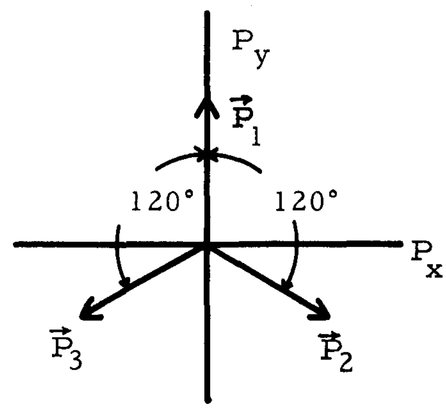


Figure 7

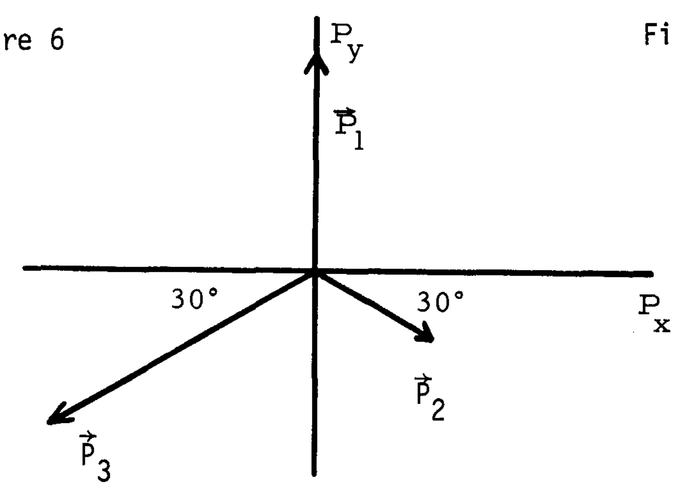


Figure 8

(c) The total linear momentum of the system is

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3.$$

In pictorial form this addition is shown in Figure 9, and you see from inspection that $\vec{P} \neq 0$. Pity.

Unless you prefer to work directly with the polygon in Figure 9 the best method to find P is to find its x and y components and add (vector) them. This will require you first to resolve the momenta of the three particles into their x and y components (refer to Figure 7):

$$p_1 = p_1\hat{j} = m_1v_1\hat{j} = (6.0\hat{j}) \text{ kg m/s},$$

$$p_2 = p_2 \cos(30°)\hat{i} + p_2 \sin(30°)(-\hat{j})$$

$$= [(8.0)(0.866)\hat{i} - (8.0)(0.500)\hat{j}] \text{ kg m/s},$$

$$p_3 = p_3 \cos(30°)(-\hat{i}) + p_3 \sin(30°)(-\hat{j})$$

$$= [(4.0)(0.866)\hat{i} - (4.0)(0.500)\hat{j}] \text{ kg m/s}.$$

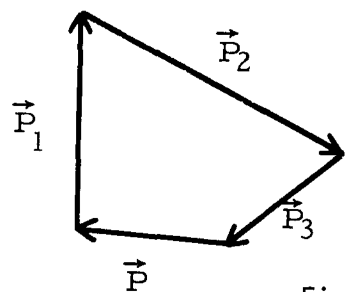


Figure 9

Use

$$p_x = p_{1x} + p_{2x} + p_{3x} = 0 + (8.0)(0.866) - (4.0)(0.866)$$

$$= 3.5\hat{i} \text{ kg m/s}.$$

Similarly

$$p_y = p_{1y} + p_{2y} + p_{3y} = [6.0 - (8.0)(0.500) - (4.0)(0.500)\hat{j}] \text{ kg m/s} = \vec{0}$$

for this case. Thus for the total linear momentum,

$$\vec{P} = [(2.00)(1.732)\hat{i}] = 3.5\hat{i} \text{ kg m/s}.$$

(d) The velocity of the center of mass is the total linear momentum of the system divided by the system's mass:

$$\vec{V}_{c.m.} = \vec{P}/M = \Sigma m\vec{v}/\Sigma m = \vec{V} \quad \text{or} \quad Mv_{x(c.m.)} = m_1v_{x1} + \cdots + m_nv_{xn}.$$

Thus $\vec{V}_{c.m.}$ or $\vec{V} = [(2.00)(1.732)\hat{i} \text{ kg m/s}]/3.8 \text{ kg} = (0.53)(1.732)\hat{i} \text{ m/s} = 0.92\hat{i} \text{ m/s}$,

$$v_{x(c.m.)} = [(2.00)(1.732) \text{ kg m/s}]/3.8 \text{ kg} = 0.92 \text{ m/s},$$

which is the magnitude of the velocity of the c.m.

C(2). A croquet ball (mass 0.50 kg) initially at rest is struck by a mallet, receiving the impulse shown in Figure 10. What is the ball's velocity just after the force has become zero? Assume the graph is a parabola.

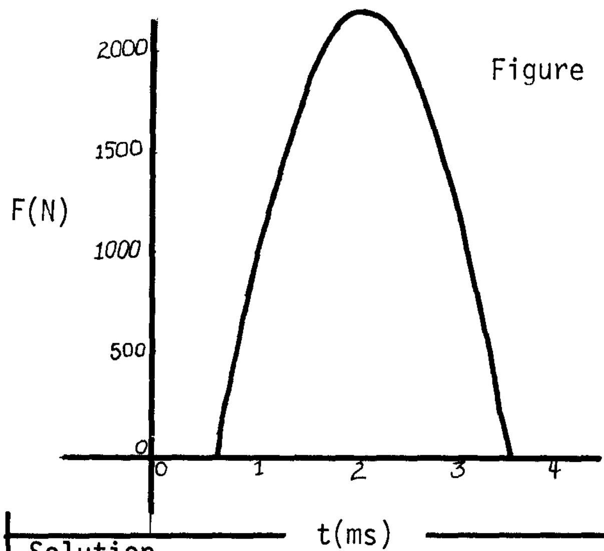
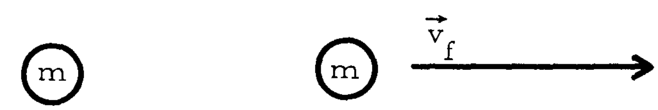


Figure 10

Figure 11



initially   after force becomes zero

Solution

This is an impulse problem. The croquet ball's initial velocity (and therefore its initial momentum) is zero, and you want to find its final velocity. Since you are given the ball's mass, you must find its final momentum and divide by the mass: $\vec{v}_f = \vec{p}_f/m$.

Note that the problem is one dimensional, and you are not told the direction the ball rolls. However, you are asked for the ball's velocity (vector), and you must just assume a direction such as "horizontally to the right," and proceed to find the speed. Start with

$$\int_{t_i}^{t_f} \vec{F}\,dt = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i,$$

and apply this equation to the croquet ball. Since $\vec{v}_i = 0$, you can solve for $\vec{v}_f$ algebraically:

$$\vec{v}_f = \frac{1}{m}\int_{t_i}^{t_f} \vec{F}\,dt.$$

You know m and the integral is the area under the force versus time graph. Here are two ways to integrate this area:

(1) Square counting: See Figure 12. Each small square has an area of (100 N)(0.200×10^{-3}) = 0.0200 N s. Now you simply count the number of squares under the graph and multiply this by the area of one square. A count accurate to 1%

gives 210 squares. Thus the total area is

$$\int_{t_i}^{t_f} F\,dt = (210 \text{ squares})(0.0200 \text{ N s/square}) = 4.2 \text{ N s}.$$

Since this is a one-dimensional problem the vector notation has been dropped:

$$v_f = \frac{1}{m}\int_{t_i}^{t_f} F\,dt \quad \text{and} \quad v_f = \frac{4.2 \text{ N s}}{0.50 \text{ kg}} = 8.4 \text{ m/s}.$$

(2) Calculus: An equation for a parabola is $y = kx^2$. However, this parabola passes through the origin, but your graph does not have F at zero when t is zero; thus the equation $F = kt^2$ will not work for you. The equation

$$(F - 2200 \text{ N}) = k(t - 2.00 \times 10^{-3} \text{ s})^2$$

will work, if you evaluate k. Pick a point such as F = 0 and $t = 0.50 \times 10^{-3}$, and plug these values into the above equation. This gives

$$k = -9.78 \times 10^8 \text{ N/s}^2.$$

Now

$$F = (9.78 \times 10^8 \text{ N/s}^2)(t - 2.00 \times 10^{-3} \text{ s})^2 + 2200 \text{ N}$$

and

$$\int_{t_i}^{t_f} F\,dt = \int_{0.50 \times 10^{-3} \text{ s}}^{3.4 \times 10^{-3} \text{ s}} [(-9.78 \times 10^8 \text{ N/s}^2)(t - 2.00 \times 10^{-3} \text{ s})^2 + 2200 \text{ N}]dt$$

$$= (-9.78 \times 10^8 \text{ N/s}^2) \int_{0.50 \times 10^{-3} \text{ s}}^{3.4 \times 10^{-3} \text{ s}} (t - 2.00 \times 10^{-3} \text{ s})^2\,dt$$

$$+ (2200 \text{ N}) \int_{0.50 \times 10^{-3} \text{ s}}^{3.4 \times 10^{-3} \text{ s}} dt.$$

A change of variable will simplify the first integral. Let $t - 2.00 \times 10^{-3} \text{ s} \equiv \tau$.

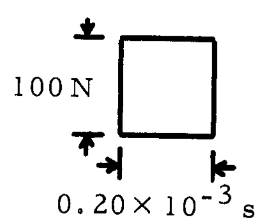


Figure 12

Then $dt = d\tau$, and changing the limits of integration gives us

$$F\,dt = (-9.78 \times 10^{8}\ \mathrm{N\ s^{2}}) \int_{-1.50 \times 10^{-3}}^{1.4 \times 10^{-3}} \tau^{2}\,d\tau + 2200\ \mathrm{N} \int_{0.50 \times 10^{-3}}^{3.4 \times 10^{-3}} dt = 4.38\ \mathrm{N\ s}.$$

Note here that this value is a bit more than the one obtained from square counting. Possibly the assumption that the graph was a parabola symmetric about 2×10^{-3} s was not correct. As before,

$$v_f = 4.38\ \mathrm{N\ s}/0.50\ \mathrm{kg} = 8.76\ \mathrm{m/s} = 8.8\ \mathrm{m/s}.$$

D(2). A child runs and leaps into a stationary wagon. The wagon can roll without friction on the level, rough driveway, but it is not headed in the direction the child was running. The wagon and child move in the direction the wagon was pointed.

(a) Is the linear momentum of the child-wagon system conserved in this process? Explain why.

(b) Are any linear-momentum components of this system conserved? Explain why.

Solution

(a) Momentum is conserved if the total linear momentum of the system does not change. Before the child jumps on the wagon she alone has some linear momentum. The total linear momentum of the system then is in the direction the child is running. When the child is aboard the rolling wagon the total linear-momentum vector of the system points in the direction the wagon is rolling. This is not the direction in which the child was running, and thus the linear momentum of the system cannot be the same before and after the child jumped on the wagon. It is impossible for two vectors to be equal if they are not in the same direction. The linear momentum of the system has not been conserved.

(b) Another way to identify momentum conservation is by the use of the equation

$$\int_{t_i}^{t_f} \vec{F}_{ext}\,dt = \Delta\vec{p}.$$

If $\vec{F}_{ext}$ is zero, then so will be $\Delta\vec{p}$.

Furthermore, if there is any direction in which some component of $\vec{F}_{ext}$ is zero, the component of $\Delta\vec{p}$ in that direction will also be zero. In your problem the wagon rolls without friction in the direction it is pointed. There can be no external forces acting on the system in this direction, and the component of the system's linear momentum in this direction is conserved. You should realize that there is a considerable external force on the system when the child lands in the wagon: the frictional force between the wheels and the rough driveway. This is the force that causes the momentum of the system to change.

An often overlooked, but not always trivial, direction in which momentum might be conserved is the vertical. The external vertical forces acting on the child-wagon system add to zero (assuming the child's c.m. moves horizontally); and the system's change in momentum in the vertical direction is thus also zero. The system's linear-momentum component in the vertical direction remains zero during the process.

PRACTICE TEST

1. Explain how conservation of momentum applies to a handball bouncing off a wall.

2. Three particles floating in space are attached to one another with springs. Their masses are 5.0 kg, 7.3 kg, and 12.2 kg, respectively. One of the particles is hit by a meteorite. The force-time graph of this collision is shown in Figure 13. Calculate the change in momentum of the three-particle system.

3. (a) Write the formula for the center of mass of any system. Explain what you would need to know about the system in order to calculate it.

   (b) Write the formula for the linear momentum of any system whose parts are moving in straight lines and not rotating. Explain what you would need to know about the system in order to calculate it.

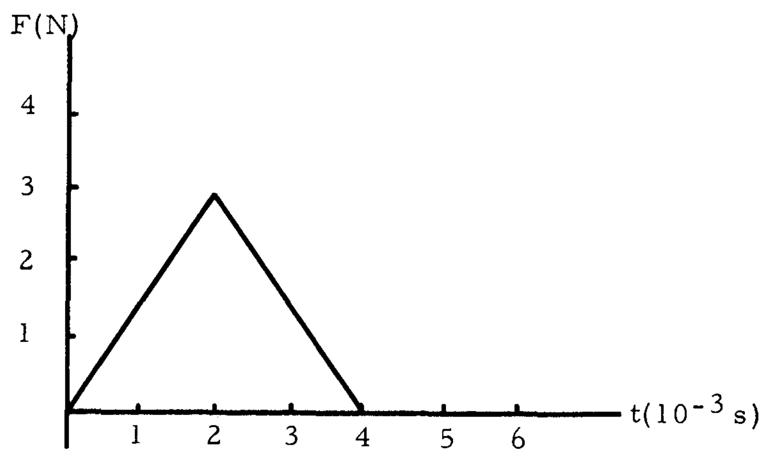


Figure 13

Practice Test Answers

1. The linear momentum of the ball is certainly not conserved. The ball's momentum has flipped direction owing to the force from the wall. If you include the wall and everything it is attached to in your system then momentum must be conserved: no external forces act. It is not easy to visualize the wall's momentum changing during the collision, but it does. Its great mass permits a very small velocity change.

2. Use $\int_{t_i}^{t_f} \vec{F}_{ext}\, dt = \Delta\vec{P}$.

The system is the three masses and the springs. The spring forces are internal. The only external force is provided by the meteorite.

$\vec{F}\, dt = 6.0 \times 10^{-3}$ Ns.

Thus, $\Delta\vec{P} = 6.0 \times 10^{-3}$ Ns in the direction of the external force.

3. (a) $\vec{r}_{c.m.} = x_{c.m.}\hat{i} + y_{c.m.}\hat{j} + z_{c.m.}\hat{k}$, where

$$x_{c.m.} = \sum_{i=1}^{N} xm / \sum_{i=1}^{N} m + \sum_{i=1}^{n} (\int x\, dm / \int dm),$$

$$y_{c.m.} = \sum_{i=1}^{N} ym / \sum_{i=1}^{N} m + \sum_{i=1}^{n} (\int y\, dm / \int dm),$$

$$z_{c.m.} = \sum_{i=1}^{N} zm / \sum_{i=1}^{N} m + \sum_{i=1}^{n} (\int z\, dm / \int dm)$$

for particles and extended bodies. You must know all the masses and positions of the particles; and the positions, shapes, and density distributions of the extended bodies.

(b) $$\vec{P} = \underbrace{\sum_{i=1}^{N} m\vec{v}}_{\text{particles}} + \underbrace{\sum_{i=1}^{n} m\vec{V}_{c.m.}}_{\text{extended bodies}}$$

You must know the masses and velocities of the particles; and the masses and velocities (of any portion) of the extended bodies.

IMPULSE AND MOMENTUM

Date ____________

pass recycle

Mastery Test Form A

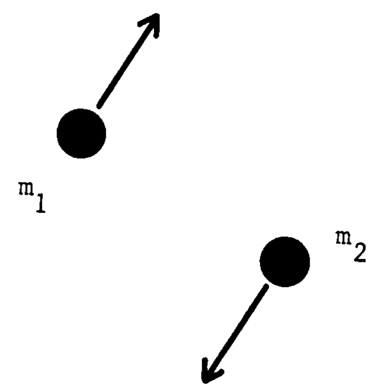
1 2 3 4

Name ______________________ Tutor ______________________

1. (a) Write the formulas for the center of mass of any system. Explain all the terms.

   (b) Write the formulas for the linear momentum of any system that has no rotation. Explain all the terms.

2.



Two particles are moving apart as shown in the figure above. The mass m_1 is 5.0 kg and m_2 is 3.00 kg; at the instant shown they are 6.0 m apart. Each has a speed of 15.0 m/s.

(a) Find the center of mass of this system at the instant shown.

(b) Find the total linear momentum of this system at the instant shown.

(c) Find the velocity of the center of mass of this system at the instant shown.

3. A 2.0-kg ball falls at a constant speed of 0.100 m/s through a viscous fluid. What is the force of the fluid on the ball?

4. Attack or defend the following statement: If a system is made large enough its linear momentum is always conserved.

IMPULSE AND MOMENTUM

Date ____________

pass recycle

Mastery Test Form B

1 2 3 4

Name ____________ Tutor ____________

1. (a) You are shown a system of particles and larger bodies in motion. What would you have to know about the system to calculate its center of mass? How would you calculate the center of mass of this system?

   (b) None of the bodies in the above system is rotating. What would you have to know about the system to calculate its linear momentum? How would you calculate the linear momentum of the system?

2. (a) Starting with a statement for the conservation of linear momentum, show that it takes an external force to accelerate the center of mass of a system with constant mass.

   (b) You are a prisoner in a 4.0-m-long boxcar whose frictionless wheels are 1.50 m from the top of a downhill grade (see Figure 1). If you can get the car to start rolling downhill, you can escape to friendly territory. The end of the car nearest the grade is stacked directly over the wheels with 1000 50-kg gold bars. The car has a mass of 40 000 kg. How many bars must you move to escape? Assume you can move the gold the full 4.0 m and ignore your mass.

3. A vertical rod is connected to a 40-kg particle (see Figure 2). The rod exerts a time-varying force on the particle, which can be calculated from the function

   $$\vec{F} = (200 + 300t)\hat{j}\ \text{N}.$$

   (a) What is the force on the particle at 0 s? Do not neglect gravity.

   (b) Calculate the particle's change of linear momentum between the first and second seconds.

   (c) Is there a time when the linear momentum of the particle is not changing? If so, calculate this time.

   (d) Is there any component of the particle's linear momentum that is always conserved? If so, what is it and why is it always conserved?

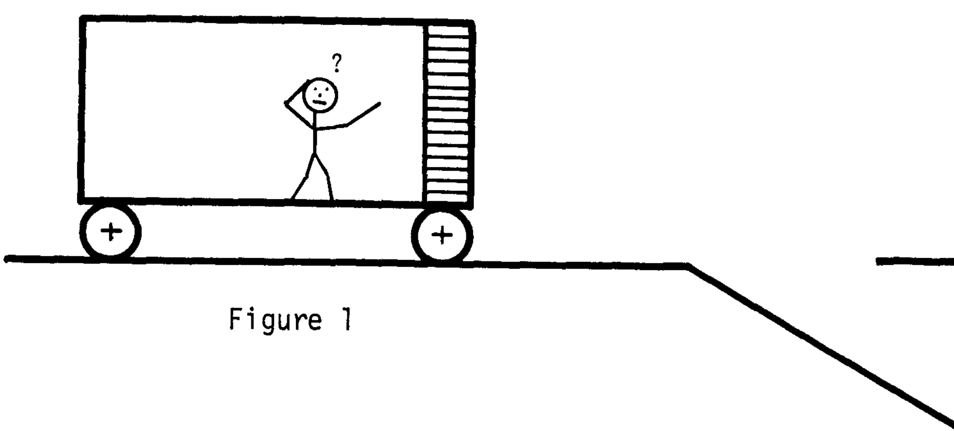


Figure 1

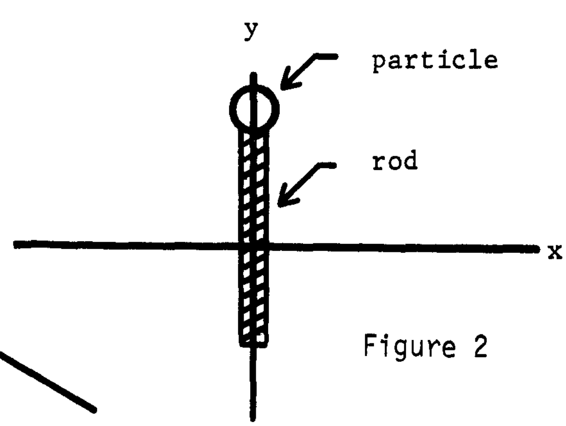


Figure 2

IMPULSE AND MOMENTUM

Date ______________

pass recycle

Mastery Test Form C

1 2 3 4

Name ______________ Tutor ______________

1. (a) Write the formula for the center of mass of any system. Explain what you would need to know about the system in order to calculate it.

   (b) Write the formula for the linear momentum of any system of nonrotating masses moving in straight lines. Explain what you would need to know about the system in order to calculate it.

2. A 30.0-kg particle is suspended by a string. The string is pulled upward and exerts a time-varying force on the particle. The magnitude of this force is given by the function

   $F = (350 + 150t^2)$ N.

   (a) What is the force on the particle at 0 s?

   (b) Calculate the change in the linear momentum of the particle between the second and third seconds.

   (c) Is the change of the particle's linear momentum per second constant? Why?

3. For which of the following systems (underlined) is linear momentum conserved? Justify your answers.

   (a) Two colliding billiard balls, rolling on a pool table.

   (b) A canoe with three nonpaddling occupants on a smoothly flowing river.

IMPULSE AND MOMENTUM

Mastery Test Form D

Date ________________

pass recycle

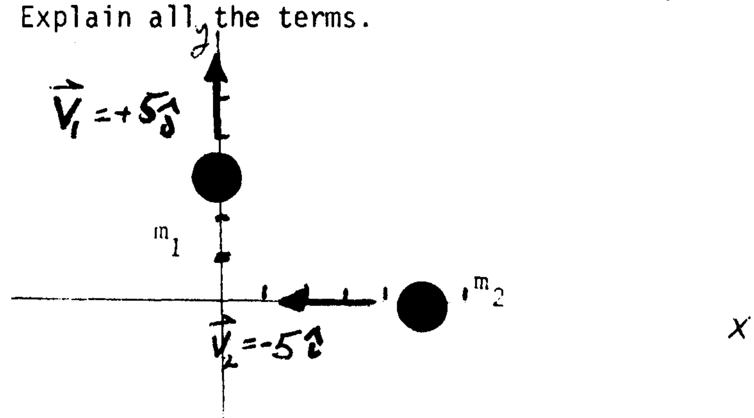
1 2 3 4

Name ______________________ Tutor ______________________

1. (a) Write the formulas for the center of mass of any system. Explain all the terms.

   (b) Write the formulas for the linear momentum of any system that has no rotation. Explain all the terms.

2.



Two particles are moving apart as shown in the figure above. The mass m_1 is 5.0 kg and m_2 is 3.00 kg; at the instant shown they are located at $3\hat{j}$ m and $5\hat{i}$ m respectively.

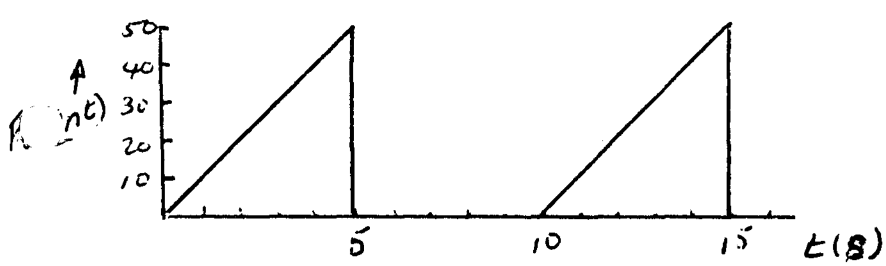
(a) Find the center of mass of this system at the instant shown.

(b) Find the total linear momentum of this system at the instant shown.

(c) Find the velocity of the center of mass of this system at the instant shown.

3. A man standing on a loading dock throws bags of sand into a passing truck. Discuss the conservation of momentum. Are all components of the linear momentum conserved?

4. Calculate the change of momentum during the period 0-10 s for a 10 kg mass acted upon by the force shown in the figure below.



Courtesy of University of Missouri-Rolla

IMPULSE AND MOMENTUM

Date ____________________

pass recycle

Mastery Test Form E 1 2 3 4

Name ______________________________ Tutor ______________________________

1. (a) Write the formula for the center of mass of any system. Explain what you would need to know about the system in order to calculate it.

   (b) Write the formula for the linear momentum of any system of nonrotating masses moving in straight lines. Explain what you would need to know about the system in order to calculate it.

2. A 30.0-kg particle is suspended by a string. The string is pulled upward and exerts a time-varying force on the particle. The magnitude of this force is given by the function

   $F = (350 + \sin\frac{\pi t}{6})\ N$

   (a) What is the force on the particle at 0 s?

   (b) Calculate the change in the linear momentum of the particle between the second and third seconds.

   (c) Is the change of the particle's linear momentum per second constant? Why?

3. A man stands on the back of a 2000 kg truck and throws 20 kg sandbags out the back with a velocity of 10 $m/_s$. How many sandbags must he throw to get the truck velocity up to 1 $m/_s$? Assume the truck wheels are frictionless. Neglect the change in the truck's mass due to the loss of sandbags.

4. What would be the effect of the man throwing sandbags off the side of the truck?

IMPULSE AND MOMENTUM

Date ____________________

pass recycle

Mastery Test Form F

1 2 3 4

Name ____________________ Tutor ____________________

1. (a) You are shown a system of particles and larger bodies in motion. What would you have to know about the system to calculate its center of mass? How would you calculate the center of mass of this system?

   (b) None of the bodies in the above system is rotating. What would you have to know about the system to calculate its linear momentum? How would you calculate the linear momentum of the system?

2. Assume you are standing (m=60 kg) standing at one end of a boxcar (m=6000 kg). If you move 5 m to the other end of the boxcar, how far will the boxcar move? Assume the wheels of the boxcar are frictionless.

3. A force $\vec{F} = (200\hat{i} + 100t\hat{j})$ N is exerted on a particle of mass 10 kg. Calculate the change in momentum of the body between the 5th and 10th seconds.

4. A man proposes to power his sail boat by mounting a fan on the top of the cabin and blowing air into the sail during still air periods. Draw a sketch and discuss the physics involved in this project.

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. System includes particles and extended bodies. The student must show some realization that the location of the c.m. is with respect to a set of axes. Two- or three-dimensional answer. If in part (b) the student introduces the troubles with nonrigid bodies, you must make a comment to him that this is too advanced for this course.	1.(a) The answer must combine the c.m. coordinates for all the masses into a single set of coordinates: either $X_{c.m.}$, $Y_{c.m.}$, $Z_{c.m.}$ or $\vec{r}_{c.m.}$

$$X_{c.m.} = \sum_{i=1}^{N} X_1 m_i / \sum_{i=1}^{N} m_i + \sum_{i=1}^{n} (\int X\, dm/\int dm),$$

for $Y_{c.m.}$ and $Z_{c.m.}$, or

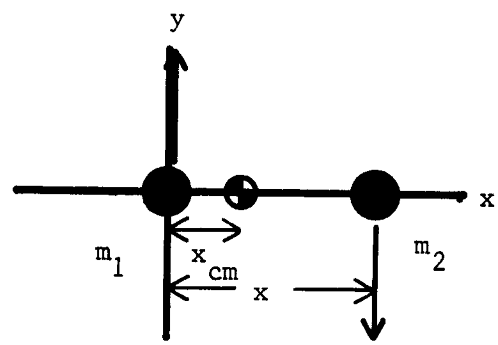
$$\vec{r}_{c.m.} = X_{c.m.}\hat{i} + Y_{c.m.}\hat{j} + Z_{c.m.}\hat{k}.$$

(b) $\vec{p} = \left(\sum_{i=1}^{N} m_i \vec{v}_i\right)_{\text{particles}} + \left(\sum_{j=1}^{N} M\vec{V}_{c.m.}\right)_{\text{extended bodies}}$.

Other forms and an answer in components are OK.

2. The student's choice of coordinates. Comment on an awkward choice, and suggest a better one. However, a poor choice will not make the problem wrong. Directions should be given for all vector quantities.

2.(a)

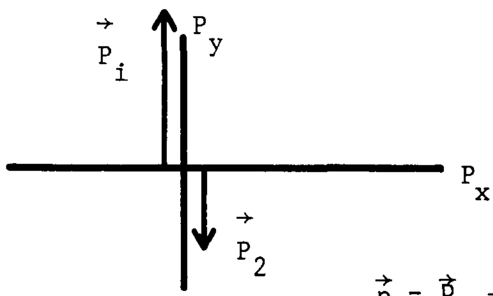


$y_{c.m.} = 0$; placement of coordinate system.

$x_{c.m.} = (0 + m_2 x)/(m_1 + m_2)$

$= (3.00\text{ kg})\ (6.0\text{ m})/8.0\text{ kg} = 2.3\text{ m}.$

2.(b)



$$\vec{p} = \vec{P}_1 + \vec{P}_2 = m_1\vec{v}_1 + m_2\vec{v}_2.$$

$$\vec{P} = m_1 v_1 \hat{j}_1 - m_2 v_2 \hat{j} = (m_1 v_1 - m_2 v_2)\hat{j}$$

$$= [(5.0 \text{ kg})(15.0 \text{ m/s}) - (3.00 \text{ kg})(15.0 \text{ m/s})]\hat{j}$$

$$= 30\,\hat{j} \text{ kg m/s}.$$

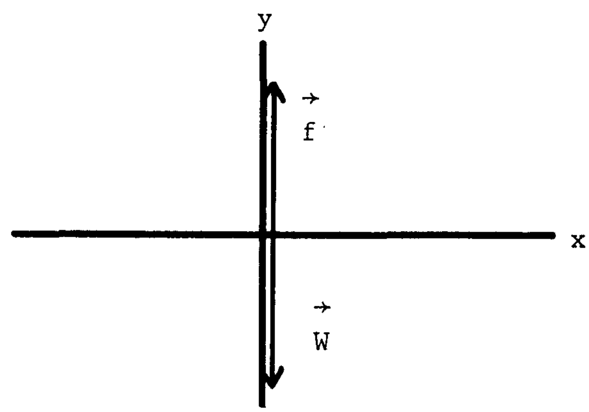
(c) $\vec{P} = M\,\vec{V}_{c.m.}$ and

$$\vec{V}_{c.m.} = \frac{\vec{P}}{M} = \frac{3.00\hat{j} \text{ kg m/s}}{8.0 \text{ kg}} = 3.75\hat{j} \text{ m/s.}$$

3.

The velocity of the ball is constant, and thus so is its linear momentum. The total external force on the ball is zero. There are two external forces acting on the ball: its weight and the force from the liquid.

$$\vec{f} = \vec{W} = m\vec{g} = (2.00)(9.8)\hat{j} \text{ kg m/s}^2 = 19.6\hat{j} \text{ N}.$$



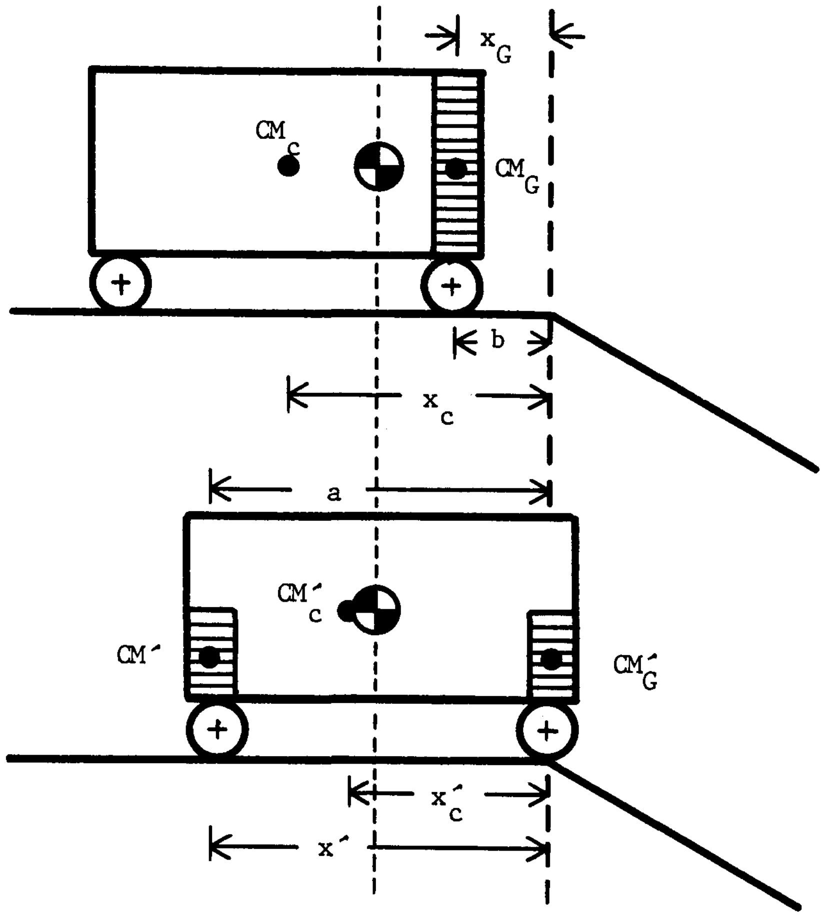
The statement is true. By increasing the size of the system, more and more of the forces become internal forces. Some students are clever enough to attack the statement successfully. You must be equally clever in analyzing their arguments. For example, if it is argued that very small external forces will cause unmeasurable momentum changes to large masses, and therefore all the forces do not have to be internal forces, you must accept this. However, you might mention that an improvement in the technology of momentumometers might make their argument wrong. Do not accept arguments based on one-particle universes.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. System includes particles and extended bodies. The student must show some realization that the location of the c.m. is with respect to an axis. Two- or three-dimensional answer. If in part (b) the student introduces the troubles with nonrigid bodies, you must make a comment to him that this is too advanced for this course.	1.(a) The answer must combine the c.m. coordinates for all the masses into a single set of coordinates: either $X_{c.m.}$, $Y_{c.m.}$, $Z_{c.m.}$ or $\vec{r}_{c.m.}$ $X_{c.m.} = \sum_{i=1}^{N} X_i m_i / \sum_{i=1}^{N} m_i + \sum_{i=1}^{n} (\int X\, dm/\int dm)$, for $Y_{c.m.}$ and $Z_{c.m.}$, or $\vec{r}_{c.m.} = X_{c.m.}\hat{i} + Y_{c.m.}\hat{j} + Z_{c.m.}\hat{k}$. You must know mass and position of all particles; and position, shape, and density distribution of the extended bodies. (b) $\vec{P} = (\sum_{i=1}^{N} m_i\vec{V}_i)_{\text{particles}} + (\sum_{j=1}^{n} M\vec{V}_{c.m.})_{\text{extended bodies}}$. You must know mass and velocity of all particles. For extended bodies you only need to know the total mass and the velocity of any part of the body.
	2.(a) Start with $\vec{F}_{ext} = M\,(d\vec{V}_{c.m.}/dt)$ or $\int \vec{F}_{ext}\, dt = \Delta\vec{P} = \Delta(M\vec{V}_{c.m.})$, and argue that if the left-hand sides of these equations are not zero, neither will the right-hand sides be zero. Recognize that the right-hand sides, which all have a change in velocity, contain the acceleration of the center of mass.
2.(b) The student should realize that here is a case where the center of mass will not accelerate. Since it is initially at rest it will continue to be at rest relative to some place outside the system. Of	(b) The c.m. of the system does not move. As the bars are moved to the left the car must roll to the right. $X_{c.m.} = (M_C X_C + M_G X_G)/(M_C + M_G)$, $X'_{c.m.} = (M_C X'_C + M'_G X'_G + M' X')/(M_C + M'_G + M')$.

What To Look For | Solutions

course, once the front wheels move over the edge of the incline a net external force acts, and the center of mass accelerates.



Neglect prisoner's mass.

$X_{c.m.} = X'_{c.m.}$, $X_C = a/2 + b$,

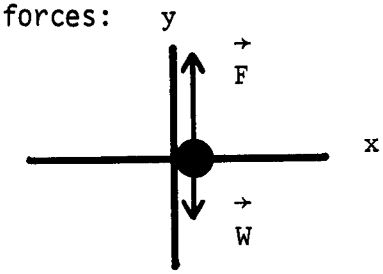
$X_G = b$, $X'_C = a/2$, $X'_G + 0$,

$X' = a$, $M_C + M_G = M_C + M'_G + M'$,

$M_C\ (a/2 + b) + M_G b = M_C\ (a/2) + 0 + M' a$.

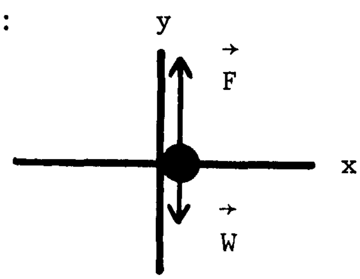
Solving for M':

$M' = (M_C b + M_G b)/a = 33\ 750$ kg.

What To Look For	Solutions
	Number of blocks moved is $n = \frac{31\ 250\ \text{kg}}{50\ \text{kg/block}} = 675$ blocks.
3. Magnitudes and directions of vectors. Recognition that this is mainly an impulse problem. In part (c) there will be an instant when the particle's momentum change is zero. It is poor practice to say that momentum is conserved then.	3.(a) Two forces:  At $t = 0$, $\vec{F} = 200\hat{j}$ N, $\vec{W} = mg(-\hat{j})$. Total force = $\vec{F} + \vec{W} = -192\hat{j}$ N. (b) Use $\int_{t_i}^{t_f} \vec{F}\,dt = \Delta\vec{p}$, $\int_{1\ s}^{2\ s} [(-192 + 300t)\hat{j}]\,dt = 258\hat{j}$ kg m/s. (c) Yes, when $\vec{F} = 0$. $\vec{F} = [(-192 + 300t)]\hat{j} = \vec{0}$ when $t = 192/300 = 0.64$ s. (d) There are no horizontal forces acting on the particle. Thus the horizontal components of the linear momentum remain constant. $\int\vec{F}\,dt = \Delta\vec{P}$ and if $\vec{F} = \vec{0}$, $\Delta\vec{P} = 0$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. System includes particles and extended bodies. The student must show some realization that the location of the c.m. is with respect to an axis. Two- or three-dimensional answer. If in part (b) the student introduces the troubles with nonrigid bodies, you must make a comment to him that this is too advanced for this course.	1.(a) The answer must combine the c.m. coordinates for all the masses into a single set of coordinates: Either $X_{c.m.}$, $Y_{c.m.}$, $Z_{c.m.}$ or $\vec{r}_{c.m.}$. $X_{c.m.} = \sum_{i=1}^{N} X_i m_i / \sum_{i=1}^{N} m_i + \sum_{i=1}^{n} (\int X\, dm/\int dm)$ for $Y_{c.m.}$ and $Z_{c.m.}$, or $\vec{r}_{c.m.} = X_{c.m.}\hat{i} + Y_{c.m.}\hat{j} + Z_{c.m.}\hat{k}$. You must know mass and position of all particles; and position, shape, and density distribution of the extended bodies. (b) $\vec{P} = (\sum_{i=1}^{N} m_i\vec{V}_i)_{particles} + (\sum_{j=1}^{n} M\vec{V}_{c.m.})_{extended\ bodies}$ You must know mass and velocity of all particles. For extended bodies you need only know the total mass and the velocity of any part of the body.
2. Magnitudes and directions of vectors. Recognition that this is mainly an impulse problem.	2.(a) Two forces:



At $t = 0$, $\vec{F} = 350\hat{j}$ N, $\vec{W} = mg(\hat{j})$.

Force: $\vec{F} + \vec{W} = 56\hat{j}$ N.

(b) Use $\int_{t_i}^{t_f} \vec{F}\, dt = \Delta\vec{P}$.

$$\int_{2\ s}^{3\ s} [(56 + 150t^2)\hat{j}]\, dt = [(56 + 950)]\hat{j} = 1006\hat{j} \text{ kg m/s}.$$

What To Look For	Solutions
3. The student must understand that his answer will depend on the existence of external forces acting on the system.	3. Depending on the viewpoint, both yes and no can be justified: (a) Yes - internal forces between the balls are involved and dominate at the moment of collision; external forces are comparatively small. No - over a slightly longer time interval, the interaction of the balls with the table must be considered; since the collision will result in some slippage of the balls on the table, they are subject to a friction force that may change the momentum of the two-ball system. (b) Yes - the net force is zero, vertical forces of water and gravity cancel, no paddling means zero horizontal force; motion of river is smooth, does not give rise to force. No - if river flows around curve.