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Trigonometry

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REVIEW MODULE

TRIGONOMETRY

This module begins with a self-check test. If you can correctly answer 90% of these test items, you do not need to study this module. (A table of trigonometric functions is given on p. 8 of this module.) The answers to these questions are given at the end of the module, i.e., immediately preceding the table.

SELF-CHECK TEST

1. Without the use of tables, convert degrees to radians and radians to degrees:

   (a) $30° =$ ________rad; (c) $225° =$ ___________rad;

   (b) $(3/4)\pi$ rad =_____°; (d) $(5/3)\pi$ rad =_______°.

2. (a) What is the maximum value for the sine of an angle, cosine of an angle, and tangent of an angle? Give at least one angle that has the maximum value for the named function.

   (b) Which angle in Problem 1 has the largest value for the sine, for the cosine, and for the tangent, respectively?

3. One acute angle of a right triangle is 37°. The length of the side opposite the angle is 12.0 cm.

   (a) What are the ratios of the lengths of the sides of this triangle?

   (b) For this triangle, find the lengths of the other two sides of the triangle. (Show your work!)

4. One acute angle of a right triangle is 40°. The length of the hypotenuse is 12.0 cm. Find the lengths of the other two sides.

5. In a right triangle the hypotenuse is $2\sqrt{3}$ and one side is 3.

   (a) Find the missing side.

   (b) What are the angles?

6. A surveyor wishes to determine the distance between two points A and B, but he cannot make a direct measurement because a river intervenes. He steps off at a 90° angle to AB a line AC, which he measures to be 264 m. He measures an angle with his transit at point C to point B. Angle BCA is measured to be 62°. With this information, calculate AB.

7. Show that, for any angle θ,
   $\sin^2 \theta + \cos^2 \theta = 1.$

RIGHT TRIANGLE

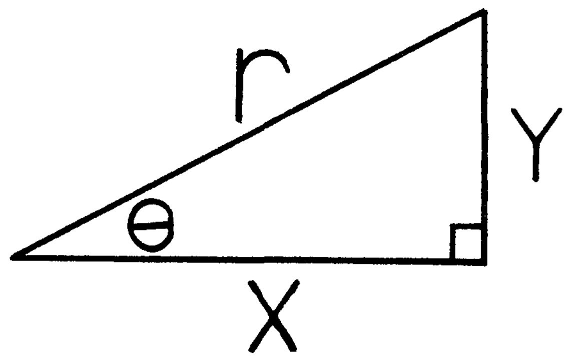
Many of the applications of physics will require you to have a thorough knowledge of the basic properties of right triangles, i.e., triangles that have one angle equal to 90°.

The trigonometric functions are defined with respect to a right triangle as follows:

$\sin \theta = y/r$

$\cos \theta = x/r$

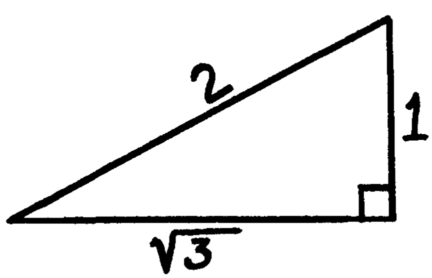
$\tan \theta = y/x$

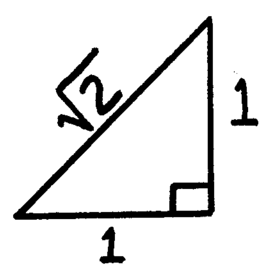


The values of the trigonometric sine, cosine, and tangent functions for a given θ can be determined from a table such as in the appendix to your text or the last page of this module. You can also get the values by use of most slide rules ("S" and "T" scales) and many electronic calculators.

The 30°-60°-90° and 45°-45°-90° Triangles

It is also useful to remember the values of the functions for θ = 30°, 45°, and 60° by means of the triangles below.





These triangles are right triangles, and you should check that the sides indeed satisfy the Pythagorean theorem. Note also that in any right triangle the longest side is the hypotenuse.

Using the basic definitions and the above triangles, one finds

$$\sin 30° = 1/2 = 0.500,$$
$$\sin 45° = 1/\sqrt{2} = \sqrt{2}/2 = 1.414/2 = 0.707...,$$
$$\sin 60° = \sqrt{3}/2 = 1.732/2 = 0.866,$$

$$\cos 30° = \sqrt{3}/2 = 1.732/2 = 0.866,$$
$$\cos 45° = 1/\sqrt{2} = \sqrt{2}/2 = 0.707,$$
$$\cos 60° = 1/2 = 0.500,$$

$$\tan 30° = 1/\sqrt{3} = \sqrt{3}/3 = 1.732/3 = 0.577,$$
$$\tan 45° = 1/1 = 1.000,$$
$$\tan 60° = \sqrt{3} = 1.732.$$

The 3-4-5 Triangle

The 3-4-5 triangle (since the sides are in the ratio of 3 : 4 : 5) is known as the 37°-53°-90° triangle:

$$\sin 37° = 0.6, \qquad \sin 53° = 0.8,$$
$$\cos 37° = 0.8, \qquad \cos 53° = 0.6,$$
$$\tan 37° = 0.75, \qquad \tan 53° = 1.33.$$

You should memorize these three special triangles so that you can compute the values of sine, cosine, and tangent for the angles involved.

ANGLES > 90°

When calculating the products of vectors (see Dimensions and Vector Addition module), it is often necessary to determine the sine and cosine of angles greater than 90°, whereas most trig tables list values only for angles less than or equal to 90°. Two alternative ways of remembering the necessary relationships are as follows:

Method I   Recall the definitions of sine and cosine for general angles:

$$\sin \theta = y/r \quad \text{and} \quad \cos \theta = x/r,$$

where x and y are the horizontal and vertical projections, respectively, of the radial distance r, as shown in the figures below for angles in the various quandrants.

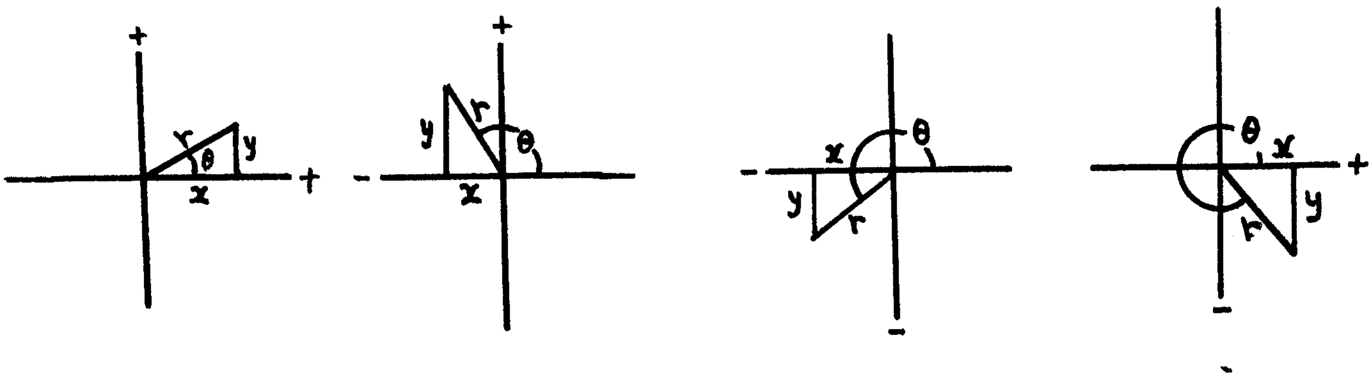


Figure 1

Method II Recall the graphs of sin θ and cos θ:

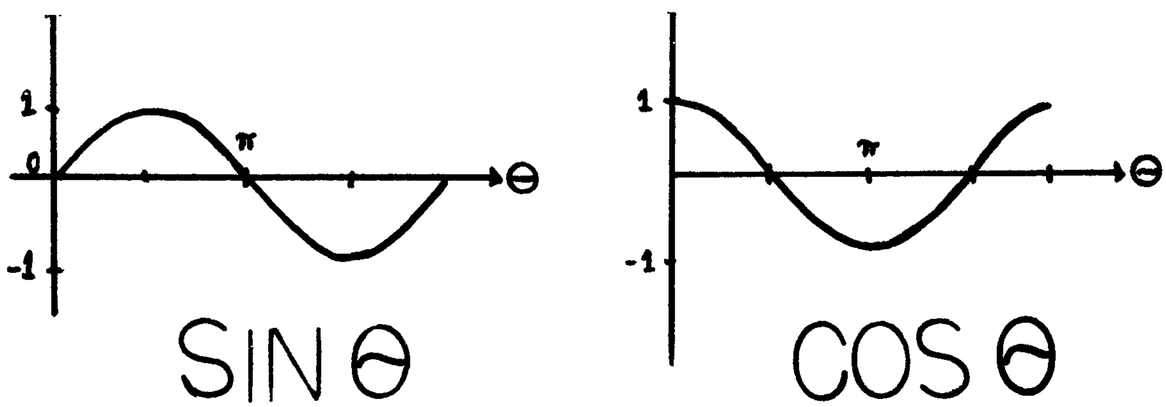


Figure 2

Example

Find sine θ and cos θ where θ = 150° (= 180° - 30°).

Solution I: Comparison of Figures 1(a) and 1(b) together with the definitions of sine and cosine, shows that sin 150° = sin 30°, cos 150° = -cos 30°. Then sin 30° and cos 30° can be looked up in a table or on a slide rule (or, for this example, easily computed).

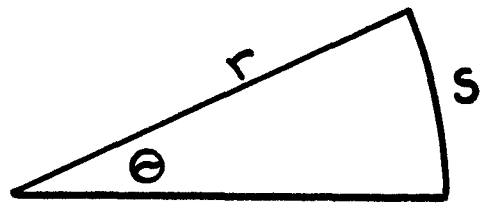
Solution II: Inspection of Figure 2(a) shows that sin 150° = sin 30°; inspection of Figure 2(b) shows that cos 150° = -cos 30°. The sine and cosine of 30° are determined as in Solution I.

RADIAN MEASURE

Many of the problems of planar and rotational motion and waves will depend upon your knowledge of radian measure of angles. Let us therefore take a look at radian measure. The number of radians in an angle at the center of a circle is equal to the (arc subtended by the angle) divided by the radius, or

$$\text{angle in radians} = \frac{\text{arc length}}{\text{radius}},$$

$$\theta = \frac{s}{r}.$$



To travel completely around a circle with a radius of 1.00 m, you will go in an arc of length 6.28 m (2π rad). But once around the circle is equal to 360°.

Derive a formula to convert back and forth between angular measurements in degrees and in radians. Use your formula to convert 45° to radian measure.

USEFUL TRIGONOMETRIC IDENTITIES

In the solution of many problems in physics you may need to use a trigonometric identity. Listed below are some of the most useful ones:

$$\sin^2\theta + \cos^2\theta = 1, \tag{1}$$

$$\sin(A \pm B) = (\sin A)(\cos B) \pm (\sin B)(\cos A), \tag{2}$$

$$\cos(A \pm B) = (\cos A)(\cos B) \mp (\sin A)(\sin B). \tag{3}$$

You can develop the relationships for the sine and cosine of 2A by letting A equal B in Eqs. (2) and (3).

PRACTICE TEST

1. Convert the following angles to radian measures and give their sine, cosine, and tangent values:

   (a) 60°;

   (b) 53°;

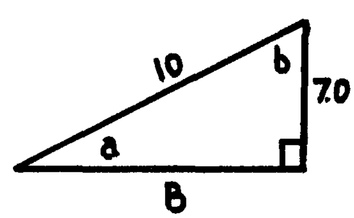
   (c) 37°.

2. Find the unknowns of triangle A:

   B = ______________.

   a = ______________.

   b = ______________.

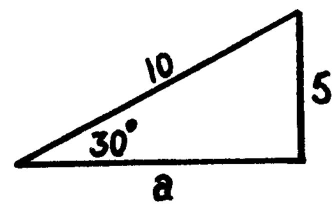


Triangle A

3. One acute angle of a right triangle is 20°. The length of the hypotenuse is 6.0 in. Use trigonometry to calculate the lengths of the two sides.

4. In a 45°-45°-90° right triangle, what is the ratio of the hypotenuse to the sides?

5. State what a in the triangle is equal to without using a trigonometry table or the Pythagorean theorem.



6. A car was traveling exactly northeast. If it went a total distance of 42.4 km, how far north had it actually gone?

Practice Test Answers

1.

	θ (radians)	sin θ	cos θ	tan θ
(a)	$\pi/3$	0.87	0.50	1.70
(b)	$53\pi/180 = 0.925$	0.80	0.60	1.30
(c)	$37\pi/180 = 0.646$	0.60	0.80	0.75

2. B = 7.0. a = 44°. b = 46°.

3. 2.05 in., 5.6 in.

4. Ratio of hypotenuse to each side = $\sqrt{2} : 1$.

5. $a = 5\sqrt{3}$.

6. 30 km.

If you have had difficulties with the Practice Test, please work through this Review Module once more, with the assistance of any general mathematics book including a chapter on trigonometry.

SELF-CHECK TEST ANSWERS

1. (a) $\pi/6$ rad; (b) 135°; (c) $5\pi/4$ rad; (d) 300°.

2. (a) $\sin\theta = 1$, $\theta = \pi/2$ or 90°; $\cos\theta = 1$, $\theta = 0$ or 0°;
   $\tan\theta \to \infty$, $\theta \to \pi/2$ or 90°.
   (b) $\sin 3\pi/4$ maximum, $\cos 30°$ maximum, $\tan 225°$ maximum.

3. (a) The sides are in the ratio of 3 · 4 : 5. (b) 20 cm, 16 cm.

4. 7.7 cm, 9.2 cm.

5. (a) $\sqrt{3}$, (b) 30°, 60°, 90°; one side = (1/2) hypotenuse.

6. 497 m.

7. $x/r = \cos\theta$, $y/r = \sin\theta$, $x^2 + y^2 = r^2$;

or

$$x^2/r^2 + y^2/r^2 = 1.$$

Thus, $\cos^2\theta + \sin^2\theta = 1$.

NATURAL TRIGONOMETRIC FUNCTIONS

Angle					Angle				
Degree	Radian	Sine	Cosine	Tangent	Degree	Radian	Sine	Cosine	Tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.748
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.332
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.43
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.30
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.08
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.64
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.29
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	