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Batelaan and Gay Reply

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Batelaan and Gay Reply: In their Comment [1] to our Letter [2], Rutherford and Grobe first disagree with our conclusion that "a tabletop Stern-Gerlach filter is feasible." Given our estimate of the throughput for such a device, $\sim$0.3 electrons/s, we were clearly not proposing it as an alternative to standard polarized electron sources ($\sim 10^{15}$ electrons/s). The main argument for "feasibility" is based on the relatively low currents required to establish the solenoidal magnetic field (5 A)—hence the use of the word "tabletop" in the abstract. In retrospect, a better statement would have been that "experimental demonstration of such a spin-splitting effect may be possible." No mention was made of space charge effects because they are negligible for the extremely low beam currents in question.

Their second objection appears to be based on a misunderstanding of the initial conditions we used in our simulations. The results presented in our paper considered *only* the extreme quantum limit, i.e., $\Delta x \Delta p = \hbar/2$ (albeit with varying values of Δp and Δx). The measure of spin separation we provide is not quantitative but graphical and is shown in Fig. 2 of Ref. [2]. We certainly agree that as one moves away from the extreme quantum limit the spin separation will disappear.

Finally, they infer a general prohibition against transverse spin separation from our statement that spin splitting is completely blurred by "transverse Stern-Gerlach magnet(s)." However, our references indicate that we were considering only the specific case of a standard Stern-Gerlach magnet discussed in most textbooks.

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[1] G. H. Rutherford and R. Grobe, preceding Comment, Phys. Rev. Lett. **81**, 4772 (1998).
[2] H. Batelaan, T. J. Gay, and J. J. Schwendiman, Phys. Rev. Lett. **79**, 4517 (1997).

Comment on "Stern-Gerlach Effect for Electron Beams"

Batelaan *et al.* [1] have reexamined the classic Bohr/Pauli edict [2] about the possibility of separating electron spins in a beam with an inhomogeneous magnetic field. The authors simulated the trajectories for a longitudinally polarized electron beam incident perpendicular to two parallel wires carrying opposite currents. The electrons' initial spatial and velocity distributions were chosen to have a Heisenberg uncertainty product close to the quantum limit $\hbar/2$. We point out the following: (1) Although we agree that the Bohr/Pauli edict is incorrect in principle, contrary to the authors' conclusion a practical spin filter device is not feasible; (2) extension of their investigation to more practical conditions shows no splitting; and (3) also in contrast to their assertion, transverse splitting is indeed possible for a suitable field configuration.

First, we note that the initial values of v_z and Δv_z chosen (10^5 and 1.7 m/s, respectively; larger for the case of Landau states, assuming they are applicable) correspond to a longitudinal energy spread of 10^{-6} eV. The authors assume the same velocity variance $\Delta v_{x,y}$ in the transverse directions, yielding a transverse energy spread of only 10^{-11} eV, or an effective temperature near 0.1 μK. The initial velocity spread inherent in any beam source of reasonable current is several orders of magnitude [3] larger than this value. If, as the authors suggest, the "low energy tail" of a beam were used to reduce the initial longitudinal velocity spread, and strict collimation were used to reduce the transverse velocity spread, essentially useless output "beams" would be realized. Furthermore, these input beam parameters would also require a geometric emittance on the order of 10^{-4} mm mrad while no mention is made of space charge effects. This is a possible theoretical inconsistency for treatment of a beam of useful current with the given parameters. This analysis is difficult to reconcile with the authors' statement that "a tabletop Stern-Gerlach electron spin filter is feasible."

Second, as the initial velocity spread is increased above the quantum limit, the authors claim that the spin separation "is still clearly evident and not marginal," although no quantitative measure is given of the degree of splitting or of a maximum velocity spread beyond which the separation is not longer significant. We propose a resolving criterion embodied in the factor R defined as the final separation between the average positions of the same-spin ensemble fractions divided by the spatial FWHM of one of the ensemble fractions. It seems reasonable to consider the separation to be clearly evident if R is larger than 1. We have extended their simulation with larger values of velocity spread, and the results are shown in Fig. 1. As expected, R is inversely proportional to Δv_z. For comparison we show in the inset two distributions for which $R = 1$. The distributions are separated but still overlapping. Note that $R \geq 1$ would require $\Delta v_z \leq 13$ m/sec, or a longitudinal energy spread of 10^{-5} eV. An energy spread around 10^{-3} eV corresponds to $\Delta v_z \approx 10^3$ m/s and no splitting ($R \approx 0.01$).

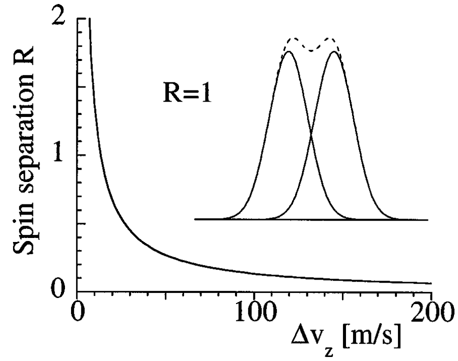

FIG. 1. The degree of final resolution R (defined in the text) as a function of the initial velocity width; all parameters as in Ref. [1].

Third, the authors dismiss the notion of transverse splitting in a Stern-Gerlach field altogether, true to the original Bohr/Pauli edict [2]. In fact, for a unidirectional field given by $B = B_0(0, 0, x)$, a somewhat simpler Stern-Gerlach-like field, transverse splitting is indeed possible under the same restrictions as for longitudinal splitting: namely, that the initial uncertainty product must be very near the quantum limit. [4]

In essence, it appears that the original Bohr/Pauli edict, while incorrect in full generality as the authors pointed out by their counterexample, yields the correct conclusion in practice, at least for beams: splitting is realized only with infeasible initial conditions, while achievable initial conditions yield no splitting.

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[3] M. Reiser, *Theory and Design of Charged Particle Beams* (Wiley, New York, 1994).
[4] G. H. Rutherford and R. Grobe, J. Phys. A (to be published)

