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Magnetization dynamics of imprinted non-collinear spin textures

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We study the magnetization dynamics of non-collinear spin textures realized via imprint of the magnetic vortex state in soft permalloy into magnetically hard out-of-plane magnetized Co/Pd nanopatterned heterostructures. Tuning the interlayer exchange coupling between soft- and hard-magnetic subsystems provides means to tailor the magnetic state in the Co/Pd stack from being vortex- to donut-like with different core sizes. While the imprinted vortex spin texture leads to the dynamics similar to the one observed for vortices in permalloy disks, the donut-like state causes the appearance of two gyrofrequencies characteristic of the early and later stages of the magnetization dynamics. The dynamics are described using the Thiele equation supported by the full scale micromagnetic simulations by taking into account an enlarged core size of the donut states compared to magnetic vortices. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4931101>]

Chiral magnetic spin textures, such as magnetic spin helices and skyrmions,^{1,2} stabilized by Dzyaloshinskii-Moriya interaction (DMI)^{3,4} in single crystals^{5–8} or at interfaces^{9–11} with inversion symmetry breaking are subject to intense fundamental research. The inertia of skyrmion lattices under external perturbations^{12–14} combined with ultra-low spin-current densities needed for coherent displacement make them appealing for application as low-energy spintronic devices.¹⁵ In general, the spin-orbit interaction governing DMI is weak compared to the Heisenberg exchange and favors chiral spin textures only within a small pocket phase with respect to temperature and magnetic bias field.^{1,2,5–11} Therefore, much attention was devoted to realizing skyrmionic textures in thin film heterostructures providing interfacial DMI^{9–11} or interlayer exchange interaction.^{16–19} The latter group, referred to as artificial skyrmions^{17,18} due to absent DMI, provides means to stabilize chiral spin textures including skyrmionic core textures at room temperature and remanence. Interestingly, topological charge/skyrmion number can be switched in a digital manner between 0 and 1,^{18,19} paving the way towards spintronics operating on topological properties of magnetic nano-objects. Whereas static properties of such spin textures are well understood, dynamic properties are yet to be investigated. Here, we probe the magnetization dynamics of chiral spin textures formed via imprinting magnetic vortices from Permalloy (Py, Ni₈₀Fe₂₀) disks into out-of-plane magnetized Co/Pd multilayer stacks.¹⁹

Coplanar waveguides (CPW) consisting of 100 nm-thick and 10 μm-wide Cu wires with an impedance $Z \approx 50 \Omega$ were fabricated onto 200 nm-thin Si₃N₄ membranes using optical lithography and sputter deposition. PMMA photoresist is spin-coated on top and patterned via electron beam lithography leaving behind circular holes with diameters of $\varnothing = (650 \pm 20)$ nm and $\varnothing = (800 \pm 20)$ nm on the CPW. Afterwards, layer stacks consisting of Pd(2)/[Co(0.4)/

Pd(0.7)]₅/Pd(*d*)/Ni₈₀Fe₂₀(40)/Pd(2) with units in nm and a Pd spacer thickness $d = 1$ nm, $d = 5$ nm were deposited via dc magnetron sputtering at room temperature (base pressure: 7×10^{-8} mbar; Ar pressure: 10^{-3} mbar, rate: 1 Å/s). The magnetic nanostructures [Fig. 1(a)] were obtained after lifting off the PMMA resist. Thicknesses of individual Co and Pd layers in the Co/Pd stack were chosen to induce a large out-of-plane magnetic anisotropy and full remanence.¹⁹ The Pd spacer thickness separating Permalloy (Py, Ni₈₀Fe₂₀) and Co/Pd stack is adjusted to provide strong ($d = 1$ nm) and moderate ($d = 5$ nm) interlayer exchange coupling. We showed in a previous work¹⁹ that systems with strong

FIG. 1. (a) Vertically stacked soft-magnetic (Py, Ni₈₀Fe₂₀) and hard-magnetic (Co/Pd) disks prepared onto a coplanar waveguide are excited with current pulses and stroboscopically recorded in MTXM. (b) Simulated spin configuration in Co/Pd multilayer stacks. (c) Visualization of in-plane magnetization component of the Py subsystem utilizing XMCD. (d) Extracted oscillations of the magnetization component that are correlated to the vortex core displacement (e).

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interlayer interaction reveal a magnetic vortex state imprinted into the Co/Pd subsystem [Fig. 1(b), top], whereas moderate coupling stabilizes magnetic donut states with either one or two circular domain walls that are referred to as donut state type I [Fig. 1(b), middle] and type II [Fig. 1(b), middle], respectively. 40 nm-thick Py disks with diameters of $\varnothing = (1100 \pm 20)$ nm and $\varnothing = (1750 \pm 20)$ nm were prepared for reference.

The magnetization dynamics were studied with magnetic transmission soft X-ray microscopy (MTXM) at the Advanced Light Source (ALS, beamline 6.1.2) in the two-bunch mode. The sample is tilted by 30° with respect to the X-ray beam propagation direction along a rotation axis coinciding with the CPW. Thus, the in-plane magnetization can be accessed utilizing X-ray magnetic circular dichroism (XMCD) as element specific contrast mechanism. We observe the typical dipolar XMCD contrast when imaging Py disks at the Fe L_3 absorption edge, which refers to a magnetic vortex state in the reference samples [Fig. 1(c)]. The magnetization dynamics in these patterned nanostructures are stroboscopically recorded synchronizing electric current pulses ($I \approx 100$ mA, $\Delta t = 1$ ns) from a pulse generator with the electron bunches orbiting in the synchrotron storage ring. The corresponding in-plane magnetic Oersted field with an amplitude of about 8 kA/m shifts the vortex core by $\approx 30\%$ of the diameter towards the edge of the disk. The annihilation of the Py vortex is observed at 12 kA/m when applying static in-plane fields.

For the reference Py sample, the in-plane oscillation of the magnetization component along the direction of the field excitation is shown in Fig. 1(d). This data can be represented as the displacement of the magnetic vortex core assuming a harmonic oscillation with the gyrofrequency ω_0 [Fig. 1(e)]. The dynamics of the normalized longitudinal magnetization component are accordingly described by

$$m_x(\varepsilon, t) = \sin[2\pi\omega_0(\varepsilon)t] \cdot \exp(-\lambda t), \quad (1)$$

with an aspect ratio ε defined as the quotient of Py film thickness $h = 40$ nm and disk diameter $\varnothing$, and phenomenological damping factor $\lambda = \alpha\gamma/M_S \approx \alpha \times 176$ GHz with Gilbert damping factor α and gyromagnetic ratio of an isolated electron $\gamma = 2\pi \times 28$ GHz/T. Using (1), the magnetization dynamics are fitted with $\omega_0(\varepsilon = 0.02) \approx 0.09$ GHz and $\omega_0(\varepsilon = 0.04) \approx 0.14$ GHz as well as $\alpha \approx 2 \times 10^{-4}$ [Fig. 1(d); only data for $\varepsilon = 0.02$ is shown], which is in agreement with previous observations.^{20,21}

The total Co thickness of the Co/Pd subsystem of 2 nm is insufficient to record a detectable signal at the Co L_3 edge due to the limited X-ray intensity of the two-bunch mode. In the following, we study the impact of the imprinted magnetic state in the Co/Pd subsystem on the magnetization dynamics of the Py spins. All images are recorded at the Fe L_3 absorption edge. Figure 2 plots the magnetization oscillations extracted from stroboscopically recorded images [Fig. 2(a)] for disks with different aspect ratios consisting of coupled [Co/Pd]/Pd/Py stacks with 1 nm- and 5 nm-thick Pd spacer. The strongly coupled system with $d = 1$ nm reveals a harmonic oscillation with one gyrofrequency similar to pure Py [Fig. 2(b)]. Samples in the donut state ($d = 5$ nm) reveal a

FIG. 2. Time dependence of the in-plane magnetization component of Py along the field direction of the magnetic field pulse measured with MTXM. (a) Stroboscopically recorded XMCD contrast patterns. (b) Strongly coupled systems ($d = 1$ nm) with two different aspect ratios ε reveal gyrofrequencies $\omega_0 \approx 0.08$ GHz. (c) Moderately coupled systems ($d = 5$ nm) possess a time-dependent gyrofrequency that increases rapidly after about one period. (d) Gilbert damping factors for coupled and reference systems.

three times larger frequency than those of strongly coupled stacks ($d = 1$ nm) [Fig. 2(c)]. Moreover, the relaxation process cannot be described with one single frequency using (1). For instance, samples with an aspect ratio $\varepsilon = 0.05$ reveal in the first ≈ 7 ns a gyrofrequency $\omega_1 \approx (0.20 \pm 0.02)$ GHz and a gyrofrequency $\omega_0 \approx (0.30 \pm 0.02)$ GHz afterwards. Furthermore, the damping is lowest for pure Py with in-plane magnetization and increases with decreasing interlayer exchange coupling [Fig. 2(d)]. These dynamics are distinct from typical vortex core dynamics in Py disks.

To understand the physical origin, we first perform micromagnetic simulations at $T = 0$ K using Nmag v.0.2,²² the HLib library²³ and typical materials parameters for Co/Pd ($h = 5$ nm, $M_S = 500$ kA/m, $A = 10^{-11}$ J/m, and $K = 200$ kJ/m³) and Py ($h = 40$ nm, $M_S = 860$ kA/m, and $A = 1.3 \times 10^{-11}$ J/m). The modeled disks possess a diameter of 400 nm. The size distribution of the tetrahedral mesh resembles a Poisson distribution with peak and mean values around 6 nm³ and 22 nm³, respectively. The standard deviation is about 40 nm³. The interlayer exchange coupling strength is varied in the range $J_i = (0.4-1)$ mJ/m². Note that the interlayer exchange coupling through the 1 nm-thick and 5 nm-thick Pd spacer is mediated by Ruderman-Kittel-Kasuya-Yoshida (RKKY)-type coupling and by spin diffusion mechanisms involving unfilled 4d and 5s-*p* Pd bands,²⁴ respectively. Landau-Lifschitz-Gilbert damping coefficients of $\alpha_{Py} = 0.05$ and $\alpha_{Co} = 0.1$ are used for efficient computation. Simulations with damping

coefficients one order of magnitude smaller than those back up our results revealing immune dynamics with similar gyrofrequencies and trajectories. Line profile of the normal magnetization component and core sizes of donut states imprinted into the Co/Pd subsystem are shown in Figs. 3(a) and 3(b), respectively. For comparison, the normal magnetization component of ordinary out-of-plane magnetized bubble domains is plotted as blue curve in Fig. 3(a). In both cases, the circular domain wall is located at $\frac{1}{\sqrt{2}}\varnothing$ providing a balanced contribution of negative and positive magnetostatic surface charges. Both core sizes and normal magnetization components can be continuously tuned by adjusting the interlayer coupling through the non-magnetic Pd spacer. Comparing numerical core size with experimentally determined values suggests that a Pd spacer thickness of $d=5$ nm mediates an interlayer exchange coupling of about 0.4 mJ/m^2 .¹⁹ While the core size of the Co/Pd layers can be as large as 100 nm for weak coupling, the Py vortex core remains unaffected with an almost thickness-independent profile [Fig. 3(c)]. This is because the vortex core dimension is determined by the unmodified exchange, while the core size of the donut state is adjusted according to the dipole interaction and thus the normal magnetization component that is tailored by the interlayer exchange coupling. This peculiarity suggests the key role of the Co/Pd core on the magnetization dynamics of the entire system.

In order to obtain further insight, the relaxation process is analytically described using the Thiele equation for magnetic domain motion²⁵

$$\mathcal{G}e_z \times \dot{\mathbf{r}} - \mathcal{K}\mathbf{r} = \mathcal{M}\ddot{\mathbf{r}}, \quad (2)$$

with $\mathbf{r} = (x, y)$. Approximating the donut state as an out-of-plane magnetized bubble domain, the coefficients represent

FIG. 3. (a) Profiles of the normal magnetization component across the Co/Pd layer obtained from micromagnetic simulations using $J_i = 0.4 \text{ mJ/m}^2$ (red curve) and $J_i = 0 \text{ mJ/m}^2$ (blue curve). (b) Numerically calculated core size as a function of interlayer exchange coupling. (c) Cross-section of the normal magnetization component through Py vortex core and top view of Co/Pd magnetization for various interlayer coupling strengths. (d) Analytical calculation of the relation between gyrofrequency and core size of the Co/Pd stack. Experimentally determined gyrofrequencies are indicated by horizontally shaded areas (red ($d = 1$ nm): $\omega_0 = (0.07 \pm 0.2) \text{ GHz}$ and $(0.08 \pm 0.2) \text{ GHz}$; blue ($d = 5$ nm): $\omega_0 = (0.29 \pm 0.02) \text{ GHz}$ and $(0.33 \pm 0.2) \text{ GHz}$). Core sizes for strong [(35 ± 6) nm] and moderate [(100 ± 12) nm] coupling obtained by micromagnetic simulations are indicated by vertical gray shaded areas.

the gyrotropic coefficient $\mathcal{G} \propto Sh$, the restoring force $\mathcal{K} \propto M_s^2 \frac{h^2}{\varnothing} \exp(\varnothing_{\text{core}})$, and the core mass $\mathcal{M} \propto \mathcal{G}^2 \varnothing_{\text{core}}$.^{26,27} S is the skyrmion number $S = \frac{1}{4\pi} \int \mathbf{m}(\partial_x \mathbf{m} \times \partial_y \mathbf{m}) dx dy$ with the normalized magnetization $\mathbf{m}$, which becomes $S = \pm \frac{1}{2}$ and $S = \pm 1$ for vortex states and bubble domains, respectively. Donut states with an interlayer exchange coupling of about 0.4 mJ/m^2 possess $S \approx \pm 0.9$ (type I) and $S \approx \pm 0.1$ (type II).¹⁹ The size of vortex cores in Py of about 6 nm lead to particle-like dynamics that can be described without the mass term ($\mathcal{M} = 0$) resulting in the well-known relationship between aspect ratio and gyrofrequency $\omega_0 = \mathcal{K}/\mathcal{G} \propto \frac{h}{\varnothing S}$.^{20,21} This simplification is no longer valid for donut states with extended cores of $\varnothing_{\text{core}} \approx 100 \text{ nm}$ [Fig. 3(b)]. The major contributions of restoring force $\mathcal{K}$ and core mass $\mathcal{M}$ cause an increase of the gyrofrequency with increasing core size $\varnothing_{\text{core}}$ or disk diameter $\varnothing$. This trend is quantitatively confirmed [Fig. 3(d)], using the exact expressions for $\mathcal{K}$, $\mathcal{M}$, and $\mathcal{G}$ given in Refs. 26 and 27 with properly modified saturation magnetization and restoring force. The saturation magnetization of the coupled systems is approximated as the mean value of both subsystems weighted by their thicknesses $M = (\frac{8}{9} \cdot 860 + \frac{1}{9} \cdot 500) \text{ kA/m}$ ($M_{\text{Py}} = 860 \text{ kA/m}$; $h_{\text{Py}} = 40 \text{ nm}$; $M_{\text{Co/Pd}} = 500 \text{ kA/m}$; $h_{\text{Co/Pd}} = 5 \text{ nm}$). The restoring force $\mathcal{K}$ originally calculated for out-of-plane magnetized bubble domains is rescaled $\mathcal{K} \rightarrow 0.64 \mathcal{K}$ taking into account reduced magnetostatic surface charges due to oblique magnetization vectors with normal magnetization components of about $0.8 M_s$ in the Co/Pd subsystem with $d = 5 \text{ nm}$ ($J_i = 0.4 \text{ mJ/m}^2$) [Fig. 3(a), red curve].

The localization of the out-of-plane magnetization components in vortex cores with sizes of about 6 nm implies a solitary motion that can be described by one gyrofrequency [Fig. 1(d)]. Similarly, this approximation holds for strongly coupled systems with core sizes $\varnothing_{\text{core}}(d = 1 \text{ nm}) \approx 35 \text{ nm}$ [Fig. 2(b)]. In contrast, donut states with extended cores $\varnothing_{\text{core}}(d = 5 \text{ nm}) \approx 100 \text{ nm}$ reveal two distinct gyrofrequencies, which

FIG. 4. Simulated relaxation of Co/Pd multilayers coupled to a Py vortex state via an interlayer exchange strength of $J_i = 0.4 \text{ mJ/m}^2$ after applying an in-plane magnetic field of 8 kA/m. (a)–(d) Snapshots. Red and blue contrasts refer to normal magnetization components. The donut state temporarily transforms into a “C” state (b)–(d) causing a domain wall vibration and (e) kinks in the trajectory of the vortex core as well as (f) a time-dependent gyrofrequency similar to those observed in experiment.

originate from a temporary deformation of the core in the first couple of nanoseconds [Fig. 4]. This instability may be caused by an interaction with the edge of the disk or by a dynamically excited vibration of the domain wall. The accordingly affected core dynamics reveal a trajectory with kinks [Fig. 4(e)] similar to those reported for bubble domains,²⁷ which justifies the use of analytical expressions derived for bubble domains with dynamic mass term to treat the motion of donut states.

In conclusion, we have presented the magnetization dynamics of imprinted non-collinear chiral spin textures recorded with transmission soft X-ray microscopy. A clear dependence of the gyrofrequency on the interlayer exchange coupling strength was observed that offers an indirect access to the core diameter via its dynamics. Moreover, understanding magnetization dynamics of skyrmionic core textures in view of field- and current-driven displacements is an essential ingredient to realize memory and logic devices.

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