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Xiang-Fa Wu
Department of Engineering Mechanics, University of Nebraska-Lincoln, xfwu@unlserve.unl.edu

Yuris A. Dzenis
Department of Engineering Mechanics, University of Nebraska-Lincoln, ydzenis@unl.edu

Kyle W. Strabala
University of Nebraska-Lincoln, kstrabala84@yahoo.com

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Wrinkling of a charged elastic film on a viscous layer

Xiang-Fa Wu,[1] Yuris A. Dzenis,[1] and Kyle W. Strabala [2]

[1] Department of Engineering Mechanics, Center for Materials Research and Analysis, University of Nebraska-Lincoln, Lincoln, NE 68588-0526, USA

[2] Department of Mechanical Engineering and Department of Mathematics and Statistics, University of Nebraska-Lincoln, Lincoln, NE 68588-0323, USA

Abstract

A thin metallic film deposited on a compliant polymeric substrate begins to wrinkle under compression induced in curing process and afterwards cooling of the system. The wrinkle mode depends upon the thin film elasticity, thickness, compressive strain, as well as mechanical properties of the compliant substrate. This paper presents a simple model to study the modulation of the wrinkle mode of thin metallic films bonded on viscous layers in external electric field. During the procedure, linear perturbation analysis was performed for determining the characteristic relation that governs the evolution of the plane-strain wrinkle of the thin films under varying conditions, *i.e.*, the maximally unstable wrinkle mode as a function of the film surface charge, film elasticity and thickness, misfit strain, as well as thickness and viscosity of the viscous layer. It shows that, in proper electric field, thin film may wrinkle subjected to either compression or tension. Therefore, external electric field can be employed to modulate the wrinkle mode of thin films. The present results can be used as the theoretical basis for wrinkling analysis and mode modulation in surface metallic coatings, drying adhesives and paints, and microelectromechanical systems (MEMS), etc.

Keywords: thin films, wrinkling, viscous flow, pattern modulation, applied mechanics

1 Introduction

Wrinkling as a common surface instability phenomenon occurs in nature and engineered film systems, which is a result of the redistribution of strain energies between the thin hard surface layers and the compliant substrates [1–3, 9, 17, 34; and references]. In such film systems, strain mismatch usually builds up during the natural or manufacturing process such as aging, cooling, and solvent evaporation, annealing, and sol-gel transition of the compliant polymeric substrates, etc. For a thin elastic film deposited on a compliant substrate with sufficient interface strength, compressive misfit strains in the film generally evoke its elastic wrinkling under small perturbation to reduce the strain energy stored in the system [4, 5, 13–16, 21, 26]. On the other hand, strain mismatch in a thin film system with weak interface strength may result in buckling delamination [10, 18]. For thin elastic film wrinkling on compliant substrate, recent theoretical investigations have made great progress in understanding the wrinkling mechanism and relevant characteristic relation that governs the wrinkle growth. By using linear perturbation, Kirchhoff plate theory, and solution to a viscous layer with forced surface perturb, Sridhar *et al.* [26] first obtained the onset of maximally unstable mode of a compressed film wrinkling on a linearly viscous substrate as a function of the misfit strain and the thickness and viscosity of the viscous substrate layer. It was found that the onset of the film wrinkling on a glass layer is the same as that for a compressively stressed free-standing film though the maximally unstable wrinkle wavelength grows with the increase of the glass layer thickness. Furthermore, by means of nonlinear von Karman plate theory and Reynolds lubrication theory, Liang *et al.* [21] and Huang and Suo [13] refined the above study and considered the relaxation of compressed finite islands and thin films on viscous layers. In the limiting cases, results obtained in their models are in a good agreement with those predicted by Sridhar *et al.* [26]. Furthermore, Huang and Suo [14] and Huang [11] extended their earlier approaches to study thin film wrinkling on viscous layer with arbitrary thickness and on viscoelastic half-space. The general conclusions are similar to those of their earlier works. Moreover, to study the 2D patterns of thin films wrinkling on compliant substrates, Huang *et al.* [15] developed a concise 2D spectral scheme to simulate the wrinkle evolution in a numerical manner. The nonlinear effect due to finite magnitudes of the wrinkle modes was further taken into account in their later investigation [16]. As a result, the wrinkle patterns obtained in the 2D spectral simulation match those observed in experiments very well [1, 4, 17]. The above studies can be used to explore the wrinkling mechanisms and related characterization and suppression in thin films. Nevertheless, in view of application, it is desirable if wrinkling of thin films can be developed in a controllable manner.

Among others, external electric field would be one of the best candidates used for wrinkling modulation. In recent pioneering study in nanolithography, researchers have formulated highly ordered surface patterns through surface instablization of thin compliant dielectric polymeric layers in electric field [6, 7, 24, 25]. Relevant theories regarding conditions of linear and nonlinear surface stabilities of the thin dielectric polymeric layers have been developed recently [22, 23, 29, 31, 32]. Full field simulations of pattern evolution in thin dielectric polymeric layers in electric field have also been conducted by Wu and Russel [30] and Wu *et al.* [31, 32] using a 2D electrohydrodynamic viscous layer model, and by Kim and Lu [19] using a 3D spectral electrohydrodynamic phase-separation model. Furthermore, Huang [12] recently developed a simple model to further examine the instability criterion of a charged thin conductive elastic film bonded on a compliant dielectric layer; and Wu and Dzenis [33] considered the electrohydrodynamic instability of an ultrathin conductive viscous layer in electric field. In the latter case, the electrostatic surface traction induced by electric field was determined by the Maxwell stress tensor [20]. These investigations have indicated that electric field did play a vital role in wrinkle evolution of thin films and can be utilized for the purpose of wrinkling modulation.

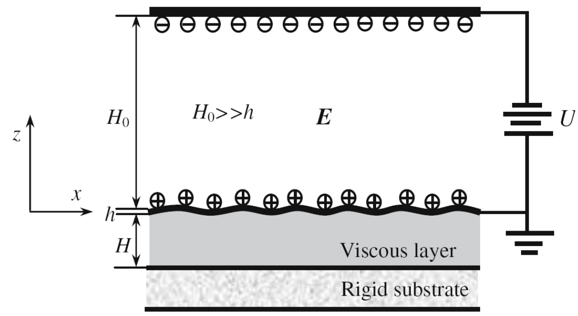

Figure 1. Wrinkling of a conductive elastic film on a viscous layer in external electric field.

Therefore, it has pragmatic value to consider the mode modulation of thin metallic film wrinkling on viscous layer in external electric field. A simple plane-strain wrinkling model is shown in Figure 1, where the upper conductive flat plate is adopted to sustain a constant far electric field. Without loss of generality, the thin conductive film is assumed being grounded. For thin film subjected to electric field, film tractions induced by surface charges vary with the density of net charges at film surfaces [20]. The charge density is a function of film profile that obeys the electrostatic law.

In an attempt to investigate the effect of surface charges on the wrinkle mode of a thin metallic film bonded on a viscous layer in electric field, in this study, we perform a linear perturbation analysis of a charged film-substrate system (see Figure 1). The characteristic relation that governs the plane-strain wrinkling is determined, *e.g.,* the critical wrinkle mode number and the wrinkle growth rate as functions of the strength of electric field, film elasticity, viscosity of the substrate layer, and system geometries, respectively. Variation of the wrinkle growth rate versus the wrinkle mode number is plotted under varying condition. Limiting cases are examined and compared with data available in literature.

2 Problem formulation and solution

Consider a charged thin metallic film on a viscous layer that is located on a flat rigid substrate, as shown in Figure 1. The system maybe modeled as the bending of a compliant beam under the coupling action of the axial stress, lateral nontrivial flow, and electrostatic pressure induced by small surface perturb. The interface between the thin film and the viscous layer is assumed ideal, *i.e.,* displacements and stresses are continuous across the interface. In the following, the inertia effect of the thin film and the viscous layer is ignored. Furthermore, since we are interested in the wrinkle initiation in the film system asymptotically close to its equilibrium state, the trivial constant forces such as those induced by the film and viscous layer gravities and the air pressure are safely neglected. The thin metallic film is considered as conductive and linearly elastic, and the viscous layer is regarded as incompressible newtonian flow.

2.1 Electrostatic tractions of a wrinkled conductive film in constant electric field

For unperturbed film, the potential of the electric field above the film may be expressed as

$$\varphi = -4\pi\sigma_0 z, \tag{1}$$

where σ_0 is the charge density at the film surface which varies with the change of the plate distance H_0 (see Figure 1), and z is the vertical coordinate from the flat film surface. For a small perturb of the thin film:

$$w = A(t)\sin(kx) \tag{2}$$

with $A(t)$ and k are, respectively, the perturb amplitude and the perturb wave number, the potential of the electrostatic field above the thin film can be expressed as [20]

$$\varphi = -4\pi\sigma_0 z + \varphi_1. \tag{3}$$

Here φ_1 is a small correction satisfying $\Delta\varphi_1 = 0$ and vanishing for $z \to \infty$. Thus, φ_1 can be further expressed as

$$\varphi_1 = CA(t)\sin(kx)\exp(-kz) \tag{4}$$

with an unknown constant C to be determined by matching the boundary condition. On the surface of the conductive film itself, the electrostatic potential must have a constant zero due to the grounding (see Figure 1), thus the perturb φ_1 given in (4) can be determined as

$$\varphi_1 = 4\pi\sigma_0 A(t)\sin(kx) \tag{5}$$

for $z = 0$. As a result, the first-order expansion of the surface electrostatic traction (Maxwell stress tensor) leads to

$$\begin{aligned} p_e(w) = -\frac{E^2}{8\pi} &\approx -2\pi\sigma_0^2 - [k\sigma_0\phi_1]_{z=0} \\ &= -2\pi\sigma_0^2 - 4\pi\sigma_0^2 kw, \end{aligned} \tag{6}$$

which is closely related to the perturb amplitude and the perturb wave number in the asymptotic sense.

2.2 Stress and velocity field in a layer of incompressible newtonian flow

The motion of the viscous layer under consideration is slow such that the inertia can be ignored, thus its equilibrium equations reduce to

$$\sigma_{ij,j} = 0 \quad (i, j = 1, 2, 3), \tag{7}$$

where σ_{ij} is the Cauchy stress tensor. For an incompressible newtonian flow, its constitutive relations are

$$\sigma_{ij} = \eta(v_{i,j} + v_{j,i}) - p\delta_{ij}, \tag{8}$$

where η is the kinematic viscosity, v_i the particle velocity of the viscous layer, p the hydrostatic pressure, and d_{ij} is the Kronecker delta sign ($d_{ij} = 1$ in the cases of $i = j$, and $d_{ij} = 0$ for other cases). The incompressibility of the viscous layer leads to

$$v_{i,i} = 0 \tag{9}$$

and

$$p = -\sigma_{ii}/3 \tag{10}$$

is the hydrostatic pressure of the flow. Under such assumptions, the flow is referred to as the Stokes flow or the creeping flow.

There exists a trivial solution to (7)–(10) that corresponds to an infinite viscous layer staying on a flat rigid substrate with constant thickness and traction-free surface. Now let us assume a small perturb to this trivial solution at surface of the viscous layer in the *x*–*z* plane. Therefore, the perturbed viscous layer is in a state of inplanar strain deformation in the *x*–*z* plane, *i.e.*, $v_x = v_x(x, z, t)$, $v_z = v_z(x, z, t)$, and $v_y = 0$. Because the viscous layer is infinite, the perturbed field has its translational symmetry along the surface such that the location of the origin of *x*-coordinate is arbitrary. Thus, the tractions acting on the flow surface are assumed to take the forms:

$$\sigma_{zz}(z = H) = q_0(t)\sin(kx), \tag{11}$$

$$\sigma_{zx}(z = H) = \tau_0(t)\cos(kx), \tag{12}$$

where H is the thickness of the viscous layer, k the perturb wave number, $q_0(t)$ and $\tau_0(t)$ are the time-dependent amplitudes of the surface tractions. The velocities of the flow at the bottom of the viscous layer are set to be zero to favor the nonsliding condition. Under above boundary conditions, the explicit solution to (7)–(10) is available in the literature [14], of which the velocities of the particles at the top surface of the viscous layer bear the forms:

$$v_z(z = H) = \bar{v}_z(t)\sin(kx), \tag{13}$$

$$v_x(z = H) = \bar{v}_x(t)\cos(kx). \tag{14}$$

Here the velocity amplitudes, $\bar{v}_z(t)$ and $\bar{v}_x(t)$, are linearly correlated to the amplitudes of the surface tractions such that

$$\bar{v}_z(t) = \frac{1}{2\eta k}[\gamma_{11}q_0(t) + \gamma_{12}\tau_0(t)], \tag{15}$$

$$\bar{v}_x(t) = \frac{1}{2\eta k}[\gamma_{21}q_0(t) + \gamma_{22}\tau_0(t)], \tag{16}$$

where the dimensionless coefficients are:

$$\gamma_{11} = \frac{1}{2}\frac{\sinh(2kH) - 2kH}{(kH)^2 + \cosh^2(kH)}, \tag{17}$$

$$\gamma_{22} = \frac{1}{2}\frac{\sinh(2kH) + 2kH}{(kH)^2 + \cosh^2(kH)} \tag{18}$$

and

$$\gamma_{12} = \gamma_{21} = \frac{(kH)^2}{(kH)^2 + \cosh^2(kH)}. \tag{19}$$

The above solution holds in the limiting case of either infinitely thick layer ($kH \to \infty$) or very thin film ($H \to 0$).

2.3 Deflection of an elastic film

For analysis of wrinkling in a thin elastic film, nonlinear plate theory is generally required in order to capture the finite deflection. While for the first-order instability analysis of wrinkle initiation, classic linear Kirchhoff plate theory is sufficient for preliminary linear perturbation analysis. In such case, the thin film on surface of the viscous layer is subjected to the inplanar membrane force N, the electrostatic surface pressure p_e, the net normal fluid pressure q, and the shear traction τ at the film bottom surface. Under the plane-strain condition, the equilibrium equations within the framework of classic linear Kirchhoff plate theory [27] lead to

$$q + p_e = -\frac{Eh^3}{12(1-\nu^2)}\frac{\partial^4 w}{\partial x^4} + N\frac{\partial^2 w}{\partial x^2} + \tau\frac{\partial w}{\partial x}, \tag{20}$$

$$\tau = \frac{\partial N}{\partial x}, \tag{21}$$

where w is the deflection of the thin elastic film, h the film thickness, E the Young's modulus, and ν is Poisson's ratio. Through Hooke's law, the membrane force N relates to the inplanar displacement u in the *x*-direction. If the thin film is initially compressed with a biaxial strain ε_0 induced by cooling for instance and the inplanar displacement is set to be zero at its initial state, the membrane force N maybe expressed as

$$N = \frac{E\varepsilon_0 h}{1-\nu} + \frac{Eh}{1-\nu^2}\frac{\partial u}{\partial x}. \tag{22}$$

Accordingly, assume the displacements of the thin elastic film have a compatible perturb such that

$$w(x, t) = A(t)\sin(kx), \tag{23}$$

$$u(x, t) = B(t)\cos(kx), \tag{24}$$

where $A(t)$ and $B(t)$ are two arbitrary small amplitudes. Substituting (23) and (24) into (22), (21), and (20), and keeping only the leading terms with either $A(t)$ or $B(t)$, one obtains the following relations:

$$N = \frac{E\varepsilon_0 h}{1-\nu} - \frac{E(kh)}{1-\nu^2}B(t)\sin(kx), \tag{25}$$

$$\tau = -\frac{E(kh)k}{1-\nu^2}B(t)\cos(kx), \tag{26}$$

$$q + p_e = -\frac{E(kh)k}{12(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2]A(t)\sin(kx). \tag{27}$$

Thus, within the framework of linear perturbation theory, there exist three linear relations that determine the forces N, τ, and $q + p_e$ through the film displacement u and w, respectively.

2.4 Governing equations of a charged elastic film on a layer of incompressible newtonian flow

From velocities (13) and (14) and displacements (23) and (24), two kinematic relations are implied:

$$\bar{v}_z = \frac{\mathrm{d}A(t)}{\mathrm{d}t} \quad \text{and } \bar{v}_x = \frac{\mathrm{d}B(t)}{\mathrm{d}t}. \tag{28}$$

Therefore, by relating (11) and (12) to (26) and (27), one can formulate the following relations of the thin film system as

$$\tau_0(t) = -\frac{E(kh)k}{1-\nu^2}B(t), \tag{29}$$

$$q_0(t) = \left\{-\frac{E(kh)}{12(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2] + 4\pi\sigma_0^2\right\} kA(t). \tag{30}$$

Consequently, substituting (28)–(30) into (15) and (16) leads to the final set of equations that governs the perturb growth of the thin film system such that

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} A(t) \\ B(t) \end{bmatrix} = \begin{bmatrix} \alpha & \gamma_{12}/\gamma_{22}\beta \\ \gamma_{12}/\gamma_{11}\alpha & \beta \end{bmatrix}\begin{bmatrix} A(t) \\ B(t) \end{bmatrix}, \tag{31}$$

where

$$\alpha = \left\{-\frac{E(kh)}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2] + 2\pi\sigma_0^2\right\}\gamma_{11}/\eta, \tag{32}$$

$$\beta = -\frac{E(kh)\gamma_{22}}{2\eta(1-\nu^2)} \tag{33}$$

and coefficients γ_{11}, γ_{12}, and γ_{22} are determined by (17)–(19). Similar relations have been derived in the literature in the case of wrinkling of charge free elastic films [14]. The general solution of (31) can be expressed as

$$\begin{bmatrix} A(t) \\ B(t) \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}\exp(st), \tag{34}$$

where A and B are the two unknown amplitudes, and s is the wrinkle growth rate. Furthermore, substitution of (34) into (31) yields an eigenvalue problem as

$$\begin{vmatrix} \alpha - s & \gamma_{12}/\gamma_{22}\beta \\ \gamma_{12}/\gamma_{11}\alpha & \beta - s \end{vmatrix} = 0, \tag{35}$$

which results in two wrinkle growth rates such that

$$s_1 = \left\{(\alpha+\beta) + \sqrt{(\alpha+\beta)^2 + 4\alpha\beta[\gamma_{12}^2/(\gamma_{11}\gamma_{22}) - 1]}\right\}/2, \tag{36}$$

$$s_2 = \left\{(\alpha+\beta) - \sqrt{(\alpha+\beta)^2 + 4\alpha\beta[\gamma_{12}^2/(\gamma_{11}\gamma_{22}) - 1]}\right\}/2. \tag{37}$$

The signs of s_1 and s_2 are determined by the wrinkle mode number k, initial film inplanar strain ε_0, surface charge density σ_0, film Young's modulus E, fluid viscosity η, and system geometries h and H. Positive sign of s_1 and s_2 accords to the exponential growth of the perturb (*i.e.*, the initial flat film is unstable); while negative sign corresponds to the exponential decay of the perturb (*i.e.*, the initial flat film is stable).

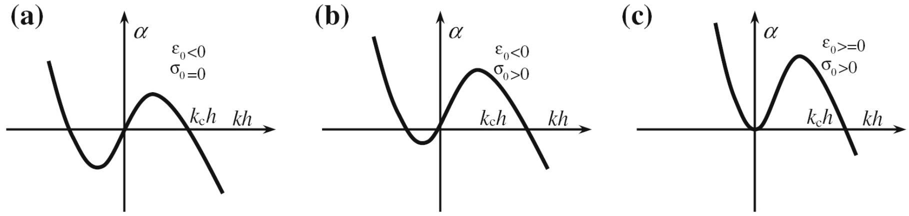

Figure 2. Schematic variation of the α-value versus the wrinkle mode number k. **a)** Charge-free elastic films in compression; **b)** Charged elastic films in compression; **c)** Charged elastic films in tension.

3 Analysis of wrinkle modes

In relation (33) it always holds $\beta < 0$. Thus, for any given wrinkle mode kH the following relation is always sustained:

$$\gamma_{12}^2/(\gamma_{11}\gamma_{22})-1=\frac{-1}{4\gamma_{11}\gamma_{22}}\frac{\sinh^2(2kH)-(2kH)^2-4(kH)^4}{\cosh^2(2kH)+(2kH)^2}<0. \tag{38}$$

Furthermore, in relations (36) and (37), when $\alpha < 0$, $s_1 < 0$ holds, *i.e.*, perturb relating s_1 decays exponentially; while for $\alpha > 0$, $s_1 > 0$ holds, *i.e.*, perturb relating s_1 grows exponentially. However, perturb relating s_2 always decays since $s_2 < 0$ always holds for any given α-value. The critical wrinkle mode number k_c can be determined by letting $\alpha = 0$ in (32) such that

$$-\frac{E(kh)}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0+(kh)^2]+2\pi\sigma_0^2=0, \tag{39}$$

corresponds to the wrinkle growth rate $s_1 = 0$.

Now, let us first consider two limiting cases of a charge-free film and an initially charged traction-free film, respectively. For a charge-free film (*i.e.*, $\sigma_0 \to 0$), the critical wrinkle mode number k_c is

$$k_c h=\sqrt{-12(1+\nu)\varepsilon_0}, \tag{40}$$

which is independent of the properties of the viscous layer and the geometries of the system [14]. In this case, when $k > k_c$, $s_1 < 0$ and the perturb decays; while $k < k_c$, $s_1 > 0$ and the perturb grows exponentially. Relation (40) indicates that the wrinkle occurs only for compressed film (*i.e.*, $\varepsilon_0 < 0$). Furthermore, for an initially charged traction-free film ($\varepsilon_0 = 0$), the resulting critical wrinkle mode number from (39) is

$$k_c h = 2\,[6\pi(1-v^2)\sigma_0^2/E]^{1/3}. \tag{41}$$

In this case, when $k > k_c$, $s_1 < 0$ and the perturb decays; however, when $k < k_c$, $s_1 > 0$ and the perturb grows exponentially. As a result, relation (41) shows that initially charged traction-free film is always unstable, and the critical wrinkle mode number is closely related to the charge density, film elasticity, as well as film thickness. Therefore, an external electric field is capable of modulating the wrinkle mode of thin metallic films.

Moreover, in a general sense, (39) gives one positive root k_c for $\alpha = 0$ in (32). The general root distribution of (39) under varying condition is illustrated in Figure 2 (all k-values with physical meanings should be positive). The normalized wrinkle growth rate $s_1\eta/E$ versus the wrinkle mode number kh for several thickness ratios H/h are plotted in Figure 3. In the numerical procedure, Poisson's ratio of the thin metallic film is selected as 0.3, and the magnitudes of the compressive and tensile strains are both selected as 0.012. For comparison, Figure 3a shows the results for a compressed charge-free film given in the literature [14]. Figures 3b and 3c show the results for a charged film in compression. It shows that surface charges contribute the wrinkle evolution. Figure 3d presents the results of a charged film in tension. It indicates that surface charges may even lead to the wrinkle of a stretched film. As a matter of fact, the wrinkle mode and its growth rate are governed through the coupling effect of the surface charges and the initial film strains.

Now let us further consider two limiting cases of the viscous layers at thick and thin thicknesses, respectively. For a very thick viscous layer (*i.e.*, $H/$

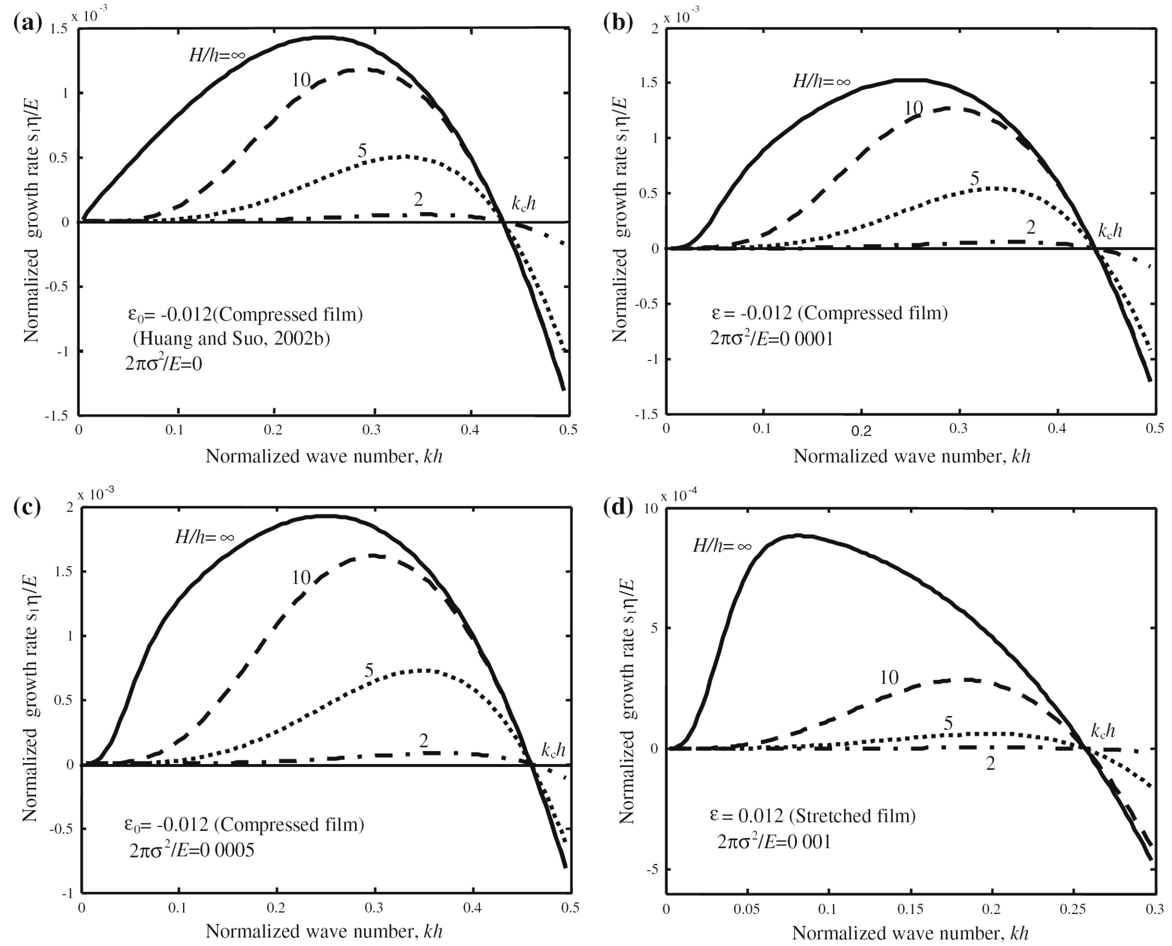

Figure 3. The wrinkle growth rate s_1 versus the mode number k with varying thickness ratio. **a)** Charge-free elastic films in compression; **b)** and **c)** Charged elastic films in compression; **d)** Charged elastic films in tension.

$h \to \infty$), the limiting wrinkle growth rate s_1 can be extracted from relation (36) such that

$$s_1\eta/E = \left\{-\frac{kh}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2] + 2\pi\sigma_0^2/E\right\}. \tag{42}$$

For a very thin viscous layer (*i.e.*, $H/h \to 0$), the corresponding wrinkle growth rate s_1 is

$$s_1\eta/E = \frac{2(kH)^3}{3}\left\{-\frac{kh}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0+(kh)^2] + 2\pi\sigma_0^2/E\right\}. \tag{43}$$

Furthermore, from Figure 3, it can be observed that the wrinkle growth rate s_1 tends to zero when the mode number k tends to both zero and k_c. Therefore, a stationary point must exist in the range (0, k_c). At this point, s_1 reaches its peak value, *i.e.*, the fastest wrinkle growth rate s_m. This value may be determined by directly letting $\partial s_1/\partial k = 0$ in (36), and the resulting k_m is the wrinkle mode number with fastest growth rate. This wrinkle mode is the one to be expected in experiment. Figure 4 shows $k_m h$ and corresponding $s_m\eta/E$ as functions of the compressive strain ε_0 for several film thickness ratios. Figures 4a and 4b are the results for charge-free films, and Figures 4c and 4d are those for charged films. From Figures 4a and 4d, it can be found that $k_m h$ grows with the increase of ε_0; while

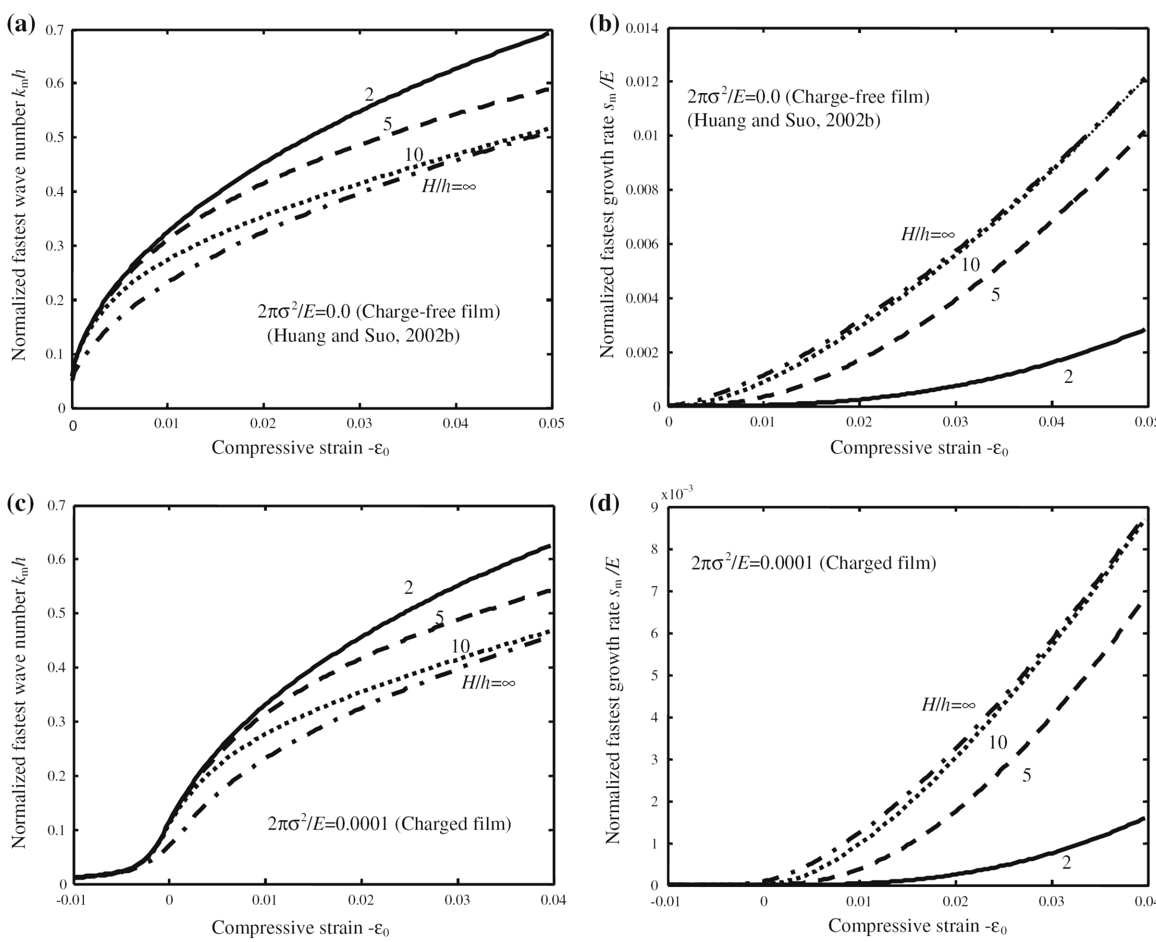

Figure 4. The fastest growth mode number k_m and corresponding wrinkle growth rate s_m versus the compressive strain with varying thickness ratio. **a)** and **b)** Charge-free elastic films in compression; **c)** and **d)** Charged elastic films in compression and in tension

it decreases with the increase of the thickness ratio H/h. The corresponding $s_m\eta/E$ increases with the increase of both ε_0 and H/h. Surprisingly, Figures 4c and 4d also indicate that surface charges may result in the wrinkling of a stretched film.

In the limiting case of very thick viscous layer ($H/h \to \infty$), s_m and corresponding k_m are determined from (42) as

$$k_m h = 2\sqrt{-\varepsilon_0(1+\nu)},$$

$$s_m\eta/E = \frac{2\sqrt{-\varepsilon_0(1+\nu)}}{3(1-\nu)} + \frac{2\pi\sigma_0^2}{E}. \qquad (44)$$

For very thin fluid layer ($H/h \to 0$), k_m relating the fastest wrinkle growth rate s_m is determined through solving the characteristic equation:

$$(kh)^3 + 8(1+\nu)\varepsilon_0(kh) + 8\pi(1-\upsilon^2)\sigma_0{}^2 = 0. \qquad (45)$$

In the limiting case of charge-free film ($\sigma_0 = 0$), relation (45) reduces to

$$k_m h = 2\sqrt{-2\varepsilon_0(1+\nu)} \qquad (46)$$

and the resulting growth rate is

$$s_m\eta/E = 16\,[-(1+\nu)H/h]^3/[9(1-\upsilon^2)]. \qquad (47)$$

4 Concluding remarks

Wrinkling of charged thin metallic films on viscous layers has been studied within the framework of linear perturbation theory. The critical wrinkle mode number has been determined as the bifurcation point of the elastic instability of the thin films. The fastest wrinkle growth rate and the corresponding mode number have been determined as functions of the film surface charge, film elasticity and thickness, mismatch strain, as well as viscosity and thickness of the viscous layer. For charged thin metallic films, the elastic wrinkle may happen not only for compressed films but also for stretched films. Therefore, external electric field can be utilized as one of the feasible manners to modulate the wrinkle mode of thin metallic films on viscous layers such as compliant polymeric layers used extensively in surface coatings, lithography, and semiconductor engineering, *etc.* General analysis and simulation of elastic instability and wrinkle pattern evolution of charged thin films in the 2D case can be developed using the present scheme and will be reported elsewhere.

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References

1. Bowden N, Brittain S, Evans AG, Hutchinson JW, Whitesides GM, (1998) Spontaneous formation of ordered structures in thin films of metals supported on elastomeric polymer. *Nature* (London) **393** (6681): 146–149
2. Cerda E, Ravi-Chandar K, Mahadevan L (2002) Thin films-wrinkling of an elastic sheet under tension. *Nature* (London) **419** (6907): 579–580
3. Chan EP, Crosby AJ (2006) Spontaneous formation of stable aligned wrinkling patterns. *Soft Matter* **2**: 324–328
4. Chen X, Hutchinson JW (2004) A family of herringbone patterns in thin films. *Scr Mater* **50**: 797–801
5. Chen X, Hutchinson JW (2004) Herringbone buckling patterns of compressed thin films on compliant substrates. *ASME J Appl Mech* **71**: 597–603
6. Chou SY, Zhuang L (1999) Lithographically induced self-assembly of periodic polymer micropillar arrays. J Vac Sci Technol B **17**(6): 3197–3202
7. Chou SY, Zhuang L, Guo LJ (1999) Lithographically induced self-construction of polymer microstructures for resistless patterning. *Appl Phys Lett* **75**(7): 1004–1006
8. Craster RV, Matar OK (2005) Electrically induced pattern formation in thin leaky dielectric films. *Phys Fluids* **17**(3), Article No. 032104
9. Genzer J, Groenewold J (2006) Soft matter with hard skin: from skin wrinkling to templating and material characterization. *Soft Matter* **2**: 310–323
10. Gioia G, Ortiz M (1997) Delamination of compressed thin film. *Adv Appl Mech* **33**: 119–192
11. Huang R (2005a) Kinetic wrinkling of an elastic film on viscoelastic substrate. *J Mech Phys Solids* **53**: 63–89
12. Huang R (2005b) Electrically induced surface instability of a conductive thin film on a dielectric substrate. *Appl Phys Lett* **87**, Article No. 151911
13. Huang R, Suo Z (2002a) Wrinkling of a compressed elastic film on a viscous layer. *J Appl Phys* **91**(3): 1135–1142
14. Huang R, Suo Z (2002b)Instability of a compressed elastic film on a viscous layer. *Int J Solids Struct* **39**(7): 1791–1802
15. Huang ZY, Hong W, Suo Z (2004) Evolution of wrinkles in hard films of soft substrates. *Phys Rev E* **70**, Article No. 030601
16. Huang ZY, Hong W, Suo Z (2005) Nonlinear analyses of wrinkles in a film bonded to a compliant substrate. *J Mech Phys Solids* **53**: 2101–2118
17. Huck WTS, Bowden N, Onck P, Pardoen T, Hutchinson JW, Whitesides GW (2000) Ordering of spontaneously formed buckles on planar surfaces. *Langmuir* **16**(7): 3497–3501
18. Hutchinson JW, Suo Z (1992) Mixed-mode cracking in layered materials. *Adv Appl Mech* **29**: 63–191
19. Kim D, Lu W (2006)Three-dimensional model of electrostatically induced pattern formation in thin polymer films. *Phys Rev B* **73**, Article No. 035206
20. Landau LD, Lifshitz EM, Pitaevskii LP (1993) *Electrodynamics of continuous media,* 2nd edition, Butterworth-Heinemann, Oxford, pp. 29–33
21. Liang J, Huang R, Yin H, Sturm JC, Hobart KD, Suo Z (2002) Relaxation of compressed elas-

tic islands on a viscous layer. *Acta Mater* **50**(11): 2933–2944

22. Pease LF, Russel WB (2002) Linear stability analysis of thin leaky dielectric films subjected to electric fields. *J Non-Newtonian Fluid Mech* **102**(2): 233–250
23. Pease LF, Russel WB (2003) Electrostatically induced submicron patterning of thin perfect and leaky dielectric films: a generalized linear stability analysis. *J Chem Phys* **118**(8): 3790–3803
24. Schaffer E, Thurn-Albrecht T, Russell TP, Steiner U (2000) Electrically induced structure formation and pattern transfer. *Nature* (London) **403**(6772): 874–877
25. Schaffer E, Thurn-Albrecht T, Russell TP, Steiner U (2001) Electrohydrodynamic instabilities in polymer films. *Europhys Lett* **53**(4): 518–524
26. Sridhar N, Srolovitz DJ, Suo Z (2001) Kinetics of buckling of a compressed film on a viscous substrate. *Appl Phys Lett* **78**(17): 2482–2484
27. Timoshenko S, Woinowsky-Krieger S (1987) *Theory of plates and shells,* 2nd edition, McGraw-Hill, New York, pp. 378–380
28. Wu L, Chou SY (2003) Dynamic modeling and scaling of nanostructure formation in the lithographically induced self-assembly and self-construction. *Appl Phys Lett* **82**(19): 3200–3202
29. Wu L, Chou SY (2005) Electrohydrodynamic instability of a thin film of viscoelastic polymer underneath a lithographically manufactured mask. *J Non-Newtonian Fluid Mech* **125**(2–3): 91–99
30. Wu N, Russel WB (2005) Dynamics of the formation of polymeric microstructures induced by electrohydrodynamic instability. *Appl Phys Lett* **86**, Article No. 241912
31. Wu N, Pease LF, Russel WB(2005) Electric-field induced patterns in thin polymer films: weakly nonlinear and fully nonlinear evolution. *Langmuir* **21**(26): 12290–12302
32. Wu N, Pease LF, Russel WB (2006) Toward large-scale alignment of electrohydrodynamic patterning of thin polymer films. *Adv Funct Mater* **16**(15): 1992–1999
33. Wu XF, Dzenis YA (2005) Electrohydrodynamic instability of thin conductive liquid film. *J Phys D Appl Phys* **38**(16): 2848–2850
34. Yoo PJ, Suh KY, Park SY, Lee HH (2002) Physical self-assembly of microstructures by anisotropic buckling. *Adv Mater* (Weinhein, Germany) **14**(19): 1383–1387