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Abstract†

The highly skewed and heavy tailed distributions used to model insurance losses (claims) raise a concern about the validity of the applications of the capital asset pricing model (CAPM) to insurance pricing when market risks are essential. This paper provides an alternative pricing model, called the Rubinstein-Leland model, which can be used to price insurance contracts. The Rubinstein-Leland model has a distribution-free feature that can fully capture the asymmetry embedded in insurance losses. Thus, this model is better able to derive fair prices for insurance policies than is the CAPM.

Key words and phrases: *co-movements, power utility function, market based pricing model*

1 Introduction

To price property/casualty insurance contracts, insurers can determine the underwriting risks by using the insurer's own (subjective) assessments of the volatility of the company's value or by using the market's (objective) assessment. To objectively determine a fair premium,

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one can apply market equilibrium pricing models such as the capital asset pricing model (CAPM), which requires information about the expected payoff and its co-movements with the market returns. The use of CAPM is justified when the assumptions of a quadratic utility function and normal distribution for returns are met. In practice, however, the models used for insurance losses use highly skewed heavy tailed asymmetric distributions, which raises a concern about the application of the CAPM to pricing insurance policies. In addition, given the unbounded nature of the loss distribution, the quadratic utility may not be appropriate.

Attempts to incorporate asymmetry into pricing insurance contracts have been made using a three-moment CAPM (Kraus and Litzenberger, 1976) and an N-moment CAPM (Kozik and Larson, 2001). Though the adoption of the N-moment CAPM could possibly capture the asymmetry characteristic of the insurance loss process, the difficulty in determining the optimal moment, N, limits the application of this model.

This paper introduces an alternate model, originally developed by Rubinstein (1976) and applied by Leland (1999), which fully captures elements of risk that may induce skewness, kurtosis, and higher moments. Leland (1999) demonstrates that this model is more applicable than the CAPM when the asset to be priced has asymmetric return outcomes.[1] Using a distribution-free feature and a power utility function, the Rubinstein-Leland asset pricing model (hereafter referred to as the R-L model) accommodates asymmetrically distributed risks that are embedded in the insurance loss process. As a result, the R-L model can, in theory, fairly price insurance policies.

The primary focus of this paper is to investigate the applicability of the R-L model in pricing non-life insurance contracts with asymmetric loss distributions. An example of the application of the R-L model is provided using state-contingent claim pricing techniques to establish hypothetical insurance policies. In addition, the results from the R-L model are compared with those from the risk-free pricing model[2] and

[1]A financial asset pricing model can be used to obtain the fair price of a security when the market reaches equilibrium (where sellers and buyers agree upon that equilibrium price). In equilibrium, whether the security is viewed as an asset or liability is not likely to affect its price. In insurance terminology, such a security is an insurance contract to be priced and is viewed as asset from the insured's perspective and as liability from the insurer's perspective. Applications of CAPM to pricing insurance contracts based on this equilibrium proposition are given in Fairley (1979), Kahane (1979), and Kozik and Larson (2001).

[2]Assuming the loss process and the market portfolio are uncorrelated, the risk-free pricing model uses the risk-free rate as the discounted factor and omits the systematic market risk.

the CAPM in order to identify the market risks and asymmetric risks in the insurance loss process.

The rest of this paper is organized as follow: Section 2 reviews the pricing mechanisms for insurance policies and highlights the R-L, CAPM, and risk-free pricing models. Section 3 demonstrates the approach for creating simple state-contingent insurance policies that are used in the application of the R-L model. The pricing results under the three models are compared in Section 4. Section 5 concludes the paper.

2 Pricing Models for Insurance Policies

Consider a one period insurance contract with a random loss L paid at the end of the period. Traditionally actuaries have priced such insurance contracts using the pure premium (expected loss) plus a loading for expenses, risk, and profits. Ignoring the expenses and profits, the traditional risk-loaded premium can be written as

$$P^{\text{trad}} = \frac{(1+\theta)\mathbb{E}[L]}{1+r} \tag{1}$$

where r is the valuation interest rate. Bühlmann (1970), Gerber (1979), and Eckhoudt and Gollier (1995) among others, have identified several so-called premium calculation principles (or criteria) for deriving the risk loading, θ. Examples include the variance principle, the standard deviation principle, the safety first (the semi-variance) principle, and the expected utility principle. Kreps (1990) introduces the reluctance premium calculation principle, which suggests that the risk loading is a linear combination of the standard deviation and variance of the losses on the policy and depends on the covariance of the policy with the existing book of policies. Because underwriting new policies adds volatility to the company's overall value, the insurer should consider this added volatility as well as the risks inherent in new policies. The risk load charged for the increased volatility in its value can be viewed as the insurer's compensation for its reluctance to underwrite new policies.

Kahane (1979, p. 223) states that "the insurer's ratemaking decision depends on his ability to estimate expected claims and on the selection of a fair risk loading." In other words, the premium is set according to the insurers' *subjective* assessments of the information associated with the underwriting and ratemaking processes.

An insurance contract can be thought of as a state contingent claim with payoffs made if the pre-specified events occur. Doherty and Garven (1986) apply a contingent claim approach to derive the fair rate of return for property-liability insurance companies. Kraus and Ross (1982) apply the arbitrage pricing theory (APT) of Ross (1976) to find the competitive premium under which arbitrage opportunities are excluded. The APT is applicable as long as the factors in the economy are identified. To fully and explicitly identify all the factors correlated with even the simplest loss in practice, however, is infeasible, thus rendering the APT impractical as an insurance pricing tool.

On the other hand, when an insurance policy is viewed as a project under consideration, a capital budgeting methodology such as the net present value (NPV) or the internal rate of return (IRR) can be applied to evaluate the project (insurance policy). Adoption of the NPV or IRR approach, however, requires a market-determined rate of return. One of the most prominent discounted cash flow models used to price an insurance policy is the Myers-Cohn model (Myers and Cohn, 1987). Under the Myers-Cohn model an appropriate discounted rate must be chosen in order to set the net present value of the contract to zero, i.e., equating the present value of cash inflows (premiums) and the present value of cash outflows (losses, expenses, profits, and taxes). In other words, the major concern is that fair premiums should reflect the expected losses (pure or net premiums) and certain loadings such as expenses, profits, and risk. The assessment of the loading for bearing underwriting risks, however, introduces several criteria based on actuarial and/or on financial models.[3]

By assuming no correlation between losses and market returns, traditional actuarial pricing models have implicitly used the risk-free rate as the discount factor. A more sophisticated approach, however, is to consider the co-movements between the market returns and insurance losses. The CAPM and R-L models provide risk-adjusted discount factors. When the asymmetry inherent in insurance losses is taken into account, the inadequacy of applying the CAPM in insurance pricing is addressed. We will review three models used for including underwriting risks in determining premiums: the risk-free pricing model, CAPM, and the R-L model. Thus, the M-C model can be applied in a more accurate basis by employing the discounted rate derived from these market-based pricing models. For the simplicity of illustration, pricing models are considered in a single-period model with losses paid at the end of the period.

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[3]Another prominent model is the National Council on Compensation Insurance (NCCI) model. Cummins (1988b) compares the Myers-Cohn and NCCI models.

Risk-Free Pricing Model

The risk-free pricing model assumes that the losses from an insurance contract are uncorrelated with the market portfolio. Consequently, systematic market risk is not reflected in the pricing of an insurance contract by discounting the future expected loss payments at a risk-free rate. This price is expressed in equation (2) below as:

$$P^{\mathrm{RF}} = \frac{\mathbb{E}[L]}{1 + r_f} \tag{2}$$

where P^{RF} is the premium of an insurance contract, L is the actual loss payment for the period (paid at the end of period), and r_f is the risk-free rate.

CAPM

In practice, insurance losses are likely to be correlated with market returns and CAPM may be used to measure market risk. Under CAPM, market risk is based on the variance-covariance relationship between the loss process and the market portfolio. Under the mean-variance framework, CAPM is derived by maximizing the investor's expected value of utility subject to the investor's wealth allocation. For an arbitrary utility function, the mean-variance model is justified by assuming that returns are normally distributed (thus third and higher moments of returns are ignored). On the other hand, for an arbitrary distribution of returns, the CAPM model is justified by assuming a quadratic utility function (third and higher moments of returns are again ignored). See Kahane (1979) and Fairley (1979) for more on the more on how the insurance CAPM is derived.

Let r_m denote the market rate of return and β_C denote the systematic risk of the underlying asset under the CAPM. The premium of an insurance contract under the CAPM, P^{CAPM}, and the required return on the insurance policy, r_L, are given by

$$P^{\mathrm{CAPM}} = \frac{\mathbb{E}[L] - \lambda \mathbb{C}\mathrm{ov}[L, r_m]}{1 + r_f} \tag{3}$$

$$\mathbb{E}[r_L] = r_f + \beta_C(\mathbb{E}[r_m] - r_f) \tag{4}$$

where $\beta_C = \mathbb{C}\mathrm{ov}[r_L, r_m]/\mathbb{V}\mathrm{ar}[r_m]$, $\lambda = (\mathbb{E}[r_m] - r_f)/\mathbb{V}\mathrm{ar}[r_m]$ and $r_L = (P^{\mathrm{CAPM}} - L)/L$. Equations (3) and (4) imply the risk-free pricing model if the insurance losses and the market portfolio are uncorrelated, as there will be no compensation for bearing market risk. Fairley (1979) found a negative correlation between the market returns and the claims of auto bodily injury policies, while Biger and Kahane (1978) suggest that underwriting returns are uncorrelated with the market return.

The CAPM has been applied in the insurance literature to insurance contracts (Fairley, 1979; Kahane, 1979; Hill, 1979; and Myers and Cohn, 1987), insurance equities (Harrington, 1983; Cummins and Harrington, 1988; and Cummins and Lamm-Tennant, 1994), and to insurance reserves (D'Arcy, 1988). Kahane (1979) also summarizes the drawbacks of applying CAPM as an insurance pricing mechanism due to the specific characteristics of the insurance loss process. In addition, Rubinstein (1973) and Brennan (1979), among others, have shown that a quadratic utility function does not satisfy desirable properties for describing investors' preferences.[4] Kraus and Litzenberger (1976) develop a three-moment CAPM under a logarithmic utility assumption and conclude that asset pricing models should incorporate not only the price of the second moment of risk aversion, but also the value of skewness preference.

Rubinstein-Leland (R-L) Model

Without knowing the distribution of L, an alternative pricing model must be used. One such model is the R-L model with its distribution-free feature. The R-L model is based on the power utility function and distribution-free asset returns. Rubinstein (1976) measures the co-movement between the asset returns and the market returns beyond a mean-variance framework, thereby making it a more appropriate way to price an insurance policy.

Given the power utility function $u(x) = x^b$, the R-L model premium of an insurance contract is given by

$$P^{\text{RL}} = \frac{1}{1+r_f}\left[\mathbb{E}[L] - \frac{\mathbb{C}\text{ov}\left[L, -(1+r_m)^{-b}\right]}{\mathbb{E}\left[(1+r_m)^{-b}\right]}\right] \tag{5}$$

[4]Desirable properties (Arrow, 1971) for an investor's utility functions are (i) positive marginal utility for wealth, i.e., nonsatiety with respect to wealth, (ii) decreasing marginal utility for wealth, i.e., risk aversion, and (iii) non-increasing absolute risk aversion (ARA).

where b is the degree of risk aversion of the power utility function[5] and $r_L = (P^{\mathrm{RL}} - L)/L$. If we assume market returns are lognormal then

$$b = \frac{1}{2} + \frac{\mathbb{E}\left[\ln(1 + r_m)\right] - \ln(1 + r_f)}{\mathbb{V}\mathrm{ar}[\ln(1 + r_m)]}. \tag{6}$$

The risk aversion parameter, b, can be related to the market excess return per unit of risk. Following Rubinstein (1976), Leland (1999) demonstrates a linear relation between risk and return for any insurance loss that is given by

$$\mathbb{E}\left[r_L\right] = r_f + \beta_R \times \left[\mathbb{E}\left[r_m\right] - r_f\right] \tag{7}$$

where β_R is systematic risk of the underlying contract, i.e.,

$$\beta_R = \frac{\mathbb{C}\mathrm{ov}[r_L, -(1 + r_m)^{-b}]}{\mathbb{C}\mathrm{ov}[r_m, -(1 + r_m)^{-b}]}. \tag{8}$$

Comparing the R-L Model and CAPM

In order to make consistent comparisons between the R-L model and CAPM, we follow the symmetry information and homogenous beliefs assumptions of CAPM.[6]

Implementing the R-L model requires no more information than under CAPM. In addition, under the assumptions of power utility and distribution-free asset return, the R-L model captures all elements of risk including skewness and kurtosis. The risk measure of the CAPM, β_C, is easier to estimate than the risk measure of the R-L model, β_R. However, β_R incorporates the effects of preferences and aversions contained in higher moments given that the typical investor has a power utility function with parameter b. In addition, β_R considers higher moments of co-movement between insurance losses and the market returns, while β_C in CAPM indicates only the second moment of co-movement between the returns of the underlying asset and the market portfolio.

Under the R-L pricing model, we use information not used in the traditional CAPM, the three-moment CAPM, and even the N-moment

[5]The degree of risk aversion of a utility function $u(x)$ is $-u''(x)/u'(x)$. For the power utility function, several authors have used different approaches to estimate the degree of risk aversion for households. For example, Friend and Blume (1975) use empirical surveys of consumer wealth allocation, Campbell (1996) uses the effects of human capital and the mean aversion character of the stocks index, while Bliss and Panigirtzoglou (2002) use option pricing methodology.

[6]The extended model that considers asymmetric information and heterogeneous risk aversion among insureds is left for future research.

($N > 3$) CAPM. Leland (1999) shows that the CAPM and the R-L model give similar results for assets that are symmetrically distributed. For asymmetrically distributed insurance losses, however, the error in using the CAPM may be substantial. Based on this logic, the difference between their beta estimates, $\beta_R - \beta_C$, from the R-L model and the CAPM model can be used as a proxy for asymmetric risks. Correspondingly, the price of asymmetric risks imbedded in an insurance contract is given by $(P^{\mathrm{RL}} - P^{\mathrm{CAPM}})$.

3 The Main Results

We will illustrate the application of the R-L model by using a lognormal market portfolio, a power utility with constant relative risk aversion (CRRA) property,[7] and a hypothetical insurance policy. It must be noted that the lognormal market portfolio is not an essential assumption underlying the R-L model, but it is required to apply formula (6) to derive the risk aversion parameter. Due to the limited access to empirical data, a hypothetical insurance policy is used. As we will see, our results suggest a larger than expected discrepancy between the premiums derived from the R-L model and CAPM if the underlying losses are highly skewed or heavily-tailed.

First we construct a market portfolio with lognormal distribution under a simple economy with six mutually exclusive states of nature. We assume that the occurrences of any state of nature in different periods are independent events and that only one state can occur in any period. The return structure of the theoretical market portfolio is presented in Table 1, which, for example, shows that the market portfolio has negative return (-6%) in state 1.[8] By design, the market returns are positively skewed and the Kolmogorov-Smirnov (K-S) test fails to reject the hypothesis of lognormal market returns. The market has a risk-free rate of 5% and the estimate of the risk aversion parameter for the power utility function is 6.56.

A state-contingent claims pricing technique is used to establish the insurance policies. An elementary state-contingent policy (hereafter

[7]After studying cross-sectional data on household asset holdings, Friend and Blume (1975) conclude that the assumption of constant relative risk aversion (CRRA) for households is a fairly accurate description of the market place. This paper directly adopts their empirical results and assumes that a power utility with CRRA property is a fairly justified utility function so that the fundamental utility assumption under the R-L model can be satisfied.

[8]We use a multiple-state example because we can explicitly identify the asymmetry in insurance payoffs. This cannot be achieved by assuming binomial states of nature.

Table 1
Market Returns
In Various States of Nature

State of Nature	Market Returns	Probability p_i
1	-6%	0.10
2	0%	0.20
3	10%	0.25
4	15%	0.15
5	24%	0.25
6	28%	0.05

called a state policy) is defined as a policy that pays a loss if and only if a certain state of nature occurs. Let L_i and p_i denote the loss payment and the state probability, respectively, for state policy i for $i = 1, 2, \ldots, 6$. The loss payment (payoff) for state policy i is assumed \$1,000, i.e.,

$$L_i = \begin{cases} 1000 & \text{with probability } p_i \\ 0 & \text{otherwise.} \end{cases}$$

Thus $\mathbb{E}[L_i] = 1000p_i$, $\mathbb{V}\text{ar}[L_i] = 10^6 p_i(1 - p_i)$, and the coefficient of skewness of L_i is $\mathbb{S}\text{kw}[L_i] = (1 - 2p_i)/\sqrt{p_i(1 - p_i)}$. As each $p_i < 0.50$, the L_i's are positively skewed. Table 2 shows these values for the six policies.

Table 2
Mean, Standard Deviation,
and Skewness of Policies

i	p_i	$\mathbb{E}[L_i]$	$\sqrt{\mathbb{V}\text{ar}[L_i]}$	$\mathbb{S}\text{kw}[L_i]$
1	0.10	100	300.00	2.67
2	0.20	200	400.00	1.50
3	0.25	250	433.01	1.15
4	0.15	150	357.07	1.96
5	0.25	250	433.01	1.15
6	0.05	217.94	50.00	4.13

Equations (2), (3), and (5) can now be used to determine the insurance premiums. Table 3 displays these premiums (P^{RF}, P^{CAPM}, and P^{RL}) as well as the standardized premium, which is the premium divided by the risk-free premium. This definition of standardized premium gives the risk loading factor that must be applied to the risk-free premium to give the required premium. In other words, it measures the extra systematic risk that the insurer is exposed to under the CAPM and R-L model. In addition, the discrepancy between the standardized premiums of the models, and especially the risk measures, β and B are also presented in Table 3.

As shown in Table 3, for policy 3 and policy 5 with the same amount of expected loss, under the risk-free pricing model, both policies are evaluated at the same premium. However, under the market-based pricing models (the CAPM or the R-L model), due to the recognition of the co-movements between market returns and insurance losses, policy 3 is evaluated at a higher premium than policy 5.

Table 3

Premium Estimates of Elementary Policies

	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5	Policy 6
P^{RF}	95.24	190.48	238.10	142.86	238.10	47.62
P^{CAPM}	192.85	318.96	259.64	114.07	64.97	1.87
P^{RL}	232.48	309.86	207.29	92.92	94.48	15.34
β_C	-8.11	-6.46	-1.33	4.05	42.72	392.56
β_R	-9.47	-6.18	2.38	8.62	24.40	33.76
Standardized Premiums						
R-F	1.00	1.00	1.00	1.00	1.00	1.00
CAPM	2.02	1.67	1.09	0.80	0.27	0.04
R-L	2.44	1.63	0.87	0.65	0.40	0.32

Notes: Standardized Premium = Premium/P^{RF}.

Notice that policies 1 and 2 are the most valuable state polices under the market-based pricing models in terms of the loading added to the risk-free premium, while policies 4, 5, and 6 are less valuable. This may be attributed to the direction of the co-movement between insurance losses and market returns. In other words, policy 1 suffers a loss in the state where the market portfolio has negative return, while policies 4, 5, and 6 show positive co-movements with the market payoffs. Under market-based models, when using insurance to diversify risks, investors prefer the insurance payoffs to be negatively correlated with the market.

Though both the R-L model and CAPM embody market risk in insurance pricing, we mentioned in Section 2 the differences between their fundamental assumptions. Recall that CAPM assumes returns are normally distributed and investors have a quadratic utility function, while the R-L model makes no assumption about returns and uses a power utility function. We will give three reasons why there is a discrepancy between their premiums.

1. The normal distribution assumption under CAPM focuses mainly on events occurring mostly in the middle range of the distribution, and it is likely to underestimate the possibility of the larger (or smaller) values of the distribution. For instance, the bulk of the probability weights fall in the range of (μ-2σ, μ+2σ). Accordingly, for a loss process with an asymmetrical distribution, the use of a mean-variance model like CAPM is likely to underestimate events in the tails of the distribution. On the other hand, with a distribution-free assumption, the R-L model takes full consideration of each possible value of the entire distribution and thereby can fairly reflect all probabilities. In other words, for the values falling in the spectrum of two extreme sides, without limiting the distribution, their probabilities can be reflected in the R-L model instead of being assigned to an approximately zero value based on a normal distribution.

   For example, for state policies 1, 5, and 6, loss payments are made in the states where the market portfolio's returns are in the left tail (state 1) and right tail (states 5 and 6). Premiums of the three state policies are smaller under CAPM than under the R-L model. This can be attributed to the above elaboration on the impact of the normal assumption of CAPM on insurance pricing when losses have an asymmetrical distribution. In contrast, the premiums of state policies 2, 3, and 4 (where the market has relatively modest returns) are higher under CAPM than under the R-L model.

2. Another factor that explains the discrepancy between the premiums under the R-L model and CAPM is the quadratic versus power utility functions. A quadratic utility function requires only the means and variances, while ignoring third or higher order moments. Thus, CAPM is likely to mis-price insurance policies that are skewed. On the other hand, the R-L model uses third and higher moments.

3. A third factor is the correlation between loss payments and market returns. Note that policy 1 is preferred while policy 6 is not because the loss payoff of policy 1 has an apparently negative correlation with the market returns while the loss payoff of policy 6 has positive correlation with the market returns. The negative correlation with the market returns can be viewed as a hedging function that provides payoff in the state of unfavorable market return. Hence such a policy is preferred by policyholders.[9] Being able to capture the higher moments of preference, the R-L rewards such a hedging function more than the CAPM. Without being able to foresee the aggregate effects of higher moments of preference and aversion due to the limitation of a quadratic utility function, the CAPM may over-penalize the aversion of the state 6 policy, thus significantly underestimating its premium compared to the R-L model.

   Furthermore, the omission of the correlation of the asset with the higher moments of market returns may cause the different notion of systematic market risk under the CAPM and the R-L model. Table 3 shows that under the CAPM the risk estimate β_C of the state 3 policy is negative, while the risk estimate β_R is positive under the R-L model. This finding further addresses the importance of considering the higher moments of co-movements.

Consistent with the findings of Kahane (1979), this study confirms the inadequacy of applying the CAPM as an insurance pricing mechanism due to the inconsistency between its underlying mean-variance assumptions and the asymmetrically distributed insurance losses. The above numerical examples illustrate that the R-L model can be a more appropriate insurance pricing mechanism, especially when the insurance losses are with asymmetry characteristic.

4 Summary and Closing Comments

This paper uses a simple example to illustrate the applications of three commonly used pricing models (the risk-free model, CAPM and the R-L model) to pricing insurance policies. We compare their results

[9]In the CAPM, the opposite co-movements can serve diversification purposes. The explanation is used to substantiate the values of higher order of opposite co-movements between the securities and the market portfolio. In other words, the valuation of opposite co-movements should go beyond the first and second orders when asymmetric character is embedded in return process.

and show that CAPM and the risk-free model tend to under-price policies. The risk-free pricing model evaluates an insurance contract without considering the implied market risks by assuming no correlation between the loss process and market returns; the CAPM assesses the risks based on a mean-variance framework, which is inconsistent with insurance loss distributions that are usually skewed and heavy tailed. The R-L model uses a distribution-free model for losses and a power utility. The R-L model seems to provide a relatively fair result for insurance losses that are highly skewed and heavily tailed.

An area for further research pertains to applying the R-L model in cases where there is information asymmetry, i.e., certain aspects of the policyholders' loss distribution may be unknown to the insurer but known to the insured (such as their risk-taking behaviors) or to cases where the insurer has an information advantage (such as data on the probability of certain hazards). Moreover, the model can be extended to consider heterogeneity between the risk aversion levels of insureds and insurers. Thus, the pricing process can recognize the heterogeneous risk aversion levels among insureds and generate prices based on the insureds' risk categories.

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