1975

Ohm's Law

Module _____
STUDY GUIDE

OHM'S LAW

INTRODUCTION

Recently you have heard many ways of reducing energy consumption in the home. One of the suggested ways is to use 60-W light bulbs rather than 100-W bulbs; another is to cut back on the use of electrical appliances. You readily identify these suggestions with decreasing the amount of "electricity" being transported through the wires coming into your home.

You were warned quite early in life not to stick a metal knife into the toaster to force out burning bread; you wither unplug the toaster or use a utensil with a wooden handle. Why? Because you were warned of the consequences of coming into electrical contact with the heating element - consequences most of us have experienced at one time.

Several of the topics covered in this module, such as electrical current and the energy associated with it, are familiar to us all. When you have finished studying this module, you will be able to relate the material to many every-day experiences.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Understand the relationship between electrical charge and matter (needed for Objectives 1 and 2 of this module)	Coulomb's Law and the Electric Field Module
*Apply the concept of electric field to charges (needed for Objectives 1 and 2 of this module)	Coulomb's Law and the Electric Field Module
*Relate electric fields and potential to work and energy (needed for Objectives 3 and 4 of this module)	Electric Potential Module
*Apply the definition of power (needed for Objective 4 of this module)	Work and Energy Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Current - Apply the definition of current or current density to problems in conductors in which these are related to electric charge.

2. Drift speed - Apply the microscopic model for conduction in a metal to problems where you are given information concerning several of the quantities (drift speed, current, current density, charge, charge density, cross-sectional area) and are asked to solve for another.

3. Ohm's law - Solve problems using the relationships among resistance, potential difference, electric current, current density, electric field, resistivity, and the physical dimensions of a conductor.

4. Electric power - Apply the relation for instantaneous power ($P = IV$) to problems dealing with resistive elements obeying Ohm's law.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 21, Sections 21.2, 21.3, 21.4, and 21.9. The definition of current is given in Eq. (21.1) and generalized in the preceding sentence. Read General Comment 1 regarding the vector notation given to the current in this equation. Current density is defined in Eq. (21.3). An explanation of drift velocity is given in Section 21.2, p. 392. For a more expanded description of the microscopic model for drift speed, you might like to refer to Elementary Classical Physics or Fundamentals of Physics.* Equations (21.2) and (21.3) give the relationship of current or current density to the properties of the charge carriers as called for in Objective 2. Read General Comment 2. The definition of resistance is given in Eq. (21.7). Read General Comment 3. Ohm's law is sometimes stated in terms of microscopic quantities as in Eq. (21.6), where the resistivity ρ is assumed constant. The relation for the power input or output of an element in an electrical circuit is given in Eq. (21.14). In Illustration 21.4, Ohm's law is substituted into this power relation to give $P = I^2R$ or V^2/R. These two equations are well known as Joule's law and hold only for elements with constant resistance. After completing these readings, study Problems A through H, and work Problems I through L and Problems 1, 3, 6, 8, and 15 in Chapter 21. Take the Practice Test and if you are still having any difficulties, work the Additional Problems before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 21)	(Chap. 21)
1	General Comment 1, Sec. 21.2	A, B	Illus.[a] 21.2	I	1, 3	2, 4
2	General Comment 2, Sec. 21.2	C, D	Illus. 21.2	J	6	
3	General Comment 3, Sec. 21.3	E, F	Illus. 21.3	K	8	
4	Sec. 21.9	G, H	Illus. 21.4, 21.5	L	15	14, 17

[a]Illus. = Illustration(s).

*Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2, p. 544.

David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974), pp. 508, 514.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read all of Chapter 27. The general definition of current is given in Eq. (27-1). Read General Comment 1 and Section 27-1. Current density is introduced in non-vector form in Eq. (27-3) for the case where the current is distributed uniformly across the conductor. Read General Comment 2 and Example 2. The concept of drift speed is introduced in Section 27-1 (p. 508), and the relation of current density or current to the properties of the charge carriers is developed at the end of this section (p. 509). A very nice discussion of the microscopic model for metals appears in Section 27-4. Read General Comment 3. The definitions for resistance and resistivity are given in Eqs. (27-5) and (27-6), respectively. Compare these equations carefully to relate the macroscopic variables to their microscopic counterparts. For a homogeneous conductor of length ℓ and cross-sectional area A, the relation for resistance in terms of resistivity and the physical dimensions of the conductor is given by Eq. (27-8).

A printing error may be noticed in Table 27-1. The missing labels are Column 1: Metal; Column 2: Resistivity at 20°C (ohm-m); Column 3: Temperature coefficient of resistivity,* (α per C°); Column 4: Density (gm/cm^3); and Column 5: Melting point (°C). Compare the voltage-current plots in Figures 27-4 and 27-6 - these distinguish between ohmic and nonohmic devices. The general relation for the power, or rate of transfer of electrical energy to a device through which charges flow, is given by Eq. (27-13). If the device obeys Ohm's law, Eqs. (27-14) and (27-15) can be useful. After completing the readings, study Problems A through H, and work Problems I through L in the study guide and Problems 1, 3, 6, 7, 12, 16, 23, 25, and Questions 12 and 14 in Chapter 27. Take the Practice Test, and if you are still having any difficulties, work the Additional Problems before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 27)	Additional Problems (Chap. 27)
1	General Comment 1, Sec. 27-1	A, B	Ex.[a] 1	I	1, 3	4
2	General Comment 2, Secs. 27-1, 27-4	C, D	Ex. 2	J	6	
3	General Comment 3, Secs. 27-2, 27-3	E, F		K	7, 12, 16	8, 9, 11
4	Sec. 27-5	G, H	Ex. 3	L	Quest.[a] 12, 14, Probs. 23, 25	24

[a]Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 28, Sections 28-1 through 28-4, 28-6 (neglect p. 400), and 28-7, Part 1. The definition of current is given by Eq. (28-1). Read General Comment 1 and p. 389. Read General Comment 2. Equations (28-2) and (28-4) relate current and current density to the properties of the charge carriers. A nice discussion of the microscopic model for conduction in a metal appears in Section 28-3. Although the derivation of Eq. (28-8) is not essential to the objectives of this module, it demonstrates the beauty of developing models for various physical phenomena. Read General Comment 3. Equations (28-5)(scalar form) and (28-6)(vector form) are the defining equations for resistivity. A general relation is developed for resistance [Eq. (28-12)], but for our purposes Eq. (28-13) will suffice. A general relation between resistance and resistivity is given in Eq. (28-10); however, Eq. (28-11) is adequate for our use. If the resistance in Eq. (28-13) is constant then the relation is known as Ohm's law. Equation (28-6) is also called Ohm's law (in microscopic variables), if ρ is constant. The general relation for the power or rate of transfer of electrical energy to a device through which charges flow is given by Eq. (28-23). If the element obeys Ohm's law (called pure resistance on p. 401) then we can use Eq. (28-24), which is called Joule's law. If time permits you will find the material in Sections 28-8 and 28-9 on thermocouples very interesting, but you will not be held responsible for it in this module.

After completing these readings, study Problems A through H, and work Problems I through L and Problems 28-4, 28-5, 28-1, 28-9, 28-12, 28-20, 28-21 in your text. Take the Practice Test and if you are still having any difficulties, work Problem 28-3 before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	General Comment 1, Sec. 28-1	A, B		I	28-4, 28-5	28-3
2	General Comment 2, Secs. 28-1, 28-3	C, D	Sec. 28-1, Example (p. 390)	J	28-1	
3	General Comment 3, Secs. 28-2, 28-4, 28-6	E, F		K	28-9, 28-12	
4	Sec. 28-7	G, H		L	28-20, 28-21	

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read Chapter 27, Sections 27-1 through 27-6. The definition of current is given by Eq. (27-1), and a very thorough discussion follows in Section 27-1. A relation for current density is given in Eq. (27-5), and a more general integral form is given in Eq. (27-7). Read General Comments 1 and 2. The concept of drift velocity is introduced in Section 27-1 (p. 536), and a very good discussion of the microscopic model for its development is given in Sections 27-4 and 27-5. The relation between current density or current to the properties of the charge carriers appears in Eqs. (27-3) and (27-6). A macroscopic statement of Ohm's law is given by Eq. (27-12), but it should be carefully noted that in the sentence following this equation the authors state that this same relation when R is not necessarily constant is the defining equation of electrical resistance for any dissipative element. Therefore in Figure 27-6 you can determine a resistance value for any three of the curves, for a given voltage or current. But only for the conductor in part (c) does the resistance remain constant as the voltage or current varries. The definition of resistivity is given in Eq. (27-17), and if the resistivity is constant we have a microscopic statement of Ohm's law. The relation between resistance and resistivity for a conductor of length L and cross-sectional area A is given in Eq. (27-14). Read General Comment 3. The general relation for the power or rate of transfer of electrical energy to or from a device through which a charge flows is given by Eq. (27-10). If the device

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 27-1, 27-2, General Comment 1	A, B		I	27-1, 27-4	27-2, 27-5
2	General Comment 2, Secs. 27-1, 27-4, 27-5	C, D	Ex.[a] 27-1	J	27-21	27-15(c)
3	General Comment 3, Secs. 27-4, 27-5	E, F		K	27-14, 27-17, 27-19	27-13, 27-15 (a), (b)
4	Secs. 27-3, 27-4	G, H		L	27-6, 27-11	27-9, 27-10

[a]Ex. = Example(s).

obeys Ohm's law the power relation is conveniently given by Eq. (27-13). This equation is a particular way of writing the conservation of energy principle for the special case in which electrical energy is transformed into internal energy.

After completing these readings, study Problems A through H, and work Problems I through L and Problems 27-1, 27-4, 27-21, 27-14, 27-17, 27-19, 27-6, and 27-11 before taking the Practice Test. If you have any difficulties, work the Additional Problems before attempting a Mastery Test.

GENERAL COMMENTS

1. Vector Notation - Current

Most physicists prefer not to treat current as a vector. A convention for labeling current is needed because charges of opposite sign move in opposite directions in an electric field. Hence for simplicity of treatment you may treat all charge carriers as positive and draw arrows (not representative of vectors) in the direction that these charges would move. This is usually termed conventional current (the direction that positive charges would move in a wire). (For currents in wires a negative charge moving in one direction is equivalent to a positive charge moving in the opposite direction. See Problem A.) Current density is a vector that at any given point in the conductor has a direction determined by the volocity vector of a positive charge moving through that point. The current is the same everywhere along a wire. This follows from conservation of charge, i.e., electrons just do not appear and disappear along the wire. This is a very simple but very important concept and will be used in problem solving in the module Direct Current Circuits.

2. Thermal Speed

We would like to stress that the average thermal speed of the random moving electrons is on the order of 10^8 times larger than the drift speed of these electrons. Even though the drift speed is surprisingly small, a change in the force that produced this motion can propagate through the conductor at nearly the speed of light (3.00×10^8 m/s). This is why, when you turn on the light switch, the light goes on "instantaneously." The speed of the electrons in a conductor subjected to a potential difference is very small. Even though the electrons in a metal have such small drift speed, the phenomenon responsible for it - the electric field - travels along the wire at nearly the speed of light.

3. Ohm's Law

It should be stressed that only when the resistance R is constant is the expression V = IR called Ohm's law. In a current-voltage (IV) plot as in Figure 1, the resistance for a given value of voltage and current is the reciprocal of the tangent of the curve at that point, and if R is independent of current and voltage, then the curve is straight line. Ohm's law is sometimes stated in terms of microscopic quantities, where the resistivity is assumed constant. Ohm's law follows from the definition of resistance when the resistance is independent of current and voltage. In this module we confine our discussion to homogeneous conductors and electrostatic fields.

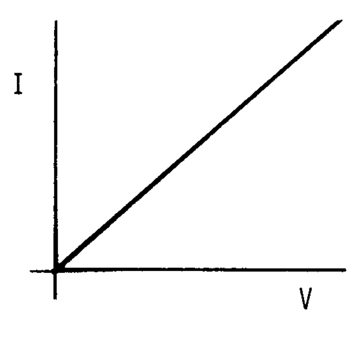

Figure 1

PROBLEM SET WITH SOLUTIONS

A(1). A current (Fig. 2) is established in a gas-discharge tube when a sufficiently high potential difference is applied across the two electrodes in the tube. The gas ionizes; electrons move toward the positive terminal and positive ions toward the negative terminal. What are the magnitude and sense of the current in a hydrogen-discharge tube in which 3.1×10^{18} electrons and 1.1×10^{18} protons move past a cross-sectional area of the tube each second?

Solution

The number of electrons passing cross-sectional area AA in 1.00 s is 3.10×10^{18}, and since the charge on an electron is -1.60×10^{-19} C, the negative charge crossing AA to the left per second is

$$I_- = (3.10 \times 10^{8}\ \text{electrons/s})(-1.60 \times 10^{-19}\ \text{C/electron}) = -5.0 \times 10^{-1}\ \text{A}.$$

The number of protons crossing AA per second is 1.10×10^{18}, and since the charge on the proton is $+1.60 \times 10^{-19}$ C, the positive charge crossing AA to the right per second is

$$I_+ = (1.10 \times 10^{18}\ \text{protons/s})(1.60 \times 10^{-19}\ \text{C/proton}) = 1.76 \times 10^{-1}\ \text{A}.$$

The negative charge moving to the left is equivalent to positive charge moving to right, thus

$$I_{total} = |I_-| + |I_+| = 6.8 \times 10^{-1}\ \text{A} \quad \text{to the right.}$$

B(1). The current in a wire varies with time according to the relation

$$I = 20 \sin(120\pi t),$$

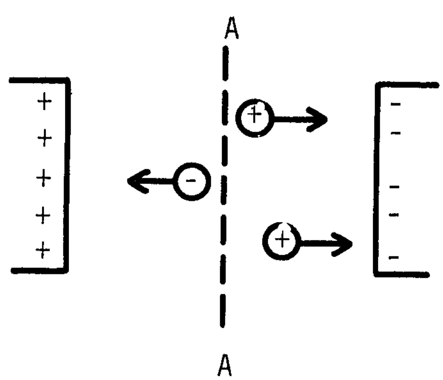

Figure 2

where I is in amperes, t in seconds, $120\pi t$ in radians, and the constants are 20 A and 120/s.

(a) How many net coulombs of charge pass a cross section of the wire in the time interval between t = 0 and t = 1/60 s?

(b) In the interval between t = 0 and t = 1/120 s?

(c) What constant current would transport the same charge in each of the intervals above?

Solution

(a) From the definition of current I = dQ/dt or dQ = I dt, thus

$$Q = \int_{t_i}^{t_f} I\,dt = \int_0^{1/60\text{ s}} 20\sin(120\pi t)\,dt = -\frac{20}{120\pi}[\cos(120\pi t)]\Big|_0^{1/60\text{ s}}$$

$$= -(1/6\pi)(\cos 2\pi - \cos 0) = -(1/6\pi)(1 - 1) = 0.$$

A plot of current versus time would look like Figure 3. Therefore at t = 0, I = 0; it increases to 20 A at t = 1/240 s and decreases to zero at t = 1/120 s. Now the current changes direction and increases to 20 A at t = 1/80 s in the opposite direction. One can therefore see that as much charge flowed to the right through our cross section during the first 1/120 s as flowed to the left through the cross section during the next 1/120 s. Thus the net charge passing the cross section in 1/60 s is 0.

(b) $$Q = \int_0^{1/120\text{ s}} 20\sin(120\pi t)\,dt = -\frac{20}{120\pi}(\cos\pi - \cos 0) = -\frac{1}{6\pi}(-1 - 1)$$

$$= \frac{2}{6\pi} = \frac{1}{3\pi} = 0.106\text{ C}.$$

(c) For a constant current I = Q/t. For 1/60 s, Q = 0, so I = 0. For 1/120 s, Q = 0.106 C, so

$$I = \frac{1/3\pi\text{ C}}{1/120\text{ s}} = \frac{40}{\pi}\text{ A} = 12.7\text{ A}.$$

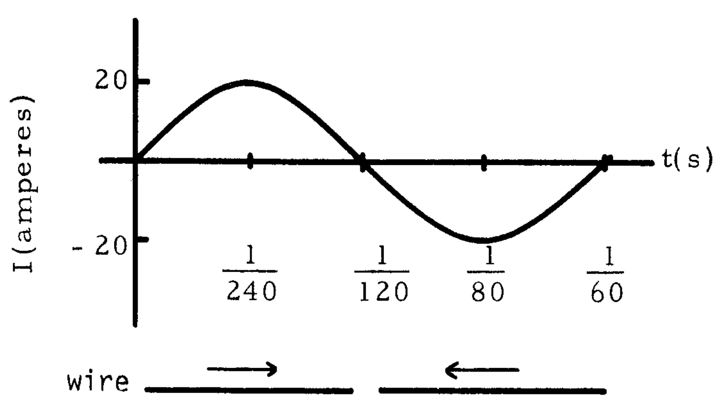

Figure 3

C(2). A small but measurable current of 1.00×10^{-10} A exists in a copper wire whose diameter is 0.250 cm. Calculate the electron drift speed.

Solution

$J = nev_d$, Current density = (Charge per unit volume)(electronic charge)(drift speed),

$v_d = J/ne$. N_0 is Avogadro's number, d is mass density, e_f is free electrons, and M is atomic weight:

$$n = \frac{N_0 d(e_f/\text{atom})}{M} = \frac{(6.0 \times 10^{26}\ \text{atoms/kg mol})(9.0 \times 10^3\ \text{kg/m}^3)(1\ \text{electron/atom})}{64}$$

$$= 8.4 \times 10^{28}\ \text{electrons/m}^3,$$

$$J = \frac{I}{\text{area}} = \frac{1.00 \times 10^{-10}\ \text{A}}{\pi(0.250/2 \times 10^2\ \text{m})^2} = 2.00 \times 10^{-5}\ \text{A/m}^2,$$

$$v_d = \frac{J}{ne} = \frac{2.00 \times 10^{-5}\ \text{A/m}^2}{(8.4 \times 10^{28}\ \text{electrons/m}^3)(1.60 \times 10^{-19}\ \text{C/electron})} = 1.48 \times 10^{-15}\ \text{m/s}.$$

D(2). A typical copper wire might have 2.00×10^{21} free electrons in 1.00 cm of its length. Suppose the drift speed of the electrons along the wire is 0.050 cm/s.

(a) How many electrons would pass through a given cross section of wire each second?

(b) How large a current would be flowing in the wire?

Solution

(a) nAv_d = (electrons/volume)(cross-sectional area)(length/second)

= [electrons/(length)(area)](area)(length/second)

= number of electrons crossing a cross-sectional area in one second

= (electrons/length)(length/second)

$= [(2.00 \times 10^{21}\ \text{electrons})/(1.00\ \text{cm})]5.0 \times 10^{-2}\ \text{cm/s} = 1.00 \times 10^{20}\ \text{electrons/s}.$

(b) $e = 1.60 \times 10^{-19}$ C/electron,

$I = (1.60 \times 10^{-19}\ \text{C/electron})(1.00 \times 10^{20}\ \text{electrons/s}) = 16.0\ \text{A}.$

E(3). A copper wire and an iron wire of the same length have the same potential difference applied to them. See Figure 4.

(a) What must be the ratio of their radii if the current is to be the same?

(b) Can the current density be made the same by suitable choices of the radii? Take

$$\rho_{Cu} = 1.70 \times 10^{-8}\ \Omega\ m, \qquad \rho_{Fe} = 1.00 \times 10^{-7}\ \Omega\ m.$$

Solution

V_{AB} = potential difference between A and B. Ohm's law, V = IR:

$$V = I\rho\ell/A = I\rho\ell/\pi r^2.$$

(a) $$\frac{(V_{AB})_{Cu}}{(V_{AB})_{Fe}} = \frac{I_{Cu}\rho_{Cu}\ell_{Cu}/\pi r_{Cu}^2}{I_{Fe}\rho_{Fe}\ell_{Fe}/\pi r_{Fe}^2}$$

where $\ell_{Cu} = \ell_{Fe}$, $(V_{AB})_{Cu} = (V_{AB})_{Fe}$, $I_{Cu} = I_{Fe}$

$$1 = \frac{\rho_{Cu}/r_{Cu}^2}{\rho_{Fe}/r_{Fe}^2} \text{ or } \frac{r_{Fe}^2}{r_{Cu}^2} = \frac{\rho_{Fe}}{\rho_{Cu}} \text{ or } \frac{r_{Fe}}{r_{Cu}} = \left[\frac{1.00 \times 10^{-7}}{1.70 \times 10^{-8}}\right]^{1/2} = 2.43.$$

(b) From above, $1 = \dfrac{I_{Cu}\rho_{Cu}r_{Fe}^2}{I_{Fe}\rho_{Fe}r_{Cu}^2}$. Therefore

$$I_{Fe}\rho_{Fe}r_{Fe}^2 = I_{Cu}\rho_{Cu}r_{Cu}^2 \text{ or } I_{Fe}\rho_{Fe}/r_{Fe}^2 = I_{Cu}\rho_{Cu}/r_{Cu}^2.$$

Multiplying both sides by $1/\pi$ and noting that $\pi r_{Fe}^2 = (\text{area})_{Fe}$ and $I_{Fe}/(\text{area})_{Fe}$ $= J_{Fe}$, we have $J_{Fe}\rho_{Fe} = J_{Cu}\rho_{Cu}$ or $J_{Fe}/J_{Cu} = \rho_{Cu}/\rho_{Fe}$.

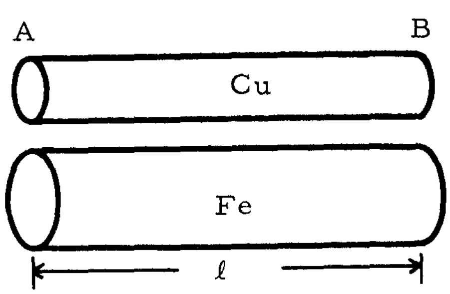

Figure 4

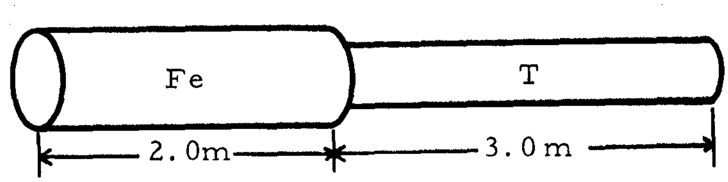

Figure 5

If $J_{Fe} = J_{Cu}$, then $\rho_{Fe} = \rho_{Cu}$. Since this is impossible, $J_{Fe} \neq J_{Cu}$. A shorter argument might be given by using the relation $E = \rho J$. Since $E = V/\ell$ is the same for both wires, J must be different for different values of ρ.

F(3). An iron wire of 1.00 m diameter and 2.00 m length is connected to a tungsten wire of 1.00 Ω resistance and 3.00 m length. A potential difference of 60 V is applied to the combined wire 5.0 in length. The resistivity of iron is 1.00×10^{-7} Ω m.

(a) What is the current flowing in the irom wire? In the tungsten wire?

(b) What is the electric field in the iron wire?

Solution

(a) See Figure 5.

$$R_{Fe} = \rho_{Fe} \frac{\ell_{Fe}}{(\text{area})_{Fe}} = \frac{(1.00 \times 10^{-7}\ \Omega\ \text{m})(2.00\ \text{m})}{\pi(5.0 \times 10^{-4}\ \text{m})^2} = 0.254\ \Omega,$$

$$R_{total} = R_{Fe} + R_T = 1.20\ \Omega, \qquad I = V/R = (60\ \text{V})(1.20\ \Omega) = 50\ \text{A in both wires.}$$

(b) $E = \rho J = 6.1$ V/m.

G(4). Comsumers of electric energy pay by the kilowatt-hour. What is the cost of operating a 10.0-Ω toaster across a potential difference of 110 V for 20 min at the rate of 3.00 cents per kilowatt-hour?

Solution

$$\text{Power} = (\text{current})(\text{potential difference}) = P = (I)(V) = V^2/R,$$

where $I = V/R$ from Ohm's law.

$$P = (110\ \text{V})^2/(10.0\ \Omega) = 1210\ \text{W} = 1.21\ \text{kW}, \qquad \text{Work} = (P)(t) = (1.21\ \text{kW})(20/60\ \text{h}) = 0.40\ \text{kWh},$$

$$\$ = (0.40\ \text{kWh})(0.030\ \text{dollar/kWh}) = \$0.012 = 1.20\ \text{cent.}$$

H(4). A clever student decides to defrost the windshield of her sports car by covering it with a transparent conductive coating. The coating is 1.40 m long, 0.30 m wide, and 5.0×10^{-4} m thick and has a resistivity of 7.0×10^{-5} Ω m. If the "defroster" is to be operated from the 12.0-V system of the car:

(a) What is the power dissipated by the defroster?

(b) How much current does the defroster utilize?

Solution

$R = \rho\ell/A = 0.65\ \Omega$. Assuming the defroster obeys Ohm's law,

(a) $P = V^2/R = 220$ W, where $V = 12.0$ V.

(b) $I = V/R = 18$ A.

Problems

I(1). The total charge that has passed a given cross section varies with time according to the relation

$$Q(t) = K(1 - e^{-t/B}),$$

where $K = 3.00 \times 10^{-5}$ C and $B = 1.00$ s.

(a) Determine the current in the wire as a function of t.

(b) If the cross-sectional area of the wire is uniform and is 3.00×10^{-5} m^2, what is the current density in the wire as a function of t?

(c) When t = 1.00 s, how does the current compare to that at t = 0 s?

(d) Draw a rough sketch of I versus t.

J(2). A 10.0-m length of Number-18 copper wire is carrying a current of 5.0 A. If the diameter of the wire were doubled, with the same current, how would the drift speed change? Support your conclusion with a short physical argument.

K(3). Number-10 copper wire can carry a maximum current of about 30 A before overheating. Its diameter is 0.260 cm. The resistivity of copper is $1.70 \times 10^{-8}\ \Omega$ m.

(a Find the resistance of a 1.00-m length of the wire.

(b) How large a voltage drop occurs along it per meter when it carries a current of 30.0 A?

L(4). You decide to heat your office by placing a steel plate (resistivity = $1.80 \times 10^{-7}\ \Omega$ m) under the rug and passing a current through it from one side to the other. If the room is 4.0 m square and you want to get 4.0 kWatts by connecting it to a 120-V power line, how thick should the plate be? Assume the current is distributed uniformly in the plate.

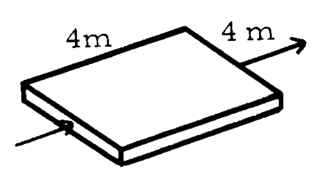

Figure 6

Solutions

I(1). (a) $I = (K/B)e^{-t/B}$.

(b) $J = (1/B)e^{-t/B}$.

(c) $I(t = 1.00\ s)/I(t = 0\ s) = 0.37$.

(d) See Figure 7 for I versus t.

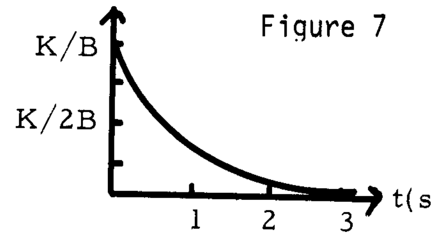

Figure 7

J(2). Decreases to 1/4; $J = I/A = nev_d$. $A = \pi d^2/4$; I and ne are constant. See how v_d changes when A changes.

K(3). (a) $3.20 \times 10^{-3}\ \Omega$. (b) 0.096 V.

L(4). Assuming Ohm's law; thickness = 5.0×10^{-8} m.

PRACTICE TEST

1. A bird whose feet are 5.0 cm apart perches on an aluminum power line that carries 1000 A. The power line is 2.54 cm in diameter, and the resistivity of aluminum is $2.80 \times 10^{-8}\ \Omega$ m. What is the potential difference between the bird's feet?

2. The current in a wire varies with time according to the relation

$$I = (4.0\ \text{A}) + (2.00\ \text{A/s}^2)t^2,$$

where I is in amperes and t is in seconds.

(a) How many coulombs pass a cross section of wire in the time interval between t = 5.0 s and t = 10.0 s?

(b) What constant current would transport the same charge in the same time interval?

3. A 100-W light bulb is constructed to operate at 115 V.

(a) What will be the current in the filament?

(b) What is the resistance of the filament coil?

(c) How much energy is radiated in one hour?

4. A potential difference V is applied to a copper wire of diameter d and length ℓ. What is the effect on the electron drift speed by (a) doubling V? (b) doubling ℓ? (c) doubling d?

Practice Test Answers

1. $\Delta V = 2.80 \times 10^{-3}$ V.
2. (a) $Q = 6.0 \times 10^2$ C. (b) $I_{const} = 120$ A.
3. (a) I = 0.87 A. (b) R = 132 Ω. (c) 3.6×10^5 J.
4. (a) v_d doubles. (b) v_d is halved. (c) v_d is unchanged.

OHM'S LAW

Mastery Test Form A

Date ______________

pass recycle

1 2 3 4

Name ______________

Tutor ______________

1. A nickel wire of length ℓ = 50 m and diameter d = 2.00 mm is joined to a wire of resistance R = 2.00 Ω to form a 100-m-long wire to which a potential difference V is applied. You wish to dissipate energy at the rate of 0.50 W in the combined wires.
   (a) What potential V is needed for this purpose?
   (b) How large is the electric field in the nickel?
   (c) How large is the drift speed of electrons in the nickel?

   Data: resistivity of nickel is 6.8×10^{-8} Ω m,
   charge of electron is 1.60×10^{-19} C,
   mass of electron is 1.00×10^{-30} kg,
   number of conduction electrons in nickel is 1.00×10^{28} electrons/m^3.

2. The current in a wire varies with time according to the relation

$$I = (1.00 \times 10^{-6}\ A)\{1 + \cos[(2\pi/s \times 10^3)t]\},$$

where I is in amperes and t in seconds. Find an expression for the charge that has passed a cross section of the wire as a function of time.

OHM'S LAW

Mastery Test   Form B

Name ______________________

Date ______________________

pass   recycle

1   2   3   4

Tutor ______________________

1. A silver wire 1.00 mm^2 in cross-sectional area carries a charge of 10.0 C uniformly over a period of 30 min. The resistivity of silver is 1.60×10^{-8} Ω m. The number of conduction electrons in silver is 6.0×10^{28} electrons/m^3.
   (a) What is the current density in the wire?
   (b) What is the electric field in the wire?
   (c) What is the potential difference across 5.0 m of the wire?
   (d) What is the power dissipated by 5.0 m of the wire?
   (e) What is the drift speed of the electrons in the silver?

2. You want to make a 1600-W heater to operate from 120 V. The heater is to be made from 4.0 m of manganin wire. The resistivity of manganin is 4.4×10^{-7} Ω m.
   (a) What should be the resistance of the wire?
   (b) What should be the wire diameter?

OHM'S LAW

Date ____________________

Mastery Test   Form C

pass   recycle

1   2   3   4

Name ____

Tutor ____________________

1. An aluminum rod of square cross section (area 16.0 mm^2) and length 2.00 m is joined to a square iron rod (area 36 mm^2) of length 0.50 m. The resistivity of aluminum is 2.80×10^{-8} Ω m, that of iron is 1.00×10^{-7} Ω m. The combined rod, 2.50 m long, carries a current of 200 A.

   (a) Find the ratio of current densities in the two rods.

   (b) Find the ratio of electric fields in the two rods.

   (c) Calculate the potential difference that must be applied to maintain the current.

   (d) Find the rate of energy dissipation in the aluminum.

   (e) The number of conduction electrons in iron is about 1.00×10^{29} electrons/m^3. The charge of an electron is 1.60×10^{-19} C, the mass is about 1.00 $\times 10^{-30}$ kg. Find the drift speed of the electrons in the iron rod.

2. The current in a wire varies with time according to the relation

   $$I = (6.0\ A)t,$$

   where I is in amperes and t in seconds. How many coulombs pass a cross section of the wire in the time interval between t = 0 s and t = 10.0 s?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) Joule's law. You are given P but need the total resistance of the combination. The resistance of the combination is the sum of the component resistances. (b) You are given ρ and A but need I. Use Ohm's law. Ask directions of E and J. (c) Relation for current density.

Solution: See Figure 8.

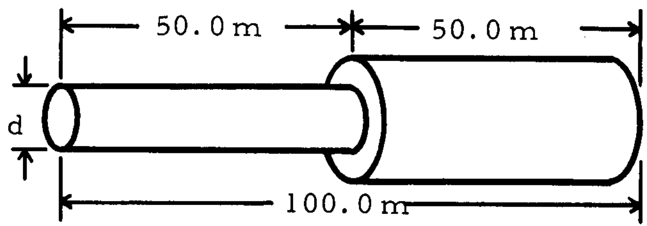

Figure 8

(a) $P = V^2/R, \quad V = \sqrt{PR}, \quad R = \rho\ell/A,$

$$R_{N_i} = \rho_{N_i}\ell_{N_i}/A_{N_i} = (6.8 \times 10^{-8}\ \Omega\ \text{m})(50\ \text{m})/[\pi(1.00 \times 10^{-3})^2\ \text{m}^2] = 1.10\ \Omega.$$

$$R_{total} = R_T = R_{N_i} + 2.00\ \Omega = 3.10\ \Omega. \qquad V = (0.50\ \text{W})(3.10\ \Omega)^{1/2} = 1.20\ \text{V}.$$

(b)

$$E = \rho J, \quad J = i/A, \quad I = V/R_T = (1.20\ \text{V})/(3.10\ \Omega) = 0.40\ \text{A},$$

$$J = (0.40\ \text{A})/[\pi(1.00 \times 10^{-3})^2\ \text{m}^2] = 1.27 \times 10^5\ \text{A/m}^2,$$

$$E = (6.8 \times 10^{-8}\ \Omega\ \text{m})(1.27 \times 10^5\ \text{A/m}^2) = 8.6 \times 10^{-3}\ \text{V/m}.$$

(c) $J = nev_d,$

$$v_d = \frac{J}{ne} = \frac{(1.27 \times 10^5\ \text{A/m}^2)}{(1.00 \times 10^{28}\ \text{electrons/m}^3)(1.60 \times 10^{-19}\ \text{C/electrons})} = 7.9 \times 10^{-5}\ \text{m/s}.$$

2. What To Look For: Definition of current. No limits of integration so we need a constant of integration.

Solution: $I = dQ/dt, \quad dQ = I\ dt,$

$$Q = \int I\ dt + \text{const} = (1.00 \times 10^{-6})\int[1 + \cos(2\pi \times 10^3 t)]\ dt + \text{const}$$

$$= (1.00 \times 10^{-6})\{t + [1/(2\pi \times 10^3)] \sin(2\pi \times 10^3 t)\} + \text{const}.$$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Determine the current that is assumed to be constant. (c) From Ohm's law V = IR. Therefore we need to solve for resistance.

Solution: (a) $I = Q/t = 10C/1800s = 5.6 \times 10^{-3}$ A,

$$J = \frac{I}{A} = (5.6 \times 10^{-3}\ A)/(1.00 \times 10^{-6}\ m^2) = 5.6 \times 10^{3}\ A/m^2.$$

(b) $E = \rho J = (1.60 \times 10^{-8}\ \Omega\ m)(5.6 \times 10^{3}\ A/m^2) = 9.0 \times 10^{-5}$ V/m.

(c) $R = \rho\ell/\text{area} = (1.60 \times 10^{-8}\ \Omega\ m)(5.0\ m)/(1.00 \times 10^{-6}\ m^2) = 8.0 \times 10^{-2}\ \Omega$,

$$V = IR = (5.6 \times 10^{-3}\ A)(8.0 \times 10^{-2}\ \Omega) = 4.5 \times 10^{-4}\ V.$$

(d) $P = VI = (4.5 \times 10^{-4}\ V)(5.6 \times 10^{-3}\ A) = 2.52 \times 10^{-6}$ W. $J = nev_d$,

$$v_d = \frac{J}{ne} = \frac{5.6 \times 10^{3}\ A/m^2}{(6.0 \times 10^{28}\ \text{electrons}/m^3)(1.60 \times 10^{-19}\ C/\text{electron})} = 5.8 \times 10^{-7}\ m/s.$$

2. What To Look For: (a) Using Joule's law.

Solution: (a) $P = V^2/R$, $R = V^2/P = (120\ V)^2/(1600\ W) = 9.0\ \Omega$.

(b) $R = \frac{\rho\ell}{\text{area}} = \frac{\rho\ell}{\pi(d/4)^2}$, where the diameter of the wire is d.

$$d^2 = \left(\frac{4}{\pi}\right)\left(\frac{\rho\ell}{R}\right), \qquad d = \left[\frac{(4)(4.4 \times 10^{-7}\ \Omega\ m)(4.0\ m)}{(3.14)(9.0\ \Omega)}\right]^{1/2} = 5.0 \times 10^{-4}\ m.$$

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) I = 200 A in both wires.
(b) Relation for E. Ask: What is the direction of E and J. Total resistance of the combination is the sum of the resistances of the two blocks. Application of Ohm's law.
(d) Joule's law.
(e) Relation between current density and drift speed.

Solution: (a) $J = I/A$, $J_{Al} = I/A_{Al} = 200\ A/(16 \times 10^{-6}\ m^2) = 1.25 \times 10^7\ A/m^2$,

$$J_{Fe} = \frac{I}{A_{Fe}} = \frac{200\ A}{36 \times 10^{-6}\ m^2} = 5.6 \times 10^6\ A/m^2, \qquad \frac{j_{Al}}{j_{Fe}} = \frac{A_{Fe}}{A_{Al}} = \frac{36 \times 10^{-6}}{16.0 \times 10^{-6}} = 2.25.$$

(b) $$E = \rho J,\ \frac{E_{Al}}{E_{Fe}} = \frac{\rho_{Al} J_{Al}}{\rho_{Fe} J_{Fe}} = \left(\frac{2.80 \times 10^{-8}}{1.00 \times 10^{-7}}\right)2.25 = 0.63.$$

(c) $$R = \rho \ell/A,\ R_{Al} = \frac{(2.80 \times 10^{-8}\ \Omega\ m)(2.00\ m)}{(16.0 \times 10^{-6}\ m^3)} = 3.5 \times 10^{-3}\ \Omega.$$

$$R_{Fe} = \frac{(1.00 \times 10^{-7}\ \Omega\ m)(0.50\ m)}{36 \times 10^{-6}\ m^3} = 1.40 \times 10^{-3}\ \Omega. \qquad R_{total} = R_T = 4.9 \times 10^{-3}\ \Omega.$$

$V = IR = (200\ A)(4.9 \times 10^{-3}\ \Omega) = 0.98\ V.$

(d) $P = I^2R = (200\ A)^2(3.5 \times 10^{-3}\ \Omega) = 140\ W.$

(e) $$J = nev_d,\ (v_d)_{Fe} = \frac{J_{Fe}}{(n_{Fe})(\ell)} = \frac{(5.6 \times 10^6\ A/m^2)}{(1.00 \times 10^{29}\ electrons/m^3)(1.60 \times 10^{-19}\ C/electrons)}$$

$$= 3.5 \times 10^{-4}\ m/s = 0.35\ mm/s.$$

2. What To Look For: Definition of current.

Solution: $I = dQ/dt$, $dq = I\ dt$,

$$Q = \int_0^{10.0\ s} 6.0t\ dt = (6.0t^2/2)\Big|_0^{10.0\ s}, \qquad a = (3.00)(100) = 300\ C.$$