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Formal Logic and Logical Form

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FORMAL LOGIC AND LOGICAL FORM

Last year Dr. F. C. S. Schiller published *Formal Logic, a Scientific and Social Problem*,—a critical text-book, he calls it in the preface, that will teach logic "in a critical spirit and with a minimum of pedantry and reverence for forms." This object is so thoroughly fulfilled—at the end of the four hundred pages the criticism has been so searching and insistent and the reverence for forms so truly a minimum that formal logic seems to be a complete ruin, and the only scientific problem left at the end is how men ever came to build it, and the only social problem what logicians will now do for a living. All that seems to be open to them is to act as guides for the curious who wish to inspect the remains of their subject.

The thesis of the book Dr. Schiller has given in the preface: "It is not possible to abstract from the actual use of the logical material and to consider 'forms of thought' in themselves, without incurring thereby a total loss, not only of truth but also of meaning." It is necessary, he says, "to pull down the pseudo-science of Formal Logic, and to show what an incoherent, worthless, and literally unmeaning structure it is, before it is possible to build up the true logic of real reasoning which starts from the act of thought and so does not lose touch with science and practical life." The object of the attack is, then, the formalism of formal logic. The reader is led through a series of chapters that follow the headings of the traditional texts one by one, and under each is shown the difficulties attendant on the usual treatment of that department. Psychological analysis shows that terms, categories, and the formal theories of judgment do not describe any processes that actually appear in thinking or any elements of thinking. The traditional laws of thought, the laws of identity, contradiction, and excluded middle cannot be taken as laws in the sense that we must think according to the forms which they set, for we usually fail of this. Nor will Dr. Schiller grant that they are laws which we ought to follow: when we say "A is A" we never mean the A's to be identical and it may turn out that the differences are essential to the purpose of the discussion. The third possibility that these laws are postulates is discussed with

peculiar results; logical laws used as postulates sometimes work and sometimes do not; whether we apply them or not rests with us and is dependent on our judgments.

"The conclusion that the 'laws of thought' are postulates and neither facts in nature, nor even necessarily applicable to all reality, will perhaps be thought to reduce their truth to the level of (more or less) successful fictions. And certainly they are not 'true' if it is the business of thought to correspond with reality. For they make no attempt to copy reality; they openly and 'arbitrarily' idealise certain features in it, and demand that reality shall conform to these ideals, although it plainly never does. However convenient then they may be, they cannot be more than fictions."

In treating the forms of judgment formal logic is mistaken in reducing any one form to another or in treating them as equivalent. Something is always lost in the meaning of the reduced form; some shade or turn that the original judgment possessed does not appear in the equivalent form. Dr. Schiller makes a careful review of the Aristotelian theory of the syllogism and finds fault with it on the old ground that either the conclusion is contained in the premises or it is not: if it is, there can be no use in drawing it, for it has been stated already; and if it is not, it cannot be inferred. Moreover the middle term in a syllogism is in practice always ambiguous because it occurs in two different contexts. That "All men love good stories" and "Smith is a man" does not make it necessary that Smith loves a good story directed at himself.

As to the symbolic logic with which modern logicians are busying themselves, Dr. Schiller points out that it has all the faults of formalism in an exaggerated degree. What his criticism would have been had he taken time to discuss that branch of logic is not in much doubt. The late M. Poincaré expressed the attitude of Schiller's school in his discussions with Mr. Bertrand Russell and M. Couturat. These logics are extensions of the Aristotelian doctrine made possible through the use of symbols. No symbol can have meaning in itself but only stands for what our interest and will make it. Moreover, M. Poincaré has remarked, in every equation of the symbolic logic the two sides stand in the equational relation by virtue of definitions of their terms, and if for each term is substituted its definition we have bare tautologies. "Logic," he says, "is sterile unless fecundated by intuition." This is in effect the same as the criticism of the syllogism in *Formal Logic*. If the conclusion is contained in the

premises why is it repeated in the conclusion? And if it is not so contained, how can it be drawn? Logical theorems seem to take the form of vast memoranda.

Writers of formal and symbolic logic have had little to say to these criticisms, as Dr. Schiller has noticed. Perhaps what delays the reply is that any man who seriously devotes himself to the intricacies of symbolic logic has little leisure or inclination to notice anything so inconsequential as a criticism of his subject. Criticism of his particular system will arouse his immediate interest, but he is apt to pay as little attention to the foundations of his subject as is the mathematician himself. There is a more serious reason why modern workers in symbolic logic should not feel called upon to reply to this literature of criticism whose main representative is Dr. Schiller's book. The truth is that the conception of logic which it attacks has been quietly discarded for some time, for so long a time that some of the younger men know it only as part of the history of the subject. That conception is the notion that logic is the science of reasoning and that it has to do with the process of thought. To this fact Mr. Hoernle called the attention of Dr. Schiller, in a sagacious review of his book, and was answered by Schiller that the old opinion lingers still in most of the colleges and in other unexpected places. The answer is, in fact, quite justified, for Mr. Hoernle expresses it as his own opinion that formal logic is either the science of reasoning or "a mere propædeutic exercise."

But surely this opinion also has vanished from at least some quarters. There is another alternative that leaves logic meaningful and still does not pretend that it deals with thought. That logic has failed as a science of thinking (for a science of thinking is no light adventure!) must surely be granted. It has long been cast up to the logician that we do not think in syllogisms; and the logician must now be prepared to take the stand that it is irrelevant to his subject whether or not we think in syllogisms. Perhaps no logical form is ever used in actual thought. The school of idealistic logicians, from Hegel to Bosanquet, in using the terminology of formal logic for their theory of judgment, have made the vast assumption that the Scholastics made before them, namely, that because we find logical forms in argument we shall find those same forms in reasoning. With the same idea in mind, Dr. Schiller decides that since formal logic has nothing to do with the thought-process it is entirely void of meaning. This is not

so clear. M. Couturat has expressed the attitude of many logicians in saying that logic does not offer rules of thought but rules of demonstration. Perhaps he should not have used the word "demonstration," which conveys the impression of the forcible conviction of an unwilling auditor, but should rather have said "rules of expression" or "rules for connected statement." At any rate he is right in that logic does not offer rules for thought or reasoning. Only God and the psychologists know how we reason; but when we express our thoughts to others it must be done in a systematic fashion that is open to investigation. A logical form may occur in thought, but, as far as we know, its appearance there is accidental; that our thinking process is governed by logical laws, whether those of formal logic or of the neo-Hegelian psychological logic that has appropriated its language, falls short a whole world of being established. Judgment, which these logicians call an act, and make the basis of the thinking process, is more nearly the result of an act which is in itself, as far as our knowledge goes, miraculous. At any rate in the non-metaphysical experience of ordinary humanity we do not pretend to predict what judgment is to follow what other on any theory of judgment. The only attempt to furnish a basis for such prediction has been the theory of the association of ideas, whose entire inefficiency Bradley holds up to public scorn.

Theories of reasoning are not logic. Logic must find its field in statement which is open to the investigation of all in a way that knowledge is not. However we think, the expression of our thought must have connection and order, and the same rules of connection and order must be in at least two minds before there is any expression at all. Aristotle (better than his disrepute) made no attempt to find logic in a necessity of thought, but sought for it exactly as we must seek for it in that agreement between two speakers which must necessarily precede discussion. If a man means anything he must say something; if he will not speak he is no better than a vegetable. We can conceive a solitary thinker brewing his conclusions somehow, as sudden intuitions or chance associations; but when he is confronted with another thinker, and desires to communicate the results of his thought, he cannot reproduce the manner in which that thought arose, for it was probably unconscious, and if not illogical, at least innocent of logical connection. Communication is not thought. Mere

mention of the terms in the order in which they occur to the thinker will not reproduce in another an identical process of thought. There is necessary a connection, a schematisation, by which the recipient can order the elements called up by words and phrases which the thought had originally suggested. This connection is more than grammar, although it must be acquired with grammar and underlies grammar. Rather it is the whole body of rules of the game of talk, which is the basis of communication. Between us and men of an entirely different culture, before any form of communication is set up, there cannot be said to be a common logic but only the vague foreshadowing of one. With the establishing of communication there grows a set of rules for intercourse. In such communication logic is crude and misunderstanding the rule. Between its vague beginnings and the intercourse of two mathematicians, whose common grounds and agreements as to method are infinitely greater and more definite, there are all degrees of elaboration, in every stage of which the logic consists in the rules of intercourse that the two parties to the discourse accept.

When we take this view we immediately see that the axioms of logic are postulates. The law of excluded middle and its companions, the law of contradiction and the law of identity, are merely those agreements as to the nature and permanence, for discussion, of affirmation and negation, representing the fundamental rules of all ordinary intercourse and connected statement. But these are not postulates that we are to use or neglect as the spirit moves us or as Dr. Schiller would have us. They are the conditions of our being understood. The question whether they will bear analysis is irrelevant; we follow them in our speech, and we are socially bound to speak that way. We find them in statements because we carry them there, and if we do without them we do so at the risk of being unintelligible. A logic stands on the same footing as a geometry. We no longer speak, with Kant, of Euclidean Geometry as *the* geometry or as *the* method in which we must order experience, but as *a* method in which we *may* order experience. So our logic is not the order in which we *must* express ourselves but an order in which we *may* express ourselves; and other logics are conceivable if not practicable. Take, for instance, the law of identity, "A boy is a boy"; we know that two different boys are involved or we should not have taken the trouble to pronounce them the same. Perhaps if we

had started at a sufficiently early period we could have had for our first postulate "A boy is not a boy" with the understanding that they were really the same or we should not have troubled ourselves to call them different. Either postulate would give a *form of expression.*

There must be formal rules if there is to be communication, and these rules constitute logic,—formal logic, if they have any formulation independent of individual situations. In fact there should be nothing so terrifying about the word "formal." Our use of words is itself formal. When we repeat a word we do not mean that the same situation is repeated but that our attention is on the sameness in situations and not their difference. The word is not the situation itself, it represents an abstraction from the situation and this abstraction is essentially formal in the worst sense of the word. We use formal grammar and logic because structure is as necessary to the expression of our thoughts as it is to the existence of physical bodies.

Dr. Schiller's part in the successful rebellion against the rationalistic idea that we are in a world that is to be deduced from a number of self-evident axioms, or fundamental principles, has led him into this vigorous denunciation of logic because somehow logic seemed the stronghold and guarantee of axioms and the whole rationalistic standpoint. What he does not see is that logical postulates have the same standing as the postulates of geometry. Geometrical postulates cannot be discarded at will without discarding the geometry which they underlie and adopting another in its place; and though this may be done, yet it would be the end of advancement in all geometry if investigators did not confine themselves to one set of postulates in the course of one argument. A tennis player could as well change at will the rules by which he plays. His penalty would be the wrath of the gallery; the penalty to violators of the rules of discourse would be complete misunderstanding.

Logic is intimately connected with language. There is logic in the language of savages as well as in the most definite language of all, the symbolism of mathematics. We acquire our logic with our language. Our neighbour's small Katharine declares herself a little boy. "But you are not a boy, you are a girl," we answer; "you can't be both." Yet the rules of intercourse, affirmation, and negation, the relation of girl-not-boy and boy-not-girl, had been acquired long ere this, or we could have spoken

to Katharine only in the monosyllables of the year before. We may call the principles of logic postulates, although we no more choose them than we choose the names of the beasts of the field or the sounds by which we indicate that we wish a room with a bath. The actual choice was a social event in which our individual will is as much concerned as in the selection of our anatomy.

Nor, if it be granted that we must have a formal logic, need we be jealous of the standing of the Aristotelian theory of the syllogism. Dr. Schiller and many other logicians have collected instances of argument that is marvellously awkward if put into the form of Barbara and Disamis; and we must grant also that the Aristotelian forms can no longer be held complete and authoritative merely because they have a kind of symmetry and finish when symbolised. The rules of Aristotle's logic served fairly well in the Athenian schools where the players in the game of dialectic were brought by common interest and respect for leaders to obey those rules; but in modern discussions, between men of different nationalities and different training, they will hardly serve; Dr. Schiller is right to this extent. For in the first place, the ideal of a reduction of all forms of inference to a few forms has only academic interest,—and academic interest hyper-refined. The completeness of logic in the sense that all valid forms of inference can once for all be tabulated is a myth. Logic is no more to be completed than is geometry or life. New forms of inference will appear constantly; and the symbolic logic of to-day is tabulating forms that are not expressible in syllogisms. The formation of a general logic is hardly to be expected from the symbolists any more than from their predecessors; the task of formulating a logic that will serve for all human intercourse is as hopeless as the task of inventing a language that will serve all nations for all time. Only a few and these the most fundamental logical laws can find even a very general application. A half hour with a student of linguistics who has worked with American-Indian languages will convince anyone that most of the conceptions of classical logic are inapplicable to a speech that very nearly lacks common nouns and tries to express in a word a situation that demands an English sentence. It is more correct to talk of logics than of logic,—the logic of each distinct culture must have a different formulation. It is with limitation of interest to a restricted field that the most complete logics arise.

In the sciences this condition has brought about specialised vocabularies and specialised structures for the use of those vocabularies; and the logic of scientific discourse becomes more complex as the method and structure of the science grow more and more definite. The logic of each science determines the place that new discoveries take in the body of that science; each of the sciences may have a method differing slightly from that of the others; though underlying them all we find the logic of common speech, whose few universal principles must be generally observed.

Of these logics of the sciences the logic of mathematics is the only one that has been given symbolic formulation. The work of Peano and Russell was possible because of the abstract nature of the subject-matter itself which admitted the use of symbolic expression. It is not true that these symbolic logics are completely expressed in symbols or can be completely expressed in symbols, for there are principles necessary to govern the operation of the symbols that will forever remain outside symbolic statement itself. In Russell's *Principia Mathematica* over fifty pages are necessary to explain the theory of types on which the logic is based. A number of paradoxes, to which the Scholastics devoted many volumes under the heading "Insolubilia,"—propositions such as "*Haec propositio est falsa,*" which being denied is true, and being affirmed, is false, led Russell to lay down the principle that no propositional function can have reference to itself in one of its own terms.[1] Now this principle must be taken as completely outside the logic for which it is assumed; for if it is within the logic, its reference to all propositions seems to include itself, and hence violate itself. To unravel this difficulty an elaborate theory is necessary before the logic can be well begun; and we can hardly feel that there can exist any truly symbolic logic in Russell's expansive understanding of the term. In every logic are innumerable difficulties that can be settled only by elaborate conventions—conventions of symbolism in logics of that character, agreements of expression in those arising more simply from analysis of linguistic forms.

Russell's logic may be adequate for the organisation of mathematical science, but it is *not* a general logic. It would be entirely inadequate in governing a parliamentary debate, for example, or delimiting a biological species. It abstracts from experience to

[1] This identical solution is proposed by Paulus Venetus, *Logica Pauli Veneti*, Venetiis, 1654, p. 84.

the same great extent as does mathematics and is useful only in mathematics for just that reason. The logic of classes, propositions, and relations in Russell is fitted to express mathematical theorems and their coherence, but not to express the uses of such a sentence as, "I wish I were in the surf." If there exists such a thing as a general logic (for it has not yet been formulated) it is altogether indefinite and is probably not expressible in symbols. Nevertheless it must be formal if we mean by formal, as Dr. Schiller would have us, "abstracted from the use of the logical material." We may paraphrase the statement quoted at the beginning of this article in some such way as: "It *is* possible to abstract from the actual use of the logical material and to consider 'forms of statement' in themselves without incurring thereby a total loss, either of truth or of meaning." The whole point lies in recognising that in expression we have the result of thought but not the thought itself, and if thought proves inscrutable we may still scrutinise statement.

The laws of statement are none the less laws if we find no instances of them in thought. Perhaps no syllogism was ever made that would stand close analysis of its terms and maintain its form. Inspection would always show the middle term susceptible of different interpretations in its two different contexts. It is the essence of usable words to be ambiguous. But that need not be taken as an indictment of the syllogism as a form of statement; it has fulfilled its particular function, whatever may happen to it; it has not forced its conclusion on its auditors, but it has offered this conclusion in an intelligible form. Other persons are not compelled to accept our individual conclusions; nor is it the function of language or of logic to force conviction. No party to an argument was ever forced to a conclusion that he was not willing to make,—forced, that is, by purely logical constraint. It is always possible to question premises *ad infinitum*—even though the questioners be fools. The function of the syllogism is to show a path of connection, not to lead one over that path. The idea of rigid proof in the sense that the forty-seventh proposition of the first book of Euclid is proved now and for all time and all men is meaningless. You can prove it only to one who has learned geometry, and learning geometry is in part accepting the rules of the mathematical game. Discussion is possible only where each is in turn a learner and endeavours to see the path the other has taken. If either

relinquishes his own path it is because the logical statement of the other gives him an insight that makes him able and willing to pronounce his own the worse way.

With the idea of rigid proof we must give up also that awe of logical form that leads Professor Royce to believe that he has discovered absolute truth in propositions (his own pet instances of the Insolubilia) whose denial involves their assertion. Denial and assertion can occur only in a logical system and paradoxes of the Roycean type are merely evidence of logical postulates and definitions that will not work together. They are demands for careful definition, not *cul-de-sacs* of reason. Truth is either to be taken as accompanying certain logical forms, and so as being a formal matter; or it must be regarded, as far as logic is concerned, as an accident of propositions. The latter horn of the dilemma is the choice of Russell, for whom, in consequence, truth represents no more than a feeling toward propositions, dictated by our interests and purposes; and he offers us as the true logic a mathematicised reasoning in which the forms of ordinary speech are hopelessly outlawed, while logical discourse becomes subject to forms so abstract and constricted that it must lose all connection with life. Dr. Schiller, on the other hand, would leave discourse so free as to be innocent of all connection and unity; it would be the capricious eloquence of temperaments such as is his, and would inevitably result in a dispersion of rationality more conclusive than was ever the confusion of Babel. But between these extremes there should lie a logic that finds its laws in the actual communications of actual men, resting upon compacts of human reason, and involving definite forms of statement which must be and are observed even by writers so independent and persuasive as Dr. Schiller.

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