Reputation Pricing: A Model for Valuing Future Life Insurance Policies

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Abstract†

The reputation of a life insurer is used to develop a model for determining the value of future life insurance policies. An $M/G/\infty$ process is used to describe the sales and terminations (due to death or maturity) of future policies. The intensity of the arrival process is assumed to depend on the company's reputation. Explicit expressions are derived for the actuarial reserves and expected profits of these future policies.

Key words and phrases: *future policyholders, expected profits, expected reserve, $M/G/\infty$ queue*

1 Introduction

When investors are interested in purchasing an insurance company, they usually seek an expert appraisal of the value of the company from actuaries, accountants, and other financial professionals. The insurance company may have diverse business interests, including different lines of products sold. As it is common for an insurance company to group similar insurance policies into portfolios (i.e., blocks of policies), the appraised value of the company should reflect the value of each

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portfolio[1] including any intangible assets[2] associated with each portfolio.

The value of a life insurance portfolio consists of two components:

(i) The value of the active portfolio, which consists of the life insurance contracts that currently exist and remain active in the portfolio and for which actuarial reserves are not equal to zero. The value of the active portfolio depends on the assets and the aggregate actuarial reserves associated with all of the contracts in the active portfolio.

(ii) The value of the future portfolio is based largely on the insurer's intangible assets: its reputation and its management/marketing strategies for attracting and maintaining new policies. We will assume new policyholders will purchase their insurance from the insurance company based on the strength of the company's reputation. Thus, to determine the value of a future portfolio, assumptions must be made about the insurer's reputation, and its attitude toward new policies.

Economists and accounts long have recognized that one of a firm's intangible assets is its name, or the reputation conveyed by its name. Economists have used game theory to study a firms reputation; see, for example, Kreps and Wilson (1982), Fudenberg and Kreps (1987), Diamond (1989), Fudenberg and Levine (1989), Kreps (1990), Kreps et al. (1992), and Hart (1995). As an example, Kreps (1990) developed a theory of the firm as a bearer of reputation and provides a simple example that demonstrates, using the ideas of the folk theorem in repeated games, how a firm's reputation can become a tradable asset. Game theoretic techniques, however, can be difficult to apply to the problem of valuing the reputation of a life insurance portfolio because of the uncertainties associated with determining the makeup of a future portfolio. Unfortunately, there is no established actuarial theory to assist in valuing an insurer's reputation.[3]

[1]In Israel, for example, experts conducting these appraisal valuations most commonly perform a separate evaluation of each portfolio.

[2]A firm's intangible assets or goodwill, which includes the firm's reputation and/or name, are usually hidden in its balance sheet. The actual value of intangible assets is known only when the firm is sold and is obtained by subtracting the value of tangible net assets from the firm's sale price.

[3]In Israel the aggregate actuarial reserves is multiplied by a loading factor to yield a value for a future portfolio that is based on the insurer's reputation. There is no actuarial guidance, however, on how the size of this loading factor is determined. For example, a private investor in Israel recently purchased the successful Israel Phoenix

In the author's opinion, when actuaries or other experts evaluate future (reputation) life insurance portfolios, they can use one of the two approaches described above for their evaluation process. The first approach is to use the insurance company's historical data to project the composition of the portfolio's future insureds. These data contain information on the date of policy inception, age, gender, mortality level, amount of insurance, type of policy, date of exit from portfolio, cause of exit, assumed mortality table, etc. By assuming these data can accurately represent the insureds in the future portfolio, one can anticipate the development of that portfolio by using deterministic methods. The second approach is to use the historical data to develop a stochastic model (such as a queueing model) of the influx and efflux of the portfolio's future insureds. In either approach, one problem will be the choice of mortality table to use. As mortality is continuously improving in most countries, the mortality table used should have built in factors that account for this improvement.

Another aspect mentioned above is the management/marketing strategy employed with respect to new policies. When investors purchase an insurance company, they often continue managing the various insurance portfolios without any restrictions on the sale of policies to future customers. In other words, the investors allow applicants for new life insurance contracts to purchase policies after they have satisfactorily completed the necessary underwriting. This approach, however, may not always be best for the investor. For example, in Israel insurance regulators require that the reserve for a life insurance portfolio be proportional to the number of policyholders insured in the portfolio. If investors do not believe the reserve requirements needed for expanding a portfolio will be available or do not believe it worthwhile to raise this money, then it may be best to restrict the sale of new policies and restrain the growth of the portfolio. In this paper we will assume there are no limits on the number policyholders accepted.

As was mentioned above, there is no established actuarial theory or model for valuing an insurer's reputation. Given an insurer's reputation, however, can we determine the value of one of its future portfolio? In the author's opinion, when actuaries or other experts evaluate future life insurance portfolios, they should use one of two approaches:

1. Use the insurance company's historical data to project the composition of the portfolio's future insureds. These data will contain

Insurance Company. Analysts on Israeli television commented that 35% of the price paid reflected the value of its life insurance portfolio, including the intangible asset based on the Phoenix's reputation.

information on the date of policy inception, age, gender, mortality level, amount of insurance, type of policy, date of exit from portfolio, cause of exit, assumed mortality table, etc. By assuming the data can represent the insureds in the future portfolio accurately, one can anticipate the development of that portfolio by using deterministic methods.

2. Alternatively, use the historical data to develop a stochastic model (such as a queueing model) of the influx and efflux of the portfolio's future insureds.

In either approach, the mortality table used should have built in factors that account for mortality improvement.

The objective of this paper is to present an actuarial model for the evaluation of a future life insurance portfolio. We will propose a dynamic stochastic model of the number of policies in force at any time[4] to describe the evolution of the future life insurance portfolio. The model assumes new policies are issued in a Poisson process and the number of policyholders decreases due to deaths and policy expirations. The rate of new policy issues is assumed to depend on the reputation of the insurer: the better the reputation, the higher the arrival rate. The number of policyholders insured (the in-force process) is allowed to increase without bounds.

Because of the Poisson process assumption, we are implicitly assuming there is an infinite population of potential policyholders. It turns out that our model can efficiently be described as an $M/G/\infty$ queue model where new customers enter the pool of insured parties by a Poisson process (M), each policyholder remains in the portfolio for random period of time that follow a general distribution (G), and the insurance portfolio has infinity capacity (∞). Using this model, we derive an expression for the prospective actuarial reserves of the portfolio t years in the future using each of the two valuation strategies.

2 The Model

Let us consider an insurance portfolio that consists of special fully continuous n-year endowment insurance policies with death benefit B_1

[4]There are not many dynamic models proposed in the actuarial literature. The first one was proposed by Ramsay (1985), who considered a birth-death model of a life insurance portfolio operating in a finite population of potential insureds. Willmot (1990) used techniques from queueing theory to analyze the claim liabilities of an insurance company and provide an example of the application to life insurance portfolio.

and survival benefit B_2, and with premiums paid for h ($h \leq n$) years. When this policy is sold to a person age x, the net annual premium, π_x, can be expressed in standard actuarial notation as

$$\pi_x = \frac{B_1 \overline{A}^{\ 1}_{x:\overline{n}|} + B_2 \, {}_nE_x}{\overline{a}_{x:\overline{h}|}} \tag{1}$$

for $x = 0, 1, 2, \ldots$. The actuarial functions are calculated using a known standard mortality with survival function ${}_tp_x$. Assuming the policyholder is alive at age $x + t$, the net premium reserve t years after the policy is issued (i.e., at age $x + t$) is $\overline{V}_x(t)$ where

$$\overline{V}_x(t) = \begin{cases} B_1 \overline{A}^{\ 1}_{x+t:\overline{n-t}|} + B_2 \, {}_{n-t}E_{x+t} - \pi_x \, \overline{a}_{x+t:\overline{h-t}|} & 0 \leq t \leq h \\ B_1 \overline{A}^{\ 1}_{x+t:\overline{n-t}|} + B_2 \, {}_{n-t}E_{x+t} & h \leq t < n \\ B_2 & t = n \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The following assumptions are needed to fully describe our model:

A.1: Each customer who applies for insurance is subject to underwriting (medical and otherwise). If the applicant is deemed insurable, then he or she is sold the special n-year endowment insurance contract described above and becomes a policyholder in the portfolio.

A.2: The mortality of a policyholder age x follows the same known survival function used to determine premiums and reserves, i.e., ${}_tp_x$. Let $T(x)$ be the future lifetime of a typical policyholder age x. Then the time spent in the portfolio is $T_n(x) = \min(T(x), n)$. The cdf of $T_n(x)$ is $G_n(s, x)$ where

$$G_n(s, x) = \mathbb{P}[T_n(x) \leq s] = \begin{cases} {}_sq_x & \text{for } s < n \\ 1 & \text{for } s \geq n. \end{cases} \tag{3}$$

and the resulting survival function is

$$\overline{G}_n(s, x) = 1 - G_n(s, x).$$

A.3: Policyholders leave the portfolio only through death or at the time of the maturity of the policy. There are no policy conversions, lapses, or cancelations.

A.4: The policyholders are mutually independent and indistinguishable, except, possibly, for their age at the issue of their respective policy.

A.5: At $t = 0$, n_x new policies are issued to policyholders age x.

A.6: The future new policyholders age x arrive in the portfolio in a homogeneous Poisson process with rate λ_x.

A.7: The size of λ_x depends on the reputation of the insurer.

A.8: Finally, there are no expenses.

3 The Main Results

Consider a new policyholder age x who joined the portfolio at time y. The net premium reserve at time $t > y$ due to this policyholder, i.e., $(t - y)$ years after joining the portfolio, is $\overline{V}_x(t - y)$. Now suppose that in the time interval $(0, t)$ we are given that k new policyholders arrived in the portfolio with the i^{th} arrival occurring at time $y_{(i)}$, where $0 < y_{(1)} < y_{(2)} < \cdots < y_{(k)} < t$. Then the expected reserve at time t given these k arrivals is

$$\sum_{i=1}^{k} \overline{V}_x(t - y_{(i)})\ {}_{t-y_{(i)}}p_x.$$

The total expected reserve at time t for policies sold to persons age x in $(0, t)$, $R_x(t)$, is thus:

$$R_x(t) = \sum_{k=0}^{\infty} \frac{e^{-\lambda_x t} (\lambda_x t)^k}{k!} \int_{y_{(1)}=0}^{t} \int_{y_{(2)}=y_{(1)}}^{t} \cdots \int_{y_{(k)}=y_{(k-1)}}^{t} \left[\sum_{i=1}^{k} \overline{V}_x(t - y_{(i)})\ {}_{t-y_{(i)}}p_x \right] f(y_{(1)}, y_{(2)}, \ldots, y_{(k)})\, dy_{(k)} \ldots dy_{(1)}.$$

From Ross (1996, Theorem 2.3.1), the conditional ordinal arrival times of a homogeneous Poisson process in $(0, t)$, given there are k arrivals, follow the same distribution as that of the order statistics of a random sample of uniform $(0, t)$ variables. Thus, the joint p.d.f. is

$$f(y_{(1)}, y_{(2)}, \ldots, y_{(k)}) = \frac{k!}{t^k} \quad \text{for} \quad 0 < y_{(1)} < y_{(2)} < \ldots < y_{(k)} < t.$$

The multiple integral can now be simplified as follows:

$$\int_{y_{(1)}=0}^{t}\int_{y_{(2)}=y_{(1)}}^{t}\cdots\int_{y_{(k)}=y_{(k-1)}}^{t}\left[\sum_{i=1}^{k}\overline{V}_x(t-y_{(i)})\,{}_{t-y_{(i)}}p_x\right]\frac{k!}{t^k}dy_{(k)}\ldots dy_{(1)}$$
$$=\int_{y_1=0}^{t}\int_{y_2=0}^{t}\cdots\int_{y_k=0}^{t}\left[\sum_{i=1}^{k}\overline{V}_x(t-y_i)\,{}_{t-y_i}p_x\right]\left(\frac{1}{t^k}\right)dy_k\cdots dy_1$$
$$=\frac{k}{t}\int_{y=0}^{t}\overline{V}_x(t-y)\,{}_{t-y}p_x dy.$$

The expression for $R_x(t)$ is now seen to be

$$R_x(t)=\sum_{k=0}^{\infty}\frac{e^{-\lambda_x t}(\lambda_x t)^k}{k!}\frac{k}{t}\int_{y=0}^{t}\overline{V}_x(t-y)\,{}_{t-y}p_x dy$$
$$=\lambda_x\int_{y=0}^{t}\overline{V}_x(t-y)\,{}_{t-y}p_x dy. \tag{4}$$

The total expected reserve at time t for all policies sold $[0,t)$, including those newly in existence at time $t=0$, is

$$R(t)=\sum_x\left[n_x\overline{V}_x(t)+\lambda_x\int_{y=0}^{t}\overline{V}_x(t-y)\,{}_{t-y}p_x dy\right]. \tag{5}$$

The reserve process $R(t)$ represents the liabilities of the insurer to its portfolio of policyholders at time t. This means that an investor who purchases the life insurance portfolio will have a commitment or obligation of amount $R(t)$ to these policyholders. If $A(t)$ represents the amount of assets the portfolio has on hand at time t, then the portfolio's surplus at time t is, $U(t)$, where

$$U(t)=A(t)-R(t). \tag{6}$$

To value the future portfolio we must perform a profit evaluation, which requires knowledge of the future expected rate of profits generated by this portfolio. To this end, we must determine the gross (profit-loaded) premium charged given assumption A.8 (there are no expenses). In practice there are typically three ways to obtain the gross premium rate that allows a profit to the insurer:

1. Use conservative estimates of the various parameters involved in the pricing process. For example, assume a lower interest rate,

higher mortality rates, and lower investment returns than are actually expected. This results in insureds paying a higher premium than they would pay if the best estimates were used.

2. Explicitly specify a profit objective then include that profit as an expense. The gross premium then can be calculated by the actuarial equivalence principle; see, for example, Bowers et al. (1997, Chapter 15). Or,

3. Increase the net premium by a loading factor.

Regardless of the approach used, given the net premium rate π_x and assuming there are no expenses, let $\pi_x^{(g)}$ and $\pi_x^{(p)}$ be the gross premium rate and the expected profit rate, respectively, for policies in the portfolio of policies sold to persons age x, i.e.,

$$\pi_x^{(g)} = \pi_x + \pi_x^{(p)}.$$

To determine the discounted expected portfolio profits we need an expression for the expected number of policyholders expected to be insured at any time t. Let $Q_x(t)$ denote the in-force process, i.e., the number of policyholders who bought their policies at age x at some time y ($y \leq t$) and are still in force at time t with $Q_x(0) = n_x$. (Unlike the models used in traditional risk theory, $Q_x(t)$ is a stochastic (queueing) process.) Thus the expected amount of profits in the time interval $(s, s + \mathrm{d}s)$ generated by the portfolio of policies that were sold to persons age x is $\pi_x^{(p)}\mathbb{E}[Q_x(s) \mid Q_x(0) = n_x]\mathrm{d}s$. If we let $\text{Profit}_x(t)$ be the discounted expected profits in $(0, t)$ from the portfolio of policies that were sold to persons age x, then

$$\text{Profits}_x(t) = \int_0^t \pi_x^{(p)}(1 + i)^{-s}\mathbb{E}[Q_x(s) \mid Q_x(0) = n_x]\mathrm{d}s \tag{7}$$

where i is the valuation rate of interest. The ultimate expected profits from the entire portfolio is

$$\text{Profits} = \sum_x \pi_x^{(p)} \int_0^\infty (1 + i)^{-s}\mathbb{E}[Q_x(s) \mid Q_x(0) = n_x]\mathrm{d}s. \tag{8}$$

Finally, we need an expression for $\mathbb{E}[Q_x(t) \mid Q_x(0) = n_x]$. Clearly $Q_x(t)$ is the number of customers at time t in an $M/G/\infty$ queue with Poisson arrivals at rate λ_x and service time distribution $G_n(s)$ given in equation (3). It is well known (e.g., Ross 1996, p. 70 and Medhi 2003, Chapter 6.10.1) that the distribution of $Q_x(t) \mid Q_x(0) = 0$ is a Poisson distribution with mean $\lambda_x(t)$ given by

$$\lambda_x(t) = \lambda_x \int_0^t \overline{G}_n(s,x)\mathrm{d}s.$$

Thus we can consider an $M/G/\infty$ queue with initial queue length $Q_x(0) = n_x$ as being artificially partitioned into two independent and disjoint sub-queues: a permanently closed queue that consists of the n_x busy servers and a permanently open (but initially empty) $M/G/\infty$ queueing system such that every newly arriving customer can only be served at the open queue. Clearly the number still remaining in the closed queue at time t is binomially distributed with parameters n_x and $\overline{G}_n(t,x)$, which gives a mean of $n_x\overline{G}_n(t,x)$. On the other hand, the expected number of customers at time t in the open queue is $\lambda_x(t)$. Thus the expected number in the queue at time t is $n_x\overline{G}_n(t,x) + \lambda_x(t)$, i.e., we have the following result:

Theorem 1. *If $Q_x(t)$ is the number of customers in an $M/G/\infty$ queue with Poisson arrivals at rate λ_x and independent service times with distribution function $G_n(s)$, then*

$$\mathbb{E}[Q_x(t) \mid Q_x(0) = n_x] = n_x\overline{G}_n(t,x) + \lambda_x \int_0^t \overline{G}_n(s,x)\mathrm{d}s. \qquad (9)$$

4 Summary and Closing Comments

This paper introduces to actuarial pricing a method for evaluating a future life insurance portfolio, which has a growth rate that depends on the reputation of the insurer. When an investor is interested in purchasing such a portfolio, the insurer must be compensated for the reputation of the portfolio. As there is no actuarial theory to assist in valuing an insurer's reputation, the common approach for actuarial practitioners in Israel, for example, is to evaluate the active portfolio which consists of the life insurance contracts that currently exist and remain active in the portfolio. This value is multiplied by a loading factor for which there have been no guidelines for determination.

To correct this state of affairs, we suggest a stochastic model for valuing the future life insurance policies. We specifically use an $M/G/\infty$ process to describe the sales and terminations of future policies and to provide expressions for the total expected reserve and the profit of the future portfolio.

In deriving the expression for the expected reserves in equation (4) we used the marginal distribution of the number of arrivals and the order statistics property of the conditioned arrival times for the homogeneous Poisson process. This technique can be applied to other processes such as the nonhomogeneous Poisson process and the Yule process because they also have the order statistics property; see, for example, Berg and Spizzichino (1999).

Other areas for further research include:

- Considering alternative models. For example, we can consider the case where the insurer intends to limit the size of the portfolio to at most c policyholders so that we have an $M/G/c$ queue with no waiting room. Thus if there are fewer than c policyholders in the portfolio, new contracts are sold (subject to underwriting approval) until there are c policyholders in the portfolio. Once there are c policyholders in the portfolio, then all applications for insurance are denied until there is a death or a policy matures.
- Using stochastic interest rates to determine present values; and
- Use insurance demand function for profit determination. For example, we can assume the demand of insurance decreases as the profit loading increases, i.e., assume that for a given reputation and insurance policy λ_x is a decreasing function of $\pi_x^{(p)}$. This is similar to the work of Kliger and Levikson (1998) and Ramsay (2005).

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