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INTRODUCTION TO QUANTUM PHYSICS

INTRODUCTION

You have probably encountered a system known as an "electric eye," which senses light from an artificial source or the sun. This information is used to open doors, count pedestrian or auto traffic, turn on lights at sunset, read holes in punched cards, and for a host of other applications. Most of these devices are based on the photoelectric effect, which is the light-induced emission of electrons from atoms.

The photoelectric effect completely baffled physicists at the time of its discovery. Einstein's explanation of this process, which won the Nobel Prize in 1921, was a major part of the twentieth-century revolution in physics known as the quantum theory. In this module we shall study the photoelectric and Compton effects, the processes that clearly demonstrate that the energy and momentum in electromagnetic waves are transferred as discrete entities called photons.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Find wavelength, given wave speed and frequency (needed for Objectives 1 through 3 of this module)	Traveling Waves Module
*Solve problems using conservation of energy (needed for Objective 2 of this module)	Conservation of Energy Module
*Solve problems involving elastic collisions (needed for Objective 3 of this module)	Collisions Module
*Relate the wavelength of electromagnetic waves to the type of radiation, and relate energy flux to energy density in the wave (needed for Objectives 1 through 3 of this module)	Maxwell's Predictions Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Quantum relations - Use the Planck-Einstein-de Broglie relations to relate the energy or momentum of a photon to the frequency or wavelength of the associated electromagnetic wave.

2. Photoelectric effect - Use conservation of energy and the quantum relations to solve problems involving photoelectric emission of electrons from metals. The relevant physical quantities are the frequency of the light, the work function of the metal, and the maximum kinetic energy of the emitted electrons.
3. Compton effect - Use conservation of energy and momentum as expressed by the Compton formula to calculate the energy or momentum of the outgoing particles in the collision of a photon with an electron at rest.

GENERAL COMMENTS

At the beginning of the twentieth century, there was a growing number of phenomena for which there was no description consistent with the physics we have studied in the preceding modules. A modification of classical physics was in order. The ideas we discuss here were developed in the first years of this century by Planck, Einstein, and de Broglie as each tried to understand these new phenomena. Their conclusions can be summarized as follows:

All energy propagates as waves. When energy or momentum is transferred between the wave and a source or absorber, the energy E and momentum p are in discrete amounts related to the frequency f and wavelength λ by the relations

$$E = hf \quad \text{and} \quad p = h/\lambda$$

where Planck's constant $h = 6.63 \times 10^{-34}$ J s. These discrete amounts of energy and momentum are called quanta; the quantum of electromagnetic radiation is called a photon. When the frequency is low and the number of photons involved very large, we use the language of classical waves. If the number of photons is small, like one, then we use the language of particle mechanics to describe the process.

The experiments that most clearly demonstrate the existence of photons are the photoelectric effect and the Compton effect. In the photoelectric effect, light incident on an atom or group of atoms may result in the ejection of an electron. The relation between the kinetic energy of the electron and the frequency of the incident light is found to be

$$hf = \phi + E,$$

which is just the conservation of energy. The energy of the absorbed photon, hf , is equal to ϕ (work function or ionization energy, the energy needed to remove an electron from an atom), plus E , the kinetic energy of the ejected electron. For single atoms, the kinetic energy given above is the energy of the ejected electron. For a solid, it is an upper bound, since the electron may lose some of its kinetic energy to other atoms as it leaves the solid. ϕ is typically of the order of 1.60×10^{-19} J, also known as an electron volt.

The Compton effect is the elastic scattering of a photon from an electron at rest. Since the photon transfers some of its energy and momentum to the electron in the collision, energy and momentum conservation together with the quantum relations require that the wave frequency must decrease and the wavelength must increase. The increase of wavelength for scattering of the photon through an angle θ from an electron of mass m is

$$\lambda' - \lambda = (h/mc)(1 - \cos \theta).$$

Notice that for $\theta = 0$ (no scattering) the photon wavelength is unchanged. As θ increases, more energy is transferred to the electron, thus energy conservation requires a larger increase in photon wavelength; when the photon bounces straight back, the wavelength change is $2h/mc = 4.85 \times 10^{-12}$ m. It follows that the wavelength shift is easily observed only for x rays (short wavelength, high photon energy) incident on electrons. Since there are no targets consisting of free electrons at rest, Compton-effect experiments are usually done by scattering from electrons found in atoms. In this case, a correction, usually negligible, must be made for the binding energy.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read the General Comments for an overview of the material of this module. Skip the discussion of blackbody radiation at the beginning of Chapter 35. Although it is historically correct that the ideas of quantum physics began with Planck's explanation of blackbody radiation, the physical ideas involved in this phenomenon are very complex and are often presented incorrectly.

Objective 1 is introduced in the context of discussing Objectives 2 and 3. As you read the suggested pages for each objective, work through the Illustrations. Then study Problems A, B, and C, and work the Assigned Problems. This should give you enough practice to understand the objectives and to work the Practice Test.

BUECHE						
Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems (Chap. 35)
		Study Guide	Text	Study Guide	Text (Chap. 35)	
1	Secs. 35.3, 35.4	A		D	10	
2	Sec. 35.3	B	Illus. ^a 35.2, 35.3	E	9, 11	12
3	Sec. 35.4	C	Illus. 35.4	F	15, 16	

^aIllus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read the General Comments for an overview of the material of this module. Skip the discussion of blackbody radiation at the beginning of Chapter 39. Although it is historically correct that the ideas of quantum physics began with Planck's explanation of blackbody radiation, the physical ideas involved in this phenomenon are very complex and are often presented incorrectly.

Objective 1 is introduced in the context of discussing Objectives 2 and 3. As you read the suggested pages for each objective, work through the Examples. Then study Problems A, B, and C, and work the Assigned Problems. This should give you enough practice to understand the objectives and to work the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems (Chap. 39)
		Study Guide	Text (Chap. 39)	Study Guide	Text (Chap. 39)	
1	Secs. 39-4, 39-5, 39-6	A		D	15	
2	Secs. 39-4, 39-5	B	Ex. ^a 2, 3	E	17, 19, 21	16, 18, 20, 22
3	Sec. 39-6	C	Ex. 4	F	25, 27	24, 26, 28

^aEx. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read the General Comments for an overview of the material of this module. Sears and Zemansky discuss the photoelectric effect only briefly, the Compton effect not at all. For this reason, we suggest that you include another text in your reading, following the Table below. For each Objective, read the text, study the respective Problem with Solution and examples, then work the corresponding Assigned Problems. Try the Practice Test before taking a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study	Text	
1	Sec. 43-4	A		D	43-15	
2	Sec. 43-4	B		E	43-5, 43-7 43-9, 43-11	43-6, 43-8, 43-10, 43-12

Frederick J. BUECHE, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

2	Sec. 35.3	B	Illus. ^a 35.2, 35.3	E	Chap. 35, Probs. 9, 11	Chap. 35, Prob. 12
3	Sec. 35.4	C	Illus. 35.4	F	Chap. 35, Probs. 15, 16	

David HALLIDAY and Robert RESNICK, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

2	Secs. 39-4, 39-5	B	Ex. ^a 2, 3	E	Chap. 39, Probs. 17, 19, 21	Chap. 39, Probs. 16, 18, 20, 22
3	Sec. 39.6	C	Ex. 4	F	Chap. 39, Probs. 25, 27	Chap. 39, Probs. 24, 26, 28

^aIllus. = Illustration(s). Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vols. 1 and 2

SUGGESTED STUDY PROCEDURE

Since your text does not cover quantum physics, you may ask your instructor for a corresponding Study Guide if you have access to one of the following texts.

Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

PROBLEM SET WITH SOLUTIONS

A(1). An FM radio station broadcasts electromagnetic waves with $f = 10^8$ Hz, $\lambda = 3.00$ m.

(a) What energy and momentum is carried by one photon?

(b) If the radiated power is 10^4 W, how many photons are emitted per second?

Solution

$$(a) E = hf = (6.63 \times 10^{-34} \text{ J s})(10^8 \text{ s}^{-1}) = 6.63 \times 10^{-26} \text{ J.}$$

$$P = h/\lambda = (6.63 \times 10^{-34} \text{ J s})/(3.00 \text{ m}) = 2.21 \times 10^{-34} \text{ kg m/s.}$$

(b) $P = RE$, where R is the rate of photon emission.

$$R = \frac{P}{E} = \frac{10^4 \text{ J/s}}{6.63 \times 10^{-26} \text{ J/ photon}} = 1.51 \times 10^{29} \text{ photons/s.}$$

Thus, it is very difficult to see individual photons.

B(2). When light of wavelength 4500 \AA ($1 \text{ \AA} = 10^{-10} \text{ m}$) falls on a metal plate, electrons are emitted with a maximum kinetic energy of $1.20 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

(a) What is the work function ϕ for the plate?

(b) What is the longest wavelength light that can cause photoemission from this plate?

Solution

(a) Photoelectric equation (energy conservation):

$$hf = \phi + E, \quad f\lambda = c,$$

$$\begin{aligned} \phi &= h\left(\frac{c}{\lambda}\right) - E = (6.63 \times 10^{-34} \text{ J s})\left(\frac{3.00 \times 10^8 \text{ m/s}}{4.5 \times 10^{-7} \text{ m}}\right) - (1.92 \times 10^{-19} \text{ J}) \\ &= 2.49 \times 10^{-19} \text{ J} = 1.56 \text{ eV.} \end{aligned}$$

(b) The energy of the emitted electron must be at least equal to zero for emission to occur:

$$hf \geq \phi, \quad hc/\lambda \geq \phi,$$

$$\lambda \geq \frac{hc}{\phi} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{1.56 \times 10^{-19} \text{ J}}, \quad \lambda_{\text{max}} = 1.28 \times 10^{-6} \text{ m} = 12\,800 \text{ \AA}.$$

- C(3). Electromagnetic waves (photons) of wavelength 0.050 \AA are scattered 60° from electrons at rest.
- (a) What is the wavelength of the scattered photon?
- (b) What is the kinetic energy of the recoil electron?

Solution

(a) $\lambda' = \lambda + (h/mc)(1 - \cos \theta)$ (Compton formula),

$$\begin{aligned}\lambda' &= 0.05 \text{ \AA} + \left(\frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} \right) (1 - \cos 60^\circ) \\ &= 0.05 \text{ \AA} + (1.21 \times 10^{-12} \text{ m}) = 0.062 \text{ \AA}.\end{aligned}$$

(b) By energy conservation: $hc/\lambda = hc/\lambda' + E$,

$$E = hc(1/\lambda - 1/\lambda'),$$

$$\begin{aligned}E &= (6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8) \left(\frac{1}{5.0 \times 10^{-12} \text{ m}} - \frac{1}{6.2 \times 10^{-12} \text{ m}} \right) \\ &= 7.7 \times 10^{-15} \text{ J} = 48\,000 \text{ eV} = 48 \text{ keV}.\end{aligned}$$

Problems

- D(1). A photon with $E = 1.00 \text{ J}$ would be easily observable as a single particle. What are the frequency and wavelength of the electromagnetic wave?
- E(2). A certain metal surface has a threshold wavelength for photoemission of λ_0 . If the incident wavelength is $\lambda_0/2$, electrons of maximum kinetic energy 1.40 eV ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$) are emitted.
- (a) What is λ_0 ?
- (b) What is the work function ϕ ?
- F(3). You have a detector that detects photons scattered 90° from a copper target. You detect photons with $\lambda_1 = 0.0300 \text{ \AA}$ and $\lambda_2 = 0.054 \text{ \AA}$.
- (a) Why are there two different wavelengths in the scattered radiation?
- (b) What is the wavelength of the incident radiation?
- (c) What is the kinetic energy of the recoil electron?

Solutions

$$D(1). \quad f = E/h = (1.00 \text{ J})/(6.63 \times 10^{-34} \text{ J s}) = 1.51 \times 10^{33} \text{ Hz.}$$

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(1.51 \times 10^{33} \text{ s}^{-1}) = 1.99 \times 10^{-25} \text{ m.}$$

It would evidently be very difficult to observe interference phenomena with such short waves, since there are no known detectors with sufficient spatial resolution.

E(2). (a) At threshold, electrons have zero energy:

$$hc/\lambda_0 = \phi.$$

For the shorter wavelength,

$$2hc/\lambda_0 = \phi + E.$$

Subtract the first equation from the second: $hc/\lambda_0 = E$,

$$\lambda_0 = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{1.4(1.6 \times 10^{-19} \text{ J})} = 8.88 \times 10^{-7} \text{ m} = 8880 \text{ \AA}.$$

$$(b) \quad \phi = hc/\lambda_0 = E = 1.40 \text{ eV.}$$

F(3). (a) The target contains electrons and copper nuclei of mass $M = 1.055 \times 10^{-25} \text{ kg}$. For electrons:

$$\Delta\lambda = h/mc = 0.0240 \text{ \AA}, \quad \Delta\lambda = \lambda' - \lambda.$$

For scattering from copper nuclei:

$$\Delta\lambda = h/Mc = 2.10 \times 10^{-7} \text{ \AA} \quad (\text{negligible shift}).$$

Thus, the 0.030-Å photons are scattered 90° from copper nuclei, and the 0.054-Å photons are scattered 90° from electrons.

(b) $\lambda_{\text{inc}} = 0.030 \text{ \AA}$ [to two significant figures; actually slightly less - see part (a)].

(c) Use energy conservation: $hc/\lambda = hc/\lambda' + E$,

$$\begin{aligned} E &= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8) \left(\frac{1}{3.00 \times 10^{-12} \text{ m}} - \frac{1}{5.4 \times 10^{-12} \text{ m}}\right) \\ &= 2.95 \times 10^{-14} \text{ J} = 184 \text{ keV.} \end{aligned}$$

PRACTICE TEST

1. The work function ϕ of tungsten metal is 4.6 eV ($1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$).
 - (a) What is the threshold wavelength for photoemission from tungsten?
 - (b) If light of wavelength 2200 Å ($1 \text{ Å} = 10^{-10} \text{ m}$) falls on a tungsten surface, what is the maximum kinetic energy (in electron volts) of the emitted electrons?

2. Photons of wavelength $\lambda = 0.040 \text{ Å}$ are Compton (elastically) scattered from electrons.
 - (a) What is the energy of the photon?
 - (b) The recoil electron has a kinetic energy of $1.40 \times 10^5 \text{ eV}$. What was the scattering angle θ of the photon? ($h = 6.63 \times 10^{-34} \text{ J s}$; $m_e = 9.11 \times 10^{-31} \text{ kg}$.)

Practice Test Answers

1. (a) 2730 Å. (b) 1.10 eV.

2. (a) $5.0 \times 10^{-14} \text{ J}$ or 311 keV. (b) $\cos \theta = -0.34$, $\theta = 110^\circ$.

INTRODUCTION TO QUANTUM PHYSICS

Date _____

Mastery Test Form A

pass recycle

1 2 3

Name _____ Tutor _____

1. Light of wavelength 350 nm shines on a metal surface whose work function is 1.80 eV ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$). The light intensity is $1.80 \times 10^3 \text{ W/m}^2$.
 - (a) What is the maximum kinetic energy of the emitted photoelectrons?
 - (b) If the intensity of the light were doubled, what would be the maximum kinetic energy of the electrons?
 - (c) If the wavelength of the incident light were doubled, what would be the kinetic energy of the emitted electrons?

2. Photons scattered backwards ($\theta = 180^\circ$) from electrons at rest are observed to have a wavelength of 0.069 \AA ($1 \text{ \AA} = 10^{-10} \text{ m}$). ($h = 6.63 \times 10^{-34} \text{ J}$; $m_e = 9.11 \times 10^{-31} \text{ kg}$.)
 - (a) What is the wavelength of an incident photon?
 - (b) What is the kinetic energy of a recoiling electron?

INTRODUCTION TO QUANTUM PHYSICS

Date _____

Mastery Test Form B

pass recycle

1 2 3

Name _____

Tutor _____

1. Light of wavelength 450 nm shines on a metal surface. The emitted electrons are seen to have a maximum kinetic energy of 0.80 eV ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$; these electrons need a retarding potential of 0.80 V to stop them).
 - (a) What is the work function of the metal?
 - (b) What is the longest wavelength that would produce emitted electrons?

2. A photon is scattered backwards ($\theta = 180^\circ$) from an electron at rest. The electron recoils with a kinetic energy of 10^5 eV ($1.6 \times 10^{-14} \text{ J}$) ($h = 6.63 \times 10^{-34} \text{ J}$; $m_e = 9.11 \times 10^{-31} \text{ kg}$).
 - (a) What is the energy of the incident photon?
 - (b) If the photon energy were doubled, what would be the change in wavelength for backward scattering ($\theta = 180^\circ$)?

INTRODUCTION TO QUANTUM PHYSICS

Date _____

Mastery Test Form C

pass recycle

1 2 3

Name _____

Tutor _____

$$h = 6.63 \times 10^{-34} \text{ J s.}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg.}$$

1. You want to build a photoelectron source using light of $\lambda = 440 \text{ nm}$. You have a choice of the following materials as emitters:

Metal	ϕ
Lithium	2.30 eV
Zinc	3.6 eV
Tungsten	4.5 eV

- (a) Which material(s) will not work at all?
 (b) Which material produces the most energetic electrons? What is their energy?
2. You have a source of photons of wavelength $\lambda = 0.0200 \text{ \AA}$ ($1 \text{ \AA} = 10^{-10} \text{ m}$). You want to produce $\lambda = 0.040 \text{ \AA}$ photons by Compton scattering from free electrons at rest; you propose to identify the $\lambda = 0.040 \text{ \AA}$ photons by measuring the energy of the recoil electron.
- (a) What is the required scattering angle?
 (b) What is the kinetic energy of the recoil electron?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Photoelectric equation. Consistent units. (c) The electron energy does not scale linearly with photon wavelength or energy. In this case, nothing comes out.

Solution: $hc/\lambda = \phi + E,$

$$E = \frac{hc}{\lambda} - \phi = \frac{6.63 \times 10^{-34} \text{ J s}}{3.5 \times 10^{-7} \text{ m}} \frac{3.00 \times 10^8 \text{ m/s}}{1.60 \times 10^{-19} \text{ J/eV}} - 1.80 \text{ eV}$$

$$= 3.55 \text{ eV} - 1.80 \text{ eV} = 1.75 \text{ eV} = 2.80 \times 10^{-19} \text{ J}.$$

(b) Same as part (a), K.E. does not depend on intensity.

(c) $E = hc/2\lambda - \phi = 1.775 \text{ eV} - 1.80 \text{ eV}$. Ugh! Not enough energy to knock out an electron. No electrons emitted.

2. What To Look For: (a) Compton formula. $\theta = 180^\circ$. $h/mc = 0.0243 \text{ \AA}$. Solve for λ .
(b) Energy conservation. Consistent units.

Solution: (a) $\lambda' - \lambda = (h/mc)(1 - \cos \theta) = 2h/mc.$

$$\lambda = \lambda' - 2h/mc = 0.069 \text{ \AA} - 0.049 \text{ \AA} = 0.0200 \text{ \AA}.$$

(b) $hc/\lambda = hc/\lambda' + E,$

$$E = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})$$

$$\times \left(\frac{1}{2.00 \times 10^{-12} \text{ m}} - \frac{1}{6.9 \times 10^{-12} \text{ m}}\right)$$

$$= 7.1 \times 10^{-14} \text{ J} = 4.4 \times 10^5 \text{ eV}.$$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: Photoelectric equation. Convert wavelength to frequency. Use consistent energy units. Set $E = 0$ in photoelectric equation. Convert electron volts to joules or vice versa.

Solution: (a) $hf = \phi + E$, $f = c/\lambda$.

$$\phi = \frac{hc}{\lambda} - E = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{(4.5 \times 10^{-7})(1.60 \times 10^{-19})} - 0.80 \text{ eV} = (2.76 - 0.80) \text{ eV} = 1.96 \text{ eV}.$$

(b) At threshold: $hc/\lambda = \phi$,

$$\begin{aligned} \lambda &= \frac{hc}{\phi} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{(1.96 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} \\ &= 6.3 \times 10^{-7} \text{ m} = 6.3 \times 10^3 \text{ \AA} = 630 \text{ nm}. \end{aligned}$$

2. What To Look For: Compton equation. Conservation of energy. Correct substitution. Multiplication by $\lambda(\lambda + 2h/mc)$. Solution of quadratic equation. Reject negative answer.

Solution: (a) $\lambda' - \lambda = (h/mc)(1 - \cos \theta) = 2h/mc$.

$$\lambda' = \lambda + 2h/mc, \quad hc/\lambda - hc/\lambda' = E, \quad 1/\lambda - 1/\lambda' = E/hc.$$

$$\frac{1}{\lambda} - \frac{1}{\lambda + 2h/mc} = \frac{E}{hc}, \quad \lambda + \frac{2h}{mc} - \lambda = \left(\frac{E}{hc}\right)\lambda\left(\lambda + \frac{2h}{mc}\right), \quad \lambda^2 + \left(\frac{2h}{mc}\right)\lambda - \frac{2h^2}{mE} = 0,$$

$$\lambda = \left[-\frac{2h}{mc} \pm \left(\frac{4h^2}{m^2 c^2} + \frac{8h^2}{mE} \right)^{1/2} \right] \left(\frac{1}{2} \right) = -\left(\frac{h}{mc} \right) \left[1 \pm \left(1 + 2\frac{mc^2}{E} \right)^{1/2} \right]$$

$$= 0.0243 \text{ \AA} \left[\left(1 + \frac{1.02 \text{ MeV}}{0.100 \text{ MeV}} \right)^{1/2} - 1 \right] = 0.057 \text{ \AA}.$$

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{5.7 \times 10^{-12} \text{ m}} = 3.5 \times 10^{-14} \text{ J} = 2.20 \times 10^5 \text{ eV}.$$

(b) $\lambda' - \lambda$ does not depend on λ . $\lambda' - \lambda = 0.049 \text{ \AA}$.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: Calculate energy of photon. Compare with work function. Use photoelectric equation to find electron energy. If negative, no emission.

Solution:

$$\frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{4.4 \times 10^{-7}} = 4.52 \times 10^{-19} \text{ J} = 2.83 \text{ eV.}$$

Metal	ϕ	E_{elec}
Lithium:	2.3 eV	0.53 eV
Zinc:	3.6 eV	No
Tungsten:	4.5 eV	No

$hc/\lambda = \phi + E_{\text{elec}}$. (a) Zinc, tungsten won't work. (b) Lithium produces most energetic electrons: $E_{\text{elec}} = 0.53 \text{ eV}$.

2. What To Look For: Compton formula. Solve for $\cos \theta$. Conservation of energy.

Solution: (a) $\lambda' - \lambda = (h/mc)(1 - \cos \theta)$.

$$\frac{h}{mc} = \frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} = 2.43 \times 10^{-12} \text{ m} = 0.0243 \text{ \AA.}$$

$$\frac{\lambda' - \lambda}{h/mc} = 1 - \cos \theta, \quad \cos \theta = 1 - \frac{\lambda' - \lambda}{h/mc} = 1 - \frac{0.040 - 0.0200}{0.0243} = 0.177.$$

Thus, $\theta = 80^\circ$.

(b) $hc/\lambda = hc/\lambda' + E$.

$$E = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})$$

$$\times \left[\frac{1}{2.00 \times 10^{-12} \text{ m}} - \frac{1}{4.0 \times 10^{-12} \text{ m}} \right] = 5.0 \times 10^{-14} \text{ J} = 3.10 \times 10^5 \text{ eV.}$$
