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Old Solutions of a New Problem

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OLD SOLUTIONS OF A NEW PROBLEM

When King David said in his haste that all men are liars, was he led to acknowledge the hastiness of his remark through reflection on its logical consequences? If he were, he showed commendable delicacy in taking for granted that we should see what the logicians insistently point out, that it must follow that he himself could not be believed. Reflection on this problem of verbal paradox has led some of the logicians, as well as the Psalmist, to wonder whether they have not made haste too rapidly. The paradox of the "Liar" is still with us; but modern writers, less hurried than David, usually state it in a form which leaves their own credibility beside the question and ponders only that of Epimenides the Cretan, our authority for the statement that all Cretans are liars. But with the most recent work in logic the comfort of so disinterested a position shows signs of forsaking us. Some of the solutions of this paradox begin to involve our right to make any statement about truth or logic and then claim that the statement itself is true or is logical. So Mr. Bertrand Russell would have us grant that no statement can contain any reference to itself, and that when we wish to assert, "All propositions have subject and predicate" or "Truth is relative," then our statements themselves cannot be propositions or be truth within the meaning of their subjects; nothing has been said, he asserts, about the statements in which our thought is couched.

The modern solutions of the "Liar" have been all offered by makers of logistic, the present custodians of formal logic, and quite properly so, since the difficulty with the paradox seems to be essentially formal. That it is a difficulty which is serious for the maker of an algebra of logic is shown by the size of the literature that it has occasioned and is still occasioning.

In the recent attempts to solve the paradox several writers have mentioned that there was a scholastic discussion of the same problem which has left us many ingenious plans for its solution. That the scholastic solutions were, several of them, identical in principle with their modern successors is not so well-

known. The solutions of Russell, Zermelo, Rüstow, Peirce, Poincaré, Herbart, and Shearman, all bear a close resemblance to one or the other of the scholastic methods, whose interest for the general student is often much greater than the solutions that have been offered with a peculiar and intricate algebra of logic in mind. Nor was the paradox unfamiliar to the ancients. Mr. Alexander Rüstow has collected a great number of references to it in ancient writers, chiefly Aristotle, Plato, and Diogenes Laërtius—for it was of a sort calculated to attract the attention of Greek lovers of eristic.

With the earlier writers the paradox was accepted as final and its speculative interest lay in the fact that unavoidable paradox could exist. Like Montaigne they asked what could be the value of a human reason that could find itself in such straits. Later writers were occupied in attempts to place the "Liar" among the Aristotelian fallacies which were supposed to care for all logical difficulties. There was no real statement of the difficulty, nor was there any attempt to solve the paradox formally. These came only with the more refined and formal logic of the scholastics. We are given to dismissing mediæval logic with little attention; and we are perhaps justified in so doing since a thorough study of the involved definitions and rules that were set up by logicians of that period would be impossible in the time allotted to the life of one man; even Prantl's monumental history amounts to little more than a collection of extracts under a limited number of headings—extracts not always representative of the views of the author quoted nor sufficient to present his theory. But this massive literature is only a reflection of the fact that the spirit for keen analysis and fine distinctions made the "Liar" an ideal subject for tests of the agility of the mediæval mind. Between the fourteenth and the sixteenth centuries each author of a logical text-book took his turn at the difficulty, and with a confidence in his results that has gone out of fashion in our more sophisticated generation heralded a solution that should dispose of the matter for all time.

It is uncertain when the name *Insolubilia* was first applied to these paradoxes. The first mediæval mention of them includes other puzzles not of this type, having in common with this only the fact that they were difficult, puzzles like those of Plato's *Euthydemus*. Gradually only those paradoxes that had the peculiar and subtle difficulty of asserting their own falsity

were included. Typical sentences that are to be found in a number of lists are as follows:

1. *Ego dico falsum.*

2. *Haec propositio est falsa.*

3. *Ponatur, quod Socrates dicat illam, "Plato dicit falsum," et Plato dicat illam, "Socrates dicit verum."*

4. *Si Deus est, aliqua conditionalis est falsa, et sit nulla alia conditionalis.*

These are from the list compiled by Albertus de Saxonia in Vienna before 1390. The first of them, "*Ego dico falsum,*" reduces to the type of the second, which is the most definite and satisfactory of all, "*Haec propositio est falsa*"; the last makes its own truth imply the falsity of a class of propositions to which it itself belongs, as does "All propositions are false." The remainder of the long list either reduces to one of these two types or is intrigued by the use of terms whose meaning is not definite; as "*Posito quod in mente Socratis sit ista, Socrates decipitur, et nulla alia, et Socrates credat illam esse veram, quaeritur, an Socrates credendo eam esse veram, decipiatur*"; for in cases like this the discussion can lose its way in disagreement over the meaning of "to be deceived." Again, in "I lie" the word *lie* need not mean that every statement is false and so need not lead to paradox at all. The real difficulty was sometimes lost sight of or dismissed without comment.

One of the earliest solutions was offered by Buridan, writing in the fourteenth century. In the *Insolubilia*, he holds, we have propositions that are both true and false, unavoidably so by logical rules. This is a contradiction of the law of excluded middle. He seems to see but one opening to reject the law. Perhaps further acquaintance with Buridan's work would show some attempt to supplement this seemingly ruthless method by showing how it would be possible to proceed without the law he discards; for Buridan knew and described a method offered by a contemporary, Hentisberus, that was more carefully planned. At the basis of this method is the doctrine of *restrictio* worked out in such detail by scholastic logic. *Restriction* means that the denotation of a term, which, as far as explicit statement goes, might be taken to be general, is often limited by the context—an approximation to the more modern and less definite universe of discourse. In the case of the *Insolubilia* this unexpressed restriction limits the denotation of a term in a proposition whose

verb is in the present tense to time immediately preceding the present instant; the instant when the sentence is begun, and not that in which it is uttered is the time represented by the verb. This is only an unusual way of stating something we are all apt to feel; namely, that no thought can really be "of itself"; before we comment on a situation there must be the situation to comment on, the expression of any thought follows by appreciable interval the suggestion of that thought. This solution disposes of the paradox "I lie," since the statement cannot apply to itself because the statement is not made until *after* it is begun. And in a proposition like "This proposition is false" there is no statement to which the sentence can refer.

This same feeling that a proposition cannot refer to itself suggested another very similar theory—one that proceeded without this reference to tense. Peter von Ailly wrote a tractate on the *Insolubilia* in which he makes a distinction between mental and verbal propositions. Written and spoken propositions are not properly true or false; they are only symbols for mental propositions which alone have real meaning and truth. The *Insolubilia* are all verbal propositions, he contended; and mental propositions cannot denote themselves. "*Nulla propositio mentalis proprie dicta potest significare, se ipsam esse veram, nec potest habere reflexionem supra se ipsam.*" We are given to understand that though the form of our words may seem to refer to itself, nevertheless the thought that leads to the expression never has that character, but always refers to some object independent of it. The rule for avoiding the paradoxes becomes, "*Pars propositionis non potest supponere pro toto.*" This is not far from the most recent solution offered by mathematical logic, the principle of the vicious circle on which Russell's theory of types is based: "Whatever contains an apparent variable must not be one of the possible values of the variable."

This method of solution was a favourite with other writers, Johannis Majoris Scotus, Olkot, and Rosetus among them. Its similarity to Russell's theory is curiously paralleled by identical criticisms, one in particular, directed at Paulus's theory by Wycliffe, whose *Logic* is less known than his historical works.

Wycliffe considers this rule that no proposition is to refer to itself and finds that it leads immediately to difficulty. What shall we do about those general propositions, like "All propositions are true or false" and "All propositions have subject and

predicate," which pretend to refer to all propositions including themselves. Yet we may mean to say that even the proposition we are using must be either true or false. The proposition "A is A" would no longer be the general statement that it pretends, for there would be one value of the "A" for which the proposition would assert nothing, namely the value "A is A." Following this criticism our theologian offers as his own contribution a theory that is rather a disappointment after his acute criticism of the other view. The paradoxical statement, "This proposition is false," he says, is neither true nor false in one sense of those terms, and so is no proposition at all. The criterion of truth in a proposition is correct representation of a situation independent of the proposition itself, and where there is no such independent reference there can be neither truth nor falsity, and so there can be no proposition. He does not go on to consider the effect of this theory on the proposition, "All propositions are true or false," where there is both independent reference and self-reference; and after all, on a little reflection, his theory of independent reference amounts to the same thing as Peter's *restrictio.*

Mansell's Aldrich offers a solution like the two just described. In the sentence, "Socrates speaks false," pronounced by Socrates himself there is nothing to which the sentence can refer, so that it is absurd and says nothing; and besides, he says, the disputes over the *Insolubilia* are great nonsense. "*Nihilque opus est plura dicere de Insolubilibus.*"

To the same class as these solutions there belongs one suggested by the unknown author of a Paris manuscript which the writer terms *cassatio.*[1] He denies that the propositions in question are propositions at all, but since he does not supplement this denial with a definition of proposition that excludes these questionable ones he offers only the beginning of a solution.

All of these solutions are of the restrictive type and make their point by declaring that propositions cannot have reference to themselves as these pretend to have. They rule out the *Insolubilia* as illegitimate and meaningless statements. But there is another type, different from these in principle, and, like the theory of restriction, having its analogue in a modern theory. This is to be found in the solution of Paulus Venetus and a number of lesser writers. They decide that we may assume that every proposition asserts its own truth, and that in addition

[1] Prantl, vol. 4, p. 41.

propositions in question assert also their own falsity. "*Hoc est falsum significat quod hoc est falsum et quod hoc est verum, sed quod hoc sit falsum et hoc sit verum est impossibile simpliciter.*" Such solutions do not resolve the paradox, exactly, but they place it in a class of false propositions on a par with "This pen has weight and has not weight"; the *Insolubilia* are shown, not only to imply contradictories, and hence to be paradoxical, but also to assert contradictories, and so to be merely false.

These represent the more noteworthy methods of subjecting the *Insolubilia* to logical rule. Other methods were attempted; but these were of rhetorical rather than logical character, like Cranston's contention that no proposition really falsifies itself when the contrary can be rationally maintained.

With the close of the scholastic period interest in the *Insolubilia* died out almost entirely. Stray references to the paradoxes contain only expressions of wonder that men could ever have been so misled by fondness for dispute as to split hairs on so vain a subject. Even formal logic neglected, and, in more modern times, the writers of logics not so formal despised, the problem—dismissing it as easily as Lotze, for instance, who said that there was no real difficulty involved when the speaker could change his statement to "I lied" instead of "I lie" and all paradox would be avoided. It was not until the rigorous formulation of modern mathematical logic forced them upon the notice of logicians that the *Insolubilia* were again recognised as significant. And they are significant; for their problem, the problem of propositions whose denial involves their assertion, is identical with the problem of the ontological proof of the existence of God. In the upshot it is the problem not only of the limits of rational discourse, but of the limits of rational thinking. Whether the judgment that "There is no truth in us" can itself be a truth in us, is a question to be determined by a close definition of truth in terms of proposition and assertion. That "this proposition is false" can be the substance of "this proposition" is a matter for strict definition of "this," and if there remains a paradox, that is, if the proposition then has contradictory results, the cure is to be sought in an amended definition, for the one that resulted in contradictories will be found to contain contradiction in its own statement.

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