Interference

INTERFERENCE

INTRODUCTION

You may have observed the sound from your radio fade in and out as you listened to some distant station. Or perhaps you have sat in a "dead" seat in a poorly designed concert hall where, despite the fact that no physical object is between you and the performer, the sound is distorted and weak. Perhaps you have held two fingers close together and looked with one eye through the narrow slit separating the fingers and observed those mysterious black lines in and parallel to the slit. These and many other similar phenomena result from the interference of two or more waves of the same frequency. This module will provide you with the basis for understanding such phenomena in terms of a wave model.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Add vectors (needed for Objective 3 of this module)	Dimensions and Vector Addition Module
*Use the wave properties of light in solving problems (needed for Objectives 1 through 4 of this module)	Wave Properties of Light Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Coherent sources - List the conditions necessary for two sources to be coherent, and apply these conditions so as to determine if two sources are coherent.

2. Interference maxima and minima - Calculate the location of amplitude maxima and minima given the positions, frequency, and relative phase of two coherent sources and the propagation speed of the wave.

3. Intensity patterns - Calculate relative intensities at various points in interference patterns resulting from two coherent sources.

4. Thin films - Relate the index of refraction n and thickness t of a thin film to maxima and minima of reflected and transmitted light (wavelength λ) and solve associated problems using this relationship among n, t, and λ.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Your readings are from Chapter 31. Read Section 31.1, study Problems A and B, and work Problem F. Then read Sections 31.3 and 31.4. You might also read Section 31.2, particularly if you have not completed the module Sound. Then study Problem C and Illustrations 31.1 and 31.2 before working Problems G, H, and I. Next read Section 31.5, study Problem D and Illustration 31.3. Work Problems J and K. Read Sections 31.9 and 31.10, and study Problem E, before working Problem L. Although you are not required to do so, you may enjoy reading Sections 31.7 and 31.8.

Try the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 31)
1	Sec. 31.1	A, B		F	Quest.[a] 11, 14
2	Sec. 31.4	C	Illus.[a] 31.1, 31.2	G, H, I	Quest. 4, Probs. 1, 5, 6
3	Secs. 31.3, 31.5	D	Illus. 31.3	J, K	Probs. 3, 7, 8
4	Secs. 31.9, 31.10	E		L	Quest. 5, 8, Probs. 9, 10, 11, 15, 21

[a]Illus. = Illustration(s). Quest. = Question(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Your readings are from Chapter 37. Read Sections 37-1 and 37-2. Pay particular attention to the discussion on pp. 707 and 708. If you do not understand the phase argument relative to Figure 37-6, reread Section 36-4 in Chapter 36. Notice that Eqs. (37-1) and (37-2) are exact for Figure 37-6 and approximate (but very accurate for D >> d) for Figure 37-5. Read Section 37-3. This is an excellent discussion of coherent and incoherent sources. Then study Problems A, B, and C, and work Problems F through I. Read Section 37-4, study Problem D and Example 2, and work Problems J and K. Then read Section 37-5, and study Examples 3 and 4 and Problem E before working Problem L. Even though you are not required to, you might enjoy reading Section 37-6, which describes the interferometer.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 37)
1	Sec. 37-3	A, B		F	Quest.[a] 3, 5
2	Sec. 37-2	C	Ex.[a] 1	G, H, I	Quest. 7, Probs. 1, 5, 9, 17, 19
3	Sec. 37-4	D	Ex. 2	J, K	Probs. 2, 5
4	Sec. 37-5	E	Ex. 3, 4	L	Quest. 9, 10, Probs. 27, 29, 31, 37

[a]Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Your readings are from Chapter 41. First read Section 41-1, an excellent discussion of coherence and interference. Study it carefully. Then study Problems A and B before working Problem F. Read Section 41-2. You may omit the last paragraph if you wish. Then study Problem C and work Problems G, H, and I. Read Section 41-3, study Problem D and work Problems J and K. Next read Sections 41-4 through 41-6, study Problem E, and work Problem L. Section 41-7 is an interesting discussion of energy conservation and interference, definitely worth reading. You may also find Sections 41-8 and 41-9 interesting even though they are not required.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	
1	Sec. 41-1	A, B	F	
2	Sec. 41-2	C	G, H, I	41-1
3	Sec. 41-3	D	J, K	41-3
4	Secs. 41-4, 41-5, 41-6	E	L	41-5, 41-9, 41-10, 41-12, 41-13

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Your readings are from Chapter 38. Read Section 38-5, study Problems A and B, and work Problem F. Then read Sections 38-1, 38-2, 38-3, and 38-6, and study Problems C and D, before working Problems G through K. Next read Sections 38-4 and 38-7, study Problem E, and work Problem L. You may find Section 38-8 enjoyable to read.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	
1	Sec. 38-5	A, B	F	
2	Secs. 38-2, 38-6	C	G, H, I	38-3, 38-5, 38-7, 38-16, 38-19
3	Sec. 38-3	D	J, K	
4	Secs. 38-4, 38-7	E	L	38-20, 38-21

PROBLEM SET WITH SOLUTIONS

A(1). List the conditions that must be satisfied if two sources are to be coherent.

Solution

The two sources must have a relative phase that does not change with time. Note that this implies that if the two are producing periodic signals, the two frequencies must be identical. (Why?)

B(1). Signals from two sources superpose at point P to give complete destructive interference. Are the two sources coherent? Why?

Solution

Yes. Complete destructive interference of two signals requires that the two signals have the same amplitude and be oppositely directed at each instant, i.e., be out of phase by π. This constant phase difference of the received signals requires the sources to have a fixed phase difference, i.e., to be coherent.

C(2). In Figure 1, S_1 and S_2 are identical (in-phase) sources emitting transverse (perpendicular to the paper) waves of frequency ν and wavelength λ. Determine the positions of all maxima and minima along the line OX.

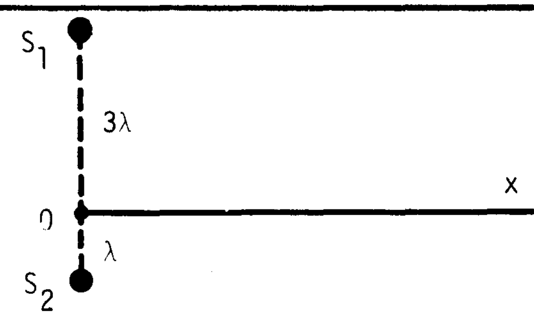


Figure 1

Solution

The pathlength difference ΔL as shown in Figure 2 is

$$\Delta L = L_1 - L_2 = \sqrt{x^2 + 9\lambda^2} - \sqrt{x^2 + \lambda^2}.$$

For maxima, $\Delta L = n\lambda$; for minima, $\Delta L = (n + 1/2)\lambda$, where n is a nonnegative integer. We treat both cases by writing $\Delta L = \alpha\lambda$, and determine x in terms of α, i.e.,

$$\sqrt{x^2 + 9\lambda^2} - \sqrt{x^2 + \lambda^2} = \alpha\lambda.$$

Solving for x gives us (after discarding negative solutions)

$$x = [\sqrt{(16 - \alpha^2)(4 - \alpha^2)}/2\alpha]\lambda.$$

Maxima: $\alpha = 1, \quad x_1 = (\sqrt{15.3}/2)\lambda = (3/2)\sqrt{5}\lambda \simeq 3.4\lambda.$

$\alpha = 2, \quad x_2 = 0.$

Minima: $\alpha = 1/2$, $x_{1/2} = (\sqrt{63 \times 15}/4)\lambda = (3/4)\sqrt{105}\lambda \simeq 7.7\lambda$.

$\alpha = 3/2$, $x_{3/2} = (\sqrt{55 \times 7}/12)\lambda = (\sqrt{385}/12)\lambda \simeq 1.60\lambda$.

The locations of these maxima and minima are shown in Figure 3 with the pathlength difference shown below the location. Note: The result given for x has answers for $\alpha \geq 4$. Clearly, these cannot be acceptable solutions since the maximum value for $L_2 - L_1$ is 2λ ($\alpha = 2$) at $x = 0$. The solutions for $\alpha \geq 4$ are extraneous values introducted in the process of solving for x.

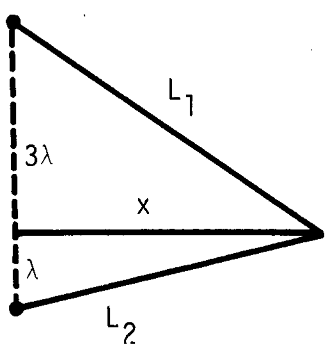


Figure 2

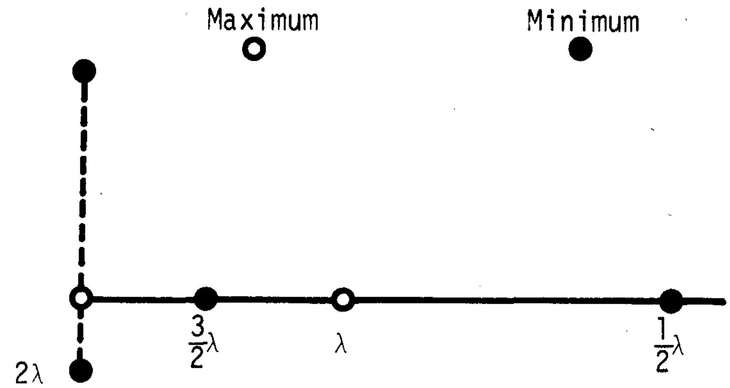


Figure 3

D(3). The width of one of the slits in a double-slit experiment is increased so that the amplitude of the light from that slit is increased by 50%. Determine the ratio of the intensity at a minimum to that of a maximum. Compare this to the result for equal slits.

Solution

The resultant amplitudes at the maximum where the two signals are in phase and at the minimum where the two are out of phase by π rad are shown in Figure 4. Since intensity is proportional to the square of the amplitude,

$$\frac{I_{min}}{I_{max}} = \left(\frac{E_{min}}{E_{max}}\right)^2 = \left(\frac{0.50}{2.50}\right)^2 = \frac{1}{25} = 0.040.$$

For identical slits the two equal signals (out of phase by π rad) exactly cancel at a minimum, and

$$I_{min}/I_{max} = 0 \quad \text{(equal width slits)}.$$

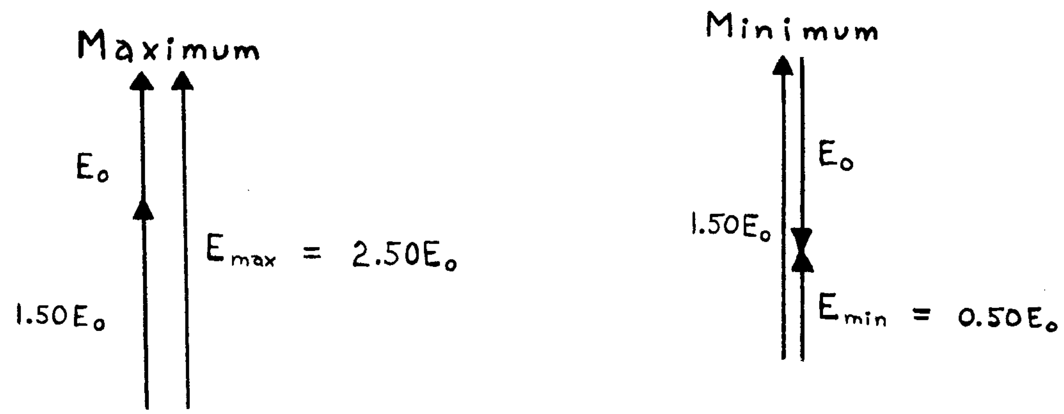


Figure 4

E(4). Light of wavelength 530 nm (5300 Å) is incident normally on an oil film (n = 1.20). Calculate the two smallest thicknesses of the film such that a person viewing reflected light would think the film had disappeared.

Solution

See Figure 5. Wavelength in the film is $\lambda_f = \lambda/n$. The distance traveled in the film is 2t. The phase change is π at the upper surface. If the two reflected rays are to interfere destructively, the phase difference must equal $(2m + 1)\pi$, $m = 0, 1, \ldots$.

Total phase difference = phase difference due to different pathlengths + phase difference due to phase change upon reflection

$$= 2\pi\left(\frac{2t}{\lambda/n}\right) + \pi = \frac{4\pi nt}{\lambda} + \pi.$$

Therefore, for destructive interference,

$$4\pi nt/\lambda + \pi = (2m + 1)\pi \quad \text{or} \quad t_n = m\lambda/2n,$$

$$t_1 = \lambda/2n = (530\ \text{nm})/2(1.20) = 220\ \text{nm}, \qquad t_2 = \lambda/n = (530\ \text{nm})/1.20 = 440\ \text{nm}.$$

Figure 5

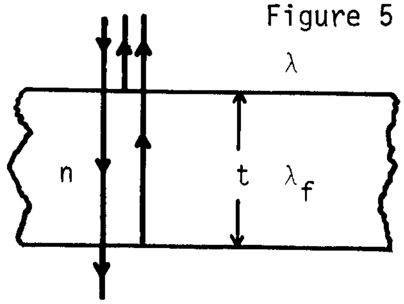
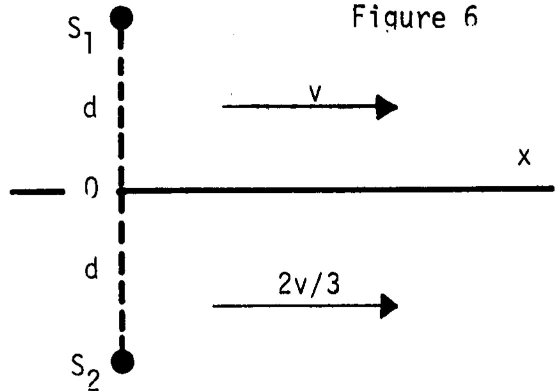


Figure 6



Problems

F(1). Consider the two coherent sources of Problem B.
(a) If the amplitude of one source is doubled with no other changes occurring, are the two still coherent?
(b) Is complete destructive interference still observed at point P? Explain.

G(2). (a) Would the transverse disturbances at the two maxima [$x = 0$, $x = (3/2)\sqrt{5}\lambda$] of Problem C have equal amplitudes?
(b) Would the two minima of Problem C be complete destructive interference?

H(2). In Figure 6, S_1 and S_2 are identical sources emitting transverse (perpendicular to the paper) waves of frequency ν. S_1 is immersed in a medium that propagates waves with a speed v, whereas the propagation speed in the medium surrounding S_2 is $2v/3$. The line OX represents a boundary between the two media. Determine the location of maxima and minima along OX if $d = v/\nu$.

I(2). Light of wavelength 510 nm (5100 Å) used in a double-slit (Young's) experiment produces interference fringes separated by 0.250 cm on a screen 0.50 m from the double slit. Calculate the slit separation.

J(3). Repeat Problem D for the case where the wider slit contributes a signal with twice the amplitude of the smaller slit.

K(3). For the conditions of Problem J, consider a point in the interference halfway between a maximum and an adjacent minimum. Calculate the ratio of the intensity at this point to that at the maximum. Compare this to the equal-width slit result.

L(4). Work Problem E for the case where the oil film is deposited on a flat, glass ($n = 1.50$) surface.

Solutions

F(1). (a) Yes. Changing the amplitude of one source does not change the phase relationship between the two.
(b) Not <u>complete</u> destructive interference. The two signals still arrive at point P oppositely directed at each instant; but one amplitude is always twice the other so that some disturbance is now observed at P.

G(2). (a) No. <u>Hint</u>: The amplitude of any real wave decreases with distance from its source. In fact, if this effect had been considered carefully, it would have caused a change in the positions of the maxima and minima found in Problem C.
(b) No. Same as part (a).

H(2). See Figure 7. <u>Maxima</u>: $x_n = (v/\nu)\sqrt{4n^2 - 1} = d\sqrt{4n^2 - 1}$,

<u>Minima</u>: $x_{n - 1/2} = (2v/\nu)\sqrt{n(n - 1)} = 2d\sqrt{n(n - 1)}$, where $n = 1, 2, \ldots$.

For $n \gg 1$, $x_n \simeq 2nd$ and $x_{n - 1/2} \simeq (2n - 1)d$.

I(2). 1.00×10^{-4} m = 0.0100 cm = 0.100 mm.

J(3). $1/9 \simeq 0.111$.

K(3). $5/9 \simeq 0.56$, 0.50.

L(4). 110 nm, 330 nm.

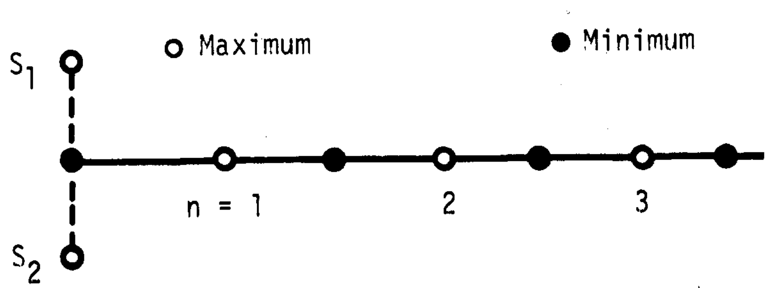


Figure 7

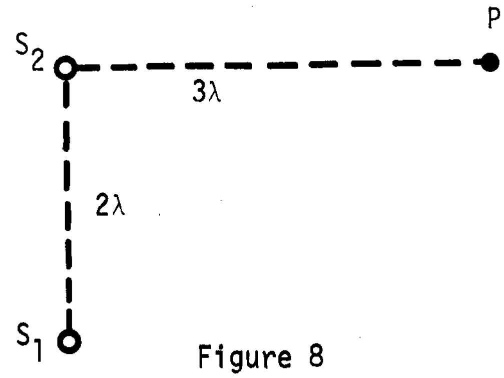


Figure 8

PRACTICE TEST

1. (a) List the condition(s) necessary for two sources to be coherent.
   (b) Two speakers are driven by different audio-oscillators. Is it possible for these two to be coherent sources? Explain.

2. A double-slit arrangement (slit separation = 0.100 mm) is used to determine the wavelength of a monochromatic beam of light. Adjacent bright fringes are separated by 0.61 cm on a screen 1.00 m away. Calculate the wavelength.

3. In Figure 8, S_1 and S_2 are in-phase coherent sources of transverse (perpendicular to the paper) disturbances. The S_1 to S_2 amplitude ratio at point P is 0.80. Determine the ratio of the intensity of the disturbance at point P to that at point P if S_1 is turned off.

4. Determine the smallest nonzero value for the thickness of a film (n = 1.40) that transmits all of the 620-nm-wavelength light that strikes the film perpendicularly.

Practice Test Answers

1. (a) Phase difference between two fixed in time.
   (b) Yes, provided two oscillators are finely tuned. Even then thermal drifts introduce slowly varying phase.

2. 610 nm.

3. 0.65.

4. 220 nm.