2012

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JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS Vol. 47, No. 4, Aug. 2012, pp. 851–872

doi:10.1017/S0022109012000269

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Geoffrey C. Friesen, Yi Zhang, and Thomas S. Zorn*

Abstract

This study tests whether belief differences affect the cross-sectional variation of risk-neutral skewness using data on firm-level stock options traded on the Chicago Board Options Exchange from 2003 to 2006. We find that stocks with greater belief differences have more negative skews, even after controlling for systematic risk and other firm-level variables known to affect skewness. Factor analysis identifies latent variables linked to risk and belief differences. The belief factor explains more variation in the risk-neutral skewness than the risk-based factor. Our results suggest that belief differences may be one of the unexplained firm-specific components affecting skewness.

I. Introduction

Under no-arbitrage assumptions, the price of an option equals the expected payoff under a risk-neutral probability distribution, discounted at the risk-free rate. It is well documented that risk-neutral densities, imputed from observed option prices, tend to be more negatively skewed than are the lognormal distributions of Black and Scholes (1973). This negative skewness is linked by a one-to-one correspondence to variation in the implied volatilities across strike prices, as measured by the "slope" of the implied-volatility curve. Risk-neutral skewness and the implied-volatility slope capture the same underlying object, though measurement of the latter is subject to a number of known biases.[1]

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[1]An earlier version of this paper entitled "Heterogeneous Beliefs and the Option Implied Volatility Smile" looked at the relation between belief differences and the slope of the option-implied volatility smile. The results were qualitatively similar to those reported in the current paper, as one might expect given the strong correlation between the volatility smile and the skew of the risk-neutral density function (Bakshi, Kapadia, and Madan (BKM) (2003)). However, the current paper focuses on risk-neutral skewness because at the firm level, the cross-sectional variation in the implied-volatility measure is vulnerable to the following criticisms: First, it is model specific; second, the smile slope

Empirical work by Dennis and Mayhew (2002), Lemmon and Ni (2008), Duan and Wei (2009), and Taylor, Yadav, and Zhang (2009) shows that skewness is related to systematic risk, market volatility, market sentiment, and a number of firm-level variables including size, liquidity, and trading volume. In addition, Dennis and Mayhew (p. 492) note that in explaining skewness, there exists an "unexplained firm-specific component more important than all other explanatory variables put together." Though yet untested, theoretical work from Shefrin (2001), Ziegler (2003), and Buraschi and Jiltsov (2006) predicts that investor belief differences will affect option prices, the slope of the implied-volatility smile, and the skewness of the risk-neutral density function.

This study tests the hypothesis that heterogeneous beliefs among investors affect skewness of the risk-neutral distribution as imputed from observed option prices. In particular, we examine whether investor belief differences are one of the unexplained components alluded to by Dennis and Mayhew (2002). We test this hypothesis, which is a key prediction of the Buraschi and Jiltsov (2006) model, using a sample of firm-level option data for 856 firms traded on the Chicago Board Options Exchange (CBOE) from 2003 to 2006. Following Dennis and Mayhew, our skewness measure is calculated from out-of-the-money (OTM) options using the methodology of BKM (2003).

Our use of firm-level stock options has an important advantage over studies that examine only index options (e.g., Buraschi and Jiltsov (2006), Han (2008)). Han finds that while institutional investor sentiment affects index option prices, individual investor sentiment does not. Lemmon and Ni (2008) confirm this result but also show that individual investors are important participants in option markets for individual stocks. Firm-level options data enable us to examine the impact of individual investors on option prices and to test the cross-sectional prediction of Buraschi and Jiltsov that risk-neutral skewness will be more negative for those assets with greater belief differences.

In our empirical analysis, we use several variables commonly used as proxies for investor belief differences. We find that stocks with greater open interest in OTM options, greater dispersion in financial analysts' earnings forecasts, greater idiosyncratic volatility, and higher trading volume have more negative skews. The skew is also more negative for small stocks and growth stocks (low earnings-to-price (EP) ratio) relative to large stocks and income stocks (high EP ratio). These results lend support to the hypothesis that the risk-neutral skewness is related to belief differences among investors.

We also examine the importance of belief differences in explaining skewness, relative to the other variables in the literature. To obtain a measure of belief differences more robust than a collection of individual proxies, we conduct a factor analysis using firm-level data. Two latent factors are identified. The 1st factor is highly loaded on firm size, open interest, and stock trading volume. We suggest that this factor can be linked to the systematic risk factor proposed by Duan and Wei (2009). The 2nd factor is highly loaded on dispersion in financial

may be affected by cross-sectional variation in the range of observed strike prices, as well as measurement error in the prices of deep out-of-the-money options.

analysts' earnings forecasts, idiosyncratic volatility, and the EP ratio, and it appears to reflect investor belief differences.

Using the latent factor for belief differences as a cleaner measure than the individual proxies, we examine the relation between belief differences and risk-neutral skewness. We find a strong relation between the skew and the belief differences factor, with stronger results than those produced in regressions using individual proxy variables. The latent factor for belief differences also explains more variation in the risk-neutral density than the risk factor. This is consistent with the finding of Dennis and Mayhew (2002) that firm-specific factors are more important than systematic risk in explaining the skew, and it suggests that belief differences may be the unexplained determinant of risk-neutral skewness described in their study.

The hypothesis that heterogeneous beliefs cause greater skewness in option prices is related to the price pressure hypothesis examined by Bollen and Whaley (2004). Specifically, it is possible that belief differences simply cause investors to trade in ways that put pressure on option prices, moving them away from the price they would have if options were priced by dynamic arbitrage. It is also possible that belief differences cause equilibrium option prices to have greater skew in the absence of price pressure. To examine this, we look at the sensitivity of skewness to dispersion in beliefs as a function of the liquidity of the option. We find that skewness is more sensitive to belief differences for the less liquid options, but that the relation between belief differences and skewness holds, after controlling for option liquidity. This suggests that the effect of belief differences on skewness goes beyond the net price pressure suggested by Bollen and Whaley.

The rest of the paper is organized as follows: Section II describes related literature regarding risk-neutral skewness. Section III develops hypotheses on the relation between heterogeneous beliefs and risk-neutral skewness. Section IV describes data and empirical methodology. Section V presents empirical results. Section VI discusses the empirical results, and Section VII concludes this study.

II. Related Literature

Some explanations of the volatility smile and risk-neutral skewness focus on violations of assumptions made by the Black-Scholes (1973) model. These violations include non-lognormally distributed returns or nonconstant volatility (Hull (1993)). Hull and White (1988) and Heston (1993) suggest that the volatility smile reflects stochastic volatility. Peña, Rubio, and Serna (1999) show that transaction costs also affect the curvature of the volatility smile. BKM (2003) show that leptokurtic returns cause the Black-Scholes model to misprice OTM and in-the-money (ITM) options.

A 2nd class of studies focuses on the importance of firm-specific and systematic factors in explaining the skews in the risk-neutral density of individual firm stock options. Dennis and Mayhew (2002) find that volatility skews are affected by liquidity and market risk measured by beta and size. Duan and Wei (2009) find that higher systematic risk leads to a steeper implied-volatility slope. Skewness may also be related to a firm's leverage. As a company's equity value declines, leverage increases, the equity becomes more risky, and its volatility increases. Toft and Prucyk (1997) find that more highly levered firms have negative skews.

Bollen and Whaley (2004) find that net buying pressure determines the shape of the volatility smile, which they attribute to arbitrage limits. For individual stock options, the shape of the implied-volatility function is directly related to net buying pressure from call option demand. Li (2007) develops a model of option pricing in an economy in which investors have different time preferences and heterogeneous beliefs about dividend growth rates. The pattern of the implied volatility depends on beliefs, time preferences, and relative wealth of investors. When pessimistic investors hold a relatively large portion of total wealth, the implied volatility exhibits a "skew"; when optimistic investors hold a relatively large portion of total wealth, the implied volatility exhibits a "smile." Han (2008) shows a significant relation between time variation in market sentiment and the slope of the volatility smile and risk-neutral skewness. Lemmon and Ni (2008) find that the time-series variation in the volatility smile slope is related to investor sentiment.

Recent theories suggest that belief differences among investors could affect option prices. Shefrin (2001) demonstrates that investor sentiment affects the pricing kernel so that the log stochastic discount factor (SDF) is U-shaped rather than monotonically decreasing, as assumed in standard asset pricing models. The U-shape in the log-SDF gives rise to a volatility smile. Ziegler (2003) shows that investor belief differences can impact equilibrium state-price densities. Buraschi and Jiltsov (2006) develop an option pricing and volume model in which agents have heterogeneous beliefs about expected returns and face model uncertainty. In their model, optimistic investors demand OTM calls, while pessimistic investors demand OTM puts. They show that heterogeneous beliefs about the dividend growth rate affect option prices and volume, and they produce a volatility smile and a negatively skewed risk-neutral density function. The implication of all 3 studies is that investor belief differences may affect option prices through the slope of the implied-volatility smile and the skewness of the risk-neutral density function.

III. Hypothesis Development

Our study seeks to examine the relation between belief differences and risk-neutral skewness. While theoretical models provide a motivation for our empirical work, our empirical findings do not depend on the validity of any particular model. The 1st proxy we consider for heterogeneous beliefs is open interest on options. The put/call open interest ratio is commonly used as a measure of investor sentiment: More put open interest indicates pessimism, while more call open interest indicates optimism.

Hypothesis 1. Stocks with greater OTM option open interest have more negative skews.

The most widely used proxy for heterogeneous beliefs in previous studies is the dispersion in financial analysts' earnings forecasts (e.g., Diether, Malloy, and Scherbina (2002), Doukas, Kim, and Pantzalis (2004)). Goetzmann and Massa (2003) show that differences in investors' beliefs are correlated with dispersion in financial analysts' opinions. Therefore, stocks with a greater dispersion in financial analyst earnings forecasts are expected to have more negative skews.

Hypothesis 2. Stocks with a greater dispersion in financial analysts' earnings forecasts have more negative skews.

Another common proxy for heterogeneous beliefs is stock idiosyncratic volatility. Shalen (1993), He and Wang (1995), and Biais and Bossaerts (1998) suggest a positive link between heterogeneous beliefs and volatility. Empirically, Harris and Raviv (1993) and Diether et al. (2002) find that stocks with a higher dispersion of opinion have higher volatility. Therefore, stocks with higher idiosyncratic volatility are expected to have more negative skews.

Hypothesis 3. Stocks with a greater idiosyncratic volatility have more negative skews.

Stock trading volume is also used as a proxy for heterogeneous beliefs by Harris and Raviv (1993), Kandel and Pearson (1995), Odean (1998), and Hong and Stein (2003). Since belief differences induce trading, stocks with more divergent beliefs among investors tend to induce greater trading volume.

Hypothesis 4. Stocks with greater trading volume have more negative skews.

Diether et al. (2002) and Baik and Park (2003) find that investors' belief differences tend to be greater for small firms than large firms. Because financial analysts and the financial press tend to follow large firms, investors may have less information about smaller firms and therefore more divergent beliefs as firm size decreases.

Hypothesis 5. Small stocks have greater negative skews than large stocks.

In the equity market, shares of companies that are growing at an above-average rate relative to the market or their industry are commonly called growth stocks. Growth stocks normally pay smaller dividends than income stocks. Growth stocks may therefore have greater heterogeneity in investor beliefs about dividend growth rates compared to income stocks. One common method to distinguish growth stocks and income stocks is the EP ratio.[2] Baik and Park (2003) find that stocks with lower EP ratios have more dispersion in financial analysts' earnings forecasts, consistent with the idea that growth stocks have greater belief heterogeneity. Thus, the 6th hypothesis:

Hypothesis 6. Growth stocks have greater negative skews than income stocks. Conversely, the EP ratio should be positively related to the skew.

[2]Another measure for the growth stock is the book-to-market (BM) ratio. Diether et al. (2002), Baik and Park (2003), and Doukas et al. (2004) find that stocks with high BM ratios have larger dispersion in financial analysts' earnings forecasts than stocks with low BM ratios, indicating that value stocks have greater belief heterogeneity than growth stocks. Therefore, we include it as a proxy for heterogeneous beliefs in the subsequent empirical analysis. However the BM ratio is also believed by some to be a priced risk factor, and we later find that its relation with the skew is more attributable to the risk component than to the belief difference component.

If multiple proxy variables are related to heterogeneous beliefs, the underlying belief differences should be identifiable using factor analysis. If a latent factor for heterogeneous beliefs is successfully identified, we can estimate the factor as the linear combination of proxy variables using factor score coefficients. Therefore, our last hypothesis:

Hypothesis 7. The latent factor for heterogeneous beliefs is negatively related to the skew.

IV. Data and Methodology

A. Data and Risk-Neutral Skewness Measure

This study uses a data set from options traded on the CBOE from 2003 to 2006.[3] The data set contains daily option data including trading date, expiration date, strike price, last-trade price, closing bid and ask quotes, volume, open interest, implied volatility, and the underlying security price. We use the risk-neutral skewness metric of BKM (2003) and follow the empirical methodology of Dennis and Mayhew (2002) to construct the skewness measure.[4]

We use trapezoidal approximation to estimate the integrals in the BKM (2003) measure of skewness with discrete option data. Using the midpoint of the closing bid and ask prices for each contract,[5] we compute the BKM skewness measure each day for the 2 maturities that are greater than 7 days but closest to 22 trading days. Equal numbers (at least 2) of OTM puts and calls for each stock for each day are used. In the sample, the average number of OTM puts/calls used for estimating the BKM measure of skewness is 3.4. A hypothetical option with 22 trading days to maturity is constructed using linear interpolation or extrapolation. We then average the daily skewness measures to obtain a weekly measure.

B. Explanatory Variables

We compute the weekly OTM open interest by averaging daily total OTM open interests over the week. Following Diether et al. (2002), the dispersion in financial analysts' earnings forecasts is measured as the standard deviation of forecasts for quarterly earnings scaled by the absolute value of the mean earnings forecast. The data on financial analysts' earnings forecasts are taken from the Institutional Brokers' Estimate System summary history data set. Only the most recent statistical summary is adopted. To ensure that the forecast is current, only the forecast period of 1 quarter (forecast period indicator (FPI) = 6) is selected. Firms with a zero mean forecast or without a standard deviation are excluded.

[3]The data are provided by deltaneutral.com. The data set is called National Best Bid Offer data set from Option Pricing Report Authority, which collects data from the CBOE. ITM option prices are checked, and bad prices are marked as irregularities in the data set.

[4]Additional computational details are available from the authors.

[5]Dennis and Mayhew (2002) discuss the bias introduced by the discreteness of the strike price interval. Most of our data have a strike price interval of $2.50 or $5.00, so bias is roughly the same for all of the observations in our sample.

Firms' financial ratios are constructed using data from the Compustat Industrial Quarterly File. Firm size is measured as the natural logarithm of market capitalization. The EP ratio is measured by quarterly earnings (Data 27) divided by the stock price. The BM ratio is the book value of common equity (Data 59) divided by the stock price. Stock return and volume data are obtained from the Center for Research in Security Prices. The idiosyncratic volatility is calculated by regressing daily returns over the last 200 days on the daily returns of the value-weighted index (including distributions) using the market model. We also obtain the beta from the regression and include it as a control variable because Dennis and Mayhew (2002) find a negative relation between the beta and the skew.

Taylor et al. (2009) show that option and stock liquidity affects the risk-neutral skew of individual firms. The daily trading volume of OTM options is used as a proxy for the liquidity of options. Taylor et al. use stock volume as the proxy for stock liquidity. Because stock volume is used in our study as a proxy for heterogeneous beliefs, we use an alternative measure of liquidity of stocks: the bid-ask spread of the stock closing prices, estimated daily as the (ask price – bid price)/ask price. Stocks with high liquidity are expected to have a less negative skew. In addition, leverage can affect the risk-neutral skewness as argued by Geske (1979) and Toft and Prucyk (1997). The leverage is defined as the debt-to-equity ratio, computed as the long-term debt (Data 51) divided by the market capitalization. All daily measures are averaged to obtain a weekly measure.

To consider how the risk-neutral skewness of individual stocks is affected by the market-wide factors, we also include a market volatility variable. To maintain consistency with our estimation of idiosyncratic volatility, market volatility is measured as the standard deviation of daily returns for the value-weighted market index (including distributions) over the last 200 days and then is averaged weekly.

C. Factor Analysis Methodology

We combine 7 proxy variables for heterogeneous beliefs to extract the underlying latent variable for heterogeneous beliefs. The factor model in matrix form is assumed to be

$$z = \Lambda f + \eta, \tag{1}$$

where z is the vector of standardized proxies, f is the factor vector with 0 means and 1 variance, Λ is the matrix of factor loading, and η is independent of the factor vector. The factor model implies that

$$\Sigma = \mathrm{Cov}(z) = \Lambda\Lambda' + \mathrm{Cov}(\eta) = \Lambda\Lambda' + \Psi. \tag{2}$$

The goal of factor analysis is to find Λ and Ψ that satisfy the factor analysis equation (2). Because the units across the 7 proxies vary by several orders of magnitude, using raw data leads to overweighting some proxies, because their scale makes them seem much more variable than other proxies. Therefore, the Spearman correlation matrix is applied, which is similar to standardizing the data by subtracting the mean and then dividing by the standard deviation.

We adopt 2 common methods for solving the factor analysis equations: the principal factor method and the maximum likelihood method.[6] We follow the common practice of rotating factors to facilitate their interpretation, which normalizes as many of the factor loadings as closely as possible to 0. We adopt the Kaiser (1958) varimax orthogonal rotation method, which ensures that factors are uncorrelated with each other. We then estimate the factor as the linear combination of proxy variables using factor score coefficients:

$$f = S^{*}z, \tag{3}$$

where S is the factor score coefficient matrix.

V. Empirical Results

A. Results Based on Proxy Variables

Table 1 presents descriptive statistics for variables. The sample includes 67,910 weekly observations of 856 unique firms during the period from Jan. 2003 to Dec. 2006. Due to the large variation in the skew, the upper and lower 5% of the skew are trimmed. The number of firms in the sample increases from 572 in 2003 to 674 in 2004, 779 in 2005, and 788 in 2006. The median skew for all 4 years is -1.484. The skew levels over our sample period are similar to those reported in Conrad, Dittmar, and Ghysels (2012). They examine skewness over the 1996–2005 period, but their results are much more negative than the average skew reported by Dennis and Mayhew (2002), derived over the earlier 1986–1996 period.[7] While not the focus of the current study, an interesting avenue for future research is to study the cause of increasing skewness of individual stock options, especially since 2003.

The average daily OTM open interest of our sample is 9,000 contracts. The average dispersion in financial analysts' earnings forecasts is 0.067. The average idiosyncratic volatility of the sample is 2.5%, and the average daily stock trading volume of about 683 thousand shares. Most stocks in our sample are of medium and large capitalization. Most small stocks do not have options or are not followed by financial analysts. The average firm size in the sample is about \$1 billion. Our sample has an average EP ratio of 0.024 and an average BM ratio of 0.298. The average daily OTM option volume in our sample is about 230 contracts, while the average bid-ask spread of the stocks in our sample is 3.4%. The median leverage is 0.04, with a large number of firms in the sample having no long-term debt. Our sample has an average beta of 1.479. During our 4-year study period, the average daily market volatility is 0.7%.

[6] A factor analysis initially requires suitable estimates of the communalities. The communality estimate for a variable is the proportion of the variance of each variable that is explained by the common factors. We choose one common approach to set the prior communality estimate for each variable to its squared multiple correlation of each variable with all remaining variables.

[7] In the paper of Conrad et al. (2012), the median skew is -1.297 for 2003, -1.399 for 2004, and -1.609 for 2005, which is much more negative than previous years: -0.742 for 2002, -0.648 for 2001, -0.562 for 2000, -0.601 for 1999, -0.464 for 1998, -0.539 for 1997, and -0.449 for 1996.

TABLE 1

Descriptive Statistics

Table 1 presents descriptive statistics for variables that are used in empirical testing. The sample includes 67,910 weekly observations of 856 unique firms during the period from Jan. 2003 to Dec. 2006. SKEW is the BKM (2003) risk-neutral skewness measure. The upper and lower 5% of SKEW are trimmed. OPEN_INTEREST is the average daily open interest on all OTM options. DISPERSION is the dispersion of financial analysts' forecasts for quarter earnings, measured by the standard deviation of forecasts scaled by the absolute value of the mean forecast. IDIOSYNCRATIC_VOLATILITY is the estimated standard deviation of daily stock returns using the market model for the past 200 days. STOCK_VOLUME is the daily stock trading volume averaged weekly. SIZE is the natural logarithm of the market capitalization. EP is the earnings-to-price ratio, computed as the quarterly earnings divided by the market price. BM is the book-to-market ratio, computed as the quarterly book value of common equity divided by the market price. OPTION_VOLUME is the average daily trading volume of OTM options. LIQUIDITY is the stock's bid-ask spread, computed as the (ask price − bid price)/ask price. LEVERAGE is the debt-to-equity ratio computed as the long-term debt divided by the market capitalization. BETA is estimated by the market model for the past 200 days using daily data. MARKET_VOLATILITY is the standard deviation of daily returns of the value-weighted index including distributions for the past 200 days.

Year	No. of Obs.	P25	Median	P75	Standard Deviation
Panel A. Dependent Variable: SKEW					
2003	14,236	−3.097	−1.623	−0.907	1.568
2004	16,730	−2.634	−1.425	−0.804	1.490
2005	17,410	−3.058	−1.613	−0.871	1.584
2006	19,534	−2.691	−1.339	−0.745	1.557
Total	67,910	−2.859	−1.484	−0.820	1.553
Variables	No. of Obs.	P25	Median	P75	Standard Deviation
Panel B. Proxy for Heterogeneous Beliefs					
OPEN_INTEREST (in millions)	67,910	0.003	0.009	0.035	0.155
DISPERSION	64,091	0.030	0.067	0.159	0.557
IDIOSYNCRATIC_VOLATILITY	67,909	0.019	0.025	0.032	0.011
STOCK_VOLUME (in millions)	67,893	0.304	0.683	1.764	6.615
SIZE	67,893	19.994	20.723	21.763	1.351
EP	66,993	−0.015	0.024	0.042	0.123
BM	65,282	0.189	0.298	0.471	0.639
Panel C. Control Variables					
OPTION_VOLUME (in thousands)	67,910	0.041	0.230	1.203	7.142
LIQUIDITY	67,893	0.026	0.034	0.045	0.015
LEVERAGE	66,104	0.000	0.004	0.123	0.668
BETA	67,910	1.090	1.479	1.936	0.628
MARKET_VOLATILITY	67,910	0.006	0.007	0.008	0.003

Table 2 presents the correlations between variables of interests. The skew is positively correlated to option volume and negatively correlated to bid-ask spread, indicating that high liquidity in either options or stocks leads to less negative skews. This is consistent with the finding of Dennis and Mayhew (2002). The skew is negatively correlated with leverage, supporting the leverage argument that stocks with high leverage have more negative skews. The negative correlation between the skew and beta is consistent with the findings of both Dennis and Mayhew (2002) and Taylor et al. (2009). The correlations between the skew and proxies for heterogeneous beliefs also have their predicted signs. The skew is negatively related to the open interest on options, dispersion in financial analysts' earnings forecast, the stock's idiosyncratic risk, trading volume, and the BM ratio, while positively related to firm size and the EP ratio. These correlations imply that firms with greater divergence in investor beliefs have a more negatively skewed risk-neutral density. Also, the skew of individual stocks is negatively related to market volatility, suggesting that the risk-neutral density for individual stock options is more negatively skewed when the market is more volatile.

Several high correlations deserve attention: firm size with option volume (0.462), liquidity (−0.395), open interest (0.549), idiosyncratic volatility (0.476),

TABLE 2

Correlations between Variables

Table 2 presents correlations between variables that are used in empirical testing. All variables are defined in Table 1. Correlations in bold are discussed in the text.

	SKEW	OPTION_VOLUME	LIQUIDITY	LEVERAGE	BETA	OPEN_INTEREST	DISPERSION	IDIOSYNCRATIC_VOLATILITY	STOCK_VOLUME	SIZE	EP	BM
OPTION_VOLUME	0.041											
LIQUIDITY	−0.116	−0.029										
LEVERAGE	−0.049	−0.028	0.019									
BETA	−0.088	−0.024	0.283	0.011								
OPEN_INTEREST	−0.035	**0.735**	−0.118	−0.021	−0.040							
DISPERSION	−0.087	−0.040	0.076	0.045	0.023	−0.037						
IDIOSYNCRATIC_VOLATILITY	−0.148	−0.067	**0.573**	0.016	0.250	−0.115	0.079					
STOCK_VOLUME	−0.076	0.624	−0.021	−0.023	0.010	**0.773**	−0.028	−0.086				
SIZE	0.181	**0.462**	**−0.395**	−0.069	−0.086	**0.549**	−0.151	**−0.476**	**0.491**			
EP	0.216	0.031	−0.253	−0.095	−0.073	0.020	−0.084	−0.288	0.005	0.206		
BM	−0.109	−0.057	0.005	**0.657**	−0.007	−0.055	0.040	0.015	−0.039	−0.152	−0.048	
MARKET_VOLATILITY	−0.046	−0.032	0.207	0.018	−0.082	−0.029	−0.006	0.273	0.014	−0.060	−0.120	0.061

and stock volume (0.491); open interest with option volume (0.735) and stock volume (0.773); idiosyncratic volatility with liquidity (0.573); and leverage with the BM ratio (0.657). These correlations indicate a possible collinearity problem in subsequent regression analysis.

We begin our cross-sectional analysis of the risk-neutral skew by running the Fama-MacBeth (1973) 2-stage regressions. For each week, we run the cross-sectional regressions of the form

$$\text{(4)} \qquad \text{SKEW}_i = \alpha + \beta_1 \text{OPTION\_VOLUME}_i + \beta_2 \text{LIQUIDITY}_i + \beta_3 \text{LEVERAGE}_i + \beta_4 \text{BETA}_i + \beta_5 \text{PROXY}_i,$$

where SKEW is the skew of the risk-neutral density for the firm, OPTION_VOLUME is the trading volume of OTM options, LIQUIDITY is the bid-ask spread on the firm stock, LEVERAGE is the debt ratio of the firm, and BETA is the firm's beta on stock returns obtained from the model. PROXY is 1 of the 7 variables representing heterogeneous beliefs: including OTM open interest, the dispersion in financial analysts' forecast, idiosyncratic volatility, stock trading volume, firm size, and the EP and BM ratios.

The cross-sectional regressions generate 208 weekly estimates of coefficients. The time-series averages of each coefficient with Newey-West (1987) *t*-statistics are reported in Table 3. For reference, column (1) includes only control variables. The skew is positively related to option volume and is negatively related to the bid-ask spread, implying that liquidity in either stocks or options reduces the skewness of the risk-neutral density. The skew is negatively related to leverage, supporting the asserted effect of leverage on option prices. The negative coefficient on beta indicates a negative relation between the skew and systematic risk, consistent with the findings of previous studies.

To test our 1st hypothesis, open interest on OTM options is added into the regression. Column (2) shows that the coefficient on open interest is significantly negative (−2.478), supporting the hypothesis that stocks with more OTM option open interest have more negative skews. Column (3) shows that the dispersion in financial analysts' forecasts is significantly and negatively related to the skew, supporting our 2nd hypothesis that stocks with greater divergence of opinion have more negative skews. Column (4) presents the significantly negative coefficient on idiosyncratic volatility, indicating that stocks with larger volatility are associated with more negative skews, supporting Hypothesis 3. This result is consistent with Taylor et al. (2009) but contrary to Dennis and Mayhew (2002).

The negative coefficient (−0.055) on stock trading volume in column (5) shows that stocks with higher volume have more negative skews. This result is consistent with Taylor et al. (2009) but again contrary to Dennis and Mayhew (2002). Both of those studies use trading volume as a proxy for liquidity, but in our regression liquidity is already controlled for using the bid-ask spread. Thus, after controlling for liquidity the stock volume captures the effect of heterogeneous beliefs. The regression results thus support Hypothesis 4. Also as predicted, small firms have more negative skews than large firms, while growth stocks (low EP ratios) have more negative skews than income stocks, as shown in columns (6) and (7), supporting Hypotheses 5 and 6, respectively. Small firms and growth

TABLE 3

Cross-Sectional Regression of Risk-Neutral Skewness

Table 3 presents the time-series averages of the coefficients from weekly cross-sectional regressions using the BKM (2003) risk-neutral skewness, described in the text, as the dependent variable. The sample includes weekly observations during the period from Jan. 2003 to Dec. 2006. Newey-West (1987) *t*-statistics (in parentheses) are reported. All variables are defined in Table 1. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
OPTION_VOLUME	0.008***	0.061***	0.044***	0.006***	0.05***	−0.017***	0.006***	−0.003*	0.05***
	(4.85)	(15.39)	(2.83)	(3.38)	(17.38)	(−8.73)	(4.00)	(−1.91)	(12.92)
LIQUIDITY	−13.421***	−15.607***	−12.037***	−8.451***	−13.934***	−4.861***	−7.049***	−13.027***	−2.565***
	(−16.04)	(−16.58)	(−13.78)	(−9.93)	(−17.03)	(−5.61)	(−8.49)	(−16.47)	(−2.99)
LEVERAGE	−0.253***	−0.240***	−0.270***	−0.248***	−0.253***	−0.234***	−0.151***	−0.124***	−0.098***
	(−7.94)	(−7.97)	(−6.95)	(−7.75)	(−8.13)	(−7.32)	(−6.44)	(−3.90)	(−3.83)
BETA	−0.19***	−0.193***	−0.203***	−0.156***	−0.167***	−0.226***	−0.173***	−0.146***	−0.177***
	(−5.99)	(−6.15)	(−6.26)	(−5.09)	(−5.75)	(−6.85)	(−6.60)	(−5.90)	(−7.15)
OPEN_INTEREST		−2.478***							−1.474***
		(−18.77)							(−9.48)
DISPERSION			−0.362***						−0.181***
			(−14.41)						(−8.41)
IDIOSYNCRATIC_VOLATILITY				−13.094***					−3.492**
				(−10.62)					(−2.60)
STOCK_VOLUME					−0.055***				−0.049***
					(−21.68)				(−15.58)
SIZE						0.216***			0.109***
						(20.98)			(9.41)
EP							3.45***		3.352***
							(23.92)		(22.45)
BM								−1.299***	−1.184***
								(−27.46)	(−25.23)
Mean adj. R^2 (%)	4.60	6.13	5.67	5.37	6.62	6.84	8.69	8.65	16.74

stocks are subject to a greater divergence of opinion among investors. The results support the argument that stocks with greater belief heterogeneity are more negatively skewed. The skew is negatively related to the BM ratio, as shown in column (8), suggesting greater negative skew among value firms.

We further combine all proxy variables into the cross-sectional regressions. The coefficients of the variables retain their predicted signs, although multicollinearity reduces the significance of the coefficient on idiosyncratic volatility. The mean adjusted R^2 increases to 16.74% from a range of 5.37%–8.69% in the regressions that included only one proxy variable.

Following Dennis and Mayhew (2002), we also estimate a pooled cross-sectional time-series regression to explore how skews of individual firms vary over time and are affected by market volatility. Market volatility and a lagged skew are included in the regression as additional explanatory variables. The results are reported in Table 4. The skew of individual firms is negatively related to market volatility. Our results are consistent with the results of Dennis and Mayhew (2002) and Taylor et al. (2009). Including the lagged skew in the analysis significantly increases the explanatory power of the regression to about 70%. The sign and significance of the estimated coefficients for the firm-specific variables remain the same as the cross-sectional estimates reported in Table 3 (exceptions are the liquidity and market volatility variables, whose coefficients become positive or insignificant due to the multicollinearity described earlier).

Table 5 presents mean and median skews for quintiles sorted on a particular proxy for heterogeneous beliefs: open interest, dispersion in financial analysts' earnings forecasts, idiosyncratic volatility, stock trading volume, size, the EP ratio, and the BM ratio. Panel A of Table 5 shows that the quintiles of greater dispersion in financial analysts' earnings forecasts and higher idiosyncratic volatility have more negative skews, while the small capitalization quintile, low EP ratio quintile, and higher BM ratio quintile have more negative skews. The skew differentials between the highest and lowest quintiles are statistically significant, consistent with the argument that firms with greater investor belief differences have more negative skews.

The skewness-ranked quintiles for open interest and stock volume do not appear as a monotonic trend but exhibit a kind of U-shape in the skew. Both open interest and trading volume are highly correlated with firm size, as indicated by the high correlation coefficients reported in Table 2. Hence, it is unclear whether or not the pattern reflects a size effect. In order to remove the size effect, we run separate regressions for open interest and stock volume on firm size and obtain residuals from each regression that are orthogonal to firm size. Stocks are then sorted into quintiles based on the residuals. Results presented in Panel B of Table 5 are consistent with Hypotheses 1 and 4.

Each of the belief proxies utilized previously is at best a noisy measure of investor beliefs. For example, stock trading volume is also used to proxy for liquidity, while size is also used to proxy for systematic risk. If the observable proxy variables for heterogeneous beliefs capture the unobservable differences in beliefs, a latent variable should be identifiable. The latent variable will be a less-noisy measure of heterogeneous beliefs than the individual proxy variables.

TABLE 4

Regression of Risk-Neutral Skewness

Table 4 presents pooled cross-sectional time-series regression results with the BKM (2003) risk-neutral skewness, described in the text, as the dependent variable. The sample includes weekly observations during the period from Jan. 2003 to Dec. 2006. Newey-West (1987) corrected estimates and *t*-statistics (in parentheses) are reported. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
INTERCEPT	−0.202***	−0.212***	−0.197***	−0.219***	−1.208***	−0.275***	−0.196***	−1.187***
	(−12.17)	(−12.59)	(−11.87)	(−13.27)	(−16.31)	(−15.90)	(−11.56)	(−12.33)
OPTION_VOLUME	0.007***	0.001**	0.001***	0.006***	−0.002***	0.001***	0.001**	0.006***
	(8.84)	(2.42)	(2.70)	(10.06)	(−3.97)	(3.02)	(2.26)	(7.86)
LIQUIDITY	−0.981***	−0.456	0.997***	−0.696***	0.978***	0.366	−0.56**	2.245***
	(−3.50)	(−1.57)	(3.15)	(−2.51)	(3.25)	(1.30)	(−2.00)	(6.60)
LEVERAGE	−0.026***	−0.025***	−0.026***	−0.027***	−0.022***	−0.017***	0.023***	0.02***
	(−4.68)	(−4.35)	(−4.72)	(−4.81)	(−3.86)	(−2.91)	(3.39)	(3.02)
BETA	−0.05***	−0.052***	−0.041***	−0.048***	−0.055***	−0.051***	−0.052***	−0.049***
	(−8.06)	(−8.22)	(−6.54)	(−7.74)	(−8.91)	(−8.23)	(−8.34)	(−7.41)
MARKET_VOLATILITY	−3.385**	−3.082*	−0.385	−2.919*	−4.836***	−2.254	−3.586**	0.464
	(−2.11)	(−1.87)	(−0.24)	(−1.82)	(−3.01)	(−1.39)	(−2.19)	(0.27)
$SKEW_{t-1}$	0.826***	0.826***	0.827***	0.825***	0.824***	0.821***	0.825***	0.805***
	(304.05)	(291.07)	(305.55)	(302.86)	(299.73)	(290.91)	(294.91)	(257.50)
OPEN_INTEREST	−0.343***							−0.302**
	(−8.00)							(−4.96)
DISPERSION		−0.046***						−0.029***
		(−6.17)						(−4.03)
IDIOSYNCRATIC_RISK			−4.498***					−1.846***
			(−10.47)					(−3.64)
STOCK_VOLUME				−0.007***				−0.008***
				(−8.41)				(−6.34)
SIZE					0.045***			0.042***
					(14.09)			(10.11)
EP						0.567***		0.665***
						(112.06)		(12.31)
BM							−0.074***	−0.065***
							(−6.96)	(−5.85)
Adj. R^2 (%)	69.36	69.06	69.38	69.37	69.41	69.43	69.11	69.32

TABLE 5

Skewness of Quintiles

Table 5 presents mean and median BKM (2003) skewness for quintiles of stocks sorted based on the stated proxies for heterogeneous beliefs. Each week stocks are assigned to 1 of the 5 quintiles based on the value for the particular proxy variable. The proxy variables are defined in Table 1.

Panel A. Quintiles of Proxy Variables for Heterogeneous Beliefs

	OPEN_INTEREST		DISPERSION		IDIOSYNCRATIC_VOLATILITY		STOCK_VOLUME		SIZE		EP		BM	
Quintiles	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median
1 (small)	−2.12	−1.73	−1.83	−1.35	−1.71	−1.35	−2.05	−1.64	−2.71	−2.56	−2.81	−2.62	−1.63	−1.20
2	−1.98	−1.46	−1.55	−1.17	−1.74	−1.29	−1.88	−1.38	−2.06	−1.56	−2.12	−1.61	−1.68	−1.21
3	−1.95	−1.36	−1.84	−1.33	−2.01	−1.46	−1.95	−1.39	−1.78	−1.27	−1.66	−1.23	−1.85	−1.35
4	−1.92	−1.37	−2.19	−1.71	−2.22	−1.67	−1.98	−1.42	−1.75	−1.24	−1.66	−1.24	−2.17	−1.68
5 (large)	−2.07	−1.54	−2.45	−2.02	−2.36	−1.95	−2.18	−1.62	−1.74	−1.30	−1.78	−1.34	−2.61	−2.25
Q5 − Q1	0.05	0.19	−0.62	−0.67	−0.65	−0.60	−0.13	0.02	0.97	1.26	1.03	1.28	−0.98	−1.04
	(2.54)	(4.35)	(−31.94)	(−29.96)	(−36.00)	(−28.26)	(−6.27)	(3.39)	(52.76)	(50.69)	(53.44)	(50.41)	(−51.86)	(−48.53)

Panel B. Control for Size

	OPEN_INTEREST		STOCK_VOLUME	
Quintiles	Mean	Median	Mean	Median
1 (small)	−1.53	−1.16	−1.47	−1.16
2	−1.73	−1.23	−1.59	−1.17
3	−1.89	−1.38	−1.83	−1.34
4	−2.26	−1.78	−2.30	−1.84
5 (large)	−2.63	−2.41	−2.85	−2.74
Q5 − Q1	−1.11	−1.24	−1.38	−1.58
	(−63.12)	(−57.36)	(−79.54)	(−68.62)

B. Results of Factor Analysis

We apply factor analysis to the Spearman correlation matrix of the 7 proxies for heterogeneous beliefs because of the different scales of the variables. The factor analysis produces 2 factors using either the principal factoring or maximum likelihood method. The factor extraction criterion is that the cumulative proportion of common variance is greater than or equal to 1. Results for the principal factoring method are presented in Panel A of Table 6. The factor pattern shows that the 1st factor is highly and positively loaded with open interest (0.75), stock volume (0.79), and size (0.91), while negatively loaded with dispersion in analysts' forecasts (−0.30), idiosyncratic volatility (−0.42), and the BM ratio (−0.31).[8] These factor loadings are consistent with the presence of an underlying size factor because large firms tend to have more option open interest and higher trading volume but less dispersion in analysts' forecasts and less idiosyncratic volatility.[9] This factor explains about 71% of common variance.

TABLE 6

Factor Analysis

Table 6 presents results of the factor analysis. Panel A presents results using the principal factoring method, and Panel B presents results using the maximum likelihood method. Each proxy variable is standardized by subtracting its mean and then dividing by its standard deviation. The prior communality estimate for each variable is set to its squared multiple correlation of each variable with all remaining variables. Varimax orthogonal rotation method is employed to rotate factors.

	Factor Pattern		Factor Score	
Variable	Factor 1	Factor 2	Factor 1	Factor 2
Panel A. Principal Factoring Method				
OPEN_INTEREST	0.75	0.37	0.20	0.32
DISPERSION	−0.30	0.38	−0.05	0.17
IDIOSYNCRATIC_VOLATILITY	−0.42	0.57	−0.04	0.31
STOCK_VOLUME	0.79	0.33	0.22	0.39
SIZE	0.91	−0.15	0.58	−0.37
EP	0.19	−0.49	0.03	−0.22
BM	−0.31	−0.15	−0.02	−0.09
Variance explained	71%	29%		
Panel B. Maximum Likelihood Method				
OPEN_INTEREST	0.72	0.45	0.11	0.40
DISPERSION	−0.31	0.30	−0.02	0.09
IDIOSYNCRATIC_VOLATILITY	−0.49	0.60	−0.05	0.36
STOCK_VOLUME	0.77	0.44	0.16	0.52
SIZE	0.97	−0.11	0.75	−0.48
EP	0.21	−0.39	0.01	−0.12
BM	−0.31	−0.09	−0.01	−0.03
Variance explained	71%	29%		

The 2nd factor is highly and positively loaded with open interest (0.37), dispersion in financial analysts' earnings forecasts (0.38), idiosyncratic volatility (0.57), and stock volume (0.33), but negatively loaded with firm size (−0.15), the EP ratio (−0.49), and the BM ratio (−0.15). This factor captures the

[8]As is standard in factor analysis, we refer to a variable as highly loaded on a factor if its loading on the factor is greater than 0.3.

[9]The correlation between firm size and factor 1 extracted using the principal factoring method and the maximum likelihood method is 0.91 and 0.97, respectively, which leads us to interpret factor 1 as a size factor.

characteristics of heterogeneous beliefs, as all the loadings of proxy variables (except the BM ratio) on this factor have their predicted signs on heterogeneous beliefs. Open interest, dispersion, idiosyncratic volatility, and stock volume have positive loadings on the 2nd factor, consistent with the prediction that stocks with greater belief differences have larger OTM option open interest, greater dispersion in financial analysts' earnings forecasts, higher idiosyncratic volatility, and more trading volume on stocks. Firm size and the EP ratio have negative loadings on the 2nd factor, consistent with the idea that small stocks and growth stocks generate more heterogeneity in investor beliefs. Our results show that the BM ratio is negatively related to heterogeneous beliefs, also implying that growth stocks have more dispersion in investor beliefs than value stocks. The 2nd factor explains about 29% of common variance.

Panel B of Table 6 presents results of the factor analysis using the maximum likelihood method. As with the principal factoring method, 2 factors are extracted. The 1st factor measures firm size, is related to systematic risks, and explains 71% of common variance, while the 2nd factor is associated with belief differences and explains 29% of common variance. To estimate a latent factor for subsequent analysis, it is necessary to assign a value for each latent factor. We use factor score coefficients to estimate each latent factor as a linear combination of the proxies.

C. Results Based on Latent Factors

The correlation between the skew and factor 1 (size factor) from the principal factoring method and the maximum likelihood method is 0.09 and 0.11, respectively. Fama and French (1992) argue that firm size is a priced risk factor. Thus, our results suggest that systematic risk may be an important determining factor for variation in firm-level risk-neutral skews as proposed by Duan and Wei (2009). The correlation between the skew and factor 2 (heterogeneous belief factor) is −0.25 and −0.24, respectively, confirming the negative relation between belief differences and the skew of risk-neutral density for individual firms: The greater the belief differences, the more negative the skew.

To tell whether the latent factor for heterogeneous beliefs does a better job of explaining risk-neutral skewness than individual proxies, we use the estimated latent factor to sort stocks into 5 quintile portfolios. The mean and median skews are reported in Table 7. Stocks with a greater magnitude of the latent factor for heterogeneous beliefs (factor 2) have more negative skew, supporting the argument that stocks with greater belief heterogeneity have more pronounced volatility smiles. The differential skews between the highest and lowest belief heterogeneity quintiles are significantly different from 0. Quintiles sorted on the latent factor 1 do not exhibit a monotonic pattern in the skew.

We also conduct a cross-sectional regression of the skew variable on latent factors and report results in Table 8. Factor 1 is positively related to the skew, consistent with previous results that small firms (or risky firms) have more negative skews. Factor 2 is negatively related to the skew, also consistent with previous results and supporting the hypothesis that the latent factor for heterogeneous beliefs is negatively related to the skew. The absolute value of t-statistics for the

TABLE 7

Skewness of Quintiles Using Latent Factors

Table 7 presents mean and median BKM (2003) skewness of stocks sorted by one of the latent factors identified and reported in Table 6. Each week stocks are divided into 5 quintiles based on the stated latent factor.

	Panel A. Principal Factoring Method				*Panel B. Maximum Likelihood Method*			
	Latent Factor 1		Latent Factor 2		Latent Factor 1		Latent Factor 2	
Quintiles	Mean	Median	Mean	Median	Mean	Median	Mean	Median
1 (small)	−2.63	−2.43	−1.51	−1.20	−2.60	−2.38	−1.49	−1.19
2	−1.95	−1.45	−1.62	−1.21	−1.96	−1.47	−1.62	−1.20
3	−1.67	−1.19	−1.78	−1.32	−1.71	−1.21	−1.80	−1.33
4	−1.72	−1.23	−2.16	−1.66	−1.73	−1.23	−2.20	−1.69
5 (large)	−1.84	−1.36	−2.72	−2.51	−1.79	−1.34	−2.69	−2.47
Q5 − Q1	0.79	1.07	−1.21	−1.31	0.81	1.04	−1.20	−1.28
	(41.37)	(39.89)	(−65.40)	(−55.23)	(41.47)	(40.40)	(−65.01)	(−54.46)

TABLE 8

Cross-Sectional Regression on Latent Factors

Table 8 presents time-series averages of the coefficients from weekly cross-sectional regressions of the BKM (2003) risk-neutral skewness on selected control variables described in Table 1, and the latent factors reported in Table 6. The sample includes weekly observations during the period from Jan. 2003 to Dec. 2006. Newey-West (1987) *t*-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Variable	Principal Factoring Method			Maximum Likelihood Method		
OPTION_VOLUME	−0.008***	0.035***	0.046***	−0.013***	0.040***	0.051***
	(3.17)	(18.40)	(15.89)	(5.94)	(18.86)	(15.01)
LIQUIDITY	−10.202***	1.98**	1.464	−8.284***	1.302	1.123
	(−10.86)	(2.21)	(1.54)	(−8.85)	(1.49)	(1.21)
LEVERAGE	−0.275***	−0.235***	−0.237***	−0.271***	−0.240***	−0.240***
	(−6.86)	(−6.74)	(−6.84)	(−6.78)	(−6.60)	(−6.67)
BETA	−0.22***	−0.166***	−0.156***	−0.224***	−0.172***	−0.158***
	(−6.31)	(−5.73)	(−5.60)	(−6.40)	(−5.74)	(−5.56)
Factor 1	0.120***		−0.061***	0.171***		−0.049***
	(7.48)		(−3.43)	(11.74)		(−2.70)
Factor 2		−0.6***	−0.628***		−0.52***	−0.553***
		(−37.75)	(−38.26)		(−35.07)	(−33.30)
Mean adj. R^2 (%)	5.07	10.55	10.74	5.40	10.35	10.60

coefficients of Factor 2 is relatively larger compared to that of proxies for heterogeneous beliefs given in Table 3.

The latent factor has a stronger relation with the skew than the individual proxy variables. The adjusted R^2 is also higher relative to the adjusted R^2 of regressions on any single proxy. These results indicate that the latent factor is a less-noisy measure of heterogeneous beliefs. Also, the heterogeneous belief factor explains more variation in the risk-neutral density of individual firms than the risk-based factor (factor 1), consistent with Dennis and Mayhew (2002), who suggest that firm-specific factors are more important than systematic risk in explaining the skew. When both factors are included in the regressions, the coefficients on factor 1 change sign and become less significant. This may be due to the multicollinearity problem: Factors 1 and 2 are both correlated with the liquidity in both options and stocks.

In addition, we conduct the pooled cross-sectional time-series regression of the skew using latent factors. Table 9 presents the regression results. The signs and significance of both factors are the same as the cross-sectional results. Market volatility becomes insignificant when the latent factor for heterogeneous beliefs is included in the regression. This may be due to the correlation between the belief differences and market volatility: Investor belief differences tend to be greater during a period of higher market volatility.

TABLE 9

Regression of Risk-Neutral Skewness on Latent Factors

Table 9 presents pooled regression results of the BKM (2003) risk-neutral skewness on selected control variables described in Table 1, and the latent factors reported in Table 6. Newey-West (1987) corrected estimates and t-statistics (in parentheses) are reported. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Variable	Principal Factoring Method			Maximum Likelihood Method		
INTERCEPT	−0.223***	−0.403***	−0.406***	−0.234***	−0.039***	−0.397***
	(−12.65)	(−20.41)	(−20.58)	(−13.18)	(−19.87)	(−20.11)
OPTION_VOLUME	−0.001	0.007***	0.007***	−0.002***	0.008***	0.007***
	(−1.48)	(12.41)	(8.56)	(−2.75)	(13.47)	(9.03)
LIQUIDITY	0.067	2.42***	2.584***	0.426	2.21***	2.437***
	(−0.21)	(7.53)	(7.87)	(−1.34)	(6.86)	(7.42)
LEVERAGE	−0.021***	−0.031***	−0.03***	−0.02***	−0.023***	−0.022***
	(−3.51)	(−4.44)	(−4.32)	(−3.37)	(−3.74)	(−3.58)
BETA	−0.055***	−0.043***	−0.044***	−0.055***	−0.044***	−0.045***
	(−8.44)	(−6.70)	(−6.76)	(−8.53)	(−6.79)	(−6.90)
MARKET_VOLATILITY	−3.979**	−0.28	−0.44	−4.183**	−0.422	−0.675
	(−2.36)	(−0.17)	(−0.26)	(−2.48)	(−0.25)	(−0.40)
SKEW_{t-1}	0.826***	0.812***	0.812***	0.825***	0.813***	0.813***
	(284.75)	(268.22)	(268.19)	(283.59)	(269.50)	(269.43)
Factor 1	0.026***		0.01	0.038***		0.013**
	(4.24)		(1.59)	(7.10)		(2.21)
Factor 2		−0.142***	−0.142***		−0.12***	−0.117***
		(−20.33)	(−20.33)		(−19.16)	(−17.76)
Adj. R^2 (%)	68.88	69.19	69.19	68.90	69.17	69.17

VI. Discussion

The hypothesis that heterogeneous beliefs cause greater skewness in option prices is related to the price pressure hypothesis examined by Bollen and Whaley (2004). Specifically, it is possible that belief differences cause equilibrium option prices to have greater skew in the absence of price pressure. It is also possible that belief differences cause investors to trade in ways that put pressure on option prices, moving them away from the price they would have if options were priced by dynamic arbitrage. To examine these 2 possibilities, we look at the sensitivity of skewness to dispersion in beliefs as a function of the liquidity of the option.

Using option trading volume as a proxy for option liquidity, we sort options into quintiles based on liquidity. Within each quintile, we test the relation between heterogeneous beliefs and the skew. From Table 10, we see that skewness is more sensitive to belief differences for the less liquid securities. But the relation between heterogeneous beliefs and skewness still holds after we control for liquidity, as shown in Tables 3 and 4. This result suggests that the effect of belief differences on skew goes beyond the net buying pressure suggested by Bollen and Whaley (2004).

TABLE 10

Sensitivity of Skewness to Heterogeneous Beliefs across Option Liquidity Quintiles

Table 10 presents the coefficients on the heterogeneous beliefs factor (latent factor 2) in the regression of risk-neutral skewness for quintiles sorted on option liquidity proxied by option volume. Quintile 1 has the highest option trading volume, while Quintile 5 has the lowest.

	Principal Factoring Method		Maximum Likelihood Method	
Quintile	Cross-Sectional Regression	Pooled Regression	Cross-Sectional Regression	Pooled Regression
1	−0.31	−0.30	−0.25	−0.23
2	−0.61	−0.58	−0.55	−0.52
3	−0.67	−0.63	−0.59	−0.57
4	−0.83	−0.74	−0.79	−0.74
5	−1.00	−0.87	−1.00	−0.92

Another issue, tangential to the main focus of our paper, is the relation between leverage and skewness, which has been disputed in the prior literature. Toft and Prucyk (1997) report a negative relation between leverage and skewness, while Dennis and Mayhew (2002) find that this relation disappears after controlling for other factors. The results throughout our analysis are generally consistent with Toft and Prucyk, supporting the hypothesis that firms with more leverage have more negative skews. One exception to this is in Table 4, where we find a positive coefficient on leverage. This particular result could be driven by the multicollinearity between leverage and the BM ratio (Table 2 reports that the correlation is 0.657).[10]

Overall, the negative relation between leverage and skewness is somewhat sensitive to model specification and how the skew is measured. We view this result cautiously, and since it is not the focus of the current paper, we leave a more definitive examination of the relation for future work.

VII. Conclusion

Buraschi and Jiltsov (2006) show that belief differences can affect the skewness of the risk-neutral density function. We find that stocks with greater belief heterogeneity have more negative risk-neutral skewness. Small stocks and stocks with low EP ratios have more negative skewness than large stocks and stocks with high EP ratios, respectively. Risk-neutral skewness is correlated with OTM option open interest, the dispersion in financial analysts' earnings forecasts, idiosyncratic volatility, and stock trading volume, all of which are widely used proxies for heterogeneous beliefs in literature. More generally, our results support the hypothesis that belief differences help explain the cross-sectional variation of risk-neutral skewness.

[10]In a previous version of the paper, we examined the slope of the implied volatility as a function of both the right (positive) skew and left (negative) skew. We found a negative relation between the leverage and the negative skew (using OTM puts), which was consistent with Toft and Prucyk (1997). However, we found a positive relation between the leverage and the positive skewness (using OTM calls). The risk-neutral skewness in the current version is constructed using both OTM puts and OTM calls and thus is a composite of these 2 results.

This study also uses factor analysis to identify a latent variable that represents heterogeneous beliefs by combining variables frequently used as proxies for heterogeneous beliefs: option open interest, the dispersion in financial analysts' earnings forecasts, idiosyncratic volatility, stock trading volume, firm size, and the EP and BM ratios. Factor analysis also identifies a factor linked to systematic risk. The belief factor explains more variation in risk-neutral skewness than the risk-based factor, which supports the argument of Dennis and Mayhew (2002) that firm-specific factors are more important than systematic factors in explaining the cross-sectional variation of skewness. The impact on option prices goes beyond the net buying pressure argued by Bollen and Whaley (2004). Our results suggest that investor belief differences may be the unexplained firm-specific factor that impacts the price of stock options.

References

Baik, B., and C. Park. "Dispersion of Analysts' Expectations and the Cross-Section of Stock Returns." *Applied Financial Economics*, 13 (2003), 829–839.

Bakshi, G.; N. Kapadia; and D. Madan. "Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options." *Review of Financial Studies,* 16 (2003), 101–143.

Biais, B., and P. Bossaerts. "Asset Prices and Trading Volume in a Beauty Contest." *Review of Economic Studies,* 65 (1998), 307–340.

Black, F., and Scholes M. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, 81 (1973), 637–654.

Bollen, N. P. B., and R. E. Whaley. "Does Net Buying Pressure Affect the Shape of Implied Volatility Functions?" *Journal of Finance*, 59 (2004), 711–753.

Buraschi, A., and A. Jiltsov. "Model Uncertainty and Option Markets with Heterogeneous Beliefs." *Journal of Finance*, 61 (2006), 2841–2897.

Conrad, J.; R. F. Dittmar; and E. Ghysels. "Ex Ante Skewness and Expected Stock Returns." *Journal of Finance*, forthcoming (2012).

Dennis, P., and S. Mayhew. "Risk-Neutral Skewness: Evidence from Stock Options." *Journal of Financial and Quantitative Analysis*, 37 (2002), 471–493.

Diether, K. B.; C. J. Malloy; and A. Scherbina. "Differences of Opinion and the Cross Section of Stock Returns." *Journal of Finance*, 57 (2002), 2113–2141.

Doukas, J. A.; C. F. Kim; and C. Pantzalis. "Divergent Opinions and the Performance of Value Stocks." *Financial Analysts Journal*, 60 (2004), 55–64.

Duan, J.-C., and J. Wei. "Systematic Risk and the Price Structure of Individual Equity Options." *Review of Financial Studies*, 22 (2009), 1981–2006.

Fama, E. F., and K. R. French. "The Cross-Section of Expected Stock Returns." *Journal of Finance*, 47 (1992), 427–465.

Fama, E. F., and J. D. MacBeth. "Risk, Return, and Equilibrium: Empirical Tests." *Journal of Political Economy*, 81 (1973), 607–636.

Geske, R. "The Valuation of Compound Options." *Journal of Financial Economics*, 7 (1979), 63–81.

Goetzmann, W. N., and M. Massa. "Index Funds and Stock Market Growth." *Journal of Business*, 76 (2003), 1–28.

Han, B. "Investor Sentiment and Option Prices." *Review of Financial Studies*, 21 (2008), 387–414.

Harris, M., and A. Raviv. "Differences of Opinion Make a Horse Race." *Review of Financial Studies*, 6 (1993), 473–506.

He, H., and J. Wang. "Differential Information and Dynamic Behavior of Stock Trading Volume." *Review of Financial Studies*, 8 (1995), 919–972.

Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *Review of Financial Studies*, 6 (1993), 327–343.

Hong, H., and J. C. Stein. "Differences of Opinions, Short-Sales Constraints, and Market Crashes." *Review of Financial Studies*, 16 (2003), 487–525.

Hull, J. C. *Options, Futures, and Other Derivative Securities*. Englewood Cliffs, NJ: Prentice Hall (1993).

Hull, J. C., and A. White. "An Analysis of the Bias in Option Pricing Caused by Stochastic Volatility." *Advances in Futures and Options Research*, 3 (1988), 29–61.

Kaiser, H. F. "The Varimax Criterion for Analytic Rotation in Factor Analysis." *Psychometrika*, 23 (1958), 187–200.

Kandel, E., and N. D. Pearson. "Differential Interpretation of Public Signals and Trade in Speculative Markets." *Journal of Political Economy*, 103 (1995), 831–872.

Lemmon, M., and S. X. Ni. "The Effects of Investor Sentiment on Speculative Trading and Prices of Stock and Index Options." Working Paper, University of Utah (2008).

Li, T. "Heterogeneous Beliefs, Option Prices, and Volatility Smiles." Working Paper, City University of Hong Kong (2007).

Newey, W. K., and K. D. West. "A Simple, Positive Semi-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, 55 (1987), 703–708.

Odean, T. "Volume, Volatility, Price, and Profit When All Traders Are Above Average." *Journal of Finance*, 35 (1998), 1887–1934.

Peña, I.; G. Rubio; and G. Serna. "Why Do We Smile? On the Determinants of the Implied Volatility Function." *Journal of Banking and Finance*, 23 (1999), 1151–1179.

Shalen, C. T. "Volume, Volatility, and the Dispersion of Beliefs." *Review of Financial Studies*, 6 (1993), 405–434.

Shefrin, H. "On Kernels and Sentiment." Working Paper, Santa Clara University (2001).

Taylor, S. J.; P. K. Yadav; and Y. Zhang. "Cross-Sectional Analysis of Risk-Neutral Skewness." *Journal of Derivatives*, 16 (2009), 38–52.

Toft, K. B., and B. Prucyk. "Options on Leveraged Equity: Theory and Empirical Tests." *Journal of Finance*, 52 (1997), 1151–1180.

Ziegler, A. *Incomplete Information and Heterogeneous Beliefs in Continuous-Time Finance.* New York, NY: Springer-Verlag LLC (2003).