1975

Relativity

STUDY GUIDE

RELATIVITY

INTRODUCTION

Seldom has a development in science captured the attention of the general populace to the extent that Einstein's special theory of relativity did. Once, after giving a public lecture, Einstein was on the way to the railroad depot when he was asked to summarize his theory in one sentence, in a way the general public could understand. His reply: "When does the station get to the train?" Many of the predictions of the theory violate common sense - lengths change, times change, masses change, depending on who is looking - but the theory has been proved correct whenever it has been tested.

At the end of the nineteenth century, Newtonian mechanics and Maxwell's electromagnetic theory seemed to explain all physical phenomena, and some scientists believed that physics as a creative science was finished! Yet there was a serious gap: the ether, the postulated medium necessary for the propagation of electric and magnetic fields, appeared to have several incompatible properties. To resolve the ether problem, Einstein reformulated theoretical physics by introducing an operational approach that made use of light signals propagating at the speed $c = 3.00 \times 10^8$ m/s relative to all inertial frames. This approach replaced the view that space and time have certain absolute properties, as assumed by Newton and his successors.

Incidentally, the "special" in special theory of relativity refers to the fact that only uniform relative motion is considered. Accelerated reference frames, such as those attached to projectiles, are not treated in the special theory of relativity, but are the subject of Einstein's general theory.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Use Cartesian coordinate systems in three dimensions (needed for Objectives 1 to 3 of this module)	Trigonometry Review
*Relate speed, distance, and time for motion in one dimension (needed for Objectives 1 to 3 of this module)	Rectilinear Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Postulates - State the two postulates of special relativity and apply them in a descriptive way to simple phenomena.

2. Lorentz-Einstein transformation - State the Lorentz-Einstein transformation equations, compare them with the Galilean transformation equations, and apply them to simple problems involving two inertial reference frames in uniform relative motion.

3. Length contraction, time dilation - Apply the length-contraction and time-dilation formulas in simple situations to find their consequences, separately or in combination.

GENERAL COMMENTS

If your text does not have a substantial introduction to Einstein's special theory of relativity, we suggest that you consult one or more of the following references:

Adolph Baker, Modern Physics and Anti-Physics (Addison-Wesley, Reading, Mass., 1970).*

Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition, Chapter 6.

Albert Einstein, Relativity (Doubleday, Garden City, N.Y., 1947).*

Anthony P. French, Special Relativity (Norton, New York, 1968).

Carl Helmholz and Burton Moyer, Berkeley Physics Course: Mechanics (McGraw-Hill, New York, 1973), Vol. 1, Chapters 10, 11.

N. David Mermin, Space and Time in Special Relativity (McGraw, New York, 1968), Chapters 1 through 12.*

B. Russell, The ABC of Relativity (New American Library, New York, 1959; revised, 1970 or Humanities, revised 1969).*

Edwin F. Taylor and John A. Wheeler, Spacetime Physics (Freeman, San Francisco, 1966).

Hugh D. Young, Fundamentals of Mechanics and Heat (McGraw-Hill, New York, 1974), second edition.

We have prepared study guides to accompany the texts by Bueche and by Young. If you use the other readings, go through the entire book fairly quickly to locate the principal points, then review the module objectives and correlate them with the readings. We have purposely not included the relativistic theory of force, mass, acceleration, and energy for the sake of brevity, but you may wish to read these topics after you complete the module. After completing the reading, study Problems A through E and work Problems F through K. Finally, check your understanding by taking the Practice Test.

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*These semipopular books are expecially worthwhile, including as they do presentations of the historical developments in physics that led to the formulation of the theory of relativity.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

An introduction to Einstein's special theory of relativity involves many qualitative understandings of apparently paradoxical relationships. To supplement the text, which concentrates on the theoretical aspects, with historical and descriptive information we strongly recommend that you read one of the semipopular books listed in the General Comments.

Read Sections 6.1 through 6.13 in Chapter 6, studying especially Illustrations 6.1 through 6.4. Then study Problems A through E and work Problems F through K. Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

Objective 1 is taken up in the first six sections. Chapter 6 begins with a statement of Einstein's two postulates called the principle of relativity (a better statement is, "Accurate identical experiments performed in any two inertial reference frames in uniform relative motion will give identical results") and the principle of constancy. Most of the paradoxical results of the theory can be traced back to the principle of constancy, as you will discover in later sections.

Some of the surprising consequences are described in Sections 6.5 and 6.6. The discussion in these two sections makes use of "thought experiments," theoretical speculation about what would or would not happen under certain experimental conditions that are plausible to imagine but very difficult to create in reality. Of course, one does not get real data from a thought experiment, but one can get more concrete insights into the workings of a theory.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 6)
1	Secs. 6.1 to 6.6	A, B			Quest.[a] 1, 2, 5, 6, 9, 10
2	Secs. 6.7 to 6.10, Appendix 8 (p. 845)	C		F, G	Probs. 16, 17
3	Secs. 6.11 to 6.13	D, E	Illus.[a] 6.1 to 6.4	H, I, J, K	Quest. 8; Probs. 1 to 9, 14

[a]Illus. = Illustration(s). Quest. = Question(s).

One difficulty with thought experiments is that one often does not know whether one has done them "right." For instance, what really happens to make the man in the car (p. 76) say that the light pulses hit the opposite ends of the car at equal times? Since he is in the middle, he might have mirrors at the ends and observe the two pulses returning to him at the same time. But then, would he still be sure that they had hit the ends simultaneously? Perhaps certain effects would cancel between the outward and return trip of each pulse. Einstein found it necessary to describe, by means of a thought experiment, how one observer would judge the simultaneity of events occurring at different places in his own inertial reference frame. This procedure justifies the example on p. 76, and is as follows:

Assume that there is an "observer" at each point in the inertial reference frame. Assume further that all the observers have identical clocks, running at exactly the same rate. To compare events, the clocks have to be set to the "same" time once, so none is permanently either early or late by a fixed amount. To set the clocks, the observer 0_1 at the origin of coordinates sends out a light pulse that is reflected by a second observer 0_2 back to 0_1. Observer 0_2 notes the time of reflection, and 0_1 notes the times of emission and return. Now, in view of the principle of constancy, the time for the pulse to travel from 0_1 to 0_2 equals the time for return, since the speed of light always has the same value. This means that the time of reflection should be midway between the times of emission and return. 0_1 signals this time to 0_2 and 0_2 can set his clock ahead or behind, depending on the observed reflection time. Einstein called the process the synchronization of clocks. Whenever coordinate and time measurements in a reference frame are mentioned, they are accomplished by means of synchronized clocks.

Sections 6.7 and 6.8 provide background for Objective 2 by describing in considerable detail the so-called Galilean transformation that relates observations in two coordinate systems in uniform relative motion according to ideas proposed by Galileo and in accord with our everyday experience. The crucial thought experiment in Section 6.9 makes use of the "light sphere," whose progress in the (x, y, z, t) system is observed by synchronized clocks reading t, and whose progress in the (x´, y´, z´, t´) system is observed by synchronized clocks reading t´. The discrepancy with the Galilean approach is evident.

The Lorentz transformation equations (Objective 2) can be derived from Eqs. (6.2) and (6.3) only by the addition of certain assumptions that are stated in Eq. (A8.1) in Appendix 8. You are not required to derive these, but you may enjoy playing with the algebra from Eq. (A8.1) on to see how far you get. You are expected to know Eqs. (6.4) and to be able also to state the inverse transformation leading from the (x´, y´, z´, t´) system to the (x, y, z, t) system.

Unlike the Galilean transformation, Eqs. (6.4) relate both x´ and t´ to x and t: the time t is not equal to the time t´. Look at it this way: At a certain instant, Observer S in the (x, y, z, t) system is just passing observer S´ in the (x´, y´, z´, t´) system; at that instant, Observer S looks at his own clock and at the clock of S´ and sees different readings. Just what time difference he sees will depend on the locations of the two observers (and their clocks) relative to the coordinate origins of the systems, as measured by the values of x and x´.

Objective 3 is treated in Sections 6.11 through 6.13. These topics really represent applications of the Lorentz transformation equations, but they are so important that we have stated them as a separate objective. You might begin with Section 6.13, which describes certain mathematical properties of the relativistic factor $\sqrt{1 - v^2/c^2}$, which is often represented by the symbol γ^{-1},

$$\gamma = (1 - \beta^2)^{-1/2}, \qquad \beta = v/c. \tag{B1}$$

Remember that β is always less than one, and that γ is always one or greater. A useful identity (check this!) is

$$\gamma^2 = 1 + \beta^2\gamma^2. \tag{B2}$$

Regarding the Lorentz contraction, the point here is that "length" signifies the distance between the end point of the rod at the same time in the system in which length is measured - the (x', y', z', t') system for the proper length L_0 and the (x, y, z, t) system for the contracted length L. In finding $L = x_2 - x_1$, therefore, we use the same time t in the (x, y, z, t) system for both x_2' and x_1' in Eqs. (6.4c) and (6.4d):

$$x_2 = x_2'/\gamma + vt, \qquad x_1 = x_1'/\gamma + vt, \qquad L = (x_2' - x_1')/\gamma = L_0(1 - \beta^2)^{1/2}. \tag{B3}$$

Thus, the proper length of the rod, L_0 (i.e., the length in the coordinate system in which it is at rest), is longer than the length L measured in any moving frame.

In Section 6.12 the analysis of time dilation proceeds in a similar fashion to length contraction, but the text does not use the term "proper time" correctly. Just replace the phrase "proper time" on p. 85 by the phrase "time on stationary clocks" and every statement will be correct. The proper time interval of a phenomenon is the time interval in a reference frame in which the phenomenon stays at the same place. Thus, every clock shows its own proper time. A precise definition is "A time interval between two events is called the PROPER TIME interval if it is measured by one clock that is present at both events. If a time interval is measured by two different synchronized clocks, each present at one of the events but not at the other, it is not a proper time interval."

The statement about moving clocks running slowly can be stated as follows: "The proper time interval $\Delta\tau$ is the shortest possible time interval between two events. The proper time interval is related to any other time interval Δt between the same pair of events by the equation

$$\Delta\tau = \Delta t/\gamma = \Delta t(1 - \beta^2)^{1/2}. \tag{B4}$$

To give you practice with proper time: In Illustration 6.3 the proper half-life of pi-mesons is 2.00×10^{-8} s, the time after which half have decayed when observed at rest. In Illustration 6.4 the proper time of the one-way trip is the two and one-half months read by the spaceship clock that is physically present at the departure of the Earth and then the arrival of Alpha Centauri. The round trip in this illustration is not really suitable for discussion with special relativity, because the spaceship cannot be in uniform motion if it is to stop and return!

TEXT: Hugh D. Young, Fundamentals of Mechanics and Heat (McGraw-Hill, New York, 1974), second edition

SUGGESTED STUDY PROCEDURE

Your readings are from Chapter 14. As you do the readings, work the problems in this study guide. Then take the Practice Test.

Young does not state Einstein's assumptions explicitly. There are two:

(1) Principle of relativity: The fundamental laws of physics are identical for any two observers in uniform relative motion (p. 376, top).

(2) Principle of constancy: The observed speed of light is independent of the motion of the source (p. 377, bottom).

Most of the paradoxical results of the theory of relativity can be traced to the principle of constancy, which is contrary to everyday experience with water waves, sound propagation, and the motion of objects thrown from moving vehicles.

In Sections 14-3 and 14-4, Young investigates the consequences of the two principles for the measurement of time intervals and lengths by differing observers in uniform relative motion. The discussion of the synchronization of clocks (p. 385), though brief, is very important. It is essential for a good understanding of why the time intervals are dilated: As $0'$ moves relative to S, the clock at $0'$ is compared, not with the clock at 0, but with many different clocks along the path of the motion of $0'$ in S. In Section 14-5, the time-dilation and Lorentz-Fitzgerald contraction results are put together to lead to the Lorentz transformation equations.

PROBLEM SET WITH SOLUTIONS

A(1). State the two postulates on which the special theory of relativity is based.

Solution

(1) Principle of relativity: The fundamental laws of physics are identical for two observers in uniform relative motion.

(2) Principle of constancy: The speed of light in vacuum is the same when measured in any inertial reference frame, independent of the motion of the source.

B(1). Use Einstein's two postulates directly to show that the hands on a moving clock advance more slowly than those on a stationary clock. (Do not make use of the Lorentz transformation equations.) Hint: Analyze a thought experiment in which light from a source travels to a mirror and back, and is observed from a second frame moving parallel to the mirror.

Solution

A frame of reference S′ moves with velocity u relative to a frame S. An observer O′ in S′ has a source of light that he directs at a mirror a distance d away, oriented so that the light is reflected back to him as shown in Figure 1. This observer measures the time interval $\Delta t'$ required for a light pulse to make the round trip to the mirror and back. The total distance as measured in S′ is 2d, the speed is c, and the time required is

$$\Delta t' = 2d/c. \qquad (1)$$

Consider how this experiment looks to an observer at O, with respect to which O′ is moving with speed u. Let the time interval observed by O for the round trip be Δt. During this time, the source moves relative to O a distance $u\,\Delta t$, as shown in Figure 1(b). The total round-trip distance as seen by O is not 2d, but

$$2\ell = 2\sqrt{d^2 + [(1/2)u\,\Delta t]^2}\ . \qquad (2)$$

According to the basic postulate of relativity, the speed of light c is the same with respect to both observers, and thus the relation in S analogous to Eq. (1)

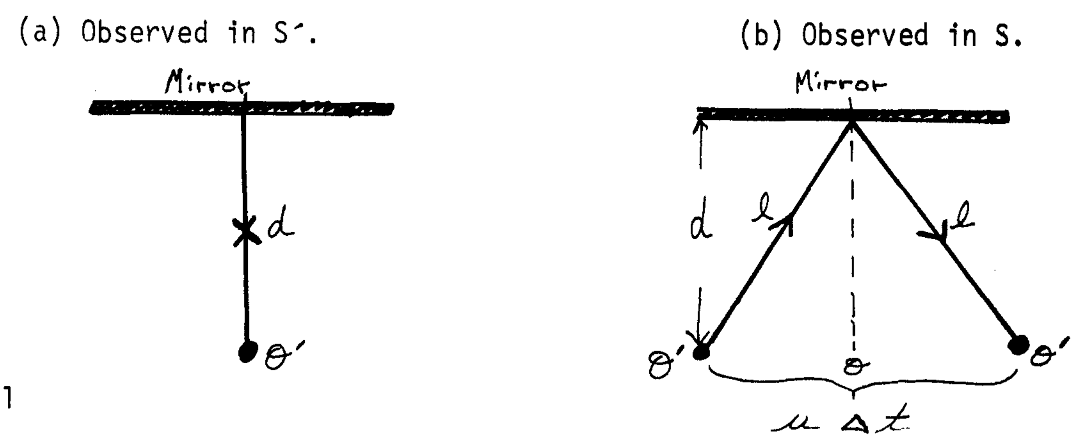


Figure 1

is

$$\Delta t = 2\ell/c = (2/c)\sqrt{d^2 + [(1/2)u\ \Delta t]^2}. \tag{3}$$

Equations (1) and (3) can be combined to obtain a relationship between Δt and $\Delta t'$ that does not contain the distance d. We solve Eq. (1) for d, substitute the result in Eq. (3), and solve for Δt. The result of this algebraic manipulation is

$$\Delta t = \frac{\Delta t'}{(1 - u^2/c^2)^{1/2}}. \tag{4}$$

We may generalize this important result: If two events occurring at the <u>same space point</u> in a frame of reference S' (in this case, the departure and arrival of the light signal at $0'$) are observed to be separated in time by an interval $\Delta t'$, then the time interval Δt between these two events as observed in the frame of reference S is <u>larger</u> than $\Delta t'$, and the two are related by Eq. (4). Thus a clock moving with S' appears to an observer in S to run at a rate that is <u>slower</u> than the rate observed in S'.

C(2). Two events take place at (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) in one coordinate system, and they take place at "primed" coordinates (x_1', y_1', z_2', t_2') in a second coordinate system. The second system, its axes parallel to corresponding axes of the first, moves relative to the first with constant speed u along their common x axes. Find the transformation equations relating the space and time intervals $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, $\Delta z = z_2 - z_1$, and $\Delta t = t_2 - t_1$, to the corresponding primed space and time intervals. You may start from the Lorentz transformation equations.

<u>Solution</u>

The Lorentz transformation equations may be used to obtain the required intervals. Suppose that event 1 occurs at position x_1, time t_1, and event 2 occurs at position x_2, time t_2, in the unprimed frame. The spatial separation in the primed system is derived in the following manner:

$$x_1' = \gamma(x_1 - \beta ct_1), \qquad x_2' = \gamma(x_2 - \beta ct_2),$$

$$x_2' - x_1' = \Delta x' = \gamma(x_2 - x_1) - \beta c(t_2 - t_1),$$

$$\Delta x' = \gamma(\Delta x - \beta c\Delta t), \quad \Delta y' = \Delta y, \quad \text{and} \quad \Delta z' = \Delta z.$$

Similarly, the separation in time is

$\Delta t' = \gamma(\Delta t - \beta\, \Delta x/c)$.

Remember, $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$, and $\beta = 1 - 1/\gamma^2$.

D(3). The μ-meson is an unstable particle whose half-life (the time after which half of an original number have decayed radioactively) when at rest is approximately 2.30×10^{-6} s. These mesons are produced in bursts of many mesons in the upper atmosphere by the impact of energetic cosmic rays.
(a) What is the half-life, as measured by observers on Earth, of μ-mesons moving through the atmosphere with a speed $u/c = 0.99$?
(b) How far do the mesons move during the time of one half-life as measured by observers on Earth?
(c) How far has the ground moved during the time of one half-life, as observed in the rest frame of the mesons? Compare with the Lorentz-Fitzgerald contraction.

Solution

(a) A burst of μ-mesons traveling at the same velocity can be used to define two events: (1) their production and (2) the decay of half the original burst. In the rest frame of the mesons, the time between these events is 2.30×10^{-6} s, and this defines the proper time interval $\Delta t'$. In the other frame, relative to which the μ-mesons move at constant velocity u, the time between the two events is given by

$$\Delta t = \frac{\Delta t'}{[1 - (u/c)^2]^{1/2}} = \frac{2.30 \times 10^{-6}}{[1 - 0.99^2]^{1/2}} = \frac{2.30 \times 10^{-6}}{[0.020]^{1/2}}$$

$$= 2.30 \times 10^{-6}/0.14 = 1.60 \times 10^{-5} \text{ s}, \quad \text{greater than } \Delta t'.$$

(b) Since the mesons are moving with the speed $u = 0.99c$, the distance Δx they travel in the time Δt is

$$\Delta x = u\, \Delta t = 0.99(3.00 \times 10^8)(1.6 \times 10^{-5}) = 4.8 \times 10^3 \text{ m}.$$

(c) In the mesons' rest frame the ground moves with speed $u = 0.99c$ in the direction opposite to the mesons' motion, and the motion occurs for the time interval $\Delta t'$; hence the distance $\Delta x'$ moved by the ground is

$$\Delta x' = u\, \Delta t' = 0.99(3.00 \times 10^8)(2.30 \times 10^{-6}) = 6.8 \times 10^2 \text{ m}.$$

Note that the answers in parts (b) and (c) are unequal. This is an example of the Lorentz-Fitzgerald contraction associated with the relative motion of the ground and meson reference frames:

$$\Delta x = \Delta x'(1 - u^2/c^2)^{1/2} = 0.14\, \Delta x.$$

There are several important points to remember: (1) In the time-dilation equation, the proper time interval $\Delta t'$ in the reference frame S' denotes the time interval between two events at the same position in that frame. (2) In any inertial reference frame S in which these two events occur at different positions, the time interval

Δt separating them is longer than the proper time interval $\Delta t'$ between the same events. (3) In the Lorentz-Fitzgerald contraction equation, the proper length L' is measured in the frame of reference in which the object is at rest. (4) In any inertial reference frame S in which the object is moving parallel to its length, the length L must be measured by noting the positions of its ends at the same time in S, and L is smaller than the proper length L'.

Note: An experimental comparison of μ-meson flux and decay rates has been carried out by using Mt. Washington, NH and sea level as the two locations. See D.H. Frisch and J.H. Smith [Am. J. Phys. 31, 342 (1963)]. Also available is a film by Frisch and Smith, "Time Dilation, an Experiment with μ-mesons," Education Development Center, Newton, MA, 1963.

E(3). The Orient Express moves past a small station in Central Europe at a speed of 1.50×10^8 m/s. It has mirrors attached to both ends. The length of the train (in its rest system) is 100 m. After it passes, the stationmaster turns on a light. The light travels to both mirrors and is reflected back to him. How much times elapses between the arrival of the two reflected light beams at the station?

Solution

We consider two events: light striking the rear mirror and light striking the front mirror. We have to provide for the motion of the train as well as the motion of the light. Let the unprimed frame be that of the stationmaster and the track (see Figure 2), and let

$x = 0$	be the position of the rear mirror when light strikes;
$x = x_0$	be the position of the front mirror when light strikes;
$t = 0$	be the time when rear mirror is struck;
$t = t_0$	be the time when front mirror is struck;
v	be the speed of the train;
L_0	be the length of the train (proper); and
$L = L_0\gamma$	be the length of the train (moving).

Since the light reflected from the front has to travel the extra distance x_0 two times (forward and back), it will take the extra time $2t_0$, which will be the answer. The time for light to reach the train's rear is immaterial for the delay we are calculating, since both beams require that extra time. Since the train is moving while the light is propagating, we have

$$x_0 = vt_0 + L \quad \text{(from motion and length of train)}$$

and

$$x_0 = ct_0 \quad \text{(from motion of light).}$$

We must solve these equations for t_0 in terms of v, which appears explicitly and concealed in $L = L_0(1 - v^2/c^2)^{1/2}$:

$$(c - v)t_0 = L = L_0(1 - v^2/c^2)^{1/2},$$

$$t_0 = \frac{L_0(1 - v^2/c^2)^{1/2}}{c\,(1 - v/c)} = \frac{L_0}{c}\left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

$$= \frac{100}{3 \times 10^2}\left(\frac{1 + 0.50}{1 - 0.50}\right)^{1/2} = 0.58 \times 10^{-6}\ \text{s},$$

$$2t_0 = 1.16 \times 10^{-6}\ \text{s}.$$

Comments: (i) Note how we have written t_0 in terms of L_0/c and the dimensionless ratio v/c. This makes it easy to check for dimensional consistency. (ii) In the train's frame, the light has traveled from back to front in the time $t_0' = L_0/c$, which is smaller than t_0. (iii) Proper time does not enter this problem because no clock is present at both events.

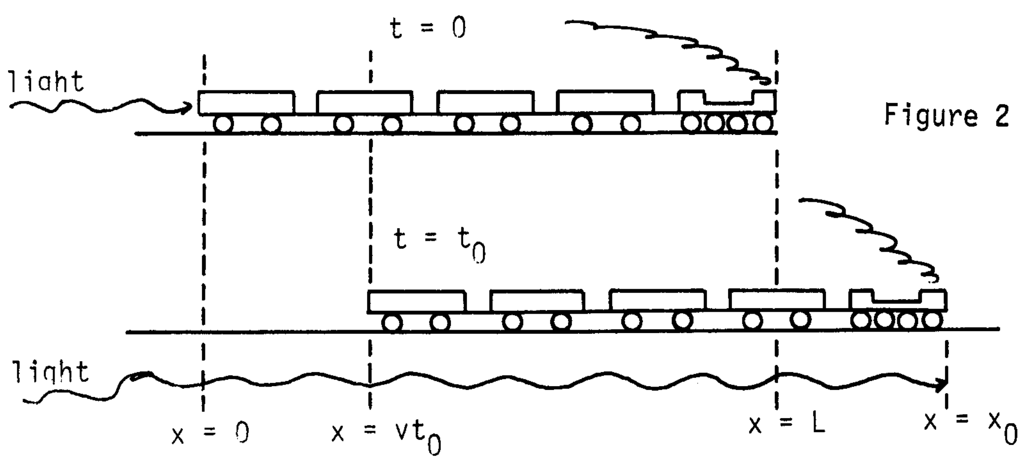


Figure 2

Problems

F(2). A very energetic proton interaction is observed at laboratory coordinates $x_1 = 0.100$ m, $y_1 = 0.100$ m, $z_1 = 0.100$ m, $t_1 = 0$ s. A mysterious second event occurs at $x_2 = 0.500$ m, $y_2 = 0.100$ m, $z_2 = 0.100$ m, and $t_2 = 1.00 \times 10^{-9}$ s.
(a) How far apart in space and time are the two events (see Problem C) as observed by a distinguished visitor passing through the laboratory on a brief inspection tour with a speed $v_x = +1.80 \times 10^8$ m/s?
(b) How fast must the visitor be moving with respect to the laboratory to observe the two events simultaneously in her reference system?
(c) How fast must the visitor be moving with respect to the laboratory to observe the two events at the same position?
(d) (Optional) What is the possibility that the proton interaction "caused" the mysterious event?

G(2). In the same experimental arrangement used in Problem F, a second energetic proton interaction and a particle decay are observed at the following positions and times:

proton: $x_1 = 0$, $y_1 = 0$, $z_1 = 0$, $t_1 = 0$.

particle decay: $x_2 = 1.00$ m, $y_2 = z_2 = 0$, $t_2 = 1.00 \times 10^{-8}$ s.

(a) How far apart in space and time are the two events for a distinguished visitor passing through the lab with a speed $v_x = 1.80 \times 10^8$ m/s?
(b) How fast must the visitor be moving with respect to the lab to observe the two events occurring simultaneously in her frame?
(c) How fast must the visitor be moving to observe the events at the same position in her frame?
(d) (Optional) What is the possibility that the decaying particle was created in the proton interaction?

H(3). David Winch rode his bicycle up a straight mountain road at a steady speed of 0.385 c. Stationary observers with synchronized clocks at the beginning and end of his 50-km course noted the times of his start and finish.
(a) What elapsed time did the observers record?
(b) What elapsed time did David record?
(c) Whose time was the proper time?
(d) What distance was the length of the course as observed by David?

I(3). A train measures 1000 m in length when stationary. The train's track goes through a tunnel that is 1100 m long when surveyed in its rest frame. The engineer does not want the train ever to be contained completely within the tunnel. Therefore he decides to go through the tunnel very fast, at 0.60 times the speed of light. That way, he figures, at least __(a)__ meters of the train will always be out in the sunlight. He tests this prediction with cameras on board the engine and the rear end of the train. The cameras are set up to take motion pictures of the surroundings at both ends of the train. The tests __(b)__ (choose "confirm" or "contradict") his prediction. The dispatcher alongside the track has a different opinion. Going fast, he asserts, will only make the situation worse, because it will make the train still shorter in comparison with the tunnel. He sets up cameras on the ground along the track (outside and inside the tunnel) to provide evidence. At the instant when the rear of the train enters the tunnel, all of his cameras take flash pictures. The camera that records the head of the train is __(c)__ (choose "inside" or "outside") the tunnel, __(d)__ meters from its exit. Do the two direct measurements by the engineer and the dispatcher: (A) confirm only one of the predictions, definitely settling the question as to whether the train is really ever entirely within the tunnel? or (B) confirm both predictions, leaving the engineer and dispatcher in disagreement? or (C) confirm neither prediction, because the train and the tunnel both shrink or expand by the same factor in any frame of reference? __(e)__. [Choose (A), (B), or (C).]

The dispatcher takes out his stop watch and determines that it takes __(f)__ seconds for the entire train to pass in front of him. The engineer determines this time interval by seeing how much time elapsed between the picture showing the dispatcher taken by the engine-mounted camera and the picture showing the dispatcher taken by the rear camera; he finds it to be __(g)__ ("longer" or "shorter"), namely __(h)__ seconds. Which of these is the "proper time interval"? __(i)__.

J(3). Two spaceships, each 120 m long in its own rest frame, meet in outer space. An observer in the cockpit at the front of one ship observes that a time of 1.80×10^{-6} s elapses while the entire second ship moves past her, heading in the opposite direction.
(a) What is the relative velocity of the two spaceships?
(b) How much time elapses on the clocks of the first ship while the front of the second ship moves from the front to the back of the first ship?

K(3). A student bit his tongue (by mistake) and opened his mouth to cry in pain 0.50 s later. A callous friend who passed by at a high speed (but did not stop to give aid and comfort) noted that 2.00 s elapsed between the two events in his reference frame.
(a) What are the space and time intervals (see Problem C) between the two events for the student in pain?
(b) What is the speed of the callous friend?
(c) What is the spatial separation of the two events in the reference frame of the callous friend?

Solutions

F(3). Hints: (a) Find the space and time intervals in the lab frame, then transform. (b) The space and time intervals in the visitor's frame must have $\Delta t' = 0$. Solve the transformation equation for v. (c) The space interval must have $\Delta x' = 0$. Solve for v. (d) The "cause" must (precede? follow?) the effect. Answers: (a) 0.275 m; 2.50×10^{-10} s. (b) $+2.25 \times 10^8$ m/s. (c) $v = 4.0 \times 10^8$ m/s $> 3.00 \times 10^8$ m/s, impossible! (d) Since an observer going faster than $v = 2.25 \times 10^8$ m/s will see the order of events reversed, such an observer cannot even agree with the laboratory observer on which event occurred first. Since a cause must precede an effect, no matter who is observing, we conclude that these two events cannot be related causally. That is true for any two events for which $\Delta x > c\,\Delta t$. Such a separation is called spacelike.

G(2). See Hints for Problem F. (a) 1.00 m, 1.00×10^{-8} s. (b) $v = 9.0 \times 10^8$ m/s, impossible! (c) 1.00×10^8 m/s. (d) Since $c\,\Delta t > \beta\,\Delta x$, $\Delta t'$ is always positive. Thus any observer can agree with every other that the first event really was first. It could be the cause of the decaying particle, although it need not be. That is true for any two events for which $\beta\,\Delta x < c\,\Delta t$. Such a separation is called timelike.

H(3). Hints: (a) Remember in which frame (David's or observers') the 50 km is laid out. (b) Apply time-dilation equation. (c) What were the two events and whose clock was present at both events? (d) Two methods: Lorentz contraction applied to "course" or David's time applied to courses' speed relative to David. Answers: (a) 4.3×10^{-4} s. (b) 4.0×10^{-4} s. (c) David's. (d) 46 km.

I(3). Hint: Use two reference frames, one for the track, tunnel, cameras on the ground, and dispatcher; the other for the train, engineer, and cameras on the train. Identify the "events" that are being discussed. Answers: (a) 120 m. (b) confirm. (c) inside. (d) 300. (e) B. (f) 0.44×10^{-5}. (g) longer. (h) 0.55×10^{-5}. (i) (f) is proper time.

J(3). (a) 0.65×10^8 m/s. (b) 1.85×10^{-6} s.

K(3). (a) $\Delta x = 0$, $\Delta y = 0$, $\Delta z = 0$, $\Delta t = 0.50$ s. (b) 2.904×10^8 m/s. (c) 5.81×10^8 m (too far to help).

PRACTICE TEST

1. State the two postulates of special relativity.

2. A cosmic-ray particle from space crosses the solar system, parallel to the plane of the system. The speed of the particle is $v = \beta c = 0.95c$. The diameter of the solar system is about 1.20×10^{13} m.
   (a) How long does it take for the particle to cross the solar system, as measured by observers in the solar system?
   (b) How long does it take to cross the solar system according to an observer riding with the particle?
   (c) What is the diameter of the solar system as measured by the observer riding with the particle?
   (d) Which observers measure a proper time for the trip? A proper distance for the diameter?

3. In a certain inertial reference frame, two explosions occur at the space-time points $(x_1, y_1, z_1, t_1) = (5.0\text{ m}, 0, 0, 0)$ and $(x_2, y_2, z_2, t_2) = (8.0\text{ m}, 1.00\text{ m}, 2.00\text{ m}, 1.00 \times 10^{-9}\text{ s})$. An observer in a different reference frame, moving with velocity $v_x = -2.40 \times 10^8$ m/s relative to the first, observes the same two explosions. Assume the origins of the two coordinate systems coincide at $t = t' = 0$.
   (a) Write the appropriate Lorentz-Einstein transformation equations to calculate the coordinates of the two events in the second (primed) system.
   (b) Calculate the positions and times of the explosions in the primed system.

Practice Test Answers

1. Identical experiments performed in any inertial reference frames give identical results. The speed of light in vacuum is independent of the motion of the source.

2. (a) 4.2×10^4 s. (b) 1.30×10^4 s. (c) 3.8×10^{12} m. (d) "Particle" observer records proper time. Solar-system observer measures proper diameter.

3. (a) $x' = \gamma(x - \beta ct)$, $y' = y$. $t' = \gamma(t - \beta x/c)$, $z' = z$ (Note: $\beta = -0.8$). (b) $x_1' = 8.33$ m. $x_2' = 13.7$ m. $t_1' = 2.22 \times 10^{-8}$ s. $t_2' = 3.72 \times 10^{-8}$ s. $y_1' = z_1' = 0$. $y_2' = 1.00$ m. $z_2' = 2.00$ m.

RELATIVITY   Date ______________

Mastery Test   Form A   pass   recycle
                                            1   2   3

Name ______________   Tutor ______________

1. (a) Briefly name the two postulates of the special theory of relativity. You will be asked to describe them orally in more detail.

   (b) A friend argues that, since the measured speed of a horse depends on the relative motion of two observers, the principle of relativity must be incorrect. How do you reply?

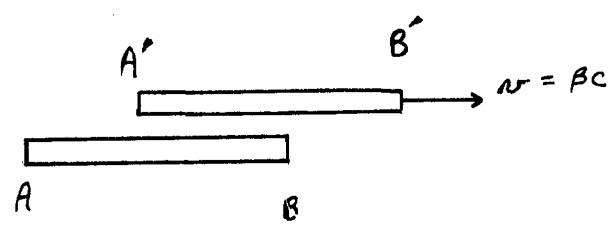
2. Figure 1 represents the top view of two meter sticks, in relative motion with speed $v = \beta c$. The picture is the view by an observer in the unprimed reference frame. At $t = t' = 0$, ends A and A′ of the two meter sticks are together. Let $\beta = 12/13 = 0.92$.

   (a) At $t' = 0$, calculate the position of end B of the unprimed meter stick, as observed by someone in the primed reference frame. Draw a sketch, similar to the one in the figure, showing the sticks at $t' = 0$, as seen by the "primed" observer.

   (b) Calculate the time t'_B when ends B and B′ are together, as observed by the "primed" observer. Sketch the situation.

   (c) How would your answers to parts (a) and (b) differ in a universe that obeyed Galilean relativity?

Figure 1



RELATIVITY Date ____________

Mastery Test Form B

pass		recycle
1	2	3

Name ____________________ Tutor ____________________

1. (a) Briefly name the two postulates of the special theory of relativity. You will be asked to describe them orally in more detail.
   (b) A flashbulb is mounted on the side of a speeding train, and an identical bulb is mounted on a pole beside the track. When the two bulbs are side by side, they are fired. Which pulse reaches the engineer first?

2. Pioneer XLVI is an unmanned space probe, traveling from Earth to Sun in the year 1992. Its uniform speed relative to the Earth is 0.60c. Assume that it leaves Earth at $t = t' = 0$. The Earth-Sun distance is 1.50×10^{11} m in the rest frame of Earth and Sun. Neglect gravitation and rotation.
   (a) At what time, according to earthbound clocks, does it reach the Sun?
   (b) According to the clock on Pioneer XLVI, how long does the trip take?
   (c) Describe how an observer riding with Pioneer XLVI would determine the distance from Earth to Sun. What distance would the observer calculate?
   (d) Describe how your answers to parts (a), (b), and (c) would differ if Galilean relativity were Nature's way of operating.

RELATIVITY

Date ____________________

Mastery Test   Form C

pass   recycle

1   2   3

Name ____________________   Tutor ____________________

1. (a) Briefly name the two postulates of the special theory of relativity. You will be asked to describe them orally in more detail.
(b) A friend argues that, since the measured speed of sound depends on the motion of the observer with respect to the source, the principle of relativity must be incorrect. How do you reply?

2. A train that is 300 m long in its rest frame is moving at a speed of $v = \beta c = 0.80c$. At $t = 0$, the engineer in the locomotive at the front of the train is adjacent to a switchman standing on the ground (event A). As the caboose at the rear of the train passes the switchman, he flashes a light signal forward (event B). The light strikes a rearview mirror beside the engineer (event C), and is reflected back to be detected by the conductor in the back of the caboose (event D), and ultimately by the switchman (event E).
(a) State which time intervals between pairs of events are proper for the conductor in the caboose.
(b) According to a system of clocks and measuring sticks on the ground, calculate the time and location of event B.
(c) According to a system of clocks and measuring sticks on the ground, calculate the time and location of event C.
(d) How would your answer to part (b) change if Galilean relativity applied?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Be sure student knows the difference between the invariance of physical laws, and, say, getting the same numbers for velocity in a given experiment.

   Solution: (a) "Laws of physics are the same in every inertial reference frame," or some equivalent version. "Speed of light is a constant in every inertial reference frame."

   (b) The relativity postulate does not say two measurements of speed must give the same number. It does say that in each frame $\vec{F} = d\vec{p}/dt$, so the horse must push with the same force to get the same momentum change.

2. What To Look For: (a) In the drawing, points A and A′ should be together, and AB should be shorter than A′B′. The solution should demonstrate ability to use the appropriate transformation equation, or the ability to reason from arguments about proper length. (b) In the drawing, points B and B′ should be together, and AB should be shorter than A′B′. Minus sign is important, since comparison of sketches in (a) and (b) shows that (b) occurred first. Ask student about the meaning of the minus sign. Problem solution should demonstrate ability to use Lorentz transformation correctly, or to reason from arguments about the proper length.

   Solution: See Figure 4.

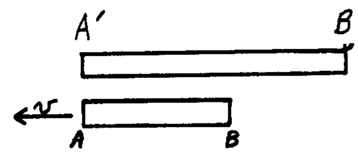


Figure 4

   $x_B = \gamma(x'_B + \beta c t'_B) = \gamma x'_B$.

   $x'_B = x_B/\gamma = (1)\sqrt{1 - \beta^2} = 0.39$ m.

   Or: A′B′ is a proper length. AB is Lorentz contracted, so $AB = A'B'\sqrt{1 - \beta^2} = 0.39$ m. Since left end is at $x'_A = 0$, right end must be at $x'_B = 0.39$ m.

   (b) See Figure 5.

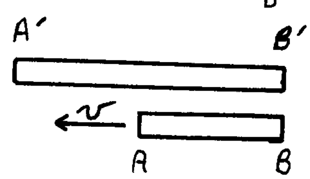


Figure 5

   $x_B = \gamma(x'_B + \beta c t'_B)$;

   $x_B = x'_B = 1.00$ m.

   $t'_B = (1/\beta c)[(x_B/\gamma) - x'_B] = -0.222 \times 10^{-8}$ s.

   Or: Primed observer says AB is 0.39 m long. At t = 0, A and A′ were together. To get B and B′ together, move AB 0.62 m to the right. That occurred at earlier time

   $t = -8/13\beta c = 0.222 \times 10^{-8}$ s.

   (c) Parts (a) and (b) would have the same answer: $x_B = x'_B = 1.00$ m at $t' = 0$. The length AB is 1.00 m.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: Student should know the postulate of constancy. Speed of source has no effect on speed of light.

   Solution: (a) See Grading Key form A, Solution 1.
   (b) Both reach the engineer at the same time.

2. What To Look For: Proper use of correct Lorentz transformation equation, or correct use of Lorentz contraction arguments, or correct use of time-dilation arguments. The distance must be less than the proper distance.

   Solution: (a) Call the Earth-Sun distance ℓ in the Earth-Sun frame. Then

$$t_s = \frac{\ell}{v} = \frac{1.50 \times 10^{11}\ \text{m}}{0.60(3.00 \times 10^8\ \text{m/s})} = 833\ \text{s}$$

   (b) $t_s' = \gamma(t_s - \beta\ell/c) = \gamma(t_s - \beta v t_s/c) = t_s(1 - \beta^2)/\sqrt{1 - \beta^2} = t_s\sqrt{1 - \beta^2}$
   $= 830(0.80) = 670$ s.

   Or: Probe "sees" a Lorentz-contracted distance. The time to traverse that distance is

   $t_s' = \ell'/\beta c$, thus

$$t_s' = \frac{\ell\sqrt{1 - \beta^2}}{\beta c} = \frac{(1.50 \times 10^{11})(0.80)}{(0.60)(3.00 \times 10^8)} = 670\ \text{s}.$$

   (c) Time the trip, and multiply by speed. The time (a proper time) is, from (b), 670 s. The Lorentz-contracted length is

   $670(0.60)(3.00 \times 10^8) = 1.20 \times 10^{11}$ m.

   (d) Answer to (a) would be unchanged. Part (b) would have same answer as (a), and answer to (c) would be 1.50×10^{11} m.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: Student should know the difference between invariance of physical law, and getting the same number in a given experiment.

   Solution: (a) See Grading Key Form A, Solution 1.
   (b) The physical properties of the medium determine the speed of sound with respect to the medium. Those properties obey physical laws that are the same in any inertial frame.

2. What To Look For: Clear understanding of condition for proper time interval. Should recognize that $x_B = 0$, $x_B' = -300$ m, in coordinate frame given here. Sign of t_B must be positive. (You may ask student why.) Alternative solution to (d) is more difficult. Student should correctly use Lorentz transformation equation.

   Solution: (a) Conductor: $\Delta t_{BD}'$ is proper.
   (b) Train frame = primed frame, with origin at front end. Ground frame = unprimed frame, with origin at switchman. Set proper time interval:

   $t_{BD}' = 2(300)/(3 \times 10^8) = 2.00 \times 10^{-6}$ s, for light to go to engineer and back.

   Then $t_{BD} = \gamma t_{BD}' = (5/3)(2)(10^{-6}) = 3.33 \times 10^{-6}$ s.

   Distance train moves: $x_{BD} = (0.80)(3 \times 10^8)(3.33 \times 10^{-6}) = 800$ m.

   $x_B = 0$. $\beta = 0.80$, $\gamma = 1.70$. $x_B' = \gamma(x_B - \beta c t_B) = -\gamma\beta c t_B$.

   $$t_B = -\frac{x_B'}{\gamma\beta c} = \frac{-(-300)}{(5/3)(4/5)(3 \times 10^8)} = 0.75 \times 10^{-6} \text{ s.}$$

   Or: Switchman sees Lorentz-contracted train, $\ell = \ell_0\sqrt{1 - \beta^2}$. Time for train to pass is $t_B = \ell/v = 300(0.60)/0.80(3 \times 10^8) = 0.75 \times 10^{-6}$ s.

   (c) Get t_B' first; then $t_C' = t_B' + \ell'/c$, t_B is proper time interval between events A and B, so

   $t_B' = \gamma t_B = 1.25 \times 10^{-6}$ s. $t_C' - t_B' = \ell_0/c = 10^{-6}$ s. $t_C' = 2.25 \times 10^{-6}$ s.

   $x_C' = 0$. $t_C = \gamma(t_C' + \beta x_C'/c) = \gamma t_C' = 3.75 \times 10^{-6}$ s. $x_C = \beta c t_C = 9 \times 10^2$ m.

   Or: Engineer argues that event B occurs after the train has passed the switchman. That takes time $t_B' = \ell_0/\beta c = 1.25 \times 10^{-6}$ s.

   $t_C' = t_B' + \ell_0/c = 2.25 \times 10^{-6}$ s.

   t_C' is proper, so $t_C = t_C'/\sqrt{1 - \beta^2} = 3.75 \times 10^{-6}$ s. $x_C = \beta c t$, as before.

   (d) $x_B = 0$, as before. t_B is time for train to pass. $t_B = \ell_0/\beta c = 1.25 \times 10^{-6}$ s.