1975

Lenses and Mirrors

LENSES AND MIRRORS

INTRODUCTION

If you have ever worn glasses, used a magnifying glass, looked through a telescope, or looked in a mirror, you have some idea of the effect that transmitting and reflecting materials have on light. When light passes from one material to another it is refracted. It is this property of light that is used in making eye glasses and magnifying glasses. The laws of reflection and refraction have immediate application in the construction of optical instruments. Two main objectives of most optical devices are to increase the light-gathering area and to provide a magnified image. Magnification is not usually the only requirement. For instance, the important characteristic of a large astronomical telescope is its diameter, which allows the telescope to gather more light, allow shorter exposures, and give higher resolution.

In this module we shall apply the laws of reflection and refraction to two types of simple devices, thin lenses and spherical mirrors. Understanding of these two simple devices sets the stage for the module Optical Instruments, which utilizes various combination of lenses and mirrors to produce the instruments' desired effects.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Solve problems using the law concerning the reflection of light rays from a surface (needed for Objective 1 of this module)	Reflection and Refraction Module
*Solve problems using Snell's law concerning a light ray passing from one substance to another (needed for Objective 2 of this module)	Reflection and Refraction Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Spherical mirrors - Solve problems involving a single spherical mirror by drawing ray diagrams and/or by applying the mirror equation. In addition you may be asked to calculate the lateral magnification.

2. Thin lenses - Solve problems involving a single thin lens by drawing ray diagrams and/or by applying the thin-lens equation. In addition you may be asked to calculate the lateral magnification.

GENERAL COMMENTS

In this module, you need to know the equations describing:

(1) The relationship between the image distance, the object distance, and the focal length for spherical mirrors.

(2) The relationship between the focal length and the radius of curvature of a spherical mirror.

(3) The lateral magnification of an object by a spherical mirror.

(4) The relationship between the image distance, the object distance, and the focal length for thin lenses.

(5) The relationship between the focal length, the radii of curvatures, and the indices of refraction of the lens material and the medium.

(6) The lateral magnification of an object by a thin lens.

You can accomplish this by memorizing the relevant equations along with the sign conventions, or you can study the derivations in order to understand them well enough to derive the equations yourself. We recommend this latter course since knowledge of the derivation of these equations can be used to consider more complex situations.

In addition, you will be expected to know and apply the rules for drawing ray diagrams for both spherical mirrors and thin lenses.

Lateral magnification is magnification in the direction perpendicular to the optical axis of the mirror or lens. Longitudinal magnification is magnification in the direction parallel to the optical axis of the mirror or lens.

To avoid confusion, note that this module uses the following notation:

n = index of refraction.

i = size of image.

o = size of object.

M = magnification.

s = object distance.

s' = image distance.

R = radius of curvature.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Sections 30.4 through 30.6 and 30.9 through 30.11 in Chapter 30. Then study Problems A and B and Illustrations 30.2 through 30.4 and 30.6 through 30.10 before working Problems C and D and Problems 3, 9, 13, and 19 in Chapter 30.

The important equations, keyed to number in the General Comments, are:

(1) Eq. (30.4) - p. 589. (2) Eq. (30.3) - p. 591. (3) Eq. 30.5) - p.
Sign conventions on p. 591, ray-diagram rules on pp. 592, 593.

(4) Eq. (30.15) - p. 601. (5) Eq. (30.14) - p. 601. (6) Eq. (30.16) - p. 602.
Sign conventions on p. 599, ray-diagram rules on p. 602.

Statement 2 in Section 30.11 is misleading, as is Figure 30.21(e). The conclusion that a ray passing through the center of a thin lens is essentially undeviated is correct; however, it is not necessarily true that "a ray through the center of the lens enters and leaves through parallel faces." Nor is it in general true that "such a ray is undeviated." For thin lenses (with the assumption of small angles, etc.) the faces are nearly (if not exactly) parallel, and the deviation of the beam is negligible.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 30)	Additional Problems (Chap. 30)
1	Secs. 30.4 to 30.6	A	Illus.[a] 30.2, 30.3, 30.4	C	Quest.[a] 1, 12, Probs. 3, 9	Quest. 2, 5, 17, Probs. 4 to 8
2	Secs. 30.9 to 30.11	B	Illus. 30.6 to 30.10	D	Quest. 11, Probs. 13, 19	Quest. 3, 4, 7 to 10, Probs. 10, 11, 12, 17, 18, 20, 21

[a]Illus. = Illustration(s). Quest. = Question(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 36, Sections 36-8 through 36-10. Answer questions 21, 24, and 30, study Problems A and B and Examples 6, 9, and 10, and work Problems C, D, and 33, 43, and 48 in Chapter 36.

The important equations, keyed to the numbers in the General Comments, are:

(1) Eq. (36-17) - p. 683. (2) Unnumbered equation preceding Eq. (36-17) - p. 683. Sign conventions on p. 683, ray-diagram rules on p. 685.

(3) Eq. (36-18) - p. 686. (4) Eq. (36-31) - p. 693. (5) Eq. (36-30) - p. 692.

(6) Eq. (36-32) - p. 693. Sign conventions on p. 688, ray-diagram rules p. 693.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text (Chap. 36)	Study Guide	Text (Chap. 36)	(Chap. 36)
1	Sec. 36-8	A	Ex.[a] 6	C	Quest.[a] 21, Prob. 33	Quest. 19, 20, Prob. 34
2	Secs. 36-9, 36-10	B	Ex. 9, 10	D	Quest. 24, 30, Probs. 43, 48	Quest. 22, 23, 25 to 29, 31, 32, Probs. 40, 41, 42, 44, 47, 52

[a]Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 39, Sections 39-3 through 39-5 and Chapter 40, Sections 40-1 through 40-4, 40-6, and 40-8. Study Problems A and B and the Examples listed in the Table. Then work Problems C and D and Problems 39-8, 40-10, and 40-17.

The important equations, keyed to the numbers in the General Comments, are:

(1) Eq. (39-9) - p. 563. (2) Eq. (39-8) - p. 563. (3) Eq. (39-6) - p. 559. Sign conventions, p. 558. Ray-diagram rules, p. 564.

(4) Eq. (40-3) - p. 575. (5) Eq. (40-2) - p. 574. (6) Eq. (40-4) - p. 576. Sign conventions, p. 558. Ray-diagram rules, p. 578.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 39-3 to 39-5	A	Examples 1 and 2 in Sec. 39-3, Example in Sec. 39-5	C	39-8	39-5, 39-6, 39-7, 39-9
2	Secs. 39-7, 40-1 to 40-4, 40-6, 40-8	B	Example in Sec. 40-2	D	40-10, 40-17	40-9, 40-11, 40-13, 40-14

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

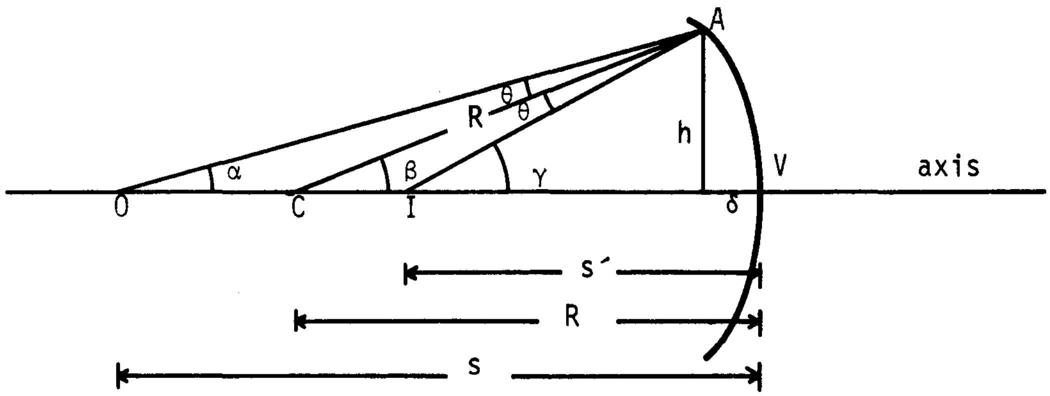
SUGGESTED STUDY PROCEDURE

Since your text does not treat spherical mirrors, you should read this study guide for the relevant development; then read Chapter 37, Sections 37-1, 37-2, 37-4. Study Problems A and B and Example 37-1, besides the problems below. Then work Problems C, D, and Problems 37-1 and 37-2 in your text, besides Problems 1 through 6 in the following discussion of spherical mirrors.

Spherical Mirrors

Consider a concave mirror of radius R as shown in Figure 1. Let there be a point source of light at O. One ray of light from O will go directly from O to V and be reflected back on itself. Pick another arbitrary ray emanating from O and hitting the mirror at A. It is reflected at A and crosses the first ray at I. Since the angle of incidence is equal to the angle of reflection, the angles made by the second ray with the radius R are equal. From the diagram we have

Figure 1



WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	This study guide	A		C, 1 to 6		
2	Secs. 37-1, 37-2, 37-4	B	Ex.[a] 37-1	D	37-1, 37-2	37-3, 37-4, 37-7, 37-8, 37-10, 37-13, 37-15, 37-18 to 37-27

[a]Ex. = Example(s).

$\tan\alpha = h/(s - \delta)$, $\tan\beta = h/(R - \delta)$, and $\tan\gamma = h/(s' - \delta)$.

Since for small angles, $\tan\alpha \simeq \alpha$,

$\alpha = h/(s - \delta)$, $\beta = h/(R - \delta)$, and $\gamma = h/(s' - \delta)$.

Looking at the geometry of the situation we also have (why?)

$\beta = \alpha + \theta$ and $\gamma = \beta + \theta$.

Eliminating θ we have $2\beta = \alpha + \gamma$. Combining this last equation with the first three and using the small-angle approximation, we get

$$\frac{h}{s - \delta} + \frac{h}{s' - \delta} = \frac{2h}{R - \delta}.$$

Since α, β, and γ are small, δ will also be small and can be neglected; thus,

$$1/s + 1/s' = 2/R. \qquad (1)$$

Now let $s \to \infty$, and Eq. (1) becomes

$$1/s' = 2/R.$$

As $s \to \infty$, the rays from O become parallel to the axis. When parallel rays strike the mirror they are focused at a point called the focus or focal point of the mirror. The distance of the focus from the mirror is called the focal length (f). Thus we can rewrite Eq. (1) as

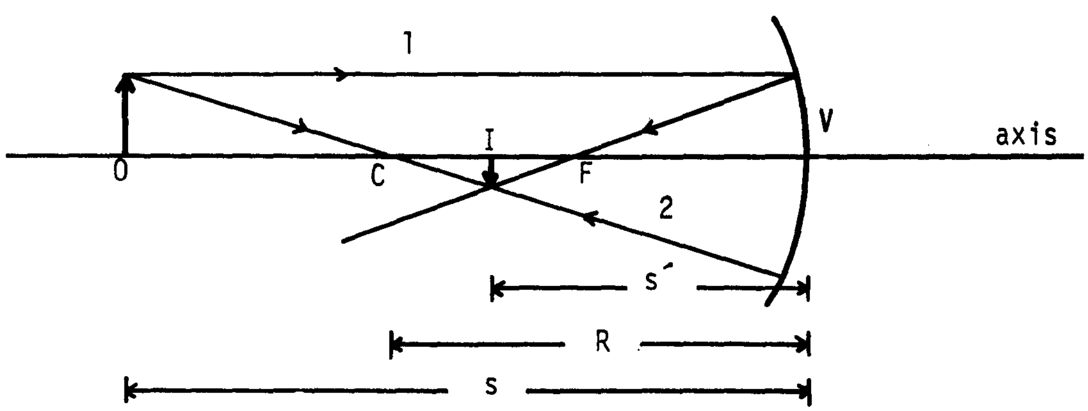
$$1/s + 1/s' = 1/f, \qquad (2)^*$$

where

$$f = R/2. \qquad (3)^*$$

Now consider an object (arrow) placed at O (see Figure 2). A ray (labeled 1) from the head of the object parallel to the axis will go through the focus. A ray [2] from the head of the object through C (the center of the spherical surface) must

Figure 2



be reflected back on itself. It can be shown that where these two rays cross all other rays emanating from the head of the object also meet, and an image of the object is formed at this distance from the mirror. Denote the distance from the mirror to this image by s´, the distance from the object (o) to the mirror by s, and we have the mirror equation

$$1/s + 1/s' = 1/f.$$

Now let us solve for the magnification of the mirror. Using similar triangles,

$$i/(R - s') = o/(s - R),$$

where i is the size of the image and o is the size of the object. The magnification of the mirror is defined as the ratio of the image size to the object size (actually lateral extent from the axis), i.e.,

$$m \equiv i/o = (R - s')/(s - R).$$

Solving for R in terms of s and s´ in Eq. (1) we have

$$R = 2ss'/(s + s').$$

Substituting this into the equation for m we have

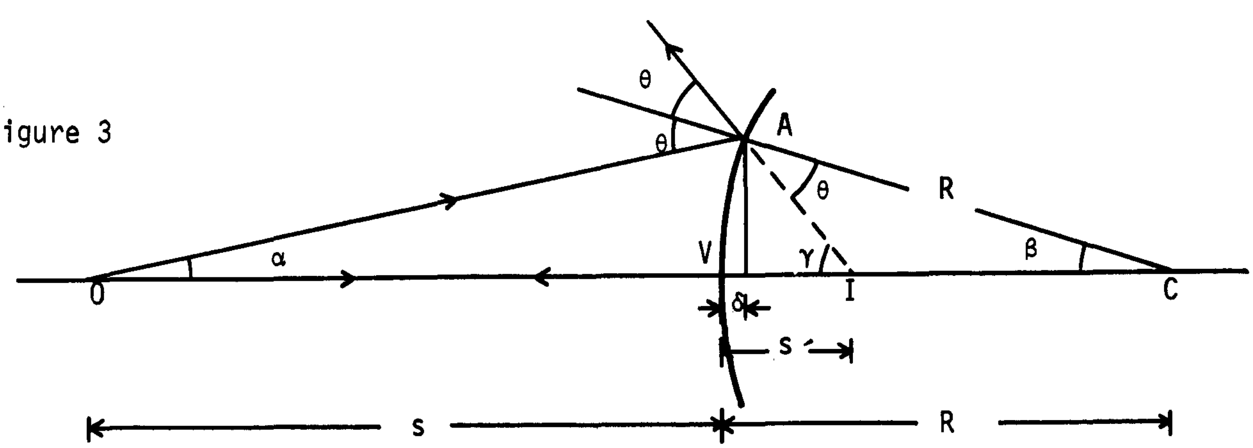
$$m = \frac{2ss'/(s + s') - s'}{s - 2ss'/(s + s')} = \frac{(2ss' - ss' - s'^2)/(s + s')}{(s^2 + s's - 2ss')/(s + s')} = \frac{ss' - s'^2}{s^2 - ss'}$$

$$= s'(s - s')/s(s - s') = s'/s. \qquad (4)^*$$

Now consider what happens when we have a convex mirror, as in Figure 3. The ray from O to A will be reflected and appear to the observer to cross the axis at I. The ray from O to V will be reflected back on itself and appear to the observer to cross the extension of the reflected ray at I. Again we have

$$\tan\alpha = h/(s + \delta), \quad \tan\beta = h/(R - \delta), \quad \text{and} \quad \tan\gamma = h/(s' - \delta).$$

Figure 3



Using the small-angle approximation, we then have

$$\alpha = h/(s + \delta), \quad \beta = h/(R - \delta), \quad \text{and} \quad \gamma = h/(s' - \delta).$$

Again using geometry we have

$$\theta = \alpha + \beta, \quad \gamma = \theta + \beta, \quad \text{or} \quad \gamma = \alpha + 2\beta, \quad \text{and} \quad \frac{h}{s' - \delta} = \frac{h}{s + \delta} + \frac{2h}{R - \delta}.$$

Again ignoring δ because of the small-angle assumption we have $1/s - 1/s' = -2/R$. Notice how this equation compares with Eq. (1). We can make the equations identical if we adopt the following conventions for spherical mirrors:

(a) The object distance is always greater than zero, i.e., $s > 0$.

(b) For the image distance,
if $s' > 0$, then light rays actually pass through the image location;
if $s' < 0$, then light rays do not pass through the image location.

(c) For the radius of curvature,
$R > 0$ when the mirror is concave to the object;
$R < 0$ when the mirror is convex to the object.

As an exercise, you should show that the absolute value of the magnification of a convex mirror is $m = s'/s$.

In order to draw ray diagrams for spherical mirrors you can use the following:

Rule 1: An incident ray parallel to the axis is reflected through the focal point of the mirror (Ray 1 in Figures 4 and 5).

Rule 2: A ray along the radius of curvature is reflected back on itself (Ray 2 in Figures 4 and 5).

Rule 3: A ray through the focal point is reflected by the mirror parallel to the axis (Ray 3 in Figures 4 and 5).

The comment in Section 37-2 on lens aberration by and large can also be applied to mirrors for essentially the same reasons. Statement 1 on p. 756 is misleading and should read, "For all practical purposes, a ray passing through the center of the lens is essentially unchanged, inasmuch as its deviation is small (the lens

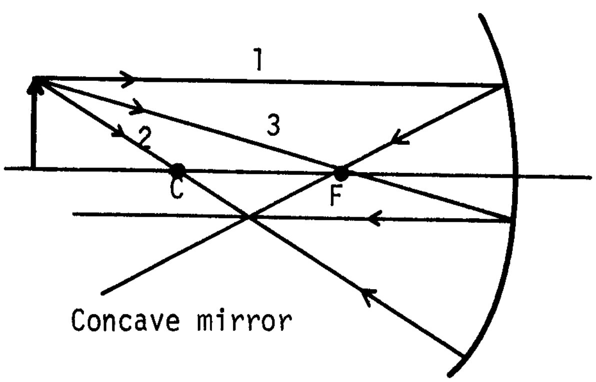


Figure 4

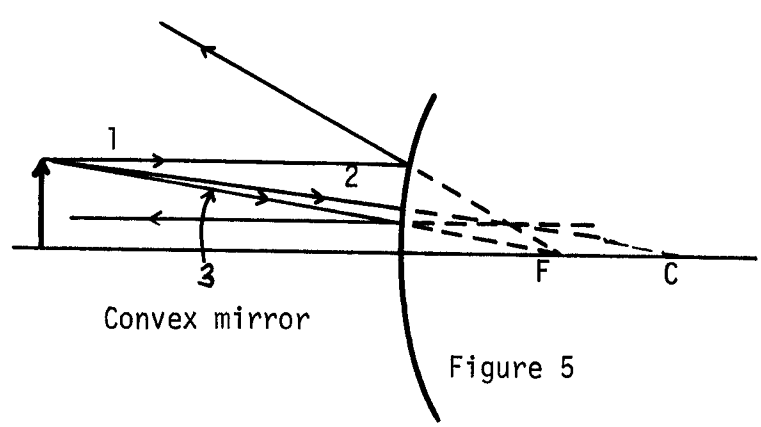


Figure 5

faces are parallel or are nearly parallel here) and it is not appreciably displaced laterally (the lens is very thin)."

The important equations keyed to the numbers in the General Comments are:

(1) Eq. (2)*. (2) Eq. (3)*. (3) Eq. (4)*. Signs and rules given above.

(4) Eq. (37-2) - p. 757. (5) Eq. (37-4b) - p. 768. (6) Eq. (37-1) - p. 753. Sign conventions given on p. 769; ray-diagram rules on p. 756.

Problems

1. In Figure 6, locate the image (a) graphically and (b) by use of the appropriate equation. Is it real or virtual? Erect or inverted? What is the lateral magnification?

2. In Figure 7, locate the image (a) graphically and (b) by use of the appropriate equation. Is it real or virtual? Erect or inverted? Lateral magnification?

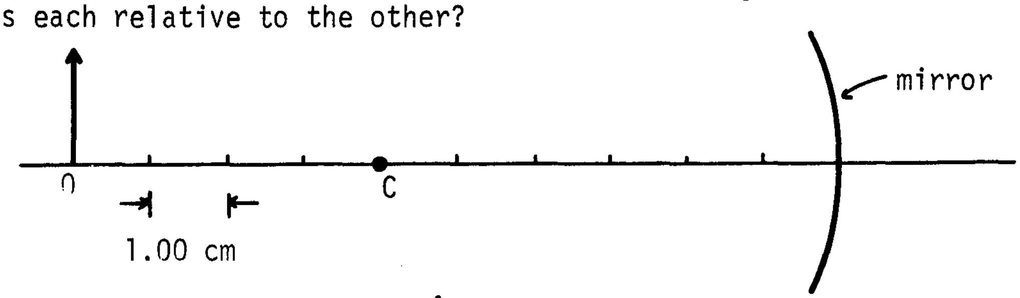
3. An object of height 6.0 mm is placed 60 cm from a concave mirror of radius of curvature 30 cm. What is the nature of the image and what is the image distance? Draw an appropriate ray diagram.

4. Where must the object be placed so that the image formed by a convex mirror is one-half as far from the mirror as the object? What is the magnification?

5. A concave mirror of radius 40 cm has an object located 50 cm in front of it. (a) Locate the image graphically. (b) Calculate the image location. (c) Calculate the magnification. (d) Is the image real or virtual?

6. Most truck drivers use both plane and convex mirrors for looking to the rear. What advantage has each relative to the other?

Figure 6



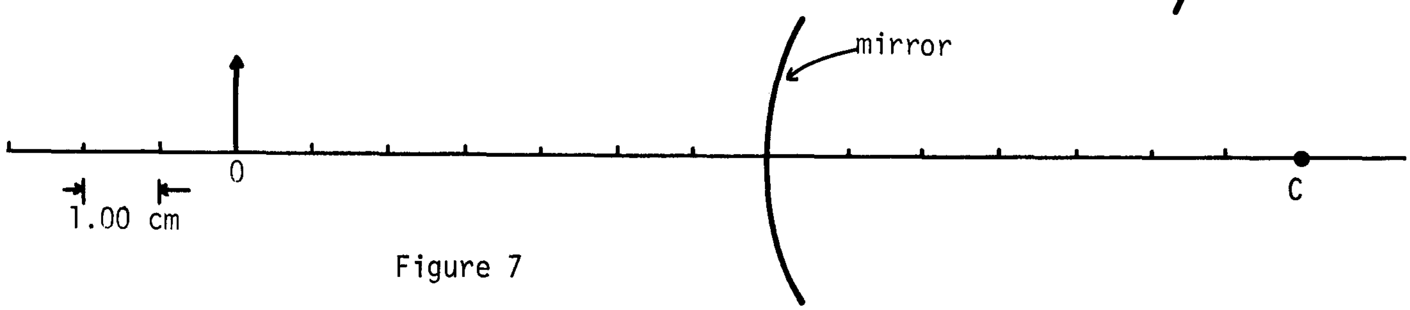


Figure 7

Answers to Problems

1. (b) 4.3 cm. Real. Inverted. 0.43. 3. 20.0 cm. Real. Inverted. 2.00 mm high. 5. (b) 33 cm. (c) 0.67. (d) Real.

PROBLEM SET WITH SOLUTIONS

A(1). Given a convex spherical mirror of radius R, discuss what happens to the image as the object starts at infinity and approaches the mirror.

Solution

For a convex mirror we have

$1/s + 1/s' = -2/R$ and $M = s'/s$. $1/s' = -2/R - 1/s$. Thus

$1/s' = -(2s + R)/sR$ and $s' = -Rs/(2s + R)$.

Since $s > 0$ and $R > 0$, $s' < 0$, the image is always virtual. As $s \to \infty$ we have

$$s' = \frac{-R}{2 + R/s} \to -\frac{R}{2}$$ and the image size goes to 0.

When $s = R$, $s' = -R^2/3R = -R/3$, and the image is one-third the size of the object. As $s \to 0$, $s' \to 0$, and the image size goes to infinity. Actually, the small-angle approximation breaks down here and our formulas are no longer valid.

B(2). Given a diverging lens of focal length f, discuss what happens to the image as the object starts at infinity and approaches the lens.

Solution

For a diverging lens we have $1/s + 1/s' = -1/f$ and $h'/h = s'/s$. $1/s' = -1/f - 1/s$. Thus

$1/s' = -(f + s)/fs$ and $s' = -fs/(f + s)$.

Since $s > 0$ and $f > 0$, $s' < 0$. Therefore the image is always virtual. As $s \to \infty$ we have

$$s' = -\frac{f}{1 + f/s} \to -f,$$ and the image size $\to 0$.

When $s = f$, we have $s' = -f^2/2f = -f/2$, and the image is one-half the size of the object. As $s \to 0$, $s' \to 0$ and the image size $\to \infty$. Actually, the small-angle approximation breaks down and our formulas are no longer valid.

Problems

C(1). A spherical mirror has a focal length of +30.0 cm. An object 1.00 cm high is located 20.0 cm in front of the mirror.
(a) Locate the image graphically.
(b) Calculate the location of the image.
(c) Calculate the radius of curvature for the mirror.
(d) Calculate the size and orientation of the image.

D(2). Where is the image of an object that is placed at the focal point of a converging lens?

Solutions

C(1). See Figure 8, for part (a). (b) -60 cm. (c) 60 cm. (d) Image virtual, upright, and 3.00 cm high.

D(2). At infinity.

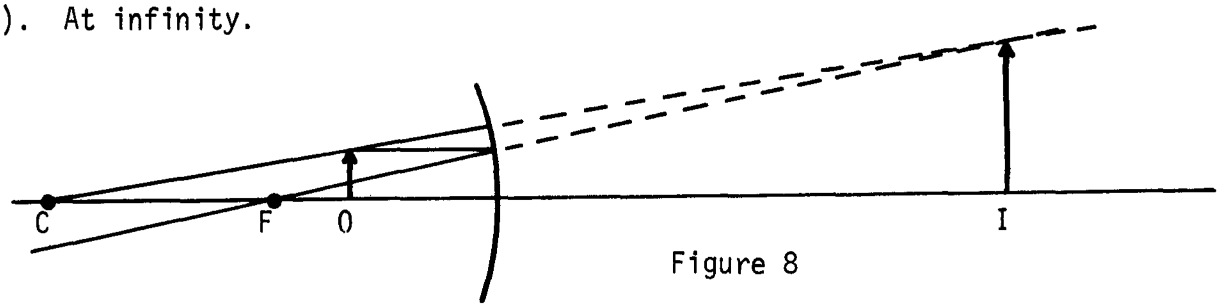


Figure 8

PRACTICE TEST

1. An object is located 20.0 cm in front of a convex mirror that forms a virtual image 5.0 cm behind the mirror.
   (a) Calculate the focal length of the mirror.
   (b) Draw a ray diagram and solve the problem graphically.

2. Explain how a magnifying glass works in terms of object and image relative to focal point.

Practice Test Answers

1. (a) -6.7 cm. (b) See Figure 9.

2. 0 < (object distance)/f < 2 for magnification > 1.

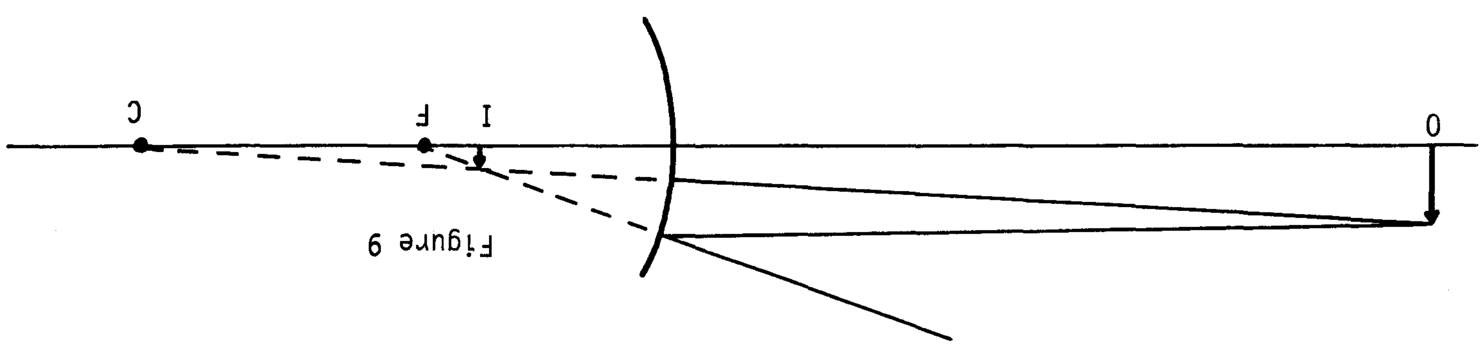


Figure 9