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Hydrodynamic models for the short-period, classical Cepheid, SU Cas

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Summary. Hydrodynamic models are constructed for the 1.95-day, classical Cepheid, SU Cas. The models cover three luminosities (479, 955 and 2239 $L_\odot$) and a range of temperatures (6200 K $\leq T_e \leq$ 6500 K), in line with observational and theoretical results from the work of previous authors. Taking the linear stability and stability of limiting pulsations of the models into consideration, it is shown that at 479 $L_\odot$ only the fundamental mode is possible; at 955 $L_\odot$ the first overtone mode is possible; at 2239 $L_\odot$ only the second overtone mode is possible. For stable limit cycles, the light and velocity curves are compared with observations by means of Fourier decomposition. The Fourier component ϕ_{21} for SU Cas is apparently greater than that of other classical Cepheids that are supposed to be fundamental mode pulsators with similar periods. However, hydrodynamic models display values of ϕ_{21} that do not differ with pulsation mode. We attribute this to a shortcoming in the F-mode models. On the other hand, these models seem to produce reliable values of ϕ_{31} and of the first-order phase lag $(\Delta\phi)_1$. We discuss the pulsation of SU Cas in the light of stellar evolution calculations, and summarize the evidence regarding its pulsation mode. We conclude that SU Cas is almost certainly not an F-mode pulsator, although a completely satisfactory overtone model has yet to be constructed.

Key words: hydrodynamics – stars: oscillations – stars: Cepheids – stars: SU Cas

1. Introduction

A number of authors (e.g. Stobie, 1975; Ivanov and Nikolov, 1976; Gieren, 1976) have suggested that some or all of the short-period Population I Cepheids with low amplitudes and sinusoidal light curves may be overtone pulsators. However, it is clear that a small amplitude and symmetric light curve are not in themselves enough to mark a star as pulsating in an overtone mode (Pel and Lub, 1973; Gieren, 1982a). Indeed, in his analysis of nine such objects, Arellano-Ferro (1984) concluded that all but one (DT Cyg) are fundamental model pulsators.

Simon and Lee (1981) fit the light curves of a large sample of classical Cepheids with Fourier series, $A_0 + \sum_{j=1}^{j_{\max}} A_j \cos(j\omega t + \phi_1)$, and formed for each star the amplitude ratios, $R_{j1} = A_j/A_1$, and phase differences, $\phi_{j1} = \phi_j - j\phi_1$. They argued that R_{21}, ϕ_{21} and ϕ_{31} could be employed as mode discriminators, a thesis which was supported by Gieren (1982b) and Antonello and Mantegazza (1984). Simon and Davis (1983) introduced the first-order phase lag $(\Delta\phi)_1$ which is the difference (in radians) between maximum light and maximum expansion velocity, defined not by the full light and velocity curves but rather by the first order terms in their respective Fourier decompositions. It was suggested by Simon (1984) that $(\Delta\phi)_1$ may also be a mode discriminator, although there is evidence that this quantity may be sensitive to amplitude (Simon, 1984) and temperature (Simon, 1985) as well.

To the present, four stars have been tabbed as overtones on the basis of their Fourier parameters. The first three are AZ Cen (Simon, 1984), DT Cyg (Simon and Moffett, 1985) and SU Cas (Simon and Lee, 1981). The overtone classification in these cases is in agreement with separate evidence presented by other authors – notably Gieren (1982a) for AZ Cen, Arellano-Ferro (1984) for DT Cyg and Gieren (1982b) for SU Cas. Recently, Antonello and Poretti (1986) found that the 2.1-day Cepheid, IR Cep, also shows the signature of an overtone pulsator.

In the present study we focus on SU Cas which has received perhaps the greatest interest of any of the short-period, sinusoidal, Population I sample. This object has been observed extensively in both light and radial velocity (Abt, 1959; Mitchell et al., 1964; Gieren, 1976; Beavers, 1979; Niva and Schmidt, 1979; Moffett and Barnes, 1980) and the question of its pulsation mode has been widely debated. The case for overtone pulsation has been presented in detail by Gieren (1982b). On the other hand, Turner and Evans (1984) classify SU Cas as a fundamental mode pulsator on the basis of its presumed membership in a poorly populated association of B and A stars.

In what follows we shall construct hydrodynamic models for this star and compare them with its observed properties. The results of these studies will allow some conclusions regarding the pulsation mode of SU Cas, while also pointing up a number of shortcomings of the hydrodynamic codes.

2. Observed properties

The observed properties of SU Cas (HD 17463) are summarized in Table 1. The pulsation period is 1.949322 days and, according to Szabados (1977), this pulsation has been stable for over 67 yr.

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Table 1. Observed properties of SU Cas

Period
1.949322 d (Szabados, 1977)
Effective temperature (Gieren 1982b)
Range: 6250 to 6667 K
Mean: 6439 K
Luminosity
479 $L_\odot$ (Turner and Evans 1984)
955 $L_\odot$ (Cox 1979)
2239 $L_\odot$ (Gieren 1982b)
Fourier parameters
Light
ϕ_{21}: 4.25 (data of Gieren 1976); 4.34 (Simon and Lee 1981)
ϕ_{31}: 2.9 (data of Gieren 1976); 3.0 (Simon and Lee 1981)
Velocity
ϕ_{21}: 5.90 (Simon and Teays 1983)
Phase Lag
$(\Delta\phi)_1$: -0.48 (Simon 1984).

The effective temperature T_e of SU Cas is rather uncertain. Gieren (1982b) has collected the temperature determinations made by different authors. These temperatures have a range from 6250 to 6667 K with a mean value of 6439 K.

The luminosity of SU Cas is even less well determined. Cox (1979) adopted a value $L = 955\,L_\odot$ while noting substantial uncertainties in the distance. Gieren (1982b) obtained a Wesselink-type radius for SU Cas and using $T_e = 6439$ K determined a luminosity $L = 2239\,L_\odot$. Finally, Turner and Evans (1984) suggested that SU Cas is a member of a poorly populated association and on the basis of the presumed distance of these stars estimated for SU Cas a luminosity $L = 479\,L_\odot$.

Fourier decomposition parameters for SU Cas were determined by Simon and Lee (1981), Gieren (1982b), Simon and Teays (1983), Simon (1984) and Moffett and Barnes (1985). The light and velocity curves were fitted to Fourier series with forms $A_0 + \sum_{j=1}^{j_{max}} A_j \cos(j\omega t + \phi_j)$ [cf. §1] and $A_0 - \sum_{j=1}^{j_{max}} A_j \sin(j\omega t + \phi_j)$, respectively. Unfortunately, the amplitude ratios $R_{j1} = A_j/A_1$ cannot meaningfully be compared with theoretical models. This is because these ratios depend significantly on the limiting pulsation amplitude, a quantity which is not accurately determined by hydrodynamic integrations. This exclusion leaves for observation-theory comparisons the Fourier phases ϕ_{21} and ϕ_{31} (light) and ϕ_{21} (velocity), and the first-order phase lag $(\Delta\phi)_1$. These quantities are displayed in Table 1.

The most accurate and comprehensive light curve for SU Cas is that given by Gieren (1976). While the data have not been published, Gieren (1982b) stated that ϕ_{21} and ϕ_{31} for his light curve were close to the values determined by Simon and Lee (1981) from different observations. We have enlarged Gieren's published light curve and measured it with a digitized coordinate gauge. Fourier decomposition of the points thus determined shows that the coefficients ϕ_{21} and ϕ_{31}, 4.25 (± 0.02) and 2.9 (± 0.1) respectively, are indeed very similar to the ones quoted in the Simon-Lee paper. We have displayed values for both sets of data in Table 1. With regard to the velocity curve of SU Cas, Table 1 contains the value of ϕ_{21} determined by Simon and Teays (1983). The quantity ϕ_{31} (velocity) has been omitted because the third-order velocity term was so small as to be completely lost in the noise of the observations.

Finally, the chemical composition of SU Cas has been discussed by Gieren (1982b). The metals are slightly deficient compared to the sun, with a CNO abundance similar to that of other Cepheids (Luck and Lambert, 1981). The helium in SU Cas may be slightly enriched.

3. Hydrodynamic models

We have used the code TGRID (Simon and Aikawa, 1986) to integrate 12 hydrodynamic models for SU Cas. Models were constructed at each of the three luminosities given in Table 1. The model characteristics are displayed in Table 2 where the columns give, in order, the model number, the mass and luminosity in

Table 2. Characteristics of hydrodynamic models[a]

Model No.	$M/M_\odot$	$L/L_\odot$	T_e	Y	Mode	PLNA (days)	η	C_Q
1	3.5	955	6339	0.28	1-O	1.921	0.011	1.0
2	3.5	955	6339	0.28	1-O	1.921	0.011	3.0
3	3.5	955	6400	0.28	1-O	1.862	0.006	1.0
4	4.0	955	6200	0.28	1-O	1.907	0.022	3.0
5	3.0	955	6500	0.38	1-O	1.933	0.020	1.0
6	3.8	955	6200	0.20	1-O	1.974	0.004	1.0
7[b]	4.0	955	6200	0.28	1-O	1.953	0.006	1.0
8[a]	3.5	955	6339	0.28	1-O	1.912	0.017	1.0
9	6.0	955	6250	0.28	F	1.965	0.0003	1.0
10	2.5	479	6200	0.28	F	1.957	0.008	1.0
11	2.3	479	6300	0.28	F	1.963	0.004	1.0
12	7.5	2239	6439	0.28	2-O	1.932	0.003	1.0

[a] All models have $Z = 0.02$, except Model 8 for which $Z = 0.01$.
[b] AMO opacity. Please see text.

solar units, the effective temperature, the helium abundance, the pulsation mode, the period (days) and growth rate (percent per period) of the corresponding linear nonadiabatic (LNA) model, and, finally, the coefficient of artificial viscosity C_Q (Stellingwerf, 1975). The opacity in all cases was the Stellingwerf (1975) analytic fit, and convection was neglected. However, in Model 7 the Stellingwerf formula was artificially increased in such a way as to enhance the contribution of the metals, producing an augmented metal opacity (AMO) model exactly as in Simon (1982). The metal abundance was always $Z = 0.02$ except in Model 8 where $Z = 0.01$.

Figure 1 shows light and velocity curves for three of the models as examples of fundamental (Model 11, Fig. 1a), first overtone (Model 3, Fig. 1b) and second overtone (Model 12,

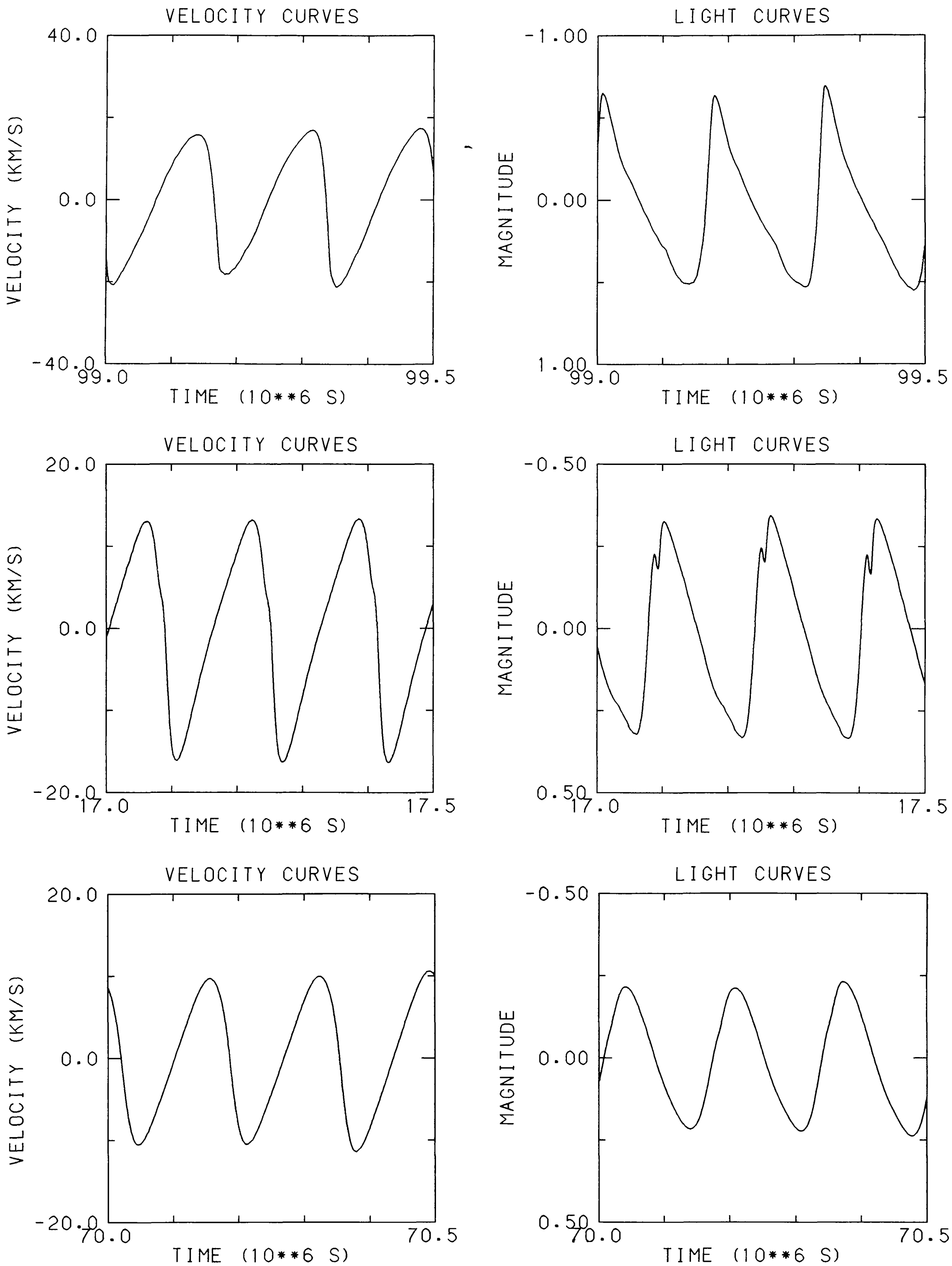

Fig. 1a–c. Light and velocity curves at the photosphere of hydrodynamic models. **a** Model 11 (the fundamental limit-cycle), **b** Model 3 (the first over-tone limit-cycle) and **c** Model 12 (the second over-tone limit-cycle). The velocities are plotted according to observational convention

Fig. 1c) pulsation, respectively. These models converged well and the dynamic zoning in TGRID was able to produce smooth curves.

4. Comparison with observations

Some limit cycle properties of the hydrodynamic models are listed in Table 3. The rows give, in order, the nonlinear period (days), the amplitude and Fourier quantities ($R_{21}, \phi_{21}, R_{31}, \phi_{31}$) for the light curve, the amplitude and Fourier quantities for the velocity curve, and, finally the first-order phase lag $(\Delta\phi)_1$. The column for the fundamental mode Model 9 is blank because the fundamental limit cycle was unstable for this model and began to switch to the first overtone.

Models 1 and 2 are identical except that the latter was integrated with a higher value of the artificial viscosity coefficient C_Q. Although Model 2 limited at a lower amplitude, as expected, and thus displayed smaller values of the amplitude dependent ratios R_{21} and R_{31}, the Fourier phase quantities ϕ_{21}, ϕ_{31} and $(\Delta\phi)_1$ were virtually identical in the two models. This result is similar to that found by Simon and Aikawa (1986) for RR Lyrae models and confirms the utility of comparing theory and observation in terms of the Fourier phases.

However, before making such comparisons one must consider the accuracy of agreement that is to be sought. If SU Cas is a unique object, then we must of course attempt to match its parameters as closely as possible. However, if, as is more likely, the star is a member of a class of similar pulsators, there will then be a range of Fourier phases satisfactory and appropriate for the star's period. For the quantity ϕ_{21} we can estimate this range using the spread in ϕ_{21} among the four suspected overtone pulsators, SU Cas, AZ Cen, DT Cyg and IR Cep. We obtain a range: $\delta\phi_{21} \sim 0.3$. We note that values of the phase ϕ_{21} for suspected overtone pulsators are truly greater than those extrapolated from short-period F-mode pulsators. Table 3, however, shows no significant difference of phase ϕ_{21} among the hydrodynamic models with different pulsation modes. This conclusion will also apply to ϕ_{21} for the theoretical velocity curves, among which there is a spread of the same size as for the light.

Turning to the quantity ϕ_{31}, we notice large deviations from the observed values in Models 7, 10, 11, and 12. The remaining models, all first overtone, should be considered acceptable since the observed value is somewhat uncertain because R_{31} is so small in SU Cas.

From the point of view of the present study, perhaps the most interesting of the Fourier parameters is the first-order phase lag, $(\Delta\phi)_1$. The observed value for SU Cas is -0.48, while that for AZ Cen is -0.56 (Simon, 1984). Examining the theoretical models in Table 3 we can discern a number of possible trends in $(\Delta\phi)_1$. The clearest of these is the tendency for $(\Delta\phi)_1$ to become smaller (i.e., more negative) with increasing temperature. This can be seen, for example, by comparing the pairs of models 2–3, 3–4 and 10–11. A similar tendency was found by Simon (1985) and Simon and Aikawa (1986) among hydrodynamic RR Lyrae models.

The first-order phase lag $(\Delta\phi)_1$ also seems sensitive to helium abundance, becoming more negative as Y falls. This is indicated by a comparison of Models 4, 5 and 6, noting the above mentioned temperature dependence of $(\Delta\phi)_1$. A look at Models 1 and 8 also reveals a possible metal sensitivity of $(\Delta\phi)_1$ with the sense that an increase in Z makes the phase lag more negative. Model 7, constructed with an augmented metal opacity (AMO) is consistent with this idea (compare, for example, Model 4). Finally, there may be a trend of algebraic decrease of $(\Delta\phi)_1$ with increasing order of the pulsation mode. Thus the first-order phase lag may be useful to discriminate among pulsation modes, provided that observations can confine the temperature and chemical abundance in a narrow range.

All the above having been said, we note from Table 3 that the second-overtone model (Model 12) shows by far the best, and the fundamental mode models (Models 10 and 11) the worst agreement with the observed value of $(\Delta\phi)_1$ for SU Cas. The first overtone models occupy a middle position in this regard.

Table 3. Fourier decomposition of hydrodynamic models[a]

Model No.	1	2	3	4	5	6	7	8	9[b]	10	11	12
PNL (days)	1.939	1.931	1.860	1.915	1.933	1.969	1.948	1.910	–	1.935	1.971	1.938
Light curves												
AMP (M_{bol})	0.72	0.54	0.68	0.65	0.69	0.74	0.60	0.83	–	1.66	1.70	0.48
R_{21}	0.30	0.25	0.30	0.31	0.23	0.33	0.30	0.37	–	0.56	0.47	0.15
ϕ_{21}	4.49	4.44	4.46	4.44	4.41	4.50	4.60	4.45	–	4.47	4.44	4.30
R_{31}	0.14	0.07	0.13	0.11	0.10	0.14	0.06	0.22	–	0.36	0.25	0.02
ϕ_{31}	3.11	3.03	2.93	2.65	3.21	3.05	3.73	2.72	–	2.54	2.48	3.87
Velocity curves												
AMP ($km\,s^{-1}$)	32.6	24.4	29.7	30.6	34.0	30.2	30.5	38.7	–	60.0	38.9	22.1
R_{21}	0.37	0.29	0.35	0.32	0.41	0.34	0.35	0.41	–	0.44	0.36	0.26
ϕ_{21}	6.02	6.08	6.04	6.04	6.01	6.14	6.23	5.98	–	5.89	6.12	6.19
R_{31}	0.14	0.09	0.12	0.12	0.18	0.12	0.14	0.17	–	0.27	0.18	0.08
ϕ_{31}	5.74	5.77	5.66	5.88	5.74	5.82	6.11	5.78	–	5.36	5.98	5.94
$(\Delta\phi)_1$	−0.29	−0.29	−0.31	−0.14	−0.24	−0.30	−0.23	−0.18	–	0.05	−0.10	−0.47

[a] Models 1–8: 1-O; Models 9–11: F: Model 12: 2-O.
[b] F-mode limit cycle is unstable

5. SU Cas and stellar evolution

Evolutionary tracks for classical Cepheids are given by Becker (1985). For the moment, let us assume that SU Cas is on the second or third crossing of the instability strip. Examining Becker's Figs. 1 and 2, we note that the first overtone models from our Table 2 will be consistent with standard stellar evolution, provided that they contain somewhat enhanced helium and/or reduced metals. This is also in line with observational indications (see §2). The second overtone model, on the other hand, has too small a luminosity for its mass. We can remedy this problem by reducing the helium and/or increasing the metals but the effect of these is to reduce the driving, moving the blue edge redward. We have calculated second-overtone LNA models which show that if one changes the composition enough to match the evolutionary tracks, the blue edge moves beyond the cool boundary of the observed temperature range for SU Cas.

Turning to the fundamental mode models, one finds similar problems. According to Fig. 2 of Becker (1985) a model with $M \sim 6.0\,M_\odot$, $L \sim 1000\,L_\odot$, $Y = 0.20$ would be in agreement with the evolutionary calculations. However, the limit cycle of our Model 9 is already unstable at $Y = 0.28$, $T_e = 6250$ K. Lowering the helium to $Y = 0.20$ would thus require for fundamental mode pulsation a temperature so cool as to be completely inconsistent with observed values for SU Cas. The F-mode Models 10 and 11 seem even worse from the standpoint of stellar evolution. Their masses are so low that drastic increases in Y and/or decreases in Z would appear necessary to put the evolutionary tracks at a luminosity as high as $479\,L_\odot$. Even given this, it is not clear that the blue loops would extend to hot enough temperatures to penetrate the instability strip. Finally, the inclusion of core overshoot and/or mass loss in the evolutionary models does not seem to alleviate these problems (see, e.g., Fig. 20 of Bertelli et al., 1985).

We close this section by considering the possibility of a first crossing for SU Cas. In the case of the low mass models (Models 11 and 12), the agreement with stellar evolution would clearly be made worse, i.e., a first crossing at $L = 479\,L_\odot$ would require composition anomalies even more drastic than those associated with later crossings. On the other hand, in the more massive Model 9 (F-mode) and Model 12 (second-overtone), the difficulties associated with chemical composition and pulsational stability would be ameliorated. The mass-luminosity relation for each of these models could then agree with that given by the evolutionary tracks, without the necessity for reducing Y or increasing Z. However, an observational constraint exists which seems to rule out a first crossing for SU Cas if its mass is $6\,M_\odot$ (Model 9) or $7.5\,M_\odot$ (Model 12). As noted in a previous section, Szabados (1977) found the period of SU Cas to be stable over a time of 67 yr. According to the calculations of Becker (1985), a $6\,M_\odot$ model will perform a first-crossing of the strip in a time $\sim 10^4$ yr. This implies, over 67 yr, a change in the pulsation period amounting to a few minutes! Such a change is almost certainly incompatible with any reasonable upper limit deduced from the analysis of Szabados. At $7.5\,M_\odot$ the crossing is, of course, faster, and the discrepancy worse.

6. Shortcomings of the hydrodynamic modeling

We have constructed 12 linear and hydrodynamic models and compared them with the observed properties of SU Cas. Unfortunately, no single model was able to satisfy all of the observational constraints.

It has been pointed out above that SU Cas displays a clearly higher value of ϕ_{21} compared with the shortest period F-mode Cepheids. However, among the hydrodynamic models, ϕ_{21} was as large for the F-mode pulsators as for the overtones. This result also seems in conflict with the well documented observation that ϕ_{21} (overtone) $>$ ϕ_{21} (fundamental) among both the double-mode Cepheids (Antonello and Mantegazza, 1984) and the RR Lyrae stars (Simon and Teays, 1982).

In our opinion, the F-mode models are probably at fault here. Models 10 and 11 of the present study give values of ϕ_{21} (light) which differ little from those of the shorter-period Los Alamos models described by Simon and Davis (1983). The run of ϕ_{21} with the period in all these models is far too flat when compared with that observed in the actual stars. Thus the agreement in ϕ_{21} between Models 10 and 11 and the light curve of SU Cas is probably fortuitous.

We may contrast this with the case of ϕ_{31}. We have examined unpublished values of this quantity from the hydrodynamic light curves of the Simon-Davis study. We find that the calculated values of ϕ_{31} match the observed ones satisfactorily, in particular giving a rather good fit to the observed run of ϕ_{31} shown in Fig. 4 of Simon and Moffett (1985).

In this connection we can also consider the first-order phase lag, $(\Delta\phi)_1$. The same F-mode, Los Alamos models (Simon and Davis, 1983) reproduce observed values of this quantity remarkably well (Simon, 1984). Thus there is some reason for accepting the reliability of theoretical values of ϕ_{31} and $(\Delta\phi)_1$ despite the obvious problems with ϕ_{21}.

At present, it is not clear why the calculations should reproduce observed values of ϕ_{31} and $(\Delta\phi)_1$, but not ϕ_{21}. Perhaps these quantities are determined at different places in the models, and some 'local' deficiency is responsible for the discrepancy in ϕ_{21}. This question certainly needs to be investigated further.

7. The pulsation mode of SU Cas

The preponderance of evidence argues strongly against fundamental mode pulsation in SU Cas. Not only are F-mode models in conflict with published stellar evolution calculations, but they also disagree drastically with observed values of ϕ_{31} and $(\Delta\phi)_1$ for SU Cas. In addition, we may quote from a recent study by Evans and Arellano-Ferro (1986), who estimated the luminosity of SU Cas from its binary companion: 'The absolute magnitude derived from the companion differs from the predicted absolute magnitude for fundamental pulsation by five times the $0^{\mathrm{m}}.2$ uncertainty in the determination. It is in much better agreement with overtone pulsation . . .'

Nonetheless it is disturbing that we have not been able to construct a satisfactory overtone model for SU Cas. Our first overtone models agree with standard evolutionary tracks and match the observed value of ϕ_{31}, while the second overtone model reproduces the observed value of $(\Delta\phi)_1$. Improved hydrodynamic modeling seems necessary before we can finally pin down SU Cas's pulsation mode.

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