1975

Reflection and Refraction

REFLECTION AND REFRACTION

INTRODUCTION

Sight is certainly one of our most important senses and depends on the interaction of electromagnetic waves in the visible portion of the spectrum with the eye. The use of materials that reflect light and that refract or "bend" light extends throughout our industrialized society.

In this module we deal with light traveling in two dimensions and encountering the boundaries between media under those conditions in which the wavelength is small compared with the size of the obstacles or apertures. Under such conditions, since diffraction and interference effects are negligible, the principal phenomena occurring at the interfaces, reflection and refraction, can be understood and the progress of a wave charted by a simple geometrical procedure: ray tracing.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Write the relations among wavelength, frequency, and velocity for a wave (needed for Objectives 1 and 3 of this module)	Traveling Waves Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Definitions - Define light ray, angle of incidence, refraction, angle of refraction, index of refraction, angle of reflection, total internal reflection, critical angle, reciprocity, and Huygens' principle.

2. Law of reflection - Use the law of reflection to solve problems involving the angles of incidence and reflection, ray paths, and/or the images formed by plane mirrors.

3. Law of refraction - (a) Use the law of refraction (Snell's law) to solve problems involving the relationships among index of refraction, wavelength, velocity of light, angles of incidence and refraction, and ray path for planar slabs of materials. (b) Use the law of refraction to find the location of the image of an illuminated object embedded in a slab of transparent material.

4. Total internal reflection - Use the concept of total internal reflection in conjunction with the laws of reflection and refraction to find the path of a ray that is internally reflected, or find the index of refraction if the ray path is given.

GENERAL COMMENTS

1. Definitions

Since the definitions of terms that you must know to master Objective 1 are scattered throughout the readings, we have collected brief definitions here. These are not meant to be necessarily complete definitions, but should serve to remind you of the full meaning and special usage of each term as you read.

Refraction: the bending of a ray of light as it passes through the boundary between two media.

Angle of refraction: the angle between the refracted ray and the normal to the boundary between two media.

Angle of incidence: the angle between the incident ray and the normal to the boundary between two media.

Angle of reflection: the angle between the reflected ray and the normal to the boundary between the two media.

Light ray: a line parallel to the direction of propagation of the light and normal to the plane wavefront. Although not entirely accurate, it is satisfactory for most ray-tracing purposes to think of a very small beam of light as equivalent to a ray.

Reciprocity: also called "optical reversibility," reciprocity means that light will follow the same ray path through a series of refractions and reflections in going from point A to point B as it will in the reverse direction, B to A.

Huygens' principle: All points on a wavefront can be considered as point sources for the production of spherical secondary wavelets. After a time t the new position of the wavefront will be the surface of tangency to these secondary wavelets.

Total internal reflection: When a ray in an optically dense medium falls on an interface with a less optically dense medium at angles of incidence greater than some critical angle, for all practical purposes no light is transmitted; it is all reflected.

Critical angle: the minimum angle of incidence at which total internal reflection appears. It corresponds to the angle of incidence for which the angle of refraction equals 90°.

Index of refraction: a property of the medium defined as the ratio of the velocity of light in vacuum to that in the medium. A material with a large index of refraction is called optically dense.

2. Dispersion

In general the texts and problems assume monochromatic light (single wavelength). However, light beams are a mixture of waves whose wavelengths extend throughout the spectrum. Although the speed of light in vacuum is the same for all wavelengths, the speed in material substances may be different for different wavelengths. A substance in which the speed of a wave varies with wavelength is said to exhibit dispersion. The index of refraction of a substance is a function of wavelength. Problem E is an example of this effect. The dispersion effect is important since it provides a means of separating (dispersing) light into its various colors in a prism spectrograph, it explains rainbows, and it is the cause of color fringing in low-quality binoculars.

3. Important Formulas

There are really only three formulas you need to memorize for this module:

Law of reflection: $\theta_1' = \theta_1$ or $\theta_i = \theta_r$.

Law of refraction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$.

Definition of index of refraction: $n = c/v$.

We strongly suggest that you do not memorize formulas for the apparent depth of an object in a pond, or the critical angle for internal reflection, etc. They can be very short derivations if you understand the principles, and we have found from past experience that students who have memorized the formulas frequently make mistakes in identification of symbols and are less able to deal with new situations.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientist and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Your text uses a different order from most texts in the presentation of the material in this module. We suggest that you read Chapter 30, Sections 30.1 through 30.8 for continuity and definitions of terms, then reread Sections 30.1 to 30.3, 30.7 and 30.8 for detailed help in mastering the objectives. Bueche does not explicitly use Huygens' principle to derive the laws of reflection and refraction, as is customary, although the discussion in Section 30.7 is based on this principle. He does give a statement of the principle in Section 32.1 on p. 632; however, the application is not particularly relevant to the present module. The principle is stated in the General Comments of this study guide. The derivations are not necessary for the applications of the laws of reflection and refraction required for mastery of this module; however, they will help your understanding. A more complete treatment can be found in Fundamentals of Physics.*

Read General Comments 1 through 3 in this study guide. Then study Problems A through G and Illustrations 30.1 and 30.5. Then solve Problems H through M. If you need more practice, you may wish to work some of the Additional Problems listed below before taking the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems (Chap. 30)
1	Secs. 30-1, 30-2, 30-3, 30-7, 30-8, General Comment 1				
2	Secs. 30-2, 30-3, General Comment 3	A, B	Illus.[a] 30.1	H, I	1
3	Sec. 30-7, General Comments 2, 3	C, D, E	Illus. 30.5	J, K	14, 15, 16
4	Sec. 30-8	F, G		L, M	

[a]Illus. = Illustration(s).

*David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974), p. 673.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 36, Sections 36-1 through 36-5 and 36-7. The text does not give much detail on the variation of n with wavelength. University Physics* has a more complete discussion that you should read if this text is available to you. However, for purposes of mastering this module you may find that the discussion in General Comment 2 and the solution to Problem E will suffice.

Read General Comments 1 through 3 in the study guide. Then study Problems A through G and Examples 1, 2, and 5 in your text. Solve Problems H through M. If you need more practice, you may work some of the Additional Problems listed below before taking the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 36)
1	Secs. 36-1 to 36-5, General Comment 1				
2	Secs. 36-2, 36-7, General Comment 3	A, B	Ex.[a] 5	H, I	1, 24 to 30, 32
3	Secs. 36-2, 36-3, 36-4, General Comments 2, 3	C, D, E	Ex. 1, 2	J, K	3, 4, 6, 8, 9, 10
4	Sec. 36-5	F, G		L, M	14 to 19, 21

[a]Ex. = Example(s).

*Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition, Chapter 38, Sections 38-5 through 38-7.

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 37, Sections 37-1 and 37-3 through 37-6. These sections give some detail on the nature of light and introduce to you the laws of reflection and refraction. Next read Chapter 38, Sections 38-1 through 38-4, 38-6, and 38-7. These sections derive the laws of reflection and refraction from Huygens' principle and define index of refraction, total internal reflection, and dispersion. Finally, read Chapter 39, Sections 39-1, 39-2, and 39-6, which discuss image formation for plane mirrors and plane refracting surfaces. Although the readings are not in the same order as the objectives, you will find the order of the text better for the first reading.

Read General Comments 1 through 3 in the study guide, and study Problems A through G. Then solve Problems H through M. If you need more practice you may work some of the Additional Problems listed in the Table below, before taking the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	
1	Secs. 37-1, 37-3 to 37-6, 38-1 to 38-4, General Comment 1			
2	Secs. 37-5, 38-1, 38-2, 39-1, 39-2, General Comment 3	A, B	H, I	37-6, 37-7, 39-2, 39-3
3	Sec. 37-5, 38-3, 38-6, 38-7, 39-6, General Comments 2, 3	C, D, E	J, K	37-8, 37-10 to 37-13, 38-1, 38-3, 38-4, 38-13, 38-15, 39-11 to 39-13
4	Sec. 38-4	F, G	L, M	38-6 to 38-12

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read Chapter 36, Sections 36-1 through 36-8. You will not be responsible for the contents of Section 36-7; however, this discussion of refraction of light from an atomic point of view should broaden your general understanding of reflection and refraction.

Read General Comments 1 through 3 in the study guide. Then study Problems A through G and Example 36-1. Solve Problems H through M in your study guide. If you need more practice, you may work some of the Additional Problems listed below, before taking the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Secs. 36-1 to 36-8, General Comment 1				
2	Secs. 36-3, 36-4, General Comment 2	A, B	Ex.[a] 36-1	H, I	36-1, 36-3, 36-4
3	Secs. 36-3, 36-5, 36-6, General Comments 2, 3	C, D, E		J, K	36-5 to 36-8, 36-11 to 36-13, 36-16, 36-19, 36-20
4	Sec. 36-8	F, G		L, M	36-17, 36-21

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(2). Two plane mirrors stand on a table adjacent to each other at an angle of 60°. See Figure 1.
(a) Trace a horizontal light ray that is reflected <u>twice</u> in this system.
(b) Compute the angle between incident ray and second reflected ray. Define clearly all quantities used.

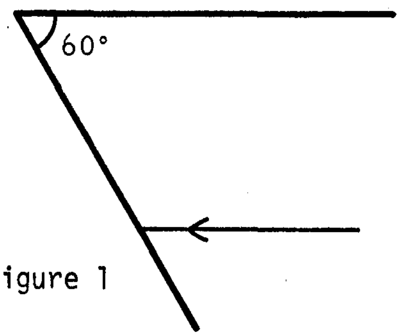


Figure 1

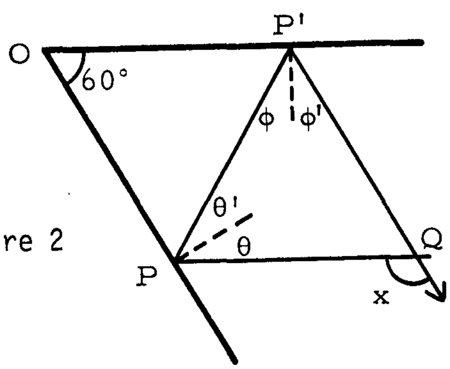
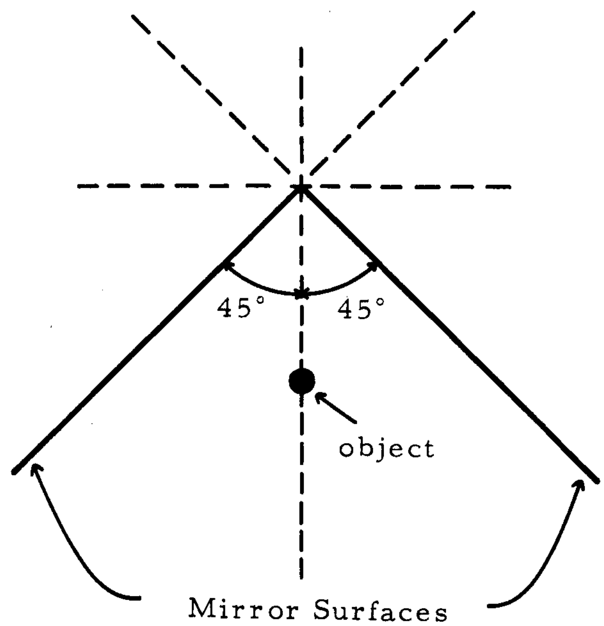


Figure 2

Figure 3



Solution

(a) See Figure 2.
(b) X is the quantity sought. By the law of reflection, $\theta = \theta'$, $\phi = \phi'$ (the dashed lines are normals). Sum the angles in triangle POP$'$:

$$60° + (90° - \phi) + (90° - \theta) = 180° \quad \underline{\text{or}} \quad \phi + \theta = 60°.$$

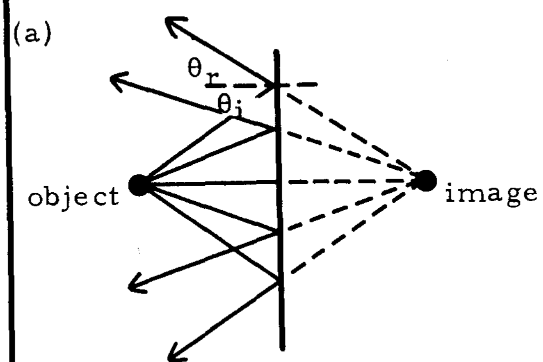
The exterior angle of triangle PP$'$Q = the sum of the opposite angles:

$$X = 2\theta + 2\phi = 120°.$$

B(2). The image formed by a plane mirror will act as an object for a second mirror. Locate in Figure 3 the three distinct images that can be seen by a suitably placed eye.

Solution

Take the right-hand mirror by itself as in Figure 4(a). Since $\theta_i = \theta_r$, the projection of any reflected ray behind the mirror intersects the normal to the mirror at a point that is as far behind the mirror as the object is in front of it: the object distance equals the image distance. We can now locate images A and B on Figure 4(b). In Figure 4(b) a few light rays have been drawn that undergo two reflections and appear to come from a point C, the third image. The image at A (and B) acts as an object for the image at C, and therefore the image lies along the normal to the extension of the mirror: AP = PC.



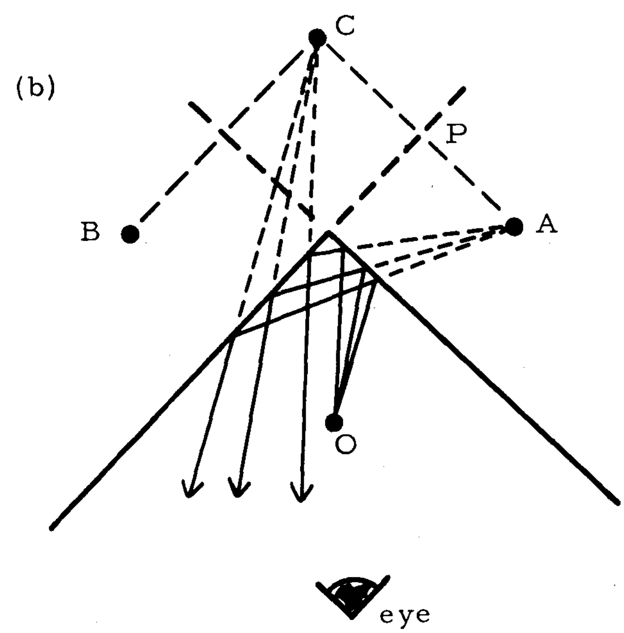


Figure 4

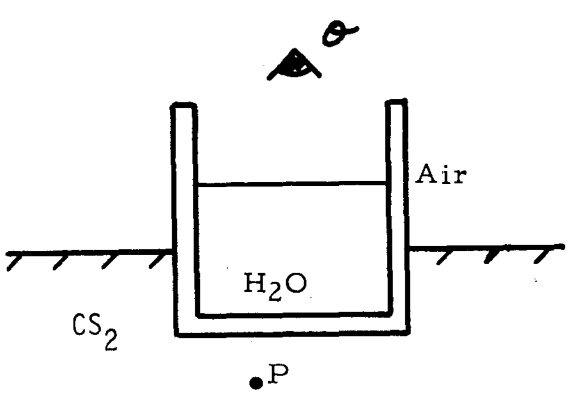
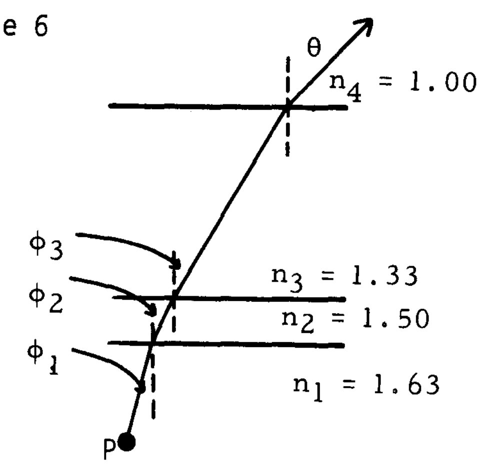


Figure 5

Figure 6



C(3). A glass of water in Figure 5 is partially immersed in a tank of carbon disulfide (CS_2). You are an observer O looking down through the water at a particle P floating in the CS_2. Some reflected light leaves the particle at a small angle ϕ with the vertical.
(a) At what angle θ does that light emerge from the water? Express your answer in terms of ϕ. $n_{water} = 1.33$; $n_{glass} = 1.50$; and $n_{CS_2} = 1.53$.
(b) If D = depth of water, does your answer to part (a) depend on D? Suppose $D \to 0$.

Solution

(a) See Figure 6. Use Snell's law at each interface:

$$(\sin\phi)/(\sin\phi_2) = n_2/n_1, \quad (\sin\phi_2)/(\sin\phi_3) = n_3/n_2, \quad (\sin\phi_3)/(\sin\theta) = n_4/n_3.$$

Multiply these three equations:

$$\frac{\sin\phi}{\sin\theta} = \frac{n_4}{n_1} \quad \text{or} \quad \theta = \arcsin\left(\frac{n_1}{n_4}\sin\phi\right) = \arcsin\,(1.63\sin\phi).$$

(b) No, θ is independent of D, even if $D \to 0$. In fact, take away the glass of water and θ is still the same. It is only necessary to consider the index of refraction of the initial and final materials in calculating the final angle of refraction, and in all cases the final angle is independent of intermediate materials. However, the apparent location of the object depends on these.

D(3). The "apparent depth" of an object immersed in an optically dense refracting medium is less than the true depth when viewed from directly above. Show that the apparent depth d' is related to the true depth d by $d' = n_2 d/n_1$, where n is the relative refractive index of the medium in which the object is immersed. See Figure 7. Note: One may assume the angles to be so small that the sine of an angle can be replaced by the angle itself.

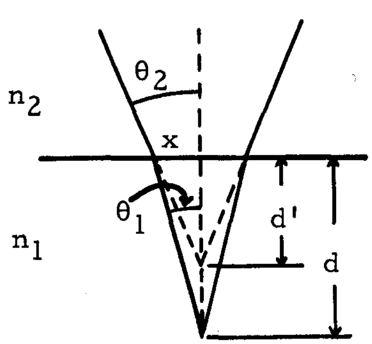
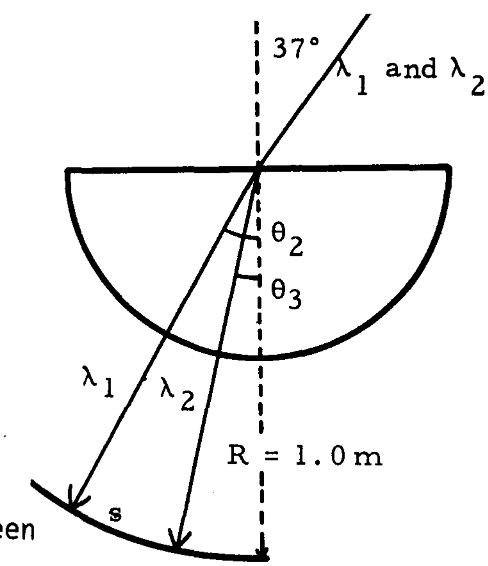


Figure 7

Figure 8



Solution

The ray starts at depth d in the medium of index n_1. By Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2,$$

and by the small-angle approximation, $\sin \theta \simeq \tan \theta \simeq \theta$ (in radians):

$$n_1\theta_1 = n_2\theta_2.$$

From the geometry of Figure 7, $\tan \theta_1 = x/d$ and $\tan \theta_2 = x/d'$. (Note that d' is found by projecting the outgoing ray backward to the normal as the eye would do, so that the light appears to come from depth d'.

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\theta_1}{\theta_2} = \frac{d'}{d} = \frac{n_2}{n_1}, \qquad d' = \frac{n_2 d}{n_1} .$$

E(3). Crown flint glass has an index of refraction that varies with the wavelength of the light passed through it from 1.66 for a wavelength of 400 nm (violet) to 1.60 for a wavelength of 700 nm (red). A narrow beam of light containing the red and violet wavelengths above falls on the center of a semicircle cut from this glass, as shown in Figure 8. Find the separation S of the two rays on a circular screen with R = 1.00 m, centered at O.

Solution

Apply Snell's law for each wavelength:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{and} \quad n_1 \sin \theta_1 = n_3 \sin \theta_3.$$

Rearranging, we find

$$\theta_2 = \text{arc sin}[(n_1/n_2) \sin \theta_1], \quad \theta_3 = \text{arc sin}[(n_1/n_3) \sin \theta_1].$$

Find

$$\theta_2 - \theta_3 = \text{arc sin}(0.60/1.60) - \text{arc sin}(0.60/1.66) = \text{arc sin}(0.375) - \text{arc sin}(0.361),$$

$$\theta = 22.0° - 21.2° = 0.8°,$$

$$s = R\,\Delta\theta = (1.00\ \text{m})\left(\frac{0.8°/57.3°}{\text{radian}}\right) = 1.40 \times 10^{-2}\ \text{m}.$$

F(4). The crooks in a typical TV drama are attempting to recover a fortune in diamonds that they earlier sunk in a chest in 8.0 m of water. As a cover for the operation they have moored a floating oil-drilling rig above the position where they sank the chest. If the dimensions of the chest are small in comparison with the rig, determine the size of rig required in order that no sailor on a passing ship can see what is going on under the surface. (Take n = 4/3 for water.)

Solution

We can consider the sunken chest as a point source of light. If the oil rig is big enough for its purpose, then all rays of light from the chest that would be refracted into the air at the surface must be blocked off by the base of the rig, and all rays striking the surface of the water outside the rig must be totally internally reflected. The rig obviously must be circular, and, if its center is moored directly above the chest, a ray of light striking the edge of the rig must do so at an angle equal to ϕ_c, the critical angle. See Figure 9.

For the minimum radius r of the rig, $n \sin\phi_c = (1)\sin\phi_2 = \sin(90°) = 1$, therefore,

$$\sin\phi_c = 1/n \quad \text{or} \quad r/(r^2 + d^2)^{1/2} = 1/n,$$

$$r^2 + d^2 = n^2r^2, \quad \text{or} \quad r^2 = d^2/(n^2 - 1).$$

This gives us

$$r = \frac{8.0\text{ m}}{(16/9 - 1)^{1/2}} = 9.1\text{ m}.$$

Alternate solution: Use Snell's law: $n \sin\phi_c = 1$; therefore, $\sin\phi_c = 1/n = 3/4$; and $\phi_c = 48.6°$. We can find r from $r = d\tan\phi_c = (8.0\text{ m})(1.13) = 9.1\text{ m}$.

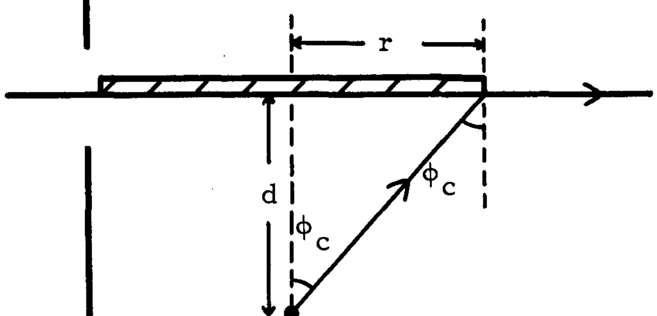
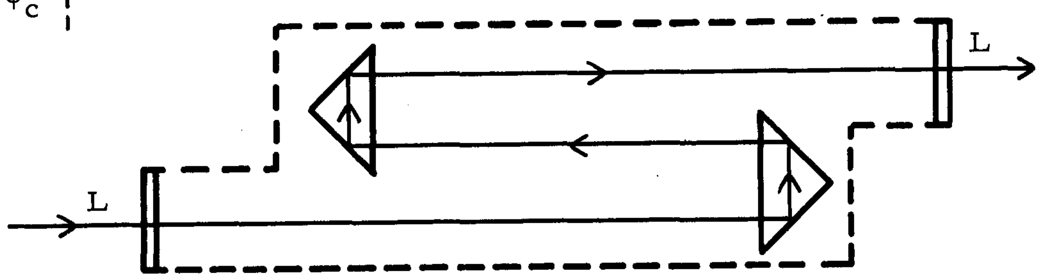


Figure 9

Figure 10



G(4). In Figure 10 is shown a light ray passing through one side of a pair of binoculars. L's represent lenses; ignore their function in this question. (a) Given that the two 90°-45°-45° prisms are included chiefly for the purpose of shortening the instrument without decreasing the optical path length between the lenses, tell whether or not prisms made of clear plastic of index n = 1.47 would be suitable. Explain, giving a quantitative argument. (b) Would the above prism in part (a) be suitable if the entire system (prisms and lenses) were immersed in water? Indicate reasoning for your answer.

<u>Solution</u>

(a) The prisms must allow total internal reflection at 45° incidence. Thus, $\theta_i = 45°$, and $\theta_i > \phi_c$, where $\sin \phi_c = 1/n$. Thus $\sin \theta_i > 1/n$, or $n > 1/(\sin 45°) = 1.414$. This condition is satisfied by a plastic of $n = 1.47$ and the prisms will work.

(b) In water the same condition becomes

$$\sin \theta_c = \frac{n_{H_2O}}{n_{plastic}} \quad \text{or} \quad \sin \theta_c = \frac{1.33}{n_{plastic}} = \frac{1.33}{1.47} = 0.905 \quad \text{or} \quad \theta_c = 65°.$$

Therefore $\theta_i < \theta_c$, and the binoculars are of little use to a scuba diver if water leaks inside them.

<u>Problems</u>

H(2). (a) Using ray paths, prove that in a calm, unpolluted lake the reflected image of a pine tree on the shore will appear upside-down to a fisherman meditating in his boat.
(b) Will the image of a fish in the same lake surface appear inverted to another fish? Explain.

I(2). Suppose that (for reasons best understood by you) you decide to photograph a street scene by using the reflection in a store window. You wish to set your camera so that point **×** will be in sharp focus; for what distance must you set your camera lens? The dimensions are as in Figure 11.

J(3). Light strikes a glass plate at an angle of incidence of 60°, part of the beam being reflected and part refracted. It is observed that the reflected and refracted portions make an angle of 90° with each other. What is the index of refraction of the glass?

K(3). A beam of light as in Figure 12 hits a parallel-sided plate of glass of index of refraction n. The thickness of the glass is 6.0 mm and the light beam is displaced a distance of 4.24 mm.

(a) What is the index of refraction of the glass?
(b) What is the angle of refraction?

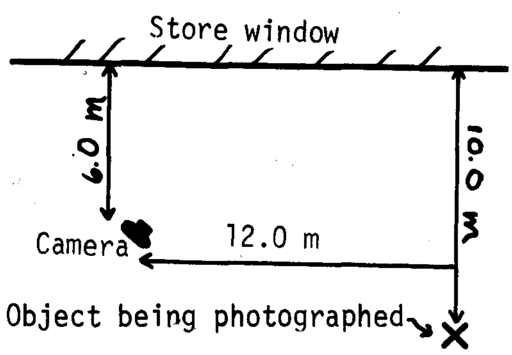


Figure 11

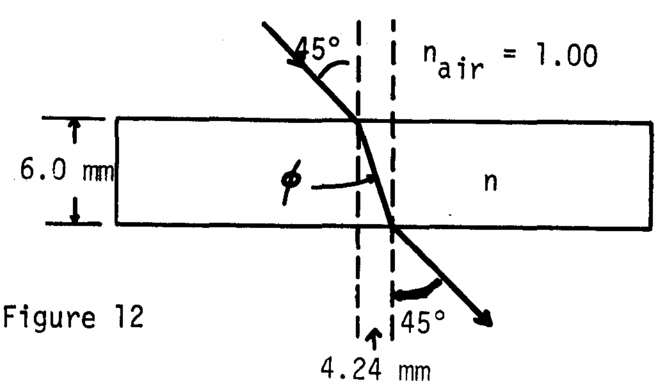


Figure 12

L(4). A beam of light shines on a glass prism as shown in Figure 13. The beam is perpendicular to the first face. Trace out the subsequent path(s) of the light beam until it has left the prism, and find the angle of refraction as it leaves the prism. The index of refraction of the glass is 1.50.

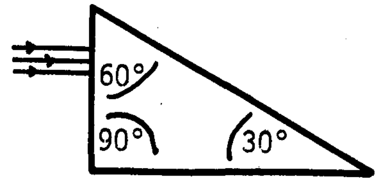


Figure 13

M(4). A point source of light is 1.50 m below the surface of a still pond of water. It appears to an observer from above the water that the light comes only from a well-defined circular area of the water surface. What is the diameter of this circle?

Solutions

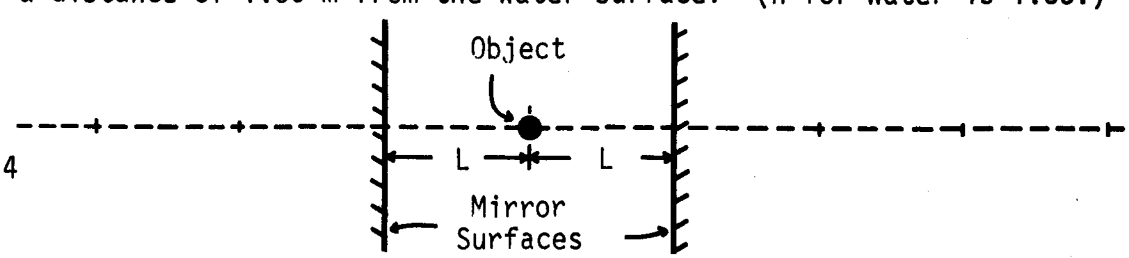
H(2). Yes. I(2). 20.0 m. J(3). 1.73.

K(3). (a) 1.22. (b) 35°. L(4). 48.6°. M(4). 3.4 m.

PRACTICE TEST

1. Define critical angle and reciprocity, and state Huygens' principle.
2. The image formed by a plane mirror will act as an object for a second mirror. Find the first four images formed by the pair of plane mirrors shown in Figure 14 (i.e., find the four images closest to the object).
3. A microscope is focused on a scratch made on the upper surface at the bottom of a small container. Water is added to the container to a depth of 3.00 mm. Through what vertical displacement must the microscope lens be raised to bring the scratch into focus again? Assume that the displacement of the lens is equal to the displacement of the image.
4. A fish looking upward toward the water-air interface sees a circular transparent hole surrounded by a mirror. What is the radius of this hole when the fish's eye is a distance of 1.00 m from the water surface? (n for water is 1.33.)

Figure 14



Practice Test Answers

1. Critical angle: the minimum angle of incidence for which total internal reflection appears, angle of incidence for which the angle of refraction equals 90°.
Reciprocity: also called "optical reversibility," light will follow the same ray path no matter in which sense it propagates. Huygens' principle: all points on a wavefront can be considered as point sources for the production of spherical secondary wavelets, the wavefront moving as the wavelets' surface of tangency.

2. ±2L, ∓4L, ±6L, ∓8L. 3. 0.75 mm. 4. r = 1.13 m.