7-2012

ECONOMIC IMPACT ON ANNUAL PROFIT OF A RETAIL INVENTORY MANAGEMENT SYSTEM WHEN USING RFID AND CONSIDERING DISCREPANCY RATIO, INACCURACY RATIO, BACKORDERS, AND FEDERAL INCOME TAX

Liyuan Zhang
University of Nebraska-Lincoln, zhangliyuan08@hotmail.com

ECONOMIC IMPACT ON ANNUAL PROFIT OF A RETAIL INVENTORY MANAGEMENT SYSTEM WHEN USING RFID AND CONSIDERING DISCREPANCY RATIO, INACCURACY RATIO, BACKORDERS, AND FEDERAL INCOME TAX

By

Liyuan Zhang

A DISSERTATION

Presented to the Faculty of

The Graduate College at University of Nebraska

In Partial Fulfillment of Requirements

For the Degree of Doctor of Philosophy

Major: Engineering

Under the Supervision of Professor Michael W. Riley

Lincoln, Nebraska

July, 2012

ECONOMIC IMPACT ON ANNUAL PROFIT OF A RETAIL INVENTORY MANAGEMENT SYSTEM WHEN USING RFID AND CONSIDERING DISCREPANCY RATIO, INACCURACY RATIO, BACKORDERS, AND FEDERAL INCOME TAX

Liyuan Zhang, Ph.D.

University of Nebraska, 2012

Adviser: Michael W. Riley

Radio Frequency Identification (RFID) emerged as an important technology with the ability to improve the efficiency and accuracy of object tracking. It has brought cost savings and added value to a wide variety of fields. Retail inventory management system is one of the beneficial fields. In order to better satisfy customers and to improve profit, retailers demand high accuracy in inventory management to service customers without redundant on-hand inventories. The RFID technology, which provides more accurate tracking, can improve retailers' profit.

Two accuracy factors were proposed to determine the difference in the profit of a retailer inventory management system caused by the RFID technology. One was Discrepancy Ratio which stood for the lost inventories during ordering. The other one was Inaccuracy Ratio which represented the difference between physical and recorded inventory. Two more factors, backorders and the Federal Income Tax, were taken into account when building economic models. Four economic models were built based on the classic reorder quantity and reorder point (R, Q) model with annual profit as the objective

function.

This research compared the impacts of the Discrepancy Ratio and Inaccuracy Ratio on annual profit by choosing women's clothes as the specific inventory. In addition, sensitivity analyses were conducted with respect to service level, fixed order cost per order, penalty cost per backorder, and percentage of shortage that became backorder.

Results showed that, every 10% reduction of the two key factors, Discrepancy Ratio and Inaccuracy Ratio, improved annual profit by even more than 20% and 10% respectively. In addition, the changes of Discrepancy Ratio and Inaccuracy Ratio affected each other. Once one factor was reduced, the influence of the other one on profits decreased correspondingly. Moreover, this research determined the joint effects of Discrepancy Ratio and Inaccuracy Ratio and service level, fixed order cost per order, penalty cost per backorder, and percentage of shortage that became backorder. Results suggested that, except for the fixed order cost per order, the desirable values of the Discrepancy Ratio and Inaccuracy Ratio led to desirable values of the other parameters and a boost of annual profits.

Acknowledgment

I would like to express my sincere gratitude to my advisor Dr. Michael Riley. His broad knowledge and constructive ideas have been of great value for me. It is an enjoyable experience for me to work with him, a very encouraging and enthusiastic professor.

I am grateful to all the professors on my committee for their insightful comments and suggestions throughout my research.

I wish to thank Dr. Jeffrey Woldstad and Dr. Erick Jones for their funding to support me with my research all through my graduate program.

I owe my thanks to my family and all my friends for their constant encouragement and support.

Table of Contents

List of Figure

List of Table

Chapter 1 Introduction

Radio Frequency Identification (RFID) is an automated identification technology, which was commercially introduced in the early 1980's. A RFID system contains four components: RFID tags, RFID antennas, RFID readers, and middleware. Figure 1.1 provides the structure of an RFID system. RFID tags have the ability to read, write, transmit, store and upload information. The antenna captures the unique information contained by an RFID tag, which is attached to an object. Then via radio waves the antenna transmits the information to the application system.

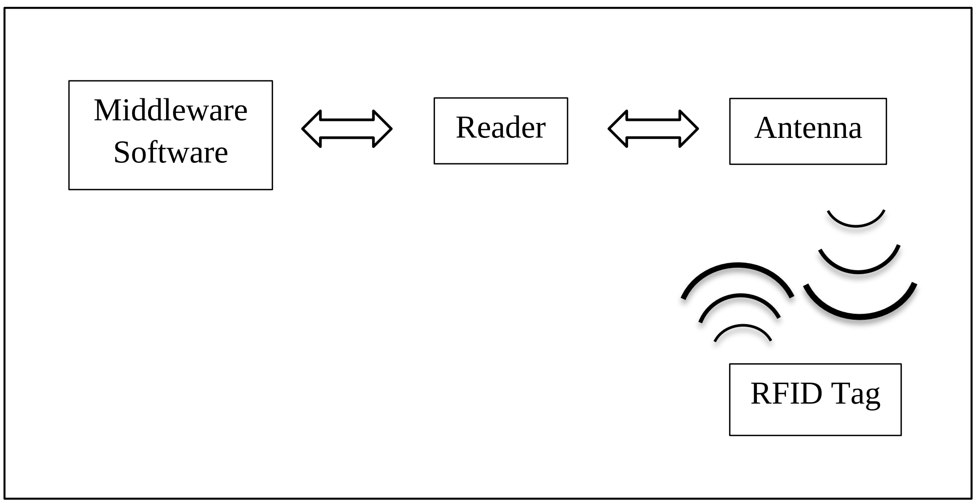


Figure 1.1 Basic structure of an RFID system

By offering an inexpensive and non-labor intensive way to track and identify objectives in real time without contact or line-of-sight, RFID technology has advantages over other data capture technologies such as a bar code system. Its applications has widely spread to various fields, including tracking baggage, identification of passports at port of entry, animal identification, health care and usage at public facilities. The for-profit companies such as Wal-Mart and Target or non-profit organizations such as

Nebraska Department of Road (NDOR) and United States Department of Defense (DOD) have gained benefits from use of RFID technology.

The retail inventory system is one of the fields which allow RFID technology to showcase its advantages. For many companies that operate inventory-carrying facilities, providing high product availability to customers at minimal operation costs is one of the key factors that determined the success of their business [1]. As a result, the inventory systems require a high level of accurate and reliable tracking information. However, there are two important issues in such inventory system, one is the inaccuracies and the other one is the discrepancies. The discrepancy emphasizes the lost inventories which occur on the way to a retailer while the inaccuracy is related to the errors between physical inventory and recorded inventory within a retailer. The existence of these two problems results in significant financial losses every year. The visibility of products provides by RFID technology can greatly improve the inventory system's accuracy and reduce the discrepancy. Even though previous researchers have identified the great improvement of RFID technology in discrepancy reduction and accuracy increase, there is still a lack of study about the joint effects of the two factors (discrepancy and inaccuracy) with respect to the inventory cost reduction and impact on corporate profit. This research project proposes two parameters, Discrepancy Ratio and Inaccuracy Ratio, to include in inventory cost models. The effects of the two parameters on the profit are considered together.

Previous researchers have shown that there are two more elements that are ignored in an RFID-enabled inventory system in the US. One is the impact of Federal Income Tax on inventory and the other one is the impact on backorders. These two factors (taxes and backorder costs) contribute a lot to the inventory costs and are also affected by the visibility of goods due to RFID. As a result, taxes and backorder costs are considered in the economic models in this research.

Continuously improving profit is the ultimate aim for retailers. High profit guarantees the retailers stay competitive. As a result, the profit is a criterion of the success of a retail inventory management system. Proper economic inventory models should be chosen to calculate the profit. Reorder point and reorder quantity model, (R, Q) model, is selected because of its wide application in inventory management and its advantages over other inventory models. Based on the classical (R, Q) model, this research build inventory models with the annual profit as the objective function. The independent variables of the models are: Discrepancy Ratio, Inaccuracy Ratio, taxes, backorder costs, and other uncertain parameters such as service level. The classical annual profit model for a retail inventory management system is:

Annual profit = Annual Sales – Annual Holding Cost – Annual Purchase Cost – Annual Ordering Cost – Annual Penalty Cost – Annual Federal Income Tax

This research chooses women's clothes as the goods for inventory model comparison because of its specific characteristics. These characteristics include: women's clothes have relatively high and uncertain customer demand during the order lead time; it can be sold in the next cycle if there are unsold women's clothes in previous replenishment cycle; women's clothes can be reordered due to the specific brands and/or styles; and its data are available from the U.S. Census Bureau on line.

The main focus of this research is to study the influence of RFID in a retail inventory management system with respect to the annual profit. Four models are built associated with the systems which combined the following situations: with and without RFID and with and without backorder. The rest of the research is organizes as follows: Chapter 2 is the literature review, which summarizes the application of RFID technology in retail inventory management system. Chapter 2 also compares different inventory models and introduced Federal Income Tax. The research objective and the research procedure are established at the end of Chapter 2. Chapter 3 proposes the methodology by dividing the retail inventory management systems into four situations and building their corresponding models on the basis of classical (R, Q) model. Assumptions, notations, as well as a step by step procedure of formula deduction for each model are explained in this chapter. Chapter 4 focuses on determining the optimal solutions of both the uncertain variables and the objective function of each model. Results are presented in Chapter 5. First relevant data for the parameters of the models are collected. Next, each pair of models is compared to define the difference of annual profits between systems with and without RFID. Finally, sensitivity analyses are completed to determine the influence of inaccuracy, discrepancy, and other uncertain parameters such as backorder and service level on annual profit. Chapter 6 is the conclusion and, summarizes the research with respect to the research objectives proposed in Chapter 2. At last, all the references are listed.

Chapter 2 Background

Chapter 2 gives the literature review of basic RFID technology as well as its application in the retail inventory management system. Then the need for research was described, which led to the research objectives. Finally, a specific research procedure was provided, and followed as the research was conducted.

2.1 Literature Review

2.1.1 RFID basics

Radio Frequency Identification (RFID) technologies, which originated from radar theories discovered by the allied forces during World War II, have been commercially available ever since the early 1980's [2]. Over the last three decades, RFID have been used for a wide variety of applications such as transportation freight tracking, retail theft prevention, expensive asset tracking and locating, automotive manufacturing, postal services, and counterfeit pharmaceutical [3, 4] to improve the efficiency of object tracking and management. Because of the recent declines in RFID cost and increases in read range and accuracy for improved design and associated signal processing, the technology is of growing interest to commerce, industry and academia [5]. Nowadays, RFID application has opened new paths to different fields, calling for new advanced techniques [6].

With the purpose of identifying or tracking objects such as products, animals, or people, RIFD technology allows data communication via electromagnetic waves from an electronic tag through a reader. Wireless data can be transferred between a stationary location and a movable object or between movable objects from a distance [7, 8]. A basic

RFID system is comprised of four functional components: tags, readers, antennas, and middleware software. The following is a list of brief introductions of each component of a basic RFID system.

1) An RFID tag is the heart of an RFID system because it stores the information of the tracked objective. Basically a tag contains an integrated electronic circuit (the chip) and a capacitor, which captures and uses energy from the antenna in order to send a signal back. Some tags include batteries, expandable memory, and sensors [9]. According to the capability, tags are divided into the two types: passive tags, which have no battery and needs to be activated by the power emitted by the reader; and active tags, which are equipped with its own battery inside and complete source of power to support their circuitry. RFID tags are made in many sizes. Passive tags are small and flat since they do not have battery inside. The smallest tag can be 0.05 x 0.05 millimeters. Figure 2.1 gives two samples of passive tags. On the contrary, active tags are bigger and are made in more shapes. Figure 2.2 shows two active tags.

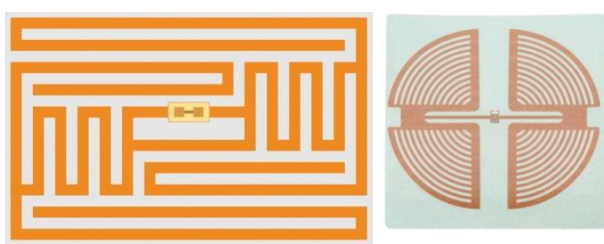


Figure 2.1 Passive RFID tags

Figure 2.2 Active RFID tags

2) An RFID antenna is a conductive part that exchanges data between tags and readers. By transmitting the energy broadcasted by the reader's signal, an antenna forms electromagnetic range to either activate the passive tags which enter in this response range or communicates with the signal coming from the active tags. Then it receives emitted information coming from tags' antenna and transmits them back to readers. Some RFID antennas are separated from their corresponding RFID readers and others are attached to the readers as an integrative component.
3) RFID middleware is system software that collects and organizes data from the readers, transfers them into meaningful information, and finally passes them to the application services [10, 11].

The advantages of RFID over other data capturing technologies such as a bar coding system are described here. First, the visibility of products provided by a RFID system improves the accuracy on the inventory level by decreasing errors between physical inventory and recorded inventory. Second, RFID eliminates human interactions in the process. This elimination leads to the reductions of labor costs and inventory inaccuracies, and simplification of the business. Third, RFID provides higher security due to the extremely unique information contained within each tag, which is almost impossible to be duplicated [12]. Fourth, RFID has higher durability compared with bar codes, whose paper or hard metal container exposes them to harsh environments and makes them vulnerable [6]. Fifth, RFID readers do not need a direct line of sight to read a tag while a bar code reader would require direct access to the bar code to capture the information.

With these advantages, the interoperable RFID has brought cost savings and added values to various fields, one of which is the retail supply chain. The next step in this research focuses on RFID application in retail inventory management system.

2.1.2 RFID application in retail inventory management system

In order to stay competitive in the market, retailers have to increase profit and improve customer satisfaction. This requires product availability to customers at a relative low costs. A high level of inventory management system is needed, which enables retailers to offer the right amount of product to the right customer at the right time without redundant on-hand inventories. As a result, many retailers rely on the performance of an inventory management system.

The standard literature on retail inventory system identify that there are many problems which significantly decreased the efficiency of inventory management. Low labor efficiency is one of the biggest issues [13]. Except for the labor efficiency, most of these issues are attributed to the product invisibility, which lead to low accuracy of inventory record. RFID technology, which provides high visibility, has been shown to be of great value in improving the efficiency of retail inventory management system.

2.1.2.1 Literatures about the RFID application in retail inventory system

Some literature about visibility is summarized below.

Gavin Chappell (2003) in Accenture® studied the effect of RFID on goods demand planning and he pointed out that the big problems in demand planning process were un-saleables, invoice accuracy, and stock out [14]. Gavin Chappell also determined in another paper (2003) that RFID had great benefits in distribution and transportation from a perspective of a retail supply chain. One of the major findings of the paper was that case-level tagging would bring the greatest number of benefits in the retail supply chain. Also he listed the issues that retailers faced in distribution and transportation, which included order fill rate, shrinkage, inventory velocity, and safety-stock inventory [15].

Keith Alexander (2002) in IBM conducted research on the impact of RFID on reducing the losses associated with product obsolescence and shrinkage [16]. The research results displayed that in a retail system, RFID could eliminate external theft, internal theft, supplier fraud as well as paper shrink.

Yun Kang (2004) summarized the causes of inventory inaccuracy, which were stock loss, transaction errors, inaccessible inventory, and incorrect product identification. He used a deterministic model to show the impact of having inventory inaccuracy on stock outs [2].

Aykut Atali (2006) pointed out that RFID technology could improve the visibility by allowing inventory replenishment to be more precise and reduce discrepancy. He built four models under different inventory inaccuracies to illustrate that RFID could reduce the total cost.

Bensoussan (2005) built an inventory cost model based on the (s, S) (s referred to the reorder point and S referred to the maximum inventory level) model to explain the imperfect information and Dehoratius (2005) also contributed to the research of inventory control under inaccurate information [17, 18]. Also other researchers Fleisch (2005) and Gaukler (2004) worked on the inventory inaccuracy within a supply chain [19, 20].

Kok (2006) conducted a break-even analysis of an RFID tag with respect to the sensitivity of inventory shrinkage [21]. The results show that the break-even prices were highly related to the value of lost inventories.

There are also many research results which provids historical data to show the issues of retail inventory management system and the improvement of the system by RFID. ECR Europe in 2001 investigated 200 companies with the dominant share of the consumer goods industry in Europe and reported that stock lost amounts to 1.75% of sales annually for the retailers, 59% of which (or, 1% of total sales) was unknown to the retailers. This means that the stores did not know where or how the products were lost [22]. DeHoratius (2004) showed that 65% of the inventory records were inaccurate at a retailer [23]. Gavin Chappell (2003) illustrated that RFID could enable increased certainty of the demand signals throughout the supply chain and improve demand planning forecast accuracy by 10-20%. The improved accuracy resulted in 10-30% decreased inventory level and 1-2% increased sales.

2.1.2.2 Summary of RFID application in retail inventory system

The previous literature review has shown that invisibility of inventory is one of the biggest issues when managing a retail inventory system. Invisibility lead to inaccuracy, thus it can be expressed by inaccuracy. This study divides the inaccuracy into two types according to the process it occurs. One is discrepancy, which records the ratio of ordering quantity that disappears during the ordering process; the other one is inaccuracy, which denotes differences between physical inventory and recorded inventory during the storage process. These two factors are the main variables that reflect the impact of implementation of RFID on retail inventory management system and are included in this research's models. Table 2.1 summarizes the literature review of the two factors, including the causes and the possible improvements by RFID. The values in this table showed that by implementing of RFID in an inventory management system, inventory inaccuracy cab be improved by 40% at most and inventory discrepancy can be reduced by almost 70%.

Table 2.1 Summary of discrepancy and inaccuracy causes

	Cause	**Possible improvements caused by implementation of RFID**
Inaccuracy	Stock loss Transaction errors Inaccessible inventory Incorrect product identification	20% - 40% [24] 10% - 30% [14, 25] 5% - 25% [26] 8% - 12% [27] 5% [28] 1% - 2% [29]
Discrepancy	Shrinkage Misplacement of inventory Transaction errors	10% - 66% [30] 40% - 50% [27] 47% [16] 11% - 18% [31] Average 16% [32] 12.3% [33] 11% [34]

One thing should be noticed that even though RFID technology has the ability to reduce inaccuracy and discrepancy, it is not the only thing to reduce them. This research aims at finding out the benefits that RFID bring to a retail inventory management system. As a result, inaccuracy and discrepancy are supposed to be reduced by RFID implementation.

2.1.3 Inventory model with respect to annual profit

For a retailer, the objective of inventory management is to have the appropriate amounts of inventories in the right place, at the right time, and at low cost. Inventory cost consists of holding cost, purchase cost, order cost, and penalty cost. The profit includes inventory sales, less the inventory cost and the Federal Income Tax for inventory.

In order to analyze the inventory profit, an appropriate inventory model should be chosen. There are a wide range of inventory models that can be applied in the retail inventory management system. Each of the different models has its own advantages and disadvantages as well as different areas of application. When managing inventory systems, questions should be regularly asked such as how often should an order be placed and how many items should be replenished? With these questions, the features of model must be considered when selecting inventory models, which are: order quantity, reorder point, review period, demand rate, and lead time [35]. Four models, that have been widely implemented and are applicable for a retail inventory management system, are briefly introduced in the following sub-section.

2.1.3.1 Economic Order Quantity (EOQ) model

The EOQ model is one of the oldest classical and the most fundamental model to minimize the inventory cost by determining the order quantity. Customer demand in an EOQ model is constant over time and a new order arrives immediately when the inventory reaches zero. Thus there is no lead time for delivery and no penalty cost for shortage.

2.1.3.2 News Vendor (NV) model

With the purpose of maximizing the inventory profit, NV model determines the optimal stocking quantity. Customer demand in this model is uncertain. NV model is for single or discrete order period. Also inventories cannot be carried across periods. Within each period, the customer demands that are above the stocking quantity are lost. Therefore, the lost sales due to product shortage exist. Back orders are assumed not to exist for these types of products (no one wants yesterday's newspaper.).

2.1.3.3 (s, S) model

In a (s, S) model, s represents the critical inventory level and S refers to the maximum quantity of each ordering. This model detects inventory level at a periodic period. At the detection point, if the inventory on hand is less than s, an order is placed, replenishing inventories to the level of S. Customer demand is also random and stock outs exist. The stock outs can be considered as back orders or lost sales or a combination of both.

2.1.3.4 (R, Q) model

The (R, Q) model is a continuous checking inventory model where R means the reorder point and Q means the reorder quantity. Once the inventory level is below R, an order of Q is triggered. Customer demand is random.

Table 2.2 summarizes the features for these five models, from which the advantages and disadvantages of each model as well as their possible applications can be compared.

From the table, it is easy to see that even though EOQ model is the most basic model for an inventory management system, it is not applicable because too many assumptions are made to simplify the real word problem, such as that the demand is a constant and the reorder point is zero. For NV model, since it only simulates single period of inventory management, it is not proper for the proposed analysis models, which have multiple and continuous order periods. The last two models have a lot of common points except that they are suitable for different kinds of inventory. This research assumes that the inventories in the model are categorized as inventory A or B, which call for the application of (R, Q) model. Therefore, the (R, Q) model is selected and applied in the following research.

Table 2.2 Features of four inventory models

Model Feature	Inventory Model			
	EOQ	**NV**	**(s, S)**	**(R, Q)**
Order Quantity	Variable	No order	Variable	Variable
Reorder Point	Zero	No reorder	Variable	Variable
Review Period	Continuous	Only for single or discrete order period	Periodic	Continuous
Demand Rate	Deterministic	Random	Random	Random
Lead Time	No lead time	No lead time	Fixed lead time	Fixed lead time
Application area	Widely used for simple inventory system	Perishable product	B and C category inventory [a]	A and B category inventory [a]

Note a: Inventories are divided into A, B, C categories. Categories A and B refer to important or perishable inventories and category C presents less important or durable inventories.

In a classical (R, Q) model, replenishment orders are triggered at the reorder point R, and the reorder quantity Q is constant over time. It is assumed that inventory management system checks the inventory continuously, and launches a replenishment order of size Q instantly once the inventory has reached the reorder point R. The reorder quantity Q arrives after a lead time T. If the customer demand during the lead time is larger than the reorder point R, there are shortages. Figure 2.2 shows the dynamic process of (R, Q) model.

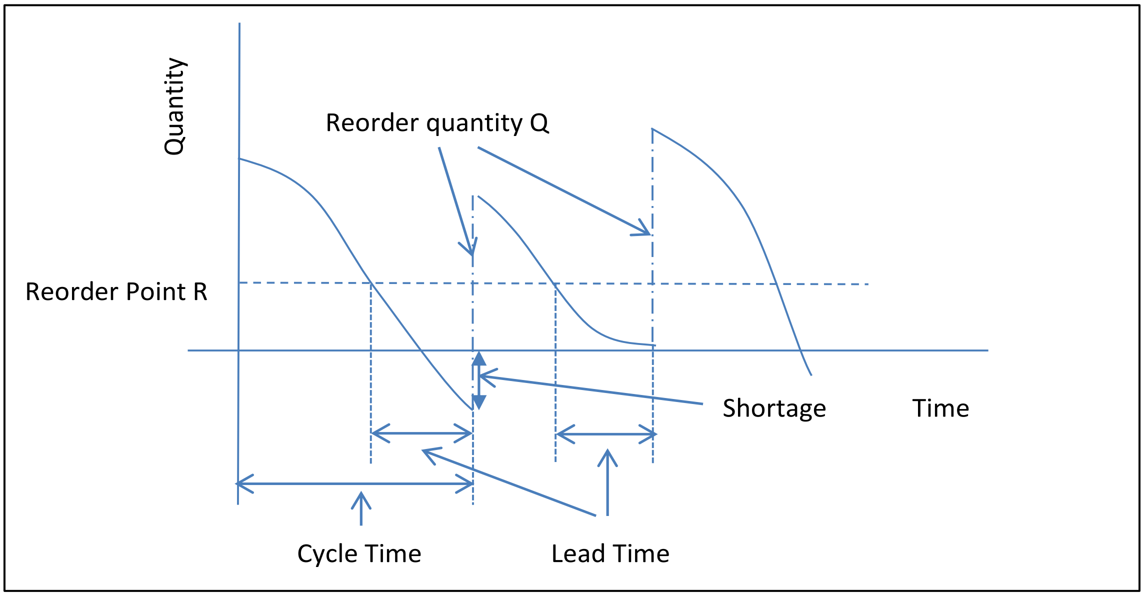


Figure 2.3 Dynamic process of (R, Q) model

2.1.4 Federal Income Tax for inventory

For a retailer, merchandise inventory should be managed well. Merchandise generally refers to the inventories that the retailer has purchased that are available for sale to customers. The retailer should balance current and future customer demand with its ability to purchase goods for resale. Merchandise tax is an important factor in this process due to its high tax burden on the retailer with large and valuable inventories.

There are two types of tax, Property Tax and Federal Income Tax. The Property Tax is similar to the tax that is charged on individual taxpayers based on the value of their real estate and personal property. The Property Tax is expressed by a percentage of a business's inventory value. This tax only applies to specific types of merchandise such as automobiles, boats, recreational vehicles, and unoccupied real estate, none of which have strong relationship with retail inventory management system. As a result, Property Tax is not taken into account in this research.

The Federal Income Tax requires retailer to claim changes in the values of their inventories when calculating business income taxes [36, 37]. This inventory tax is typically based on the total sales of the inventoried item in the prior year. The retailer needs to account for its gains by purchasing and selling merchandise [38]. To determine the Federal Income Tax, the value of inventory must be established, identified and then evaluated.

2.1.4.1 Establish the inventory value

When inventory is used to support the sale of goods, it is important to determine what kind of goods the inventory contains and how the inventory value is established. From the perspective of the Internal Revenue Service (IRS), the types of merchandise that should be counted in a company's inventory included raw materials, work-in-process, finished goods and purchased goods. For a retailer, the inventory refers to the purchase goods.

2.1.4.2 Identify the inventory value

After establishing the quantity of inventory on hand, the next step is to identify its cost which can be accepted by IRS. There are two ways that IRS prefers to identify the inventory value if the quantity of inventory was large. One is First-In-First-Out (FIFO) and the other one is Last-In-First-Out (LIFO). Under the FIFO method, it is assumed that the first purchased goods are sold first. This tend to result in the situation that the last purchased ones, values at the most recent purchase prices, stay until the end of the year. On the contrary, under the LIFO method, the assumption is that the last purchased good is the first one sold. In this way, the inventory at the end of the year consists of the early purchased ones, values at the earliest prices. The difference of the two methods depends on the change of prices. Since in this research the price of inventory is constant, either FIFO or LIFO is applicable.

2.1.4.3 Evaluate the inventory value

There are three inventory valuation methods which are generally used for a retailer's income tax purposes: the cost method, the lower of cost or market, and the retail method.

Under the cost method, the cost of purchased merchandise contains any transportation, shipping, or other costs incurred in obtaining the merchandise as well as the discounts. This method is not acceptable because the costs are charged directly to the cost of merchandise during the current period rather than being allocated to ending inventory.

Under the lower of cost or market method, the market value of each item on hand is compared with the cost of each item. Then the lower of the two is determined as the inventory value of goods. This method does not apply for tax purpose if the purchased merchandise, which are to be sold to customers, are under a firm sales contract or the inventories are identified under the LIFO method.

Under the retail method, the approximate cost of inventory is determined by reducing the average margin, which is expressed as a percentage of the sale price from the total retail sale price of each item. This method is a quick and easy way to evaluate inventory values. In addition, it is also the most commonly applied inventory evaluation method to retailers, whose inventory contains only purchased goods in stock.

After considering the identification and the applicable area for each method, the retail method is selected in this research to determine the value of inventory, which is further used to calculate the Federal Income Tax [39-40].

2.2 Research Questions

2.2.1 Needs for research

When considering the impact of RFID in retail inventory management system, people always have two questions of interest. First, what aspects does RFID affect in the retail inventory management system? Second, how do these effects impact the retailer's profits?

For the first question, previous literatures show that the visibility of RFID is its significant feature that affected the performance of a retail inventory management system. RFID enables the system to reduce the discrepancy between the ordered quantity and actually arrived inventories, and the inaccuracy between physical inventory and recorded inventory. These two factors, discrepancy and inaccuracy, are the best reflection of the improvement created by RFID's visibility. Therefore, these two factors are used as the measures of merit when studying the effect of RFID in retail. In addition, literature review displays how much RFID technology improves the reduction of discrepancy and inaccuracy. Specifically, a discrepancy ratio and an inaccuracy ratio are defined and used to quantify the impact of using RFID on retail inventory measurement.

For the second question, it is well known that for any retailer the ultimate target is profit, which guarantees the retail's competitive status in the market. With this aim, previous case studies are conducted to test the influence of discrepancy and inaccuracy on annual profit by different economic assessment methods, such as break even analysis or stochastic process analysis.

However, the question still persists: how does RFID affect the retail inventory management system according to annual profit if both discrepancy ratio and inaccuracy ratio are taken into account? Thus, in-depth and comprehensive profit models are required. These models are built on the basis of classical (R, Q) model, which imitates the retail inventory process under uncertain customer demand. Additionally, the research models contain two more elements that assist them in revealing the real world problem better than previous models. One is Federal Income Tax for inventory, which is also a significant factor that contributes to the reduction of annual profit but is ignored in previous research. Two is the consideration of mixture of backorders and lost sales.

2.2.2 Research objective

This research project aims at finding out, from a comprehensive perspective, how RFID's visibility affects the retail inventory management system with respect to annual profit. The overall objective of this research is to determine the impacts of two important factors, Discrepancy Ratio and Inaccuracy Ratio, on the annual profit of a retail inventory management system. The main goal of this research is reached by meeting the following specific objective questions.

1) What is the change in annual profit of a retail operation as a function of the changes in the Discrepancy Ratio and Inaccuracy Ratio, based on classical (R, Q) model?
2) What are the optimal solutions of the inventory (R, Q) models, including the

optimal values of R, Q, and optimal solution of objective function?

3) How do Discrepancy Ratio and Inaccuracy Ratio influence the optimal values of R and Q and how did they affect the difference between annual profits according to different models?
4) How do the ratio of backorders over stock out, service level, and penalty cost influence the difference between the profit models with and without RFID?

2.3 Research Procedure

The research project builds four models to compare the effect of RFID on an inventory management system with and without backorders. The objective function of each model is the annual profit. In each scenario, two models are compared to find out whether the how the two factors of RFID system, discrepancy ratio and inaccuracy ratio, affect annual profits. Following is the detailed procedure of this research.

1) On the basis of classical (R, Q) model, build four inventory models as a function of two uncertain variables: Discrepancy Ratio and Inaccuracy Ratio;
2) Solve each model function for the optimal solution of the two independent variables: R and Q, under a given customer service level;
3) Find the optimal solution of annual profit with the optimal values of R and Q gained by step 2 for each model;
4) Collect data to represent the customer demands and analyze these data;
5) In each scenario, compare the annual profits for the two corresponding models based on the data;

6) Conduct sensitivity analysis with respect to Discrepancy Ratio and Inaccuracy Ratio for each pair of models with other parameters as constant, and
7) Conduct sensitivity analysis with more changeable parameters such as service level and penalty cost.

Chapter 3 Methodology

In Chapter 3, some background materials were displayed for modeling, including the comparison of a retail inventory management system with and without the RFID system. In addition, the backorder problem was discussed. Based on the background materials, four different symbolic mathematically models were built with the annual profit being the dependent variable. Detailed descriptions were given for each model. In addition, the optimal values of reorder point, R, and reorder quantity, Q, for each model were determined with changing values of the two uncertain variables: Discrepancy Ratio and Inaccuracy Ratio.

3.1 Modeling Preparation

In this section, the rationale for model developments was discussed. Understanding of each model's components was a key element in comparing models.

3.1.1 Comparison of a retail inventory management system with and without RFID

This sub-section summarized the differences between a retail inventory management system with and without RFID with annual profit as the decision variable. The classical annual profit model for a retail enterprise with an inventory management system is shown as Equation 1, which contains six components with all variables in dollars.

Annual profit = Annual Sales – Annual Holding Cost – Annual Purchase Cost – Annual Ordering Cost – Annual Penalty Cost – Annual Federal Income Tax (1)

Previous research had shown that RFID enabled retail goods to have a higher visibility in their inventory management system [30]. The improved visibility allowed an inventory management system to reduce the amount of inventory discrepancy and to increase inventory accuracy [41, 42]. In this research, visibility and accuracy of inventory were the two main factors which reflected the impact of RFID on a retail inventory management system and were taken included in the classical inventory model. Visibility was addressed by using the Discrepancy Ratio. The Discrepancy Ratio was the fraction of order quantity that had disappeared. The inventory accuracy was monitored by the Inaccuracy Ratio, which was the percentage of mistakenly recorded inventory over the recorded inventory. This research considered these two factors in comparison of retail inventory management systems. Table 3.1 displays the effects of the two factors on each component of the annual profit. In this table, Y, which stands for Yes, means that this main factor affects this variable of profit directly while N, which stands for No, has the opposite meaning. It has to be noticed that N does not mean that RFID has no effect on this variable; it just shows that this main factor that RFID system brought does not directly influence the annual profit variable.

Besides these inventory visibility and accuracy, there were other factors that gave rise to the differences of annual profit between RFID-enabled and non-RFID system. For example, the purchasing and operating of the RFID system itself had costs that impacted the annual profit of a retailer. In addition, RFID systems could critically improve the efficiency of inventory management by auto reading the in and out inventories without manpower; in this way, RFID reduced the labor cost [41]. Also, RFID improved the inventory accuracy (measured by the Discrepancy Ratio) between order quantity and actually receiving quantity. The reduction in discrepancy led to the reduction of order quantities directly and consequently helped reduce the lead time [43]. In this research, the

RFID costs and cost reductions were included in the analysis. Assumptions were made to explain the additional costs that were added to annual profit model.

Table 3.1 Effect of Discrepancy Ratio and Inaccuracy Ratio on component of annual profit

Component of Annual Profit	Detailed Variable	Main Factors between RFID-enabled and non-RFID system	
		Discrepancy Ratio	Inaccuracy Ratio
Annual Sales	Sale inventory amount	Y	Y
	Sale price	N	N
Annual Holding Cost	Misplacement/Inaccessibility	Y	Y
	Annual inventory amount	Y	N
	Asset visibility	Y	Y
	Labor cost	N	N
	Inventory write-off	Y	Y
	Obsolescence cost	Y	Y
	Inventory accuracy	N	Y
Annual Purchase Cost	Forecasting error	N	Y
	Unit price	N	N
Annual Ordering Cost	Lead time	N	N
	Annual order fill rate	Y	N
	Un-saleable inventory	Y	Y
	Receiving and check-in time	Y	N
	Near-real time in-transit visibility	Y	N
	Putaway and replenishment rate	Y	Y
	Shipping rate	Y	Y
	Transaction error	Y	N
Annual Penalty Cost	Out-of-stock amount	Y	Y
	Punishment cost	Y	Y
	Shrinkage	Y	N
Annual Federal Income Tax	Beginning inventory amount	Y	Y
	Purchase amount	Y	Y
	Cost-to-retail percentage	N	N

The variables in each annual profit model that were directly influenced by changes in inventory visibility and accuracy were discussed in the following sections.

3.1.2 Backorder discussion

In an inventory management problem, shortage cost had a significant impact on the annual profit. Shortage cost was the cost of not having an item when there was a demand for that item. The shortage cost came from how the retailer might respond to the demand if it was known. The shortage was classified as lost sales or backorders. A lost sales cost could be more than just the loss of the price of an item, there might be a loss of consumer good will which could impact potential future sales. In modeling, one could consider all the shortage was lost forever or all the shortage was backordered [44]. The mixture of the backorders and lost sales made the inventory problem very complex. In order to model the problem more clearly, the models were built in a step by step way. For each system, non-RFID and RFID-enabled, first a model with all shortage lost was built. Based on this model, a more complex model, which included both the backorder and lost sales, was built. As a result, there were four models in total. The four models were divided into two scenarios according to the component of the shortage. The situation with all shortages were lost sales was named scenario 1, which contained model 1 and model 2. The situation with backorders was named scenario 2, which included model 3 and model 4. Table 3.2 shows the constructions of the four models.

Table 3.2 Construction of four models

		Visibility	
		Non-RFID	**RFID-enabled**
Shortage	**All lost sales without backorder – scenario 1**	Model 1	Model 2
	Mixture of lost sales and backorder – scenario 2	Model 3	Model 4

3.2 No backorder without RFID and no backorder with RFID

Model 1(no backorder without RFID) defined the situation that all shortages resulted in all lost sales and no backorder if a demand existed for an item that was not in inventory or could not be found in inventory. Model 2 was no backorder with RFID system.

3.2.1 Model 1: No backorder without RFID

Model 1 was the basic model for the other three models. It simulated the inventory management system without RFID and there was no backorder. This sub-section gives the detailed descriptions of this model, including assumptions, notations, and a step by step modeling process with equations.

3.2.1.1 No backorder without RFID model assumption

Assumptions were made before conducting the model with the purpose of making the model simple and easy to be formulated. Based on these assumptions, the annual profit model equation was build.

1) Customer demand during lead time follows a continuous distribution;
2) Inventory on hand is checked constantly;
3) Sale price, annual holding cost, purchase cost, ordering cost and shortage cost for each inventory are fixed values;
4) Discrepancy Ratio is defined as the fraction of ordering quantity that has disappeared, as a result it has the value between 0 and 1;
5) Inaccuracy Ratio is defined as the percentage of mistaken recorded inventory over the recorded inventory. According to empirical record[1-4], its value is set to between -1 and 1;

6) Records of the new arrived inventories at the beginning of each cycle time are correct;
7) Labor cost is ignored;
8) Replenishment lead time is a constant;
9) Inventories that were not sold in previous cycle are added into the following period without product deterioration;
10) Discount according to the existence of RFID system is disregarded;
11) Penalty cost is proportional to the quantity of shortage;
12) Shortages are all lost sales;
13) The retail inventory method is used to calculate the Federal Income Tax, and
14) Type one service level is used, which describes the probability of no stockout within an order.

3.2.1.2 No backorder without RFID model notation

The following notations were used to represent the variables and parameters in the model formula.

i: the tab of models, i = 1, 2, 3, and 4;

π_i : expected annual profit in dollar of model i;

π_i^* : the optimal value in dollar of π_i ;

α : Discrepancy ratio;

θ : Inaccuracy ratio;

R_i: recorded reorder point;

R_i^*: the optimal value of recorded reorder point R_i;

Q_i: reorder quantity;

Q_i^*: the optimal value of reorder quantity Q_i;

x: random variable representing demand during lead time;

f(x): Probability Density Function (PDF) characterizing x;

F(x): Cumulative Distribution Function (CDF) characterizing x;

D: the expected value of x;

λ : the expected annual customer demand;

SL: service level;

SL_0: target service level;

L: lead time in year;

T: cycle time in year;

N: the expected order times per year;

r: selling price per unit in dollar;

h: annual holding cost per unit in dollar;

c: purchase cost per unit in dollar;

K: fixed order cost per order in dollar;

p: penalty cost per shortage in dollar;

i: tax ratio;

S: annual sales in dollar;

E(S): the expected value of S in dollar;

H: annual holding cost in dollar;

E(H): the expected value of H in dollar;

C: annual purchase cost in dollar;

E(C): the expected value of C in dollar;

O: annual ordering cost in dollar;

E(O): the expected value of O in dollar;

P: annual penalty cost in dollar;

E(P): the expected value of P in dollar;

I: annual Federal Income Tax for inventory in dollar, and

E(I): the expected value of I in dollar.

3.2.1.3 No backorder without RFID model formula

Expressions for each element of annual profit in Equation 1 are expanded here, with detailed procedures. Before expanding each element, here are some common equalities to aid one's understanding.

1) The quantity of arrived inventory by the end of lead time is not Q but (1-α)Q;
2) The actual quantity of inventory on hand at the reorder point is not R but (1+θ)R;
3) The expected annual customer demand $\lambda = \frac{D}{L}$;
4) The expected order times per year N

   = the expected annual demand / actual reorder quantity per time

   $= \frac{D}{L} / ((1-\alpha)Q) = \frac{D}{(1-\alpha)QL}$;
5) Service level = Probability of demand during lead time being less than or equal to the quantity of inventory on hand at the reorder point = P (x $\leq$ R) = F (R) according to the definition of cumulative distribution function.

6) Random variable x and its expected value D are independent. As a result, the expected value of the product of a function of x and a function of D equal to the product of the expected value of the function of x and the expected value of the function of D.

Now, the six elements of annual profit are explained.

3.2.1.3.1 Annual Sales

Annual sales equal the sales during one cycle time multiplied by the expected order times per year. During one cycle time T, the sold items are the items sold before reordering plus the minimum of items on hand at the reorder point and customer demand during the lead time. As a result, the annual sales are explained with Equation 2.

$$S = r * [(1-\alpha)Q - (1+\theta)R + \min(x, (1+\theta)R)] * N \quad (2)$$

In Equation 2, the factor $\min(x, (1+\theta)R) = (1+\theta)R - \max(0, (1+\theta)R - x)$. Also, x is the only random variable with its corresponding PDF f(x) and CDF F(x). As a result, the expected value of this factor is calculated by Equation 3.

$$E(\min(x, (1+\theta)R)) = E((1+\theta)R - \max(0, (1+\theta)R - x))$$

$$= (1+\theta)R - \int_0^{(1+\theta)R} ((1+\theta)R - x) f(x)dx$$

$$= (1+\theta)R - \int_0^{(1+\theta)R} (1+\theta)Rf(x)dx + \int_0^{(1+\theta)R} xf(x)dx$$

$$= (1+\theta)R - (1+\theta)R\int_0^{(1+\theta)R} f(x)dx + \int_0^{(1+\theta)R} xf(x)dx$$

$$= (1+\theta)R - (1+\theta)R * F((1+\theta)R) + \int_0^{(1+\theta)R} xf(x)dx \quad (3)$$

Substituting the factor $\min(x, (1+\theta)R)$ and N leads to the expected annual sales as shown by Equation 4.

$$E(S) = r * [(1-\alpha)Q - (1+\theta)R + (1+\theta)R - (1+\theta)R*F((1+\theta)R) + \int_0^{(1+\theta)R} xf(x)dx] * \frac{D}{(1-\alpha)QL}$$

$$= r * [(1-\alpha)Q - (1+\theta)R*F((1+\theta)R) + \int_0^{(1+\theta)R} xf(x)dx] * \frac{D}{(1-\alpha)QL} \quad (4)$$

3.2.1.3.2 Annual Holding Cost

Annual holding cost is the product of the annual holding cost per unit, h, and the average amount of inventory on hand, which is $\frac{(1-\alpha)Q}{2}$. Since there is no random variable, the expected value equals itself, as shown by Equation 5,

$$E(H) = h * \frac{(1-\alpha)Q}{2} \quad (5)$$

3.2.1.3.3 Annual Purchase Cost

Expected annual purchase cost, shown by Equation 6, equaled the purchase cost per unit, c, multiplied by the expected annual customer demand, $\frac{D}{L}$.

$$E(C) = c * \frac{D}{L} \quad (6)$$

3.2.1.3.4 Annual Ordering Cost

Expected annual ordering cost, as shown in Equation 7, was the product of fixed order cost per order, K, and the expected order times per year, $\frac{D}{(1-\alpha)QL}$.

$$E(O) = K * \frac{D}{(1-\alpha)QL} \quad (7)$$

3.2.1.3.5 Annual Penalty Cost

Annual penalty cost equals the penalty cost per shortage, times the shortage during the lead time, and times the order times per year, N. During one lead time, L, the shortage items are the maximum of items on hand at the reorder point and customer demand during the lead time minus the on hand inventory. As a result, the annual penalty cost is explained with Equation 8.

P = p * [max ((1+θ)R, x) - (1+θ)R] * N (8)

The shortage during the lead time is max ((1+θ)R, x) - (1+θ)R = max (0, x - (1+ θ)R) has x as the only random variable. This factor represents the shortage amount for one cycle time. The factor is defined with M by Equation (9).

M = max (0, x - (1+θ) R) (9)

The expected value of the shortage is written as:

E [max ((1+θ)R, x) - (1+θ)R] = E [max (0, x - (1+θ)R)]

$$= \int_{(1+\theta)R}^{\infty} (x-(1+\theta)R) f(x)dx = \int_{(1+\theta)R}^{\infty} xf(x)dx - \int_{(1+\theta)R}^{\infty} (1+\theta)Rf(x)dx$$

$$= \int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R \int_{(1+\theta)R}^{\infty} f(x)dx$$

$$= \int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R*[1 - \int_{0}^{(1+\theta)R} f(x)dx]$$

$$= \int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R*[1 - F((1+\theta)R)]$$

$$= \int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R) \quad (10)$$

Then, the expected annual penalty cost is then written by Equation 11.

$$E(P) = p * [\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R)] * \frac{D}{(1-\alpha)QL} \quad (11)$$

3.2.1.3.6 Annual Federal Income Tax for Inventory

According to the Retail Inventory Method, during each cycle time, the federal income tax was the product of the tax ratio, i, and the ending inventory cost of this cycle time. The ending inventory cost equaled the beginning inventory cost plus the purchases minus the product of sales and the cost-to-sale percentage, where the cost-to-sale percentage was the ratio of purchase cost over selling price, which was c/r [41].

At the beginning of a cycle time, the inventory on hand is the inventory left from the previous cycle time, which is the maximum of inventory on hand minus the customer demand and zero during last lead time. The inventory on hand can be expressed by max $((1+\theta)R - x, 0)$. The purchases that are available for sale in one cycle time is $(1-\alpha)Q$. The sales for one cycle time had already been calculated in annual sales.

The annual federal income tax, as shown by Equation 12, equals the tax in one cycle time multiplied by the expected order times per year.

$$I = i * [c * \max((1+\theta)R - x, 0) + c * (1-\alpha)Q - \frac{c}{r} * \frac{S}{N}] * \frac{D}{(1-\alpha)QL} \quad (12)$$

After putting Equation 4 into it, Equation 12 leads to the expected annual federal income tax, as seen by Equation 13.

$$E(I) = i * c * [\int_0^{(1+\theta)R} ((1+\theta)R - x) f(x)dx + (1-\alpha)Q - \frac{1}{r} * r * \{(1-\alpha)Q - (1+\theta)R*F((1+\theta)R) + \int_0^{(1+\theta)R} xf(x)dx\}] * \frac{D}{(1-\alpha)QL} \quad (13)$$

After simplification, the equation for the expected annual federal income tax is shown by Equation 14.

$$E[I] = i * c * [2(1+\theta)R*F((1+\theta)R) - 2\int_0^{(1+\theta)R} xf(x)dx]* \frac{D}{(1-\alpha)QL} \quad (14)$$

By combining Equations 4, 5, 6, 7, 11, and 14, the final equation for the annual profit of model 1 is obtained, which is Equation 15.

$$\pi_1 =$$

$$r * [(1-\alpha)Q - (1+\theta)R*F((1+\theta)R) + \int_0^{(1+\theta)R} xf(x)dx] * \frac{D}{(1-\alpha)QL} -$$

$$h * \frac{(1-\alpha)Q}{2} - c * \frac{D}{L} - K * \frac{D}{(1-\alpha)QL} -$$

$$p * [\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R)] * \frac{D}{(1-\alpha)QL} -$$

$$i * c * [2(1+\theta)R*F((1+\theta)R) - 2\int_0^{(1+\theta)R} xf(x)dx]* \frac{D}{(1-\alpha)QL} \quad (15)$$

3.2.2 Model 2: No backorder with RFID

Model 2 described the inventory management system with RFID but there was still no backorder. On the basis of model 1, additional assumptions and notations were added to the model and modified formulas were provided.

3.2.2.1 No backorder with RFID model assumption

Besides no backorder without RFID model's assumptions, there were three more assumptions for model 2, which were given below.

1) The fixed cost for the RFID system, like the cost for readers and antennas, was ignored. It was assumed that the equipment had already set up in the retail store.
2) Cost related to the RFID system was the cost of RFID tags, each of which was attached to one item;
3) The cost of a RFID tag was a constant, which was a percentage of the inventory purchase cost.

3.2.2.2 No backorder with RFID model Notation

There was only one extra notation compared with the notation for no backorder without RFID model. Symbolically s was the tag price per unit.

3.2.2.3 No backorder with RFID model formula

There was only one difference between the expected annual profit equations for no backorder with and without RFID models (model 1 and model 2), which was the annual purchase cost. Since there was additional cost associated with the RFID tag for each unit, the expected annual purchase cost was rewritten as by Equation 16 and then the final equation for the annual profit of no backorder with RFID model was expressed by Equation 17.

$$E[C] = (c + s) * \frac{D}{L} \quad (16)$$

$$\pi_2 =$$

$$r * [(1-\alpha)Q - (1+\theta)R*F((1+\theta)R) + \int_0^{(1+\theta)R} xf(x)dx] * \frac{D}{(1-\alpha)QL} -$$

$$h * \frac{(1-\alpha)Q}{2} - (c + s) * \frac{D}{L} - K * \frac{D}{(1-\alpha)QL} -$$

$$p * [\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R)] * \frac{D}{(1-\alpha)QL} -$$

$$i * c * [2(1+\theta)R*F((1+\theta)R) - 2\int_0^{(1+\theta)R} xf(x)dx]* \frac{D}{(1-\alpha)QL} \quad (17)$$

3.3 Backorder without RFID and backorder with RFID

The two models, backorder with and without RFID, defined the situation that part of shortages was backordered. Model 3, backorder without RFID, and model 4, backorder with RFID, were built in the following subsection.

3.3.1 Model 3: Backorder without RFID

In Model 3, backorders were allowed but the inventory management system did not have the assistance of RFID. Based on the descriptions for the no backorder without RFID model, incremental descriptions for this model were provided using the assumptions, notations, and modeling procedure.

3.3.1.1 Backorder without RFID model assumption

On the basis of assumptions for the no backorder without RFID model, seven more assumptions were added to this model. The assumptions were as follows.

1) A fraction of shortage became backorder;
2) The penalty for backorders was different from the penalty for shortages. Usually it was smaller than the penalty for shortage;
3) Backorders were supplemental to the ordinary order items and arrived with the ordinary items;
4) Discrepancy also existed in backorders;
5) All backorders were sold to customers immediately when they arrived, so there was no extra holding cost for backorders;
6) Backorders had the same sale price as normal items, and
7) Backorders that were lost during ordering became lost sales.

3.3.1.2 Backorder without RFID model Notation

Besides the notations for no backorder without RFID model, two added notations for the model were:

b: percentage of shortage that becomes backorder, and

p': penalty cost per backorder.

3.3.1.3 Backorder without RFID model formula

The difference between the annual profit formulas of no RFID models with and without backorder mainly existed in the shortage cost, which affected the annual sales, annual purchase cost, annual penalty cost, and the federal income tax. Since all backorders were sold to customers at the moment they arrived, the annual holding cost did not change. Also, the backorders had no influence on the expected order times per year, N, which led to no change in the annual order cost.

3.3.1.3.1 Annual Sales

Equation 9 gave the expression to the amount of shortage in one cycle time when there was no backorder. If b percent of these shortages were backordered, with the addition of the existence of the discrepancy, $1-\alpha$ of the backorders were sold at the price of r. As a result, the annual sales were expressed with Equation 18.

$$S = r * [(1-\alpha)*(Q + b*M) - (1+\theta)R + \min(x, (1+\theta)R)] * N \quad (18)$$

After putting the expression of M as well as the Equations 3 and 10 into Equation 18, the expected annual sales were given by Equation 19.

$$E(S) = r * [(1-\alpha)Q + b(1-\alpha) *(\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R * F((1+\theta)R)) -$$

$$(1+\theta)R + (1+\theta)R - (1+\theta)R*F((1+\theta)R) + \int_{0}^{(1+\theta)R} xf(x)dx] * \frac{D}{(1-\alpha)QL}$$

$$= r * [(1-\alpha)Q - b(1-\alpha)(1+\theta)R + (b-b\alpha-1)(1+\theta)R* F((1+\theta)R) + b(1-\alpha)$$

$$\int_{(1+\theta)R}^{\infty} xf(x)dx + \int_{0}^{(1+\theta)R} xf(x)dx\,] * \frac{D}{(1-\alpha)QL} \quad (19)$$

3.3.1.3.2 Annual Holding Cost

The expected annual holding cost for backorder without RFID model was exactly the same as the one for the no backorder without RFID model, which was given by Equation 5.

3.3.1.3.3 Annual Purchase Cost

In addition to the annual purchase cost of the no backorder without RFID model, the extra purchase cost for the expected backorder was added to the annual profit for the backorder without RFID model. The total expected annual purchase cost was then given by Equation 20.

$$E[C] = c * \frac{D}{L} + c * (b*M) * \frac{D}{(1-\alpha)QL}$$

$$= c * [\frac{D}{L} + b*(\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R * F((1+\theta)R)) * \frac{D}{(1-\alpha)QL}]$$

(20)

3.3.1.3.4 Annual Ordering Cost

Annual ordering cost for the backorder without RFID model equaled the one for the no backorder without RFID model, which was expressed by Equation 7.

3.3.1.3.5 Annual Penalty Cost

Even though b percent of shortage was backordered, only (1-α) percent could be truly sold to the waiting customers with a penalty cost p'. The other α percent was lost during ordering and finally became lost sales again with a penalty cost, p per unit. As a result, the annual penalty cost was

P = [p * (1 - (1- α)b)*M + p’ * (1- α)b*M] * N

With the expression of M given by Equation 9, the expected annual penalty cost was given by Equation 21.

$$E[P] = [p - (1-\alpha)b * (p - p')] * [\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R * F((1+\theta)R)] * \frac{D}{(1-\alpha)QL} \quad (21)$$

3.3.1.3.6 Annual Federal Income Tax for Inventory

For the federal income tax, there were two differences compared with the no backorder without RFID model. First the purchases that were available for sale in one cycle time were (1- α)(Q + b*M). Second was the sales, which were calculated by Equation 18. The annual federal income tax was calculated by Equation 22.

$$I = i * [c * \max((1+\theta)R - x, 0) + c * (1-\alpha)(Q + b*M) - \frac{c}{r} * \frac{S}{N}] * \frac{D}{(1-\alpha)QL} \quad (22)$$

After putting expressions of M and S (given by Equation 9 and 18) into it, Equation 22 leads to the expected annual federal income tax, as seen by Equation 23.

$$E(I) = i * c * [\int_{0}^{(1+\theta)R} ((1+\theta)R - x) f(x)dx + (1-\alpha)(Q + b*M) - \frac{1}{r} * r * \{(1-\alpha)(Q + b*M) - (1+\theta)R*F((1+\theta)R) + \int_{0}^{(1+\theta)R} xf(x)dx\}] * \frac{D}{(1-\alpha)QL} \quad (23)$$

After simplification, the equation for the expected annual federal income tax was shown by Equation 24, which was the same as the federal income tax for no backorder without RFID model. This was reasonable since the backorders were sold immediately without being kept on hand and there should be no tax fee for these transient inventories.

$$E[I] = i * c * [2(1+\theta)R*F((1+\theta)R) - 2\int_{0}^{(1+\theta)R} xf(x)dx\,]* \frac{D}{(1-\alpha)QL} \quad (24)$$

The combination of Equations 19, 5, 20, 7, 21, and 24 was the formula for the annual profit of model 3, as shown with Equation 25.

$$\pi_3 =$$

$$r * [(1-\alpha)Q - b(1-\alpha)(1+\theta)R + (b-b\alpha-1)(1+\theta)R* F((1+\theta)R) + b(1-\alpha)$$

$$\int_{(1+\theta)R}^{\infty} xf(x)dx + \int_{0}^{(1+\theta)R} xf(x)dx\,] * \frac{D}{(1-\alpha)QL} -$$

$$h * \frac{(1-\alpha)Q}{2} -$$

$$c * [\frac{D}{L} + b*(\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R)) * \frac{D}{(1-\alpha)QL}] -$$

$$K * \frac{D}{(1-\alpha)QL} -$$

$$[p - (1-\alpha)b * (p - p')] * [\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R)] *$$

$$\frac{D}{(1-\alpha)QL} -$$

$$i * c * [2(1+\theta)R*F((1+\theta)R) - 2\int_{0}^{(1+\theta)R} xf(x)dx\,]* \frac{D}{(1-\alpha)QL} \quad (25)$$

3.3.2 Model 4: Backorder with RFID

The inventory system simulated by backorder with RFID model was under the control of the RFID system and b percent of shortage in this model was backorder.

The assumptions and notations for this model were the combinations of all assumptions and notations of previous 3 models. The expected annual profit formula of this model was very similar to the formula of the backorder without RFID model with the only difference that the purchase cost contained the cost of RFID. The cost per unit was no longer c but c + s where s referred to the cost of RFID tag. In this way, the final equation for the annual profit of model 4 was given by Equation 26.

$$\pi_4 =$$

$$r * [(1-\alpha)Q - b(1-\alpha)(1+\theta)R + (b-b\alpha-1)(1+\theta)R* F((1+\theta)R) + b(1-\alpha) \int_{(1+\theta)R}^{\infty} xf(x)dx + \int_{0}^{(1+\theta)R} xf(x)dx] * \frac{D}{(1-\alpha)QL} -$$

$$h * \frac{(1-\alpha)Q}{2} -$$

$$(c + s) * [\frac{D}{L} + b*(\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R)) * \frac{D}{(1-\alpha)QL}] -$$

$$K * \frac{D}{(1-\alpha)QL} -$$

$$[p - (1-\alpha)b * (p - p')] * [\int_{(1+\theta)R}^{\infty} xf(x)dx - (1+\theta)R + (1+\theta)R* F((1+\theta)R)] * \frac{D}{(1-\alpha)QL} -$$

$$i * c * [2(1+\theta)R*F((1+\theta)R) - 2\int_{0}^{(1+\theta)R} xf(x)dx]* \frac{D}{(1-\alpha)QL} \qquad (26)$$

3.4 Chapter Summary

After providing explanation of profit equation models, this chapter built four models on the basis of classical (R, Q) model. These models were separated according to the status of RFID and backorder. The objective function of these models was the annual profit. The impact of RFID on the models was reflected in the value of two critical factors, which were Discrepancy Ratio and Inaccuracy Ratio. Annual profit formulas, containing the two factors as uncertain variables, were deduced based on a series of assumptions. Final formulas for each model were correspondingly given by Equations 15, 17, 25, and 26. Chapter 4 would focus on solving the optimal solutions of these models given a certain service level.

Chapter 4 Optimal Solutions of Models

Four models were built in Chapter 3 in the form of equations with maximizing annual profit as the objective function under a given customer service level. In order to compare each pair of models in terms of annual profit, the optimal solutions of the objective function were found. This chapter described the optimal solutions for each model. Before determining the optimal solutions for the annual profit, the optimal values of the reorder point, R, and the reorder quantity, Q, were found. The following four sections provided the procedures for the optimization of the objective functions for the four models.

The aims of the four models were the same, which were to maximize the annual profit in order to satisfy a target service level. By the definition, the service level (SL) was the cumulative distribution function of customer demand at a certain point, R, which could be written as SL = P (x $\leq$ R) = F (R). Once a service level was given as SL_0, by considering the equivalent demand distribution, the optimal reorder point was found by Equation 27.

$R^* = F^{-1}(SL_0)$ (27)

The optimal reorder points were the same for each model. By including R^* into the expressions of the annual profit model, there was only one independent variable, reorder quantity, Q, whose change affected the change of the optimal value for annual profit. The expressions of the four models with the substitution for R^* were written as Equations 28 to 31. Before giving the equations, some notations in these equations were defined as given below.

π_i : the objective function of the ith model, which equals to Annual Sales – Annual

Holding Cost – Annual Purchase Cost – Annual Ordering Cost – Annual Penalty Cost – Annual Federal Income Tax;

π_1: the profit of model no backorder without RFID in dollar;

π_2: the profit of model no backorder with RFID in dollar;

π_3: the profit of model with backorder without RFID in dollar;

π_4: the profit of model with backorder with RFID in dollar;

α and θ: discrepancy ratio and inaccuracy ratio;

Q: reorder quantity;

x: random variable representing demand during lead time L with f(x) as PDF, F(x) as CDF, and D as the expected value;

r, h, c, K, s, p, p': selling price per unit, annual holding cost per unit, purchase cost per unit, fixed order cost per order, tag price per unit, penalty cost per shortage, and penalty cost per backorder, all in dollar;

b: percentage of shortage that becomes backorder, and

i: tax ratio.

Equations 28, 29, 30, and 31were as follows.

$\pi_1 =$

$$r * [(1-\alpha)Q - (1+\theta)F^{-1}(SL_0) * F((1+\theta)F^{-1}(SL_0)) + \int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx] * \frac{D}{(1-\alpha)QL} -$$

$$h * \frac{(1-\alpha)Q}{2} - c * \frac{D}{L} - K * \frac{D}{(1-\alpha)QL} -$$

p * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $(1+\theta)F^{-1}(SL_0)$ + $(1+\theta)F^{-1}(SL_0)$ * $F((1+\theta)\ F^{-1}(SL_0))$] *

$\frac{D}{(1-\alpha)QL}$ -

i * c * [2 $(1+\theta)\ F^{-1}(SL_0)$ * $F((1+\theta)\ F^{-1}(SL_0))$ - 2 $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$]* $\frac{D}{(1-\alpha)QL}$

(28)

$\pi_2 =$

r * [$(1-\alpha)$Q - $(1+\theta)F^{-1}(SL_0)$ * $F((1+\theta)\ F^{-1}(SL_0))$ + $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] *

$\frac{D}{(1-\alpha)QL}$ -

h * $\frac{(1-\alpha)Q}{2}$ - (c + s) * $\frac{D}{L}$ - K * $\frac{D}{(1-\alpha)QL}$ -

p * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $(1+\theta)F^{-1}(SL_0)$ + $(1+\theta)F^{-1}(SL_0)$ * $F((1+\theta)\ F^{-1}(SL_0))$] *

$\frac{D}{(1-\alpha)QL}$ -

i * c * [2 $(1+\theta)\ F^{-1}(SL_0)$ * $F((1+\theta)\ F^{-1}(SL_0))$ - 2 $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$]* $\frac{D}{(1-\alpha)QL}$

(29)

$\pi_3 =$

r * [$(1-\alpha)$Q – b$(1-\alpha)$ * $(1+\theta)F^{-1}(SL_0)$ + (b-bα-1) * $(1+\theta)F^{-1}(SL_0)$ * $F((1+\theta)\ F^{-1}(SL_0))$ + b$(1-\alpha)$ $\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ + $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{(1-\alpha)QL}$ -

h * $\frac{(1-\alpha)Q}{2}$ -

c * [$\frac{D}{L}$ + b*($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+ θ)$F^{-1}(SL_0)$ + (1+ θ)$F^{-1}(SL_0)$ * F((1+ θ) $F^{-1}(SL_0)$)) * $\frac{D}{(1-\alpha)QL}$] -

K * $\frac{D}{(1-\alpha)QL}$ -

[p - (1- α)b * (p – p')] * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+ θ)$F^{-1}(SL_0)$ + (1+ θ)$F^{-1}(SL_0)$ * F((1+ θ) $F^{-1}(SL_0)$)] * $\frac{D}{(1-\alpha)QL}$ -

i * c * [2 (1+ θ)$F^{-1}(SL_0)$ * F((1+ θ) $F^{-1}(SL_0)$) - 2 $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{(1-\alpha)QL}$

(30)

π_4 =

r * [(1- α)Q – b(1- α) * (1+ θ)$F^{-1}(SL_0)$ + (b-b α -1) * (1+ θ)$F^{-1}(SL_0)$ * F((1+ θ) $F^{-1}(SL_0)$) + b(1- α) $\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ + $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{(1-\alpha)QL}$ -

h * $\frac{(1-\alpha)Q}{2}$ -

(c + s) * [$\frac{D}{L}$ + b*($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+ θ)$F^{-1}(SL_0)$ + (1+ θ)$F^{-1}(SL_0)$ * F((1+ θ) $F^{-1}(SL_0)$)) * $\frac{D}{(1-\alpha)QL}$] -

K * $\frac{D}{(1-\alpha)QL}$ -

[p - (1- α)b * (p – p')] * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+ θ)$F^{-1}(SL_0)$ + (1+ θ)$F^{-1}(SL_0)$ *

F((1+θ) $F^{-1}(SL_0)$)] * $\frac{D}{(1-\alpha)QL}$ -

i * c * [2 (1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$) - 2$\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{(1-\alpha)QL}$

(31)

In order to maximize the objective function, the optimal value of Q (Q^*) needed to be determined by differentiation. Then the optimal annual profit could be gained by putting Q^* into the expressions. The following four subsections gave the differentiation of the annual profit with respect to Q as well as the optimal annual profit of each model.

4.1 Optimal Solution of No Backorder without RFID Model

Equation 28 was the updated expression of the annual profit for model 1. The optimal value of Q was determined before ordering. As result, its calculation depended on the situation without discrepancy. By letting the derivative of π_1 without α with respect to Q equal to 0 according to Equation 28, the optimal value of Q was determined as follow.

$\frac{d\pi_1}{dQ} = 0 =$

r * [(1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$) - $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{L}$ * $\frac{1}{Q^2}$ -

h * $\frac{1}{2}$ + K * $\frac{D}{L}$ * $\frac{1}{Q^2}$ +

p * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)$F^{-1}(SL_0)$ + (1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$)] *

$\frac{D}{L} * \frac{1}{Q^2}$ +

i * c * [2 $(1+\theta)$F^{-1}(SL$_0$)* F($(1+\theta)$ F^{-1}(SL$_0$)) - 2$\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{L} * \frac{1}{Q^2}$

Then, $Q^2 = \frac{2D}{hL}$ * [K +

p * ($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $(1+\theta)$F^{-1}(SL$_0$) + $(1+\theta)$F^{-1}(SL$_0$) * F($(1+\theta)$ F^{-1}(SL$_0$))) +

i * c * (2 $(1+\theta)$F^{-1}(SL$_0$)* F($(1+\theta)$ F^{-1}(SL$_0$)) - 2$\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$) +

r * ($(1+\theta)$F^{-1}(SL$_0$) * F($(1+\theta)$ F^{-1}(SL$_0$)) - $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$)]

Finally, $Q_1^* = \{ \frac{2D}{hL}$ * [K +

p * ($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $(1+\theta)$F^{-1}(SL$_0$) + $(1+\theta)$F^{-1}(SL$_0$) * F($(1+\theta)$ F^{-1}(SL$_0$))) +

i * c * (2 $(1+\theta)$F^{-1}(SL$_0$)* F($(1+\theta)$ F^{-1}(SL$_0$)) - 2$\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$) +

r * ($(1+\theta)$F^{-1}(SL$_0$) * F($(1+\theta)$ F^{-1}(SL$_0$)) - $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$)]$\}^{1/2}$ (32)

As a result, the optimal annual profit for model 1 is as following:

$\pi_1^* =$

r * [(1-α) Q_1^* - $(1+\theta)$F^{-1}(SL$_0$) * F($(1+\theta)$ F^{-1}(SL$_0$)) + $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] *

$\frac{D}{(1-\alpha)Q_1^* L}$ -

h * $\frac{(1-\alpha)Q_1^*}{2}$ - c * $\frac{D}{L}$ - K * $\frac{D}{(1-\alpha)Q_1^* L}$ -

p * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)$F^{-1}(SL_0)$ + (1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$)] *

$\frac{D}{(1-\alpha)Q_1^* L}$ -

i * c * [2 (1+θ)$F^{-1}(SL_0)$* F((1+θ) $F^{-1}(SL_0)$) - 2 $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{(1-\alpha)Q_1^* L}$

(33)

where Q_1^* is expressed by Equation 32.

4.2 Optimal Solution of No backorder with RFID Model

Equation 29 provided the updated expression of the annual profit for model 2. The only difference between equation 28 for model 1 and equation 29 for model 2 was that the factor of the purchase cost for model 2 contained the cost of the RFID tag. But this different factor did not include Q, which needs to be determined by differentiation. Thus, the derivation of π_2 without α with respect to Q of model 2 was exactly the same with the one of model 1. This meant that the optimal reorder quantity of model 2 equaled the optimal reorder quantity of model 1, which was written as Q_2^* = Q_1^*.

Therefore, the optimal annual profit for model 1 was expressed by Equation 34, where Q_2^* = Q_1^* was gained by Equation 32.

π_2^* =

r * [(1-α) Q_2^* - (1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$) + $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] *

$\frac{D}{(1-\alpha)Q_2^* L}$ -

$$h * \frac{(1-\alpha)Q_2^*}{2} - (c + s) * \frac{D}{L} - K * \frac{D}{(1-\alpha)Q_2^* L} -$$

$$p * [\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - (1+\theta)F^{-1}(SL_0) + (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0))] *$$

$$\frac{D}{(1-\alpha)Q_2^* L} -$$

$$i * c * [2 (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0)) - 2\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx]* \frac{D}{(1-\alpha)Q_2^* L}$$

(34)

4.3 Optimal Solution of Backorder without RFID Model

The optimal reorder quantity of model 3 was attained by letting the derivative of π_3 without α with respect to Q equal to 0 according to Equation 30. This optimal value was expressed as following.

$$\frac{d\pi_3}{dQ} = 0 =$$

$$r * [b* (1+\theta)F^{-1}(SL_0) - (b-1) * (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0)) - b$$

$$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - \int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx] * \frac{D}{L} * \frac{1}{Q^2} -$$

$$h * \frac{1}{2} +$$

$$c * b*[\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - (1+\theta)F^{-1}(SL_0) + (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0))] *$$

$$\frac{D}{L} * \frac{1}{Q^2} +$$

K * $\frac{D}{L}$ * $\frac{1}{Q^2}$ +

[p - b*(p – p')] * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)F^{-1}(SL$_0$) + (1+θ)F^{-1}(SL$_0$) * F((1+θ)

F^{-1}(SL$_0$))]* $\frac{D}{L}$ * $\frac{1}{Q^2}$ +

i * c * [2 (1+θ)F^{-1}(SL$_0$) * F((1+θ) F^{-1}(SL$_0$)) - 2 $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$]* $\frac{D}{L}$ * $\frac{1}{Q^2}$

Then, Q^2 = $\frac{2D}{hL}$ * [c*b*($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)F^{-1}(SL$_0$) + (1+θ)F^{-1}(SL$_0$) *

F((1+θ) F^{-1}(SL$_0$))) +

K + (p - b*(p – p')) * ($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)F^{-1}(SL$_0$) + (1+θ)F^{-1}(SL$_0$) *

F((1+θ) F^{-1}(SL$_0$))) +

i * c * (2 (1+θ)F^{-1}(SL$_0$) * F((1+θ) F^{-1}(SL$_0$)) - 2 $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$) +

r * (b(1-α) F^{-1}(SL$_0$) - (b- 1) F^{-1}(SL$_0$) * SL$_0$ - b* $\int_{F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $\int_0^{F^{-1}(SL_0)} xf(x)dx$)]

Hence, Q_3^* = { $\frac{2D}{hL}$ * [c*b*($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)F^{-1}(SL$_0$) + (1+θ)F^{-1}(SL$_0$) *

F((1+θ) F^{-1}(SL$_0$))) +

K + (p - b*(p – p')) * ($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)F^{-1}(SL$_0$) + (1+θ)F^{-1}(SL$_0$) *

F((1+θ) F^{-1}(SL$_0$))) +

i * c * (2 (1+θ)F^{-1}(SL$_0$) * F((1+θ) F^{-1}(SL$_0$)) - 2 $\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$) +

r * (b* (1+θ)F^{-1}(SL$_0$) - (b-1) * (1+θ)F^{-1}(SL$_0$) * F((1+θ) F^{-1}(SL$_0$)) - b*

$\int_{F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $\int_{0}^{F^{-1}(SL_0)} xf(x)dx$)]}$^{1/2}$ (35)

Then by replacing Q in equation 30 with Q_3^* given by Equation 35, the optimal annual profit for model 3 was obtained:

$\pi_3^* =$

r * [(1-α) Q_3^* – b(1-α) * (1+θ)F^{-1}(SL_0) + (b-bα-1) * (1+θ)F^{-1}(SL_0) * F((1+θ) F^{-1}(SL_0)) + b(1-α) $\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ + $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{(1-\alpha)Q_3^* L}$ -

h * $\frac{(1-\alpha)Q_3^*}{2}$ -

c * [$\frac{D}{L}$ + b*($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)F^{-1}(SL_0) + (1+θ)F^{-1}(SL_0) * F((1+θ) F^{-1}(SL_0))) * $\frac{D}{(1-\alpha)Q_3^* L}$] -

K * $\frac{D}{(1-\alpha)Q_3^* L}$ -

[p - (1-α)b*(p – p')] * [$\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)F^{-1}(SL_0) + (1+θ)F^{-1}(SL_0) * F((1+θ) F^{-1}(SL_0))] * $\frac{D}{(1-\alpha)Q_3^* L}$ -

i * c * [2 (1+θ)F^{-1}(SL_0) * F((1+θ) F^{-1}(SL_0)) - 2 $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$] * $\frac{D}{(1-\alpha)Q_3^* L}$

(36)

4.4 Optimal Solution of Backorder with RFID Model

To find the optimal reorder quantity of model 4, the derivative of π_4 without α with respect to Q was set to be 0 according to Equation 31. Then the optimal value of Q_4 was calculated as following.

$$\frac{d\pi_4}{dQ} = 0 =$$

$$r * [b * (1+\theta)F^{-1}(SL_0) + (b-1) * (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0)) + b \int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx + \int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx] * \frac{D}{L} * \frac{1}{Q^2} -$$

$$h * \frac{1}{2} +$$

$$(c + s) * b*[\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - (1+\theta)F^{-1}(SL_0) + (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0))] * \frac{D}{L} * \frac{1}{Q^2} +$$

$$K * \frac{D}{L} * \frac{1}{Q^2} +$$

$$[p - b*(p - p')] * [\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - (1+\theta)F^{-1}(SL_0) + (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0))] * \frac{D}{L} * \frac{1}{Q^2} +$$

$$i * c * [2 (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0)) - 2\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx] * \frac{D}{L} * \frac{1}{Q^2}$$

Then, $Q^2 = \frac{2D}{hL} * [(c + s) * b * (\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - (1+\theta)F^{-1}(SL_0) + (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0))) +$

K + (p - b*(p – p')) * ($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)$F^{-1}(SL_0)$ + (1+θ)$F^{-1}(SL_0)$ *

F((1+θ) $F^{-1}(SL_0)$)) +

i * c * (2 (1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$) - 2 $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$) +

r * (b(1+θ)$F^{-1}(SL_0)$ - (b-1) (1+θ) $F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$) – b *

$\int_{F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $\int_{0}^{F^{-1}(SL_0)} xf(x)dx$)]

Hence, Q_4^* = { $\frac{2D}{hL}$ *

[(c + s) * b * ($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)$F^{-1}(SL_0)$ + (1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$)) +

K + (p - b*(p – p')) * ($\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx$ - (1+θ)$F^{-1}(SL_0)$ + (1+θ)$F^{-1}(SL_0)$ *

F((1+θ) $F^{-1}(SL_0)$)) +

i * c * (2 (1+θ)$F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$) - 2 $\int_{0}^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx$) +

r * (b(1+θ)$F^{-1}(SL_0)$ - (b-1) (1+θ) $F^{-1}(SL_0)$ * F((1+θ) $F^{-1}(SL_0)$) – b *

$\int_{F^{-1}(SL_0)}^{\infty} xf(x)dx$ - $\int_{0}^{F^{-1}(SL_0)} xf(x)dx$)]}$^{1/2}$ (37)

After replacing Q in Equation 31 with Q_4^* given by Equation 37, the optimal annual profit for model 4 was obtained:

$\pi_4^* =$

r * [(1-α) Q_4^* – b(1-α) * (1+θ)$F^{-1}(SL_0)$ + (b-bα -1) * (1+θ)$F^{-1}(SL_0)$ * F((1+θ)

$F^{-1}(SL_0)) + b(1-\alpha)\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx + \int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx] * \frac{D}{(1-\alpha)Q_4^{*}L}$ -

$h * \frac{(1-\alpha)Q_4^{*}}{2}$ -

$(c+s) * [\frac{D}{L} + b*(\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - (1+\theta)F^{-1}(SL_0) + (1+\theta)F^{-1}(SL_0) * F((1+\theta)$

$F^{-1}(SL_0))) * \frac{D}{(1-\alpha)Q_4^{*}L}]$ -

$K * \frac{D}{(1-\alpha)Q_4^{*}L}$ -

$[p - (1-\alpha)b*(p - p')] * [\int_{(1+\theta)*F^{-1}(SL_0)}^{\infty} xf(x)dx - (1+\theta)F^{-1}(SL_0) + (1+\theta)F^{-1}(SL_0) *$

$F((1+\theta) F^{-1}(SL_0))] * \frac{D}{(1-\alpha)Q_4^{*}L}$ -

$i * c * [2 (1+\theta)F^{-1}(SL_0) * F((1+\theta) F^{-1}(SL_0)) - 2\int_0^{(1+\theta)*F^{-1}(SL_0)} xf(x)dx] * \frac{D}{(1-\alpha)Q_4^{*}L}$

(38)

4.5 Chapter Summary

On the basis of the four models that Chapter 3 built, this chapter provided the optimal solutions of these models. The four equations expressing the profit were differentiated with respect to the order quantity, Q. The optimization of the objective function, which was annual profit, asked for the solutions of optimal values of reorder point R^* and reorder quantity Q^* first. Then by replacing the R and Q with R^* and Q^* respectively in each model, the optimal solutions of the annual profit were determined.

The optimal solutions of R in each model were the same, with the expression given by Equation 27. The optimal values of Q for each model were gained after putting R^* into their corresponding models and setting the derivative of π with respect to Q to be 0. The optimal values of Q were given by Equations 32, 32, 35, and 37. Finally, the optimal annual profits were obtained, which were separately expressed by Equations 33, 34, 36, and 38.

Chapter 5 Results

In this chapter, results of the research were stated. This research used the following procedure.

1) Develop mathematical inventory cost models, which included choosing an appropriate inventory type to study, collecting data that described the selected inventory, and analyzing the inventory data;
2) Compare the performance measure (profit) of the four models with respect to chosen inventory type; and
3) Conduct sets of sensitivity analyses with different critical parameters.

5.1 Model Development

5.1.1 Inventory selection

The mathematical models developed were derivatives of well-known inventory models. The research of this dissertation added the impact of new inventory monitoring technology, the use of evaluation parameters to the existing models, and Federal Income Tax. Women's clothes were chosen to be the particular goods to test and evaluate the enhanced inventory models in this research due to their special characteristics.

First, women's clothes were typically categorized as important inventories in the ABC system (category A or B) with relatively high demand during a certain period, as compared to category C inventories with less frequent demands, such as furniture or electronics. Thus, the reorder point and order quantity (R, Q) model appropriately described the behavior of the replenishment procedures typically used in retail sales according to the results of different inventory model comparison in Chapter 2.

Second, women’s clothes had uncertain customer demand during the order lead time. In addition, unlike perishable goods such as foods or some beverages, unsold women’s clothes in one replenishment cycle still could be sold in the next cycle. As a result, the assumptions for the four models in Chapter 3 of customer demand during lead time followed a continuous distribution and the inventories that were not sold in previous cycle were added into the following period without product deterioration, were reasonable.

Third, customers might reorder women’s clothes due to the specific brands and/or styles. Brand loyalty was not as common with goods such as office supplies and general merchandise. Also, RFID technology had been successfully used to tag clothing for inventory control [45]. Hence, women’s clothes were suitable as an inventory type in this dissertation’s research of inventory control using the four models developed.

Fourth, data that were used by the retailer’ profit models for women’s clothes were available from 2012 U.S. Census Bureau [46]. The use of relatively current data supports the value of the models’ results being useful in the practices of today’s retail businesses that sell a popular inventory item.

5.1.2 Data collection

U.S. Census Bureau provided the monthly retail trade from year 1992 to 2012. The historical data were collected from 12,000 retail businesses every five years, by using a mail-out/mail-back survey form [47]. Then after being stratified by major kind of business and estimated sales, 3300 of the 12000 firms, whose sales were above applicable size cutoffs, were selected with certainty and reported for all their retail establishments.

The Retail Inventories and Inventories/Sales Ratios form in Excel [46] shows the estimates of monthly retail and food services sales by kind of business from year1992. In this form, the monthly sales of women's clothes were listed with the North American Industry Classification System (NAICS) code 44812. These data come from 3300 firms, with the unit of millions of dollars. Table 5.1 shows the monthly sales data of women's clothes for all firms from the website form.

Table 5.1 Monthly sales data of women's clothes for 3300 firms (millions of dollars)

Year	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**
Jan	1,874	2,125	1,787	1,724	1,568	1,698	1,725	1,747	1,683
Feb	1,993	2,006	1,971	1,812	1,837	1,812	1,770	1,885	1,993
Mar	2,405	2,444	2,562	2,395	2,196	2,352	2,198	2,509	2,673
Apr	2,667	2,763	2,560	2,439	2,349	2,226	2,510	2,619	2,709
May	2,755	2,876	2,554	2,538	2,555	2,428	2,516	2,690	2,812
June	2,425	2,554	2,449	2,359	2,378	2,200	2,256	2,407	2,567
July	2,375	2,540	2,250	2,174	2,074	2,104	2,224	2,305	2,385
Aug	2,660	2,629	2,540	2,328	2,355	2,337	2,329	2,409	2,643
Sep	2,562	2,625	2,420	2,372	2,278	2,269	2,245	2,356	2,660
Oct	2,757	2,715	2,574	2,315	2,415	2,363	2,424	2,443	2,651
Nov	2,948	2,926	2,801	2,570	2,594	2,494	2,533	2,567	2,826
Dec	4,419	4,174	4,143	3,697	3,667	3,568	3,633	3,644	3,878
Year	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
Jan	1,948	1,989	2,058	2,266	2,331	2,522	2,588	2,543	2,270
Feb	2,156	2,162	2,094	2,400	2,445	2,522	2,635	2,753	2,384
Mar	2,673	2,783	2,623	2,912	3,096	3,200	3,417	3,362	2,898
Apr	2,804	2,683	2,728	3,019	3,167	3,384	3,439	3,405	3,206
May	2,750	2,759	2,817	3,000	3,155	3,343	3,654	3,587	3,249
June	2,510	2,482	2,539	2,772	3,038	3,184	3,305	3,180	2,898
July	2,313	2,262	2,452	2,589	2,735	2,919	2,981	2,943	2,682
Aug	2,663	2,540	2,600	2,638	2,865	2,952	3,232	3,107	2,891
Sep	2,397	2,375	2,629	2,767	2,959	3,151	3,200	3,021	2,879
Oct	2,618	2,533	2,795	2,983	3,117	3,179	3,339	3,039	3,042
Nov	2,790	2,742	2,952	3,082	3,340	3,387	3,658	3,251	3,209
Dec	3,865	3,970	4,311	4,458	4,752	4,973	4,889	4,160	4,172

The U.S. Census Bureau also provided the Average Annual Expenditures of All Consumer Units by Selected Major Types of Expenditure [48] in the Excel format. Then the average monthly expenditure of women's clothes can be obtained by dividing the average annual expenditures by twelve (12 months per year). Table 5.2 gives the average monthly expenditure of women's clothes in the US in dollars per person from year 1992 to 2009.

Table 5.2 Average monthly expenditure of women's clothes (dollars / person)

Year	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**
Expenditure	48.83	47.17	46.00	46.58	50.58	47.83	45.67	45.67	50.58
Year	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
Expenditure	46.83	48.92	44.08	52.58	52.75	52.42	52.25	49.75	46.75

5.1.3 Data analysis

In this subsection, all data used in the models were discussed. Various assumptions were made and data was aggregated or simplified so that financial calculations in the models could be made.

5.1.3.1 Lead time (L)

Since the data provided by U.S. Census Bureau were monthly sales, in order to make use of these data and ensure the feasibility of the research, lead time of women's clothes was assumed to be one month. This assumption was also reasonable because monthly ordering cycles were common in retail. As a result, L = 1/12 years.

5.1.3.2 Customer demand (x)

A group of 3300 retail firms were selected to form the census data. Then for one retail establishment, the monthly sales should be the number listed in the data base divided by 3300because it was the data from 3300 different large sales volume companies. Thus the monthly sales of women's clothes for one retailer were estimated obtained. Assuming that the lead time of women's clothes was exactly one month; then the women's clothes sales of one retailer during the lead time was estimated. Table 5.3 shows the data of women's clothes for one retailer from year 1992 to 2009 (thousand dollars).

Table 5.3 Monthly estimated sales data of women's clothes for one retailer (thousand dollars)

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Jan	567.88	643.94	541.52	522.42	475.15	514.55	522.73	529.39	510.00
Feb	603.94	607.88	597.27	549.09	556.67	549.09	536.36	571.21	603.94
Mar	728.79	740.61	776.36	725.76	665.45	712.73	666.06	760.30	810.00
Apr	808.18	837.27	775.76	739.09	711.82	674.55	760.61	793.64	820.91
May	834.85	871.52	773.94	769.09	774.24	735.76	762.42	815.15	852.12
June	734.85	773.94	742.12	714.85	720.61	666.67	683.64	729.39	777.88
July	719.70	769.70	681.82	658.79	628.48	637.58	673.94	698.48	722.73
Aug	806.06	796.67	769.70	705.45	713.64	708.18	705.76	730.00	800.91
Sep	776.36	795.45	733.33	718.79	690.30	687.58	680.30	713.94	806.06
Oct	835.45	822.73	780.00	701.52	731.82	716.06	734.55	740.30	803.33
Nov	893.33	886.67	848.79	778.79	786.06	755.76	767.58	777.88	856.36
Dec	1339.09	1264.85	1255.45	1120.30	1111.21	1081.21	1100.91	1104.24	1175.15
Year	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
Jan	590.30	602.73	623.64	686.67	706.36	764.24	784.24	770.61	687.88
Feb	653.33	655.15	634.55	727.27	740.91	764.24	798.48	834.24	722.42
Mar	810.00	843.33	794.85	882.42	938.18	969.70	1035.45	1018.79	878.18
Apr	849.70	813.03	826.67	914.85	959.70	1025.45	1042.12	1031.82	971.52
May	833.33	836.06	853.64	909.09	956.06	1013.03	1107.27	1086.97	984.55
June	760.61	752.12	769.39	840.00	920.61	964.85	1001.52	963.64	878.18
July	700.91	685.45	743.03	784.55	828.79	884.55	903.33	891.82	812.73
Aug	806.97	769.70	787.88	799.39	868.18	894.55	979.39	941.52	876.06
Sep	726.36	719.70	796.67	838.48	896.67	954.85	969.70	915.45	872.42
Oct	793.33	767.58	846.97	903.94	944.55	963.33	1011.82	920.91	921.82
Nov	845.45	830.91	894.55	933.94	1012.12	1026.36	1108.48	985.15	972.42
Dec	1171.21	1203.03	1306.36	1350.91	1440.00	1506.97	1481.52	1260.61	1264.24

In order to get the customer demand of women's clothes during the lead time using limited available source, a necessary and reasonable assumption was made. Since the average monthly expenditures of all consumer units by type of expenditure were available from the U.S. Census Bureau (Table 5.2), it was assumed that the selling price of women's clothes was represented by the monthly expenditure. In this way, the customer demand during a lead time was expressed by the number of customers during a lead time. The traditional customer demand was the expected number of women's clothes that were sold during a lead time. But in this research, the customer demand was the number of customers who come to buy women's clothes during a lead time. This assumption was reasonable due to the consistency of the unit of annual sales, which was still dollars. In addition, other prices or costs that were relative to the customer demand were percentages of the selling price. As a result, the annual profit was in the unit of thousand dollars.

After dividing the monthly sales data for women's clothes (in Table 5.3) by the average monthly expenditure (in Table 5.2), the monthly customer demand of women's clothes with the unit of thousand persons, as shown in Table 5.4 from year 1992 to 2009 was generated. There were 216 data points in total.

The customer demand data was a time series. A time series was comprised of data that can have a secular trend, seasonal component, cyclical component, and random component. The secular trend reflected the smooth and regular movement, showing the continuous growth or decline of series over a long period of time. The seasonal and cyclical variations reflected the periodically up-and-down movements of a series separately in short-term and long-term. The random component represented the unpredictable movements [49].

Table 5.4 Estimated monthly customer demand of women's clothes for one retailer (thousand persons)

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Jan	11.63	13.65	11.77	11.21	9.39	10.76	11.45	11.59	10.08
Feb	12.37	12.89	12.98	11.79	11.00	11.48	11.75	12.51	11.94
Mar	14.92	15.70	16.88	15.58	13.16	14.90	14.59	16.65	16.01
Apr	16.55	17.75	16.86	15.87	14.07	14.10	16.66	17.38	16.23
May	17.10	18.48	16.82	16.51	15.31	15.38	16.70	17.85	16.85
June	15.05	16.41	16.13	15.35	14.25	13.94	14.97	15.97	15.38
July	14.74	16.32	14.82	14.14	12.42	13.33	14.76	15.30	14.29
Aug	16.51	16.89	16.73	15.14	14.11	14.81	15.45	15.99	15.83
Sep	15.90	16.86	15.94	15.43	13.65	14.37	14.90	15.63	15.94
Oct	17.11	17.44	16.96	15.06	14.47	14.97	16.08	16.21	15.88
Nov	18.29	18.80	18.45	16.72	15.54	15.80	16.81	17.03	16.93
Dec	27.42	26.82	27.29	24.05	21.97	22.60	24.11	24.18	23.23
Year	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
Jan	12.60	12.32	14.15	13.06	13.39	14.58	15.01	15.49	14.71
Feb	13.95	13.39	14.39	13.83	14.05	14.58	15.28	16.77	15.45
Mar	17.30	17.24	18.03	16.78	17.79	18.50	19.82	20.48	18.78
Apr	18.14	16.62	18.75	17.40	18.19	19.56	19.94	20.74	20.78
May	17.79	17.09	19.36	17.29	18.12	19.33	21.19	21.85	21.06
June	16.24	15.38	17.45	15.97	17.45	18.41	19.17	19.37	18.78
July	14.97	14.01	16.86	14.92	15.71	16.88	17.29	17.93	17.38
Aug	17.23	15.73	17.87	15.20	16.46	17.07	18.74	18.92	18.74
Sep	15.51	14.71	18.07	15.95	17.00	18.22	18.56	18.40	18.66
Oct	16.94	15.69	19.21	17.19	17.91	18.38	19.36	18.51	19.72
Nov	18.05	16.99	20.29	17.76	19.19	19.58	21.22	19.80	20.80
Dec	25.01	24.59	29.63	25.69	27.30	28.75	28.35	25.34	27.04

In order to analyze the distribution of customer demand data, decomposition of this time series was conducted. By the use of decomposition function of Minitab software, random component was extracted after the trend and seasonal factors were removed (cyclical component is ignored by this software). Table 5.5 shows the decomposition result of the last twelve data from the latest year (2009). All the adjusted data resulting from decomposition function are attached as Appendix A.

Table 5.5 Decomposition of customer demand (year 2009)

SERIES	ORIGINAL	TREN	DETR	SEAS	DESE	FITS	RESI
205	14.71	18.81	0.78	0.75	19.63	14.10	0.61
206	15.45	18.83	0.82	0.79	19.58	14.87	0.59
207	18.78	18.85	1.00	1.01	18.62	19.02	-0.24
208	20.78	18.87	1.10	1.03	20.23	19.39	1.39
209	21.06	18.89	1.11	1.06	19.85	20.04	1.02
210	18.78	18.91	0.99	0.97	19.32	18.39	0.40
211	17.38	18.93	0.92	0.89	19.45	16.92	0.46
212	18.74	18.95	0.99	0.98	19.13	18.56	0.18
213	18.66	18.97	0.98	0.95	19.56	18.10	0.56
214	19.72	18.98	1.04	1.00	19.66	19.04	0.68
215	20.80	19.00	1.09	1.06	19.59	20.18	0.63
216	27.04	19.02	1.42	1.50	18.04	28.52	-1.48

The third column represented the trend data, showing the continuous growth of this series over a long period of time. This was the main part of the series data. The latest data 19.02286 customers, which came from December 2009, was used in further research.

The fifth column was the seasonal component. The average of every twelve month in a particular year equaled one. The seasonal component was removed in further research.

The last column residuals referred to the random component, which followed the Normal Distribution according to the identification of Individual Distribution Identification function of Minitab software [50]. Furthermore, mean and standard deviation (s.d.) of the random component were calculated, as 0.00379 and 1.28141 respectively.

The original data in the second column was the trend component multiply by the seasonal component plus the random factor. The software identified the customer demand as Normally distributed, with the mean 19.02286 + 0.00379 =19.02667 (thousand persons) and standard deviation 1.28141. The customer demand, x, was written as x ~ N (19.02667, 1.28141).

5.1.3.3 Service level (SL)

Service level for clothes differs between different retailers. From reported sources, the service level varies from 30% to almost 99% [51, 52].

5.1.3.4 Selling price per unit (r)

Using the assumption at the beginning of this subsection, the selling price of women's clothes was reasonably represented by the average monthly clothing expenditure. As a result, the average monthly expenditure of the latest year (year 2009 in Table 5-2) was used here, r = 46.75 (dollars/person).

5.1.3.5 Annual holding cost per unit (h)

Timme in 2003 [53] pointed out that the average holding cost as a percentage of inventory was 10 percent in U.S. companies. As a result, in this research, the annual holding cost per unit was h = 10% * r = 10% * 46.75 = 4.675 (dollars/person).

5.1.3.6 Purchase cost per unit (c)

Retail markup refers to a percentage added to the cost to obtain the retail selling price . The selling price is found as follows.

purchase cost (c) * (1+markup) = selling price (r) [54]

Normal markup in the clothing industry was reported as usually 50% to 60% [55]. The average of 55% is used in this research. As a result, purchase cost c = r / (1+markup) = 46.75 / (1+55%) = 30.16 (dollars/person).

5.1.3.7 Fixed order cost per order (K)

The lump-sum fixed order cost accounts for about 3% of the annual sales [56]. The annual sales of year 2009 were used here, which was the sum of the twelve data under year 2009 column in Table 5-3 (10842.42 dollars), to calculate the fixed order cost. Then the lump-sum fixed order cost equaled 325.27 dollars. Since there was no less than 1 order per year, the fixed order cost per order should be no more than 325.27 dollars. In this way the fixed order cost per order K was determined as: max (K) = 325.27 dollars/order.

5.1.3.8 Penalty cost per shortage (p)

Regardless of other factors such as the reputation of the retailer and the permanent loss of customers, the penalty cost equaled to the selling price of the item [57], which meant that p = r = 46.75 (dollars/person).

5.1.3.9 Penalty cost per backorder (p')

Penalty cost for backorders was smaller than penalty cost for shortage; otherwise retailers would prefer not to make the backorder. Then max (p') = p = 46.75 (dollars/person).

5.1.3.10 Percentage of shortage that becomes backorder (b)

Different retailers had different policies on this percentage of shortage based on the balance of inventory cost and revenue; this was because this sensitive parameter might impact on the retailers' profits. As a result, the change of b should be between 0 and 100%, with 0 referring to the situation that there was no backorders and 100% presenting that all shortages were backorders.

5.1.3.11 Tag price per unit (s)

Large volume retailers who implemented RFID system kept trying to reduce the price of RFID to reduce the input and increase the profit. Recently, the price of a single electronic tag in large amount had reduced to be 0.15percentage of inventory purchase cost [58]. Thus in this research, s = 0.15% * c = 0.15% * 30.16 = 0.05 (dollars/person).

5.1.3.12 Tax ratio (i)

The effective Federal Income Tax rate for wholesale and retail trade was 14.2 percent [59]. Therefore, i = 14.2%.

5.1.3.13 Data summary

Table 5.6 is the summary of all data that are needed in the following model analysis.

Table 5.6 Data summary

Parameter	Notation	Unit	Possible Range	Average
Annual Profit of model i	π_i	thousand dollars	-	-
Discrepancy ratio	α	percentage	0 ~ 70	-
Inaccuracy ratio	θ	percentage	-40 ~ 40	-
Lead time	L	year	-	1/12
Customer demand	x	thousand persons	Normal Distribution, s.d. = 1.28141	19.02667
Service level	SL	percentage	30 ~ 99	65
Selling price per unit	r	dollars/person	-	46.75
Annual holding cost per unit	h	dollars/person	-	4.675
Purchase cost per unit	c	dollars/person	-	30.16
Fixed order cost per order	K	dollars/order	0 ~ 325.27	162.64
Penalty cost per shortage	P	dollars/person	-	46.75
Penalty cost per backorder	P’	dollars/person	0 ~ 46.75	23.38
Backorder over shortage	b	percentage	0 ~ 100	50
Tag price per unit	s	dollars/person	-	0.05
Tax ratio	i	percentage	-	14.2

5.2 Model Comparison

Wolfram Mathematica 8.0® was used for the programming and calculating model output values.

In order to analyze the effect of RFID on inventory management costs and ultimately retail profit two key factors, the Discrepancy Ratio and Inaccuracy Ratio were used to assess and compare the output of the models. The Discrepancy Ratio related to inventory lost during ordering and the Inaccuracy Ratio related to accrual and reported inventory difference. Table 5-6 listed all of the models' parameters, their ranges and average values.

For model 1 and 3 systems, which had no RFID, the values of Discrepancy Ratio and Inaccuracy Ratio were equal to their maximum values. Discrepancy Ratio was 70% and Inaccuracy Ratio was -40% or 40%.

5.2.1 No backorders without RFID and no backorders with RFID

For system without backorders and RFID system, when Discrepancy Ratio (α) was 0.7 and Inaccuracy Ratio (θ) was -0.4, the profit of this system (π_1) = 1902.46; when α = 0.7 and θ = 0.4, $\pi_1^{'}$ = 1694.79. For system without backorders but has RFID system (π_2), α and θ changed within a certain range (see Table 5-6) due to RFID system.

5.2.1.1 π_2 changed according to α

In order to analyze the change of π_2 to α, θ was chosen at different levels. In this research, θ was evaluated every 10 percent, which was the set of {-0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4}. At each level, the change of π_2 was analyzed where α changed from 0 to 0.7.

When θ equaled 0.4, for instance, the change of π_2 to α is shown by Figure 5.1.

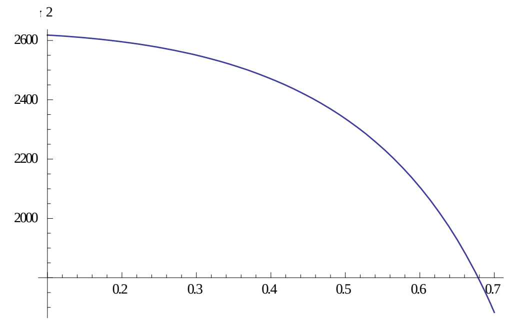


Figure 5.1 The change of π_2 to α when θ equals 0.4

In Figure 5.1, the curve represented the change of the profit of model with RFID π_2 to the Discrepancy Ratio α and the two horizontal lines represented the profits of system without RFID when θ equaled -0.4 and 0.4 respectively. It was easily to see from the curve that when α decreased, π_2 increased. This made sense because when the discrepancy during transportation decreased, more inventories were ensured to arrive. The smaller discrepancy reduced the loss as well as increased the inventories on hand, thus the total profit increased. Also, with the decrease of Discrepancy Ratio, the growth rate of the profit reduced. The change of π_2 with respect to α followed a convex curve but not a straight line.

In addition, there were intersections of this curve and the two horizontal lines, which meant that system with RFID made lower profit when α was beyond a certain point (0.678). This was due to the cost of RFID tags attached to inventories.

Figure 5.2 gives the collection of all changes of π_2 to α when θ is at different levels.

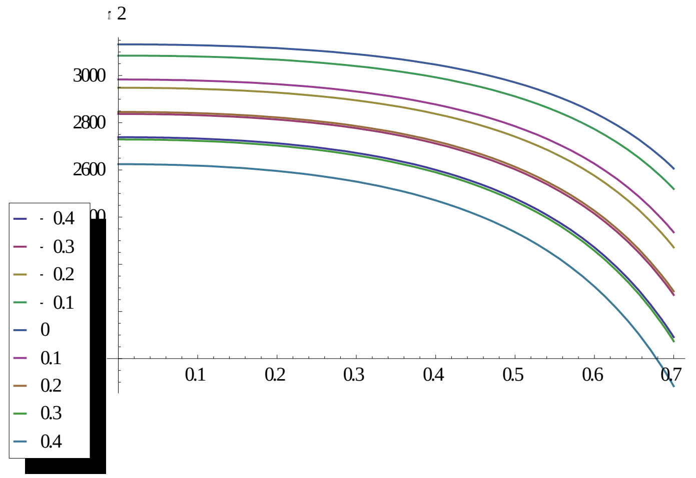


Figure 5.2 The change of π_2 according to the change of α from 0 to 0.7

The curves in Figure 5.2 showed that when θ was at different levels, π_2 had the same trend when α changes. In addition, when θ was smaller than a certain point (0.5), the system with RFID had higher profit all the time, no matter what α was. This was because that smaller θ guaranteed the accuracy of records, which improved the profit.

5.2.1.2 π_2 changed according to θ

The value of α was chosen at different levels when analyzing the change of π_2 to θ. In this research, α was evaluated every 10 percent, which followed the set of {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7}. At each level, the change of π_2 was analyzed where θ changed from -0.4 to 0.4.

Figure 5.3 gives an example of the change of π_2 to θ when α equals 0.7.

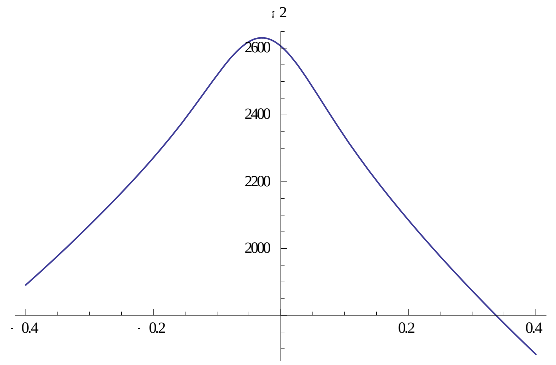


Figure 5.3 The change of π_2 to θ when α equals 0.7

In Figure 5.3, the curve represented the change of the profit of model with RFID π_2 to the Inaccuracy Ratio θ and the two horizontal lines represented the profits of system without RFID when θ equaled -0.4 and 0.4 respectively. Obviously when θ increased from -0.4 to 0.4, π_2 ascended to a maximum point when θ was around zero and then descended. The curve was axial symmetry when θ was around zero. This was because the reduction of the absolute value of θ made sure that fewer mistakes were made during recording. This improved accuracy of records improved the profit for sure. The reason why π_2 reached its maximum value when θ was not exact zero was studies in the subsection sensitivity analysis.

Also there were intersections of this curve and the two horizontal lines, which meant that system with RFID made lower profit when the absolute value of the Inaccuracy Rate θ was larger than 0.336.

Figure 5.4 gives the collection of all changes of π_2 to θ when α was at different levels.

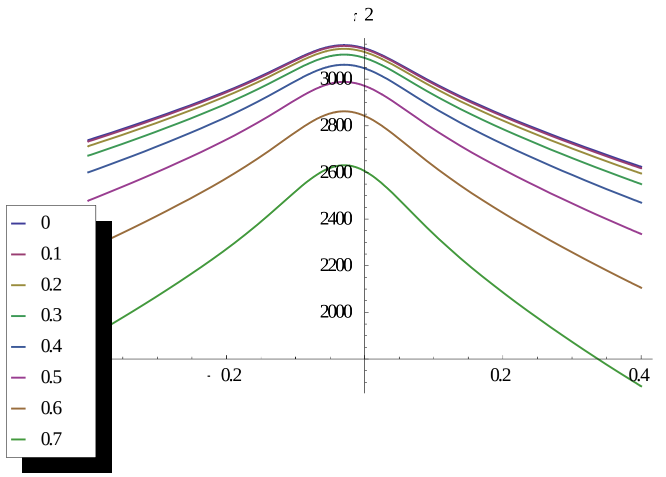


Figure 5.4 The change of π_2 according to the change of θ from -0.4 to 0.4

Figure 5.4 illustrated that π_2 had the same trend according to the change of θ when α was at different levels. What was more, when α was smaller than 0.7, the system with RFID had higher profit all the time, no matter what θ was. This was because that smaller α guaranteed the low discrepancy in transportation, which improved the profit.

Table 5.7 gives the summary of all possible annual profits of system without backorder but with RFID system (π_2) according to different α and θ. Table 5.8 and 5.9 show the improvements of profit π_2 with different α and θ respectively.

Table 5.7 The possible annual profits of π_2 with different α and θ (thousand dollars)

Inaccuracy Ratio (θ)	Discrepancy Ratio (α)							
	0	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**
-40%	2738.66	2732.9	2712.72	2671.94	2600.28	2479.19	2271.61	1891.05
-30%	2837.2	2831.98	2813.72	2776.81	2711.95	2602.36	2414.49	2070.06
-20%	2948.21	2943.61	2927.5	2894.96	2837.76	2741.12	2575.45	2271.72
-10%	3084.52	3080.67	3067.21	3040.03	2992.24	2911.5	2773.09	2519.34
0	3132.17	3128.59	3116.06	3090.75	3046.25	2971.08	2842.2	2605.92
10%	2983.28	2978.88	2963.45	2932.29	2877.51	2784.96	2626.31	2335.44
20%	2845.75	2840.58	2822.48	2785.91	2721.64	2613.04	2426.88	2085.58
30%	2729.15	2723.33	2702.96	2661.81	2589.49	2467.29	2257.81	1873.76
40%	2624.35	2617.95	2595.54	2550.28	2470.72	2336.29	2105.85	1683.37

Table 5.8 The possible improvements of annual profit π_2 with different α

Inaccuracy Ratio (θ)	Discrepancy Ratio (α)							
	0	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**
-40%	2738.66	2732.9	2712.72	2671.94	2600.28	2479.19	2271.61	1891.05
Improvement		0.21%	0.74%	1.53%	2.76%	4.88%	9.14%	20.12%
-30%	2837.2	2831.98	2813.72	2776.81	2711.95	2602.36	2414.49	2070.06
Improvement		0.18%	0.65%	1.33%	2.39%	4.21%	7.78%	16.64%
-20%	2948.21	2943.61	2927.5	2894.96	2837.76	2741.12	2575.45	2271.72
Improvement		0.16%	0.55%	1.12%	2.02%	3.53%	6.43%	13.37%
-10%	3084.52	3080.67	3067.21	3040.03	2992.24	2911.5	2773.09	2519.34
Improvement		0.12%	0.44%	0.89%	1.60%	2.77%	4.99%	10.07%
0	3132.17	3128.59	3116.06	3090.75	3046.25	2971.08	2842.2	2605.92
Improvement		0.11%	0.40%	0.82%	1.46%	2.53%	4.53%	9.07%
10%	2983.28	2978.88	2963.45	2932.29	2877.51	2784.96	2626.31	2335.44
Improvement		0.15%	0.52%	1.06%	1.90%	3.32%	6.04%	12.45%
20%	2845.75	2840.58	2822.48	2785.91	2721.64	2613.04	2426.88	2085.58
Improvement		0.18%	0.64%	1.31%	2.36%	4.16%	7.67%	16.36%
30%	2729.15	2723.33	2702.96	2661.81	2589.49	2467.29	2257.81	1873.76
Improvement		0.21%	0.75%	1.55%	2.79%	4.95%	9.28%	20.50%
40%	2624.35	2617.95	2595.54	2550.28	2470.72	2336.29	2105.85	1683.37
Improvement		0.24%	0.86%	1.77%	3.22%	5.75%	10.94%	25.10%

When comparing values in each row in Table 5.8(θ was constant), the profit decreased when the value of α increased. This was because the higher the value of α was, the more inventories would be lost during ordering process, which caused a lot of loss. As a result, the lower the profit was. In addition, when comparing the improvements in each row in Table 5.8, the larger the α was, the larger the improvement of the profit was with respect to 10% change of α. Especially when α decreased from 70% to 60%, the profit increased by more than 20%. Also, when comparing the improvements in each column in Table 5.8, it was said that when the absolute value of θ decreased, the profit improvements of every 10% change of α became smaller. This meant that the smaller the absolute value of θ was, the smaller the effect of α on profit was.

Table 5.9 The possible improvements of annual profit π_2 with different θ

Discrepancy Ratio (α)	Inaccuracy Ratio (θ)								
	-40%	-30%	-20%	-10%	0	10%	20%	30%	40%
0	2738.66	2837.2	2948.21	3084.52	3132.17	2983.28	2845.75	2729.15	2624.35
Improvement	3.60%	3.91%	4.62%	1.54%		4.99%	4.83%	4.27%	3.99%
1%	2732.9	2831.98	2943.61	3080.67	3128.59	2978.88	2840.58	2723.33	2617.95
Improvement	3.63%	3.94%	4.66%	1.56%		5.03%	4.87%	4.31%	4.03%
20%	2712.72	2813.72	2927.5	3067.21	3116.06	2963.45	2822.48	2702.96	2595.54
Improvement	3.72%	4.04%	4.77%	1.59%		5.15%	4.99%	4.42%	4.14%
30%	2671.94	2776.81	2894.96	3040.03	3090.75	2932.29	2785.91	2661.81	2550.28
Improvement	3.92%	4.25%	5.01%	1.67%		5.40%	5.25%	4.66%	4.37%
40%	2600.28	2711.95	2837.76	2992.24	3046.25	2877.51	2721.64	2589.49	2470.72
Improvement	4.29%	4.64%	5.44%	1.81%		5.86%	5.73%	5.10%	4.81%
50%	2479.19	2602.36	2741.12	2911.5	2971.08	2784.96	2613.04	2467.29	2336.29
Improvement	4.97%	5.33%	6.22%	2.05%		6.68%	6.58%	5.91%	5.61%
60%	2271.61	2414.49	2575.45	2773.09	2842.2	2626.31	2426.88	2257.81	2105.85
Improvement	6.29%	6.67%	7.67%	2.49%		8.22%	8.22%	7.49%	7.22%
70%	1891.05	2070.06	2271.72	2519.34	2605.92	2335.44	2085.58	1873.76	1683.37
Improvement	9.47%	9.74%	10.90%	3.44%		11.58%	11.98%	11.30%	11.31%

When comparing values in each row in Table 5.9 (α was constant), the profit went up to a highest value when θ increased from -40% to 0 and then went down when θ kept increasing to 40%. This was because the smaller the absolute value of θ was, the more accurate the recorded inventories were, which increased the accuracy of inventory recording. As a result, the profit increased due to the improved accuracy. When comparing the improvements in each column in Table 5.9, as the Discrepancy Ratio increased, the profit was more sensitive to the Inaccuracy Ratio. Especially when α was 70%, the profit increased by more than 10% by improving θ by 10%. Moreover, when comparing the improvements in each row in Table 5.9, when the absolute value of θ decreased, the profit improvements of every 10% change of θ became larger. This meant that the smaller the absolute value of θ was, the larger the effect of θ on profit was.

5.2.2 Backorders without RFID and backorders with RFID

For system with backorders but without RFID system, when Discrepancy Ratio (α) was 0.7 and Inaccuracy Ratio (θ) was -0.4, the profit of this system (π_3) = 1711.99; when $\alpha = 0.7$ and $\theta = 0.4$, $\pi_3^{'} = 1694.79$.

For system with backorders and RFID system (π_4), α and θ change within a certain range (see Table 5-6) due to RFID system.

5.2.2.1 π_4 changed according to α

Inaccuracy Ratio, θ, was also evaluated every 10 percent and at each level the change of profit π_4 was analyzed where Discrepancy Ratio, α, changed from 0 to 0.7.

When θ equaled 0.4, the change of π_4 to α is illustrated by Figure 5.5. Figure 5.5 had the same pattern as seen in Figure 5.1. The comparison of π_3 and π_4 according to the change of α had the same conclusion with the comparison of π_1 and π_2.

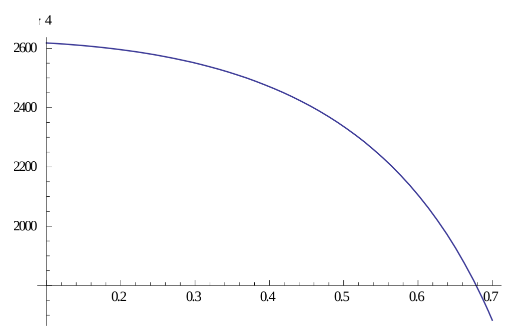


Figure 5.5 The change of π_4 to α when θ equals 0.4

Figure 5.6 gives the collection of all changes of π_4 to α when θ is at different levels. Figure 5.6 showed that, even though the increment speed of π_4 according to the decrease of α was different when θ was at different levels, the curves followed the same trend.

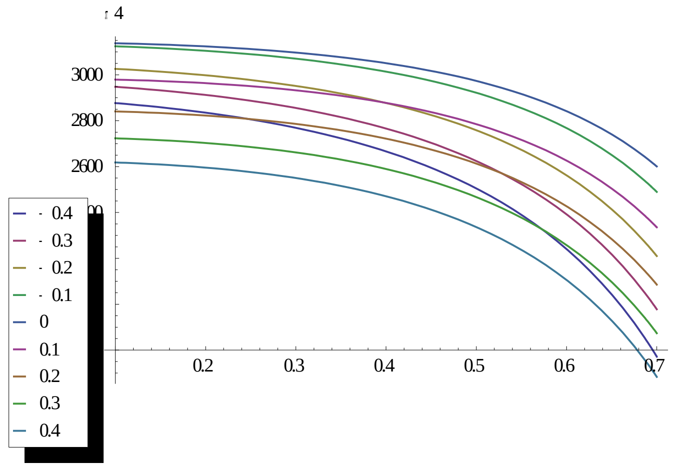


Figure 5.6 The change of π_4 according to the change of α from 0 to 0.7

5.2.2.1 π_4 changed according to θ

When analyzing the change of π_4 according to the change of θ from -0.4 to 0.4, α was evaluated every 10 percent. Figure 5.7 shows an example of the change of π_4 to θ when α equals 0.7.

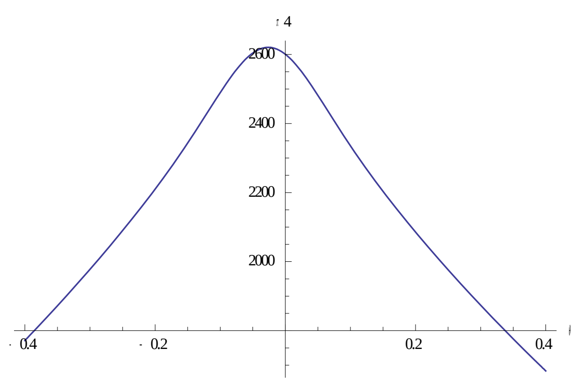


Figure 5.7 The change of π_4 to θ when α equals 0.7

Figure 5.7 had the same pattern with Figure 5-3. Thus, the comparison of π_3 and π_4 according to the change of θ had the same conclusion with the comparison of π_1 and π_2.

Figure 5.8 gives the collection of all changes of π_4 to θ when α is at different levels.

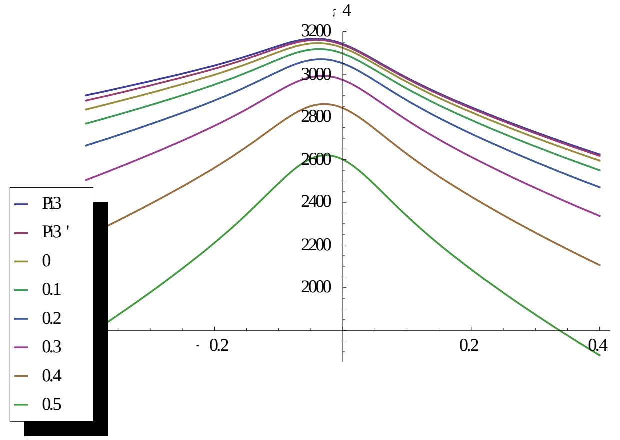


Figure 5.8 The change of π_4 according to the change of θ from -0.4 to 0.4

Figure 5.8 showed that, even though the curves were not axial symmetry when α equaled to larger values, the curves followed almost the same trend and had their maximum values when θ was around zero.

Table 5.10 The possible annual profit of π_4 with different α and θ (thousand dollars)

Inaccuracy Ratio (θ)	Discrepancy Ratio (α)							
	0	10%	20%	30%	40%	50%	60%	70%
-40%	2901.56	2873.52	2827.54	2755.92	2645.84	2474.23	2194.95	1700.3
-30%	2967.89	2945.01	2906.31	2844.99	2749.76	2600.27	2355.8	1921.41
-20%	3041.16	3024.21	2993.83	2944.27	2865.93	2741.55	2536.58	2170.47
-10%	3133.65	3123.44	3102.64	3066.71	3008.09	2913.17	2754.7	2469.17
0	3142.23	3137.39	3123.4	3096.36	3046.74	2971.77	2838.97	2596.49
10%	2983.65	2979.2	2963.72	2932.5	2877.64	2784.99	2626.19	2335.1
20%	2845.75	2840.58	2822.48	2785.91	2721.64	2613.04	2426.88	2085.58
30%	2729.15	2723.33	2702.96	2661.81	2589.49	2467.29	2257.81	1873.76
40%	2624.35	2617.95	2595.54	2550.28	2470.72	2336.29	2105.85	1683.37

Table 5.10 gives the summary of all possible annual profits of system with backorder and RFID system (π_4) according to different α and θ. This table gave the same results with Table 5.7.

5.3 Sensitivity Analysis

Service level, fixed order cost per order, penalty cost per backorder, and percentage of shortage that became backorder were the four parameters that were not constant. In this subsection, sensitivity analysis was conducted to each of them, studying their impacts on the annual profit.

5.3.1 Service level

Service level was reported to change from 30% to 99%. Each model was analyzed according to the change of service level.

5.3.1.1 No backorders without RFID

When Discrepancy Ratio (α) was 0.7 and Inaccuracy Ratio (θ) was -0.4, Figure 5.9 showed the change of profit of system 1 (π_1) along with the change of SL. It was easy to see from this figure that with the increase of service level, the profit increased as well. This made sense because when θ = -0.4 it meant that the recorded inventories were much more than the actual inventories on hand, which could not satisfy the customers when the service level was low. When service level increased, the correspondingly raised reorder point improved the customer satisfaction, as well as improved the profit.

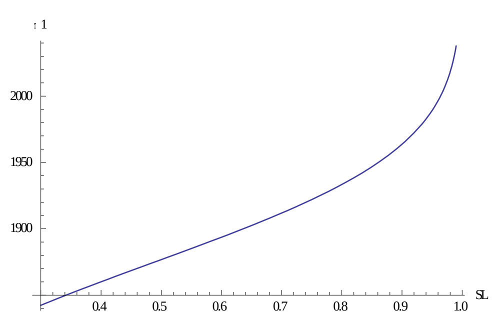


Figure 5.9 No backorders without RFID system (θ = -0.4) VS service level

When $\alpha = 0.7$ and $\theta = 0.4$, Figure 5.10 showed the change of π_1, which decreased when service level increased. This made sense because when $\theta = 0.4$ it meant that the actual inventories on hand were much more than the recorded inventories. The extra inventories could satisfy customers even when the service level was low. When the reorder point increased according to the increase of service level, these extra inventories became redundancy, causing the increase of inventory purchasing cost and holding cost.

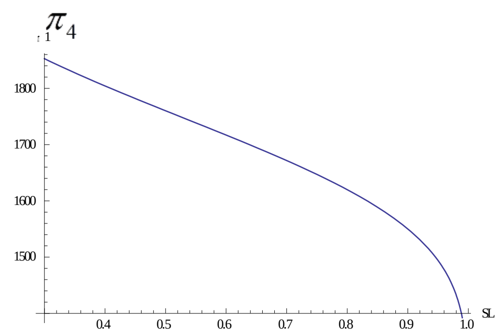


Figure 5.10 No backorders without RFID system (θ = 0.4) VS service level

5.3.1.2 No backorders with RFID

In order to study the influence of service level on the annual profit of system with RFID but without backorders, α and θ were chosen at certain levels respectively.

5.3.1.2.1 π_2 changes according to service level and α

Inaccuracy Ratio, θ, was fixed at different levels, as shown in the set of {-0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4}. At each level, the change of profit π_2, was analyzed where service level changed from 0.3 to 0.99 and α changed according to the set {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7}. Figure 5.11 showed the change of π_2 when θ equals -0.4. The horizontal axis represented service level while the vertical axis represented the annual profit. The eight curves in different colors refer to the change of π_2 at different α levels.

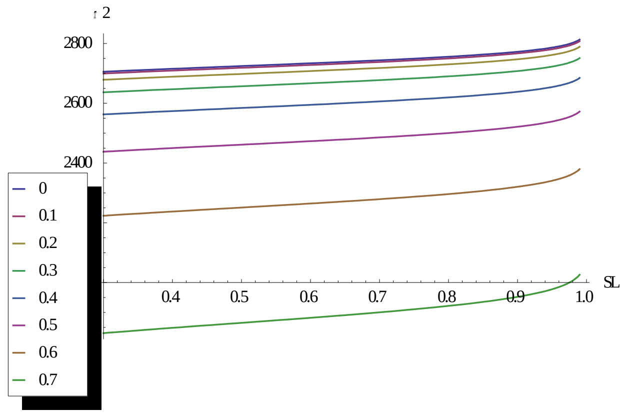


Figure 5.11 The change of π_2 VS the change of service level and α when θ = -0.4

Figures 5.12 to 5.19 showed the change of π_2 to the change of service level at different α levels when θ was at different level, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, and 0.4 respectively.

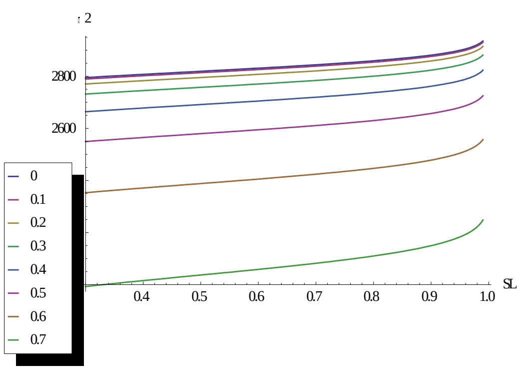


Figure 5.12 π_2 VS SL, α when θ=-0.3

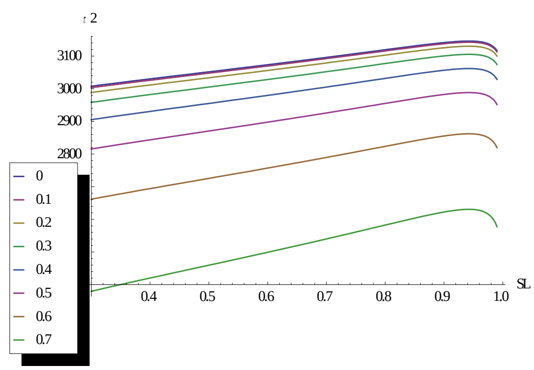


Figure 5.14 π_2 VS SL, α when θ=-0.1

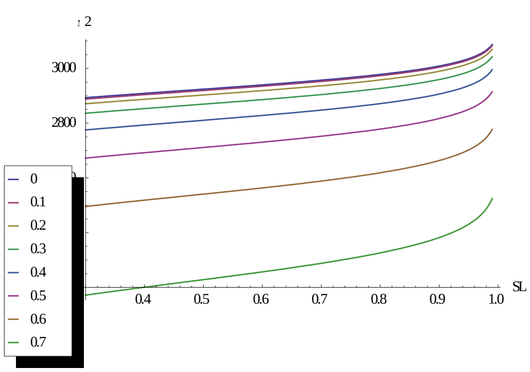


Figure 5.13 π_2 VS SL, α when θ=-0.2

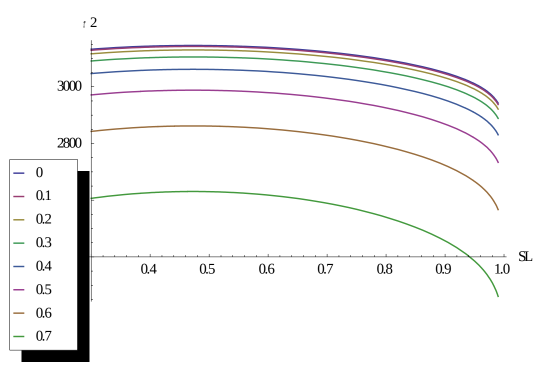


Figure 5.15 π_2 VS SL, α when θ=0

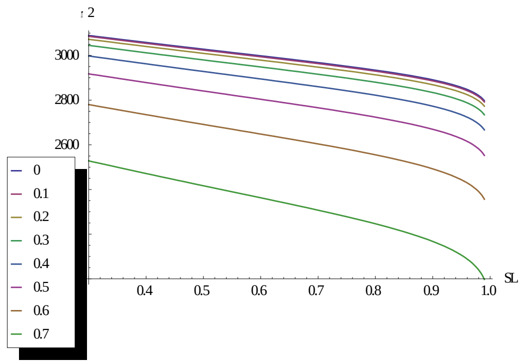


Figure 5.16 π_2 **VS SL,** α **when** θ**=0.1**

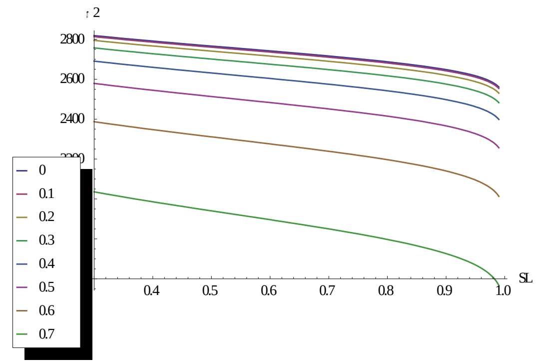


Figure 5.18 π_2 **VS SL,** α **when** θ**=0.3**

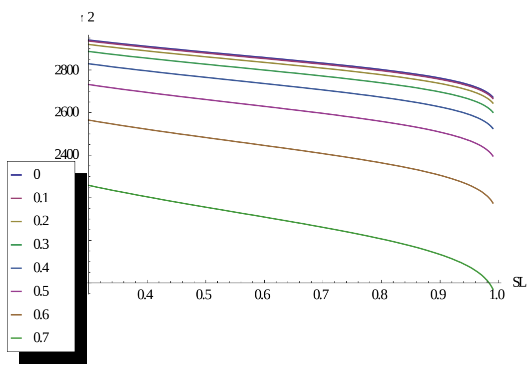


Figure 5.17 π_2 **VS SL,** α **when** θ**=0.2**

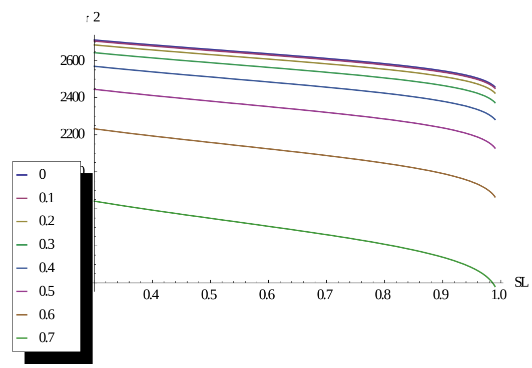


Figure 5.19 π_2 **VS SL,** α **when** θ**=0.4**

Conclusions that come from Figures 5.11 to 5.19 were summarized as below. First, when comparing the nine figures, the profit was in proportion to service level at each α level when θ was equal to or smaller than -0.2. On the contrary, when θ was equal to or larger than 0.1, the profit was in inverse proportion of service level at each α level. When θ was between -0.2 to 0.1, the profit followed a parabolic curve, increasing first then decreasing as service level increases. When service level was small, the correspondingly low reorder point cannot provide enough inventories to satisfy the customers. But when service level increased, the excess inventories became burdens and reduced the profit.

This conclusion also explained why the curves of the change of π_2 and π_4

according to the change of θ shifted to the left of the vertical axis. When the service level equaled to 0.65, it was good enough to satisfy the customers. Even if the inventories on hand were less than the recorded inventories (θ is less than 0), they could still meet the customer demand. When θ became larger, the profit reduced due to the extra inventories. In order to illustrate the effect of service level on the shift of the change of the profit to θ, service level was assumed to be 0.5 as an example. Then change of π_2 to θ when α was at different levels was showed in Figure 5.20, where the curves shifted to be axial symmetry comparing with Figure 5.4 where service level was 0.65.

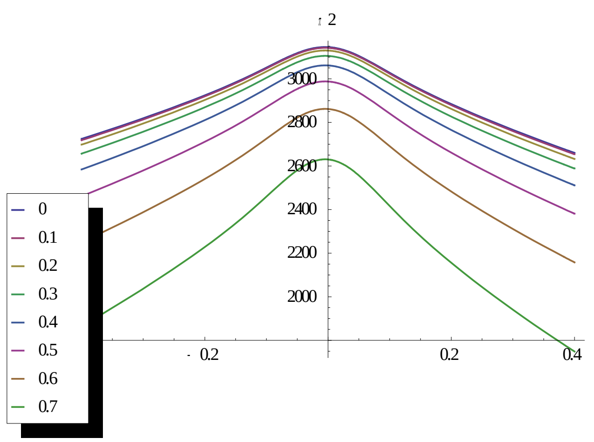


Figure 5.20 The change of π_2 to θ when service level is 0.5

Secondly, in each figure (θ was constant), it showed that the curve that represented α equaled 0 had the highest profit. The higher α was, the lower the probability of missing inventories during the shipment was, and the higher the profit was.

Thirdly, the comparison of all figures showed that the difference between the profits at different α levels (at the same service level) decreased when the absolute value of θ decreased. When θ was around 0, the system had the smallest difference between the profits at different α levels. This indicated that when the absolute value of θ became smaller, which also meant that the impact of θ reduced, the influence of α reduced.

5.3.1.2.2 π_2 changes according to service level and θ

Discrepancy Ratio, α , was fixed at different levels consistent with the set of {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7}. At each level, the change of profit π_2 was analyzed where service level changed from 0.3 to 0.99 and θ changed according to the set {-0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4}. Figure 5.21 showed the change of π_2 when α equaled 0. The horizontal axis represents service level while the vertical axis represents the annual profit. The nine curves in different colors refer to the change of π_2 at different θ levels. Figures 5.22 to 5.28 gave the change of π_2 to the change of service level and θ when α was at other levels.

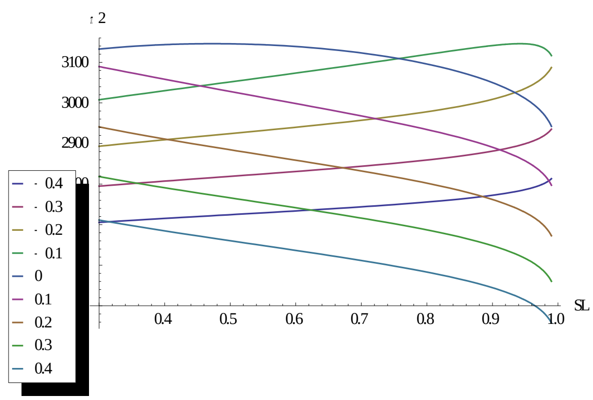


Figure 5.21 The change of π_2 VS the change of service level and θ when α = 0

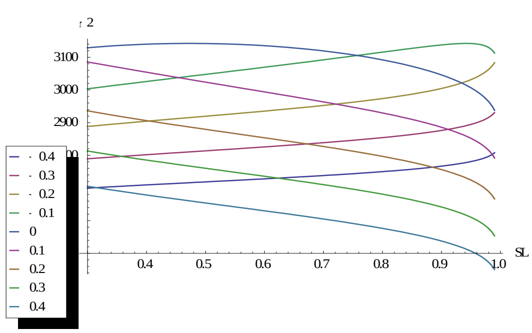


Figure 5.22 π_2 VS SL, θ when α =0.1

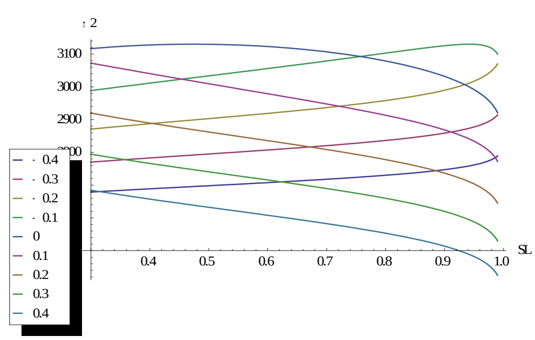


Figure 5.23 π_2 VS SL, θ when α =0.2

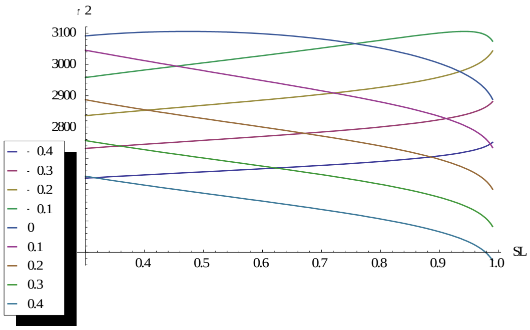


Figure 5.24 π_2 **VS SL,** θ **when** α **=0.3**

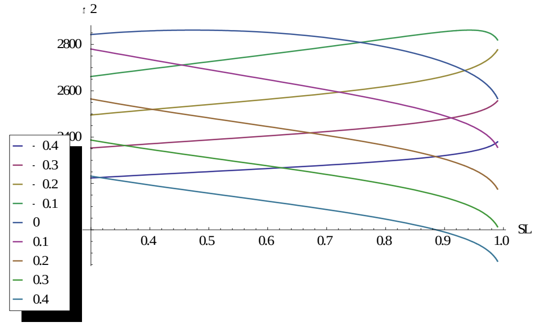


Figure 5.27 π_2 **VS SL,** θ **when** α **=0.6**

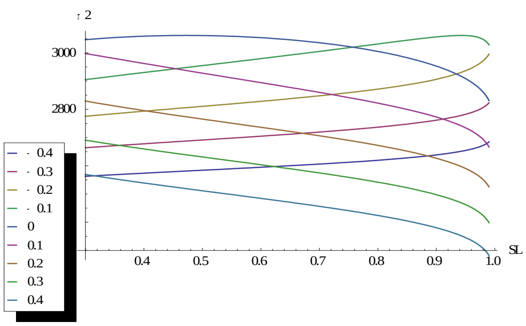


Figure 5.25 π_2 **VS SL,** θ **when** α **=0.4**

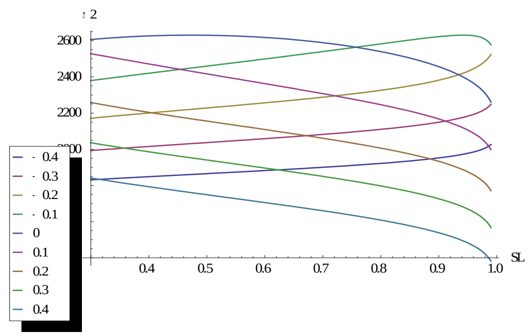


Figure 5.28 π_2 **VS SL,** θ **when** α **=0.7**

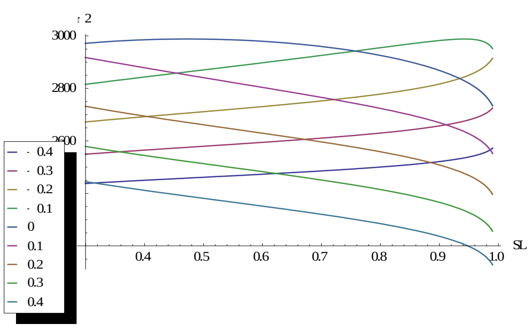


Figure 5.26 π_2 **VS SL,** θ **when** α **=0.5**

Conclusions that come from Figures 5.21 to 5.28 were summarized below. First, when comparing the eight figures, the profit increased when Discrepancy Ratio, α, reduced. This was due to the improved discrepancy ratio, which guaranteed the low disappearance of inventories during the shipment and therefore reduces the losses and increased the profit.

Secondly, in each figure (α was constant), it showed that the curves that represent

Inaccuracy Ratio, θ, equaled 0 and -0.1 had the highest profit. The lower the absolute value of θ was, the more accurate the inventory record was, and the higher the profit was.

Thirdly, in each figure, when θ was smaller than 0, the profit increased while service level increased at each α level. In contrast, when θ was equal to or larger than 0, the profit decreased while service level increased at each α level. This agreed with previous conclusion.

Fourthly, the comparison of all figures showed that the difference between the profits at different θ levels (at the same service level) decreased when α decreased. When α equaled 0, the system had the smallest difference between the profits at different θ levels. When α approached 0, which also meant that the impact of α reduced, the influence of θ reduced.

5.3.1.3 Backorders without RFID

Figure 5.29 showed the change of the profit of the system with backorder without RFID (π_3) along with the change of SL when α = 0.7 and θ = -0.4. Figure 5.30 showed the change of π_3 along with the change of SL when α = 0.7 and θ = 0.4. The trends are the same as the trends of the system with no backorders and no RFID. This means that whether the system contains backorders doesn’t affect the influence of service level on the profits.

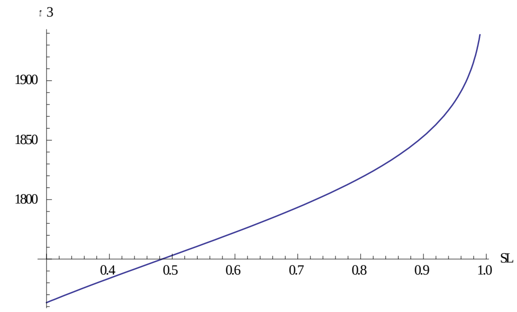


Figure 5.29 Backorders without RFID system (θ = -0.4) VS service level

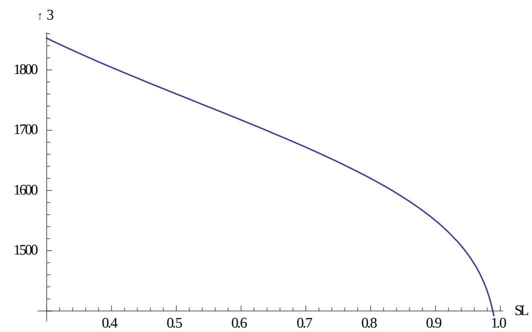


Figure 5.30 Backorders without RFID system (θ = 0.4) VS service level

5.3.1.4 Backorders with RFID

The same method was used to analyze the influence of service level on the annual profit of system with backorders and RFID, as the method used to analyze the system with RFID but without backorders.

In Appendix B, the first nine figures showed the change of profit π_4 according to the change of service level and Discrepancy Ratio α when Inaccuracy Raito θ was at different levels. The following eight figures illustrated the change of π_4 according to the change of service level and θ when α is at different levels. The same conclusions were summarized from these figures as the conclusions that were made for the system with RFID but no backorders. The effect of service level on the profit was not influenced by the existence of backorders.

5.3.2 Fixed order cost per order

Fixed order cost per order changed from 0 to 325.27 dollars. Figure 5.31 and 5.32 gave the change of the profit of system without backorder and RFID with respect to the change of fixed order cost per order. The horizontal axis referred to fixed order cost and vertical axis represented the profit π_1. It was easy to see that with the increase of fixed order cost, the profit reduced.

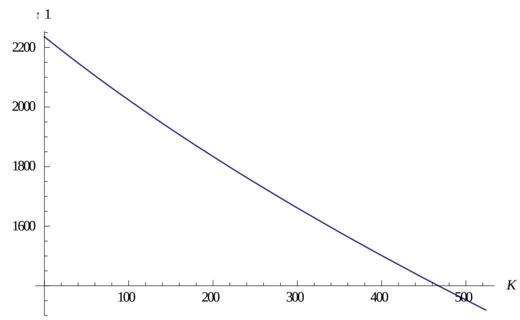


Figure 5.31 No backorders without RFID system (θ = -0.4) VS fixed order cost

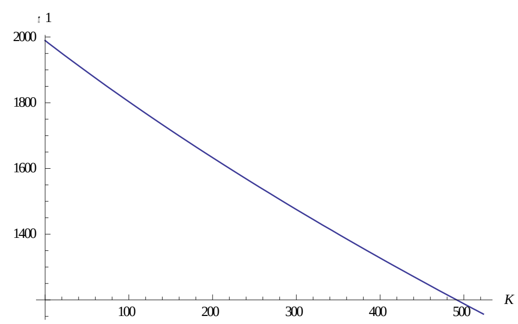


Figure 5.32 No backorders without RFID system (θ = 0.4) VS fixed order cost

Figure 5.33 and 5.34 gave the change of the profit of the system with backorder but without RFID (π_3) according to the change of fixed order cost per order. The figures also showed that the profit decreased while the fixed order cost increased.

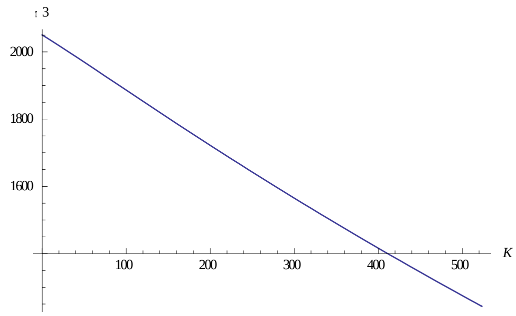


Figure 5.33 Backorders without RFID system (θ = -0.4) VS fixed order cost

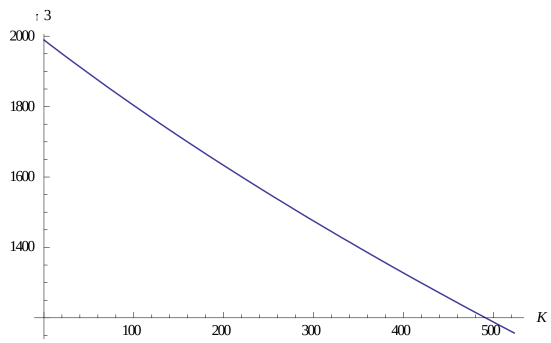


Figure 5.34 Backorders without RFID system (θ = 0.4) VS fixed order cost

Appendix C gave figures of the influence of fixed order cost on the profit of systems with RFID. All figures led to the conclusion that, no matter whether the system included backorder, and no matter how the two key factors α and θ changed, the total profit reduced when the fixed order increased.

5.3.3 Penalty cost per backorder

Both systems 1, without backorder and RFID, and system 2, without backorder but with RFID, had no backorders. As a result, in order to study the influence of penalty cost per backorder on the profit, this subsection separates the systems into two new groups. Group 1 contains systems 1 and 3. In this group, these two systems had no RFID. Group 2 contains systems 2 and 4 which had RFID system. For each group, the impact of penalty cost per backorder on the profit was analyzed. Penalty cost per backorder (p') varied from 0 to 46.75.

5.3.3.1 Group 1: No RFID without backorders VS with backorders

The two systems in this group did not have RFID. As a result, the two key factors α and θ had no effect on the profit.

When the Discrepancy Ratio (α) = 0.7 and Inaccuracy Ratio (θ) = -0.4, the profit of no RFID and no backorder system (π_1) = 1902.46. The profit of no RFID but with backorder system (π_3) changed with the change of penalty cost per backorder (p'). Let p' change from 0 to 46.75. Then the change of π_3 was reflected by Figure 5.35. With the increase of penalty cost per backorder, the profit decreased. This made sense because with all other parameters staying the same, the larger the penalty was, the larger the annual penalty cost was, and the smaller the profit was. The reason why π_3 was smaller than π_1 was that the discrepancy ratio α was really high in these no RFID systems. As a result, the existence of backorder, α percentage of which lost in the way of ordering, increased the loss of inventory and therefore reduced the profit. As a result, if there was no RFID and the inaccuracy ratio was negatively high, the inventory management system should not consider backorders.

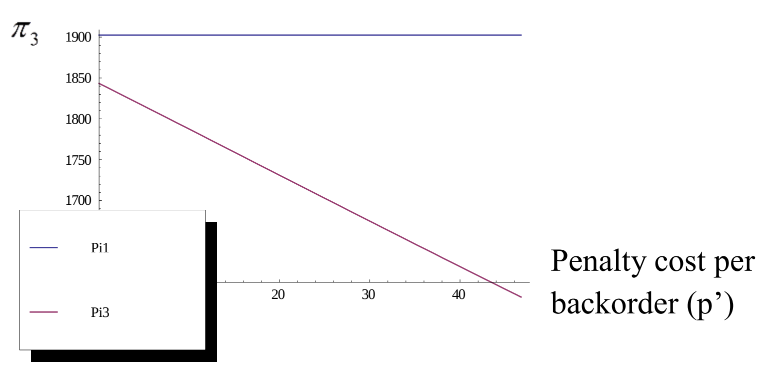


Figure 5.35 The change of π_3 according to the change of p' (θ = -0.4)

When $\alpha = 0.7$ and $\theta = 0.4$, $\pi_1^{'} = 1694.79$. Figure 5.36 showed the change of π_3 with the change of p'. It was easy to see from Figure 5.36 that π_3 was the same with $\pi_1^{'}$. This was because the inaccuracy ratio was positively high, which meant that the actual inventories on hand was far more than the recorded inventories. In this situation, the customer demand could be well satisfied and there was no shortage. As a result, the profit stayed the same no matter if there was backorder or not. When $\theta = 0.4$, the effect of penalty cost per shortage on the profit disappeared.

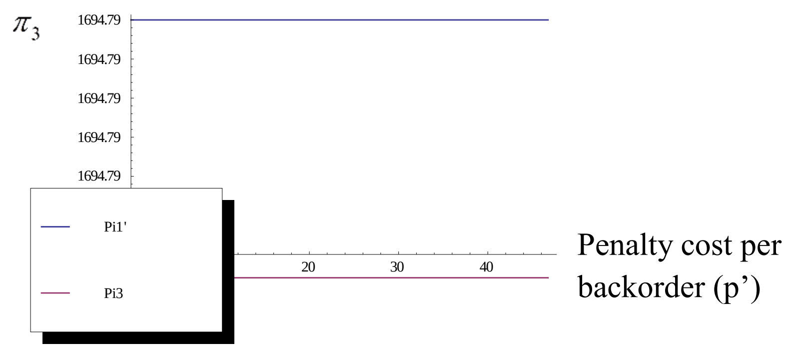


Figure 5.36 The change of π_3 according to the change of p' ($\theta = 0.4$)

5.3.3.2 Group 2: RFID without backorders VS with backorders

Both system 2 and 4 had RFID and their profits change according to both Discrepancy Ratio α and Inaccuracy Ratio θ. In order to simplify the analysis, α and θ were fixed at special values. Table 5.9 showed the combination of different levels of α and θ. Twenty five combinations were chosen for the following analysis. Number 1 to 9 in Figure 5.9 represented the twenty five combinations with different α and θ. For example, combination 1 meant α equaled 0 and θ equaled -40. For each combination, the profit of system 2 and the effect of penalty cost per back order on the profits of system 4 were analyzed.

Table 5.11 Different combination of α and θ

Discrepancy Ratio (α)	Inaccuracy Ratio (θ)				
	-0.4	**-0.2**	**0**	**0.2**	**0.4**
0	1	6	11	16	21
0.2	2	7	12	17	22
0.4	3	8	13	18	23
0.5	4	9	14	19	24
0.7	5	10	15	20	25

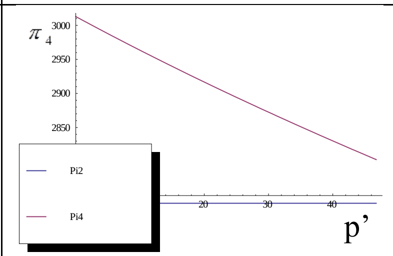
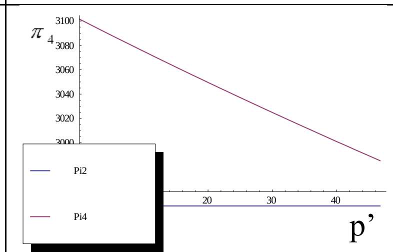
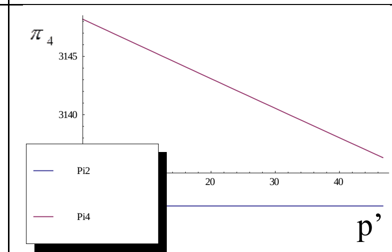
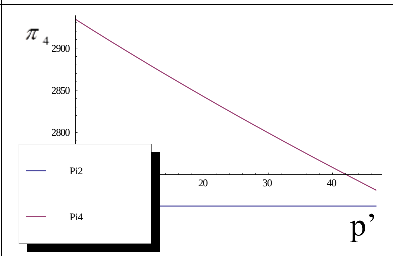
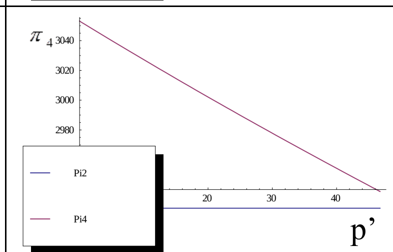
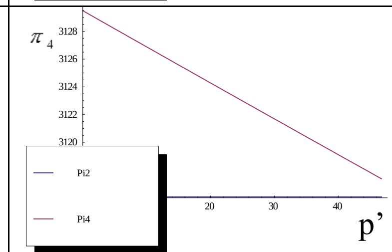
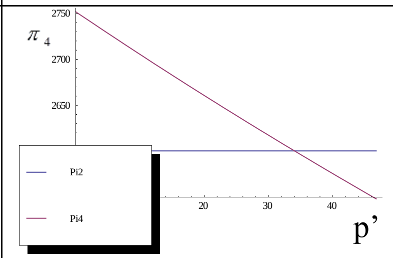
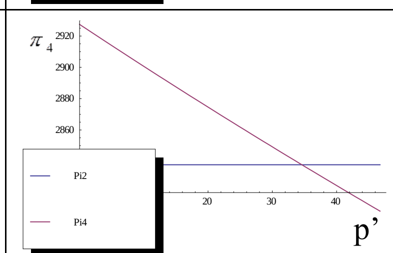
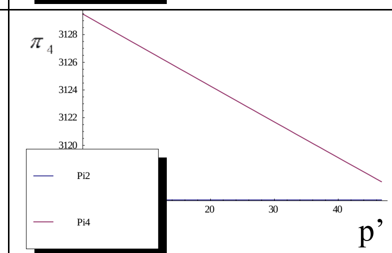
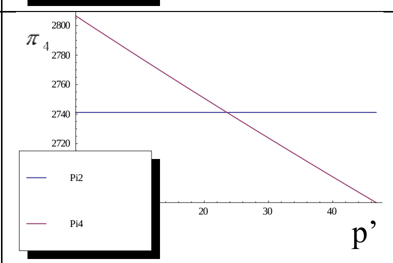
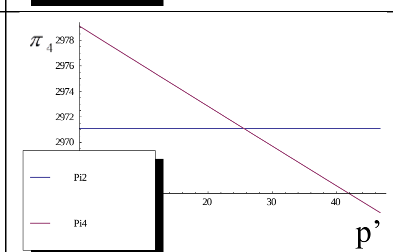
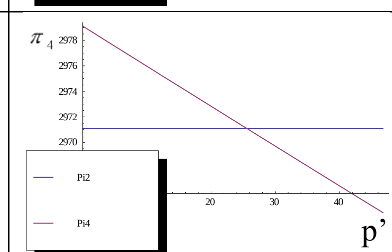
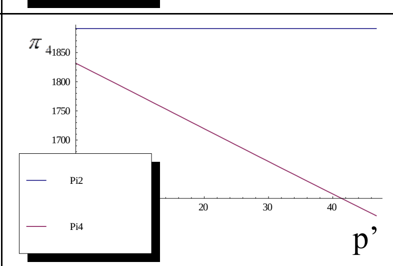
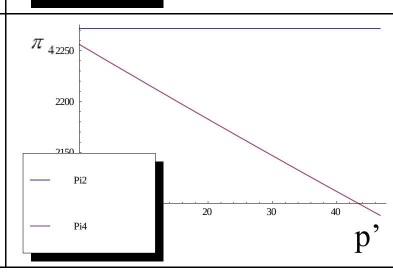
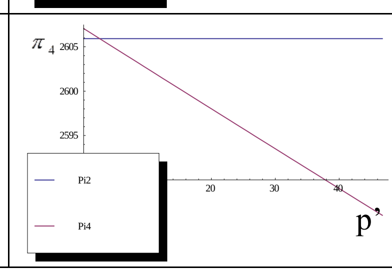
In Table 5.11, the effects of penalty cost per back order on the profits of system 4 for the first fifteen combinations were showed with different figures. For combination 16 to 25, since θ was larger than 0, which meant that actually on hand inventories could satisfy customer demand, and there was no shortage. Therefore, the profit of system 4 was the same with the profit of system 2 regardless of the penalty of backorder.

Conclusions that come from Table 5.12 were summarized as following. First, regardless of α and θ, with the increased penalty cost of each backorder the profit decreased for each combination.

Secondly, when comparing each column, it was easy to see that: when α was 0, π_4 was larger than π_2; when α was 0.7, π_4 was smaller than π_2; and when α was in between, the curves that represented π_4 and π_2 had a crossover. When α became larger, more inventories were lost during shipment. Thus, the loss due to these missing inventories offset the profit that came from the backorders. As a result, when α cannot be controlled within a low range, it was not a wise decision to make a backorder. Also, when α became larger, in order to make profit from backorders, the penalty cost per backorder should be small.

Thirdly, when comparing each row, it was easy to see that the slope of π_4 curve became smaller when θ increased from -0.4 to 0. With the increase of θ, the influence of penalty cost per backorder became smaller. When θ was closer to zero from -0.4, more inventories were available to meet the customer demand. Thus, there was less shortage. In this way, the effect of backorders was reduced.

Table 5.12 π_4 VS p' for each combination of α and θ

Alpha	Theta		
	-0.4	-0.2	0
0			
0.2			
0.4			
0.5			
0.7			

5.3.4 Percentage of shortage that became backorder

The four systems were divided into the same two groups when analyzing the percentage of backorder over shortage. The percentage of shortage that became backorder changed from 0 to 100%

5.3.4.1 Group 1: No RFID without backorders VS with backorders

When $\alpha = 0.7$ and $\theta = -0.4$, $\pi_1 = 1902.46$. The change of π_3 according to the change of percentage of shortage that became backorders (b) was shown by Figure 5.37. The figure showed that with the increase of b, the profit decreased. Since α was large, the larger the percentage of shortage became backorders, the more these backorders were lost during the shipment, and thus the less the profit was. The high discrepancy ratio was also the reason that made π_3 become smaller than π_1. Specially, when b equaled zero, which meant that there was no backorder, the profits of system 1 and 3 were the same.

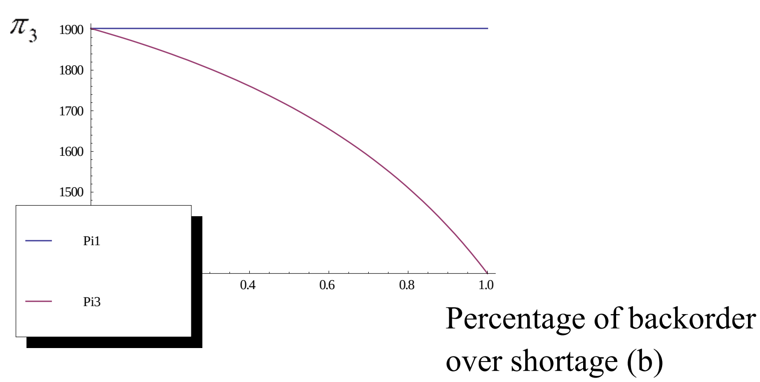


Figure 5.37 The change of π_3 according to the change of b (θ = -0.4)

When $\alpha = 0.7$ and $\theta = 0.4$, $\pi_1' = 1694.79$. Figure 5.38 showed the change of π_3 with the change of b. The figure showed that π_3 stayed the same with π_1'. With the high inaccuracy ratio, the inventories on hand to meet customer demand took away the effect of backorders on the profit.

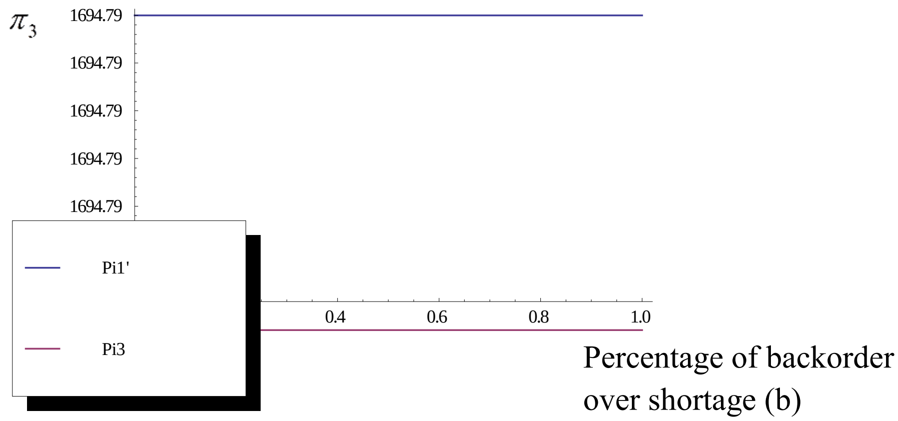


Figure 5.38 The change of π_3 according to the change of b (θ = 0.4)

5.3.3.2 Group 2: RFID without backorders VS with backorders

The same combinations of α and θ were used here as shown in Table 5-11. The last ten combinations were also discarded because of the large Inaccuracy Ratio θ. The influences of b on π_4 for each combination were analyzed, comparing with π_2, and were shown in Table 5.13.

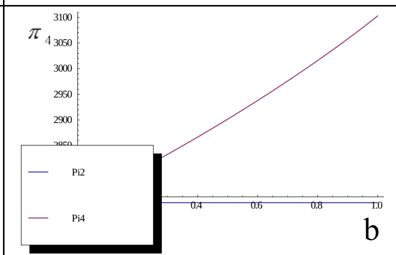
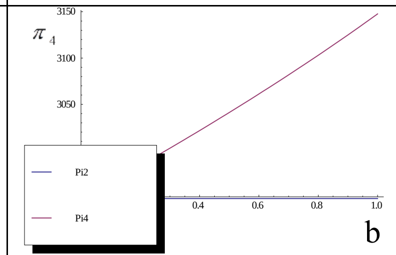
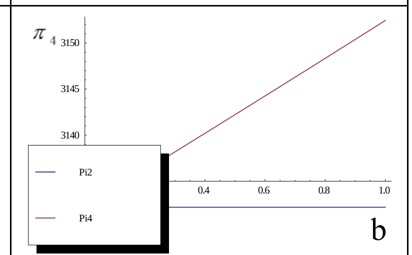
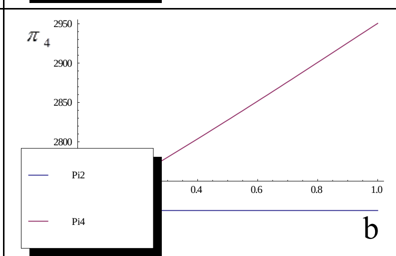
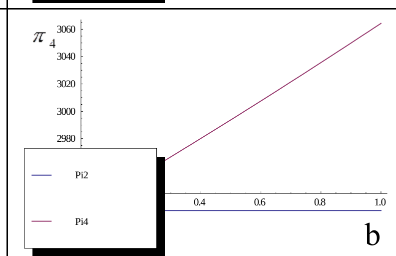
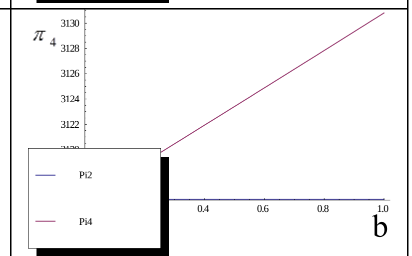
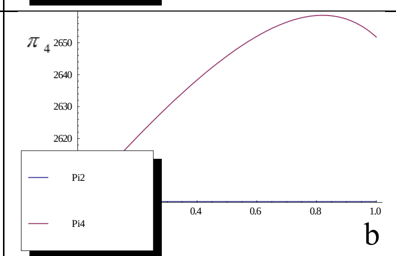
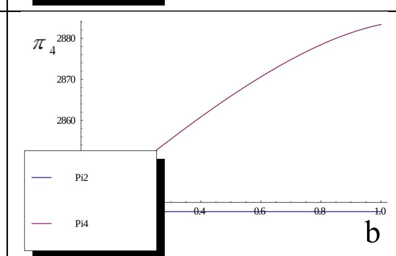
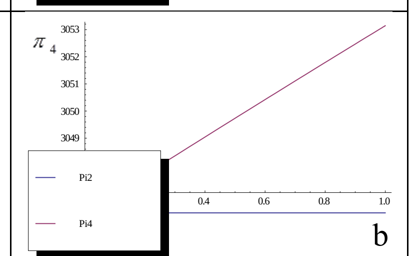
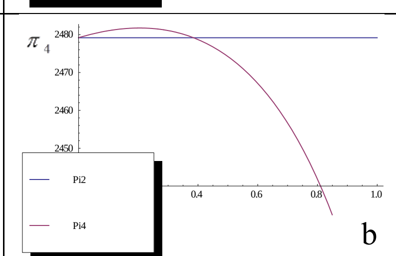
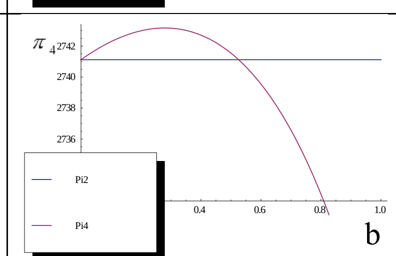
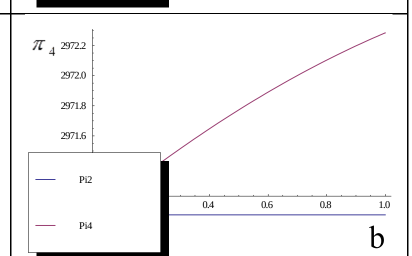
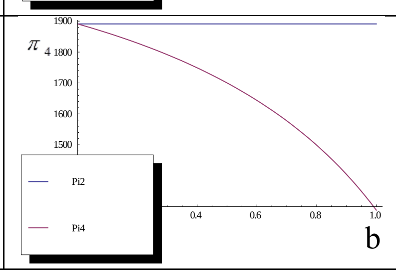
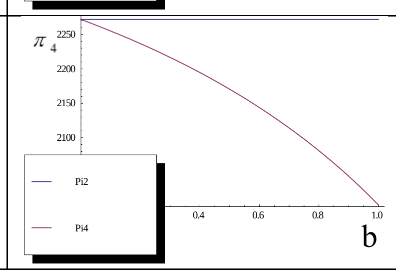
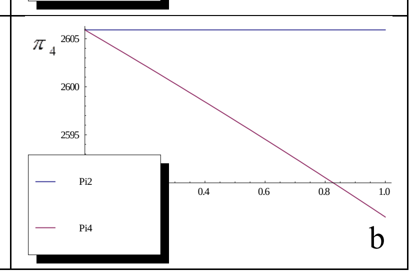
Conclusions that come from Table 5.13 were summarized as below.

First, when b was zero, π_4 equaled to π_2 regardless of α and θ because when b was zero, there was no backorder.

Secondly, when comparing each column, it was easy to see that: when α was small, π_4 increased while b increased; when α was large, π_4 deceased while b increased; and when α was in between, the curves that represents π_4 increased first and then decreased and have crossover with π_2. When α increased, more inventories were lost during the shipment. As a result, when the percentage of backorder became larger, the loss due to these missing inventories offset the profit caused by the backorders.

Thirdly, when comparing each row, the slopes of the π_4 curve became smaller. The increased θ reduced the effect of backorder percentage.

Table 5.13 π_4 VS b for each combination of α and θ

Alpha	Theta		
	-0.4	-0.2	0
0			
0.2			
0.4			
0.5			
0.7			

5.3 Chapter Summary

First this chapter set up the models by selecting typical inventory, which was women's clothes, collecting data from U.S. Census Bureau, and analyzing these data. Then each pair of models were compared to find out the effects of Discrepancy Ratio (α) and Inaccuracy Ratio (θ) on the annual profit by using Wolfram Mathematica 8.0. The comparison results were summarized by Table 5-7 and 5-8. Finally, sensitivity analyses were conducted with respect to four parameters, which were service level, fixed order cost per order, penalty cost per backorder, and percentage of shortage that became backorder. For each sensitivity analysis, the change of the parameter was combined with the change of α and/or θ to determine the joint effect on the annual profit for each model. Conclusions were made for each sensitivity analysis.

Chapter 6 Conclusion and Discussion

RFID was commercially introduced in the early 1980's and it became a mature technology with lots of advantages over other automatic identification technologies such as barcodes. The RFID market was already a multimillion dollar industry and its applications were limitless. A wide range of emerging applications in the RFID field were being improved, one of which was in retail inventory management system.

With the aim of increasing the annual profits, retailers needed to provide the right amount of goods to the right customer in time. This required high accuracy in inventory management. Problems that existed in current inventory management system were the discrepancy during the shipment in each reorder cycle and the inaccuracy between recorded and on hand inventories. RFID technology, which enhanced the visibility of goods tracking, improved the accuracy situation in retail inventory management system.

This research proposed two factors, Discrepancy Ratio (α) and Inaccuracy Ratio (θ). The Discrepancy Ratio referred to disappeared inventories the between the ordered quantity and actually arrived inventories, and the Inaccuracy Ratio represented the missing inventories between physical and recorded inventory. These two main factors reflected the anticipated effect of RFID on the inventory management. It is possible interventions, other than RFID, could also improve inventory management. All enhancements would however be expected to reduce the Discrepancy and Inaccuracy Ratio within the same ranges of changes expected to be made by RFID. With the application of RFID technology, α and θ could be reduced and the profit could be increased. The causes of the two factors and the possible improvements they brought to the profit of a retail inventory management system were listed in Table 2.1.

The purpose of this research was to compare inventory management systems with and without RFID with respect to the annual profit. Models were built on the basis of the basic (R, Q) model, where the annual profit was the objective function and the two main factors, α and θ, as well as other uncertain parameters, such as the service level, were the input variables. Two more factors that were considered in the profit model were backorders and Federal Income tax. The changes of these two factors were also reported to have influence on the profit of a retailer's inventory system. Retailers had high Federal Income tax due to their large and valuable inventories. The tax was usually ignored in previous research of inventory management. This research considered tax in the economic models to model the inventory management system more comprehensively.

According to the specific objective questions proposed in Chapter 2, Chapter 6 summarized the research in the following aspects.

1) The profit model development and optimization for retail inventory management systems at different RFID and backorder levels.
2) The profit model comparison according to Discrepancy Ratio and Inaccuracy Ratio.
3) The sensitivity analysis of the profit models with respect to service level, fixed order cost per order, penalty cost per backorder, and percentage of shortages that became backorders.

After summarizing the research work, research limitation and future work were provided at the end of this chapter.

6.1 Research Summary

6.1.1 Profit model development

In this research, the annual profit composed with annual sales and different annual costs. The profit model was presented as below.

Annual profit = Annual Sales – Annual Holding Cost – Annual Purchase Cost – Annual Ordering Cost – Annual Penalty Cost – Annual Federal Income Tax

Four mathematical models for annual profit were developed using optimized engineering economy equations with and without RFID and backorders. The four models were respectively: model 1, system without RFID and without backorders; model 2, system with RFID but without backorders; model 3, system without RFID but with backorders; and model 4, system with RFID and backorders. The profits of the four models were shown with equations 15, 17, 25, and 26 in Chapter 3. The four developed models expanded the inventory theories and the new formulations were the basis for conducting the research project.

In Chapter 4, the research determined the optimal solutions of the objective functions of the four models by finding the optimal values of the reorder point, R, and the reorder quantity, Q. The optimal values and solutions were given by Equations 27 to 38. The first objective was met and the relationship between profit and inventory management parameters was expressed in useable equations. The developed equations allows the calculation of profit using a known distribution of purchases with time but adjustable for levels of discrepancy in inventory totals, levels of inaccuracies in inventory levels, levels of penalty cost per backorder, levels of percentage of backorders over shortages, and levels of service.

In conclusion optimizing models that allow for investigating numerous scenarios related to the expected positive impacts of RFID, were developed, useable and provided useful insight into relationships between variables. Outputs of the models lead to an improved understanding of the relationships between visibility of inventory and annual profit.

6.1.2 Profit model comparison with respect to Discrepancy Ratio and Inaccuracy Ratio

Both α and θ were evaluated every 10% in this research. Discrepancy Ratio, α, changed from 0 to 0.7 and Inaccuracy Ratio, θ, changed from -0.4 to 0.4. The possible values of other parameters were listed in Table 5.6.

Model 1 and model 2 were compared because they both had no backorders. Model 1 had no RFID. The two factors, α and θ, stayed at their maximum values in this system. As a result, the profit of model 1 did not change. In model 2, with implemented RFID, α and θ changed and the profit changed accordingly.

Figure 5.2 and 5.4 illustrated the change of system 2 profit with the change of α and θ respectively. The two figures gave the result that only when θ was smaller than 0.3 and α was 0.7, the profit of model 1 was larger than the profit of model 2. Table 5.7 showed the numerical profits of corresponding to the two figures. The conclusions were: first, the profit decreased when the value of Discrepancy Ratio, α, increased. The total annual profit of women's clothes reached its maximum value as \$3132.17*1000 for one retail outlet when α was zero. Second, the larger the α was, the larger the improvements of the profit were with respect to a 10% change of α. Third, the profit increased firstly until reaching it maximum value when Inaccuracy Ratio, θ, was approximately zero and then decreased. The profit reached its maximum value as \$3132.17*1000 when θ was zero. Fourth, the larger α was, the more of an impact θ had on the profit.

In the retail practice, first, for retail inventory management systems with RFID, if θ was larger than or equal to 0.3 and α was equal to 0.7, the retail inventory management system do not need RFID.

Second, in order to improve annual profits of a retailer inventory management system with RFID, the Discrepancy Ratio and Inaccuracy Ratio should be reduced. The smaller the two factors were, the higher the profit. This is because the reduction of both Discrepancy Ratio and Inaccuracy Ratio increased the tracking visibility and accuracy, which reduced the lost and mistaken inventories and finally improved the profits. As a result, a retail inventory management system could increase its profit by reducing Discrepancy Ratio and Inaccuracy Ratio with RFID system. It may be possible to reduce the Discrepancy Ratio or Inaccuracy Ratio by methods other than RFID, but RFID was emphasized in this research because it has been reported that RFID has the capability to increase inventory visibility.

Third, when a retail inventory management system with RFID had a high Discrepancy Ratio, decreasing of Discrepancy Ratio resulted in large profit improvements. For example, when the Discrepancy Ratio was reduced from 0.7 to 0.6, the profit had more than a 20% increase. When a retail inventory management had a low Discrepancy Ratio, a decrease of Discrepancy Ratio resulted in small profit improvements. For example, when the discrepancy ratio was reduced from 0.1 to 0, the profit increased by 0.2%. This meant that the higher the Discrepancy Ratio was, the more its impact on the profit was. As a result, it was more effective to improve annual profit by reducing the Discrepancy Ratio when this ratio was high. Hence having the correct inventory visibility provides product availability to the customer and more can be sold thus making more profit.

Fourth, when a retail inventory management system with RFID had high absolute values of Inaccuracy Ratio, a decrease of the absolute value of Inaccuracy Ratio resulted in small profit improvements. For example, when the absolute value of Inaccuracy Ratio was reduced from 0.4 to 0.3, the profit had average of 6% increase. When a retail inventory management had low absolute value of Inaccuracy Ratio, a decrease of the absolute value of inaccuracy ratio resulted in large profit improvements. For example, when the Inaccuracy Ratio was reduced from 0.1 to 0, the profit increased by more than 10%. This meant that the lower the absolute value of Inaccuracy Ratio was, the more its impact on the profit was. As a result, it was more effective to improve annual profit by reducing the absolute value of Inaccuracy Ratio when this value was low. This was because of the definition of Inaccuracy Ratio, which was based on the recorded inventories but not actual inventories. If Inaccuracy Ratio was defined as the percentage of mistaken recorded inventories over actual inventories, the result would be that it was more effective to improve annual profit by reducing the absolute value of Inaccuracy Ratio when the absolute value was high.

Fifth, Discrepancy Ratio and Inaccuracy Ratio had effects on each other. When one of the two factors was large, the effect of the other one on annual profits was large. Therefore, a retail management system should focus on reducing one ratio if it had difficulties in controlling both ratios.

Model 3 and model 4 were compared because they both had no backorder. The values of α and θ stayed at their maximum values in system 3 without RFID and changed in system 4 with RFID.

Figure 5.6 and 5.8 summarized the change of system 4 profit with respect to of α and θ, and Table 5.8 listed the different outcomes of profit with different α and θ. The same conclusions were obtained when comparing of models 3 and 4 as was found in comparing models 1 and 2.

6.1.3 Sensitivity analysis

6.1.3.1 Service level

For systems without backorders or RFID, the profit increased with the increase of service level when Inaccuracy Ratio, θ, was -0.4, and the profit decreased while service level increased when θ was 0.4. Profit was low because when θ = -0.4, the actually on hand inventories were too small to satisfy the customers when the service level was low. When service level increased, the reorder point was raised and the customer satisfaction was improved then the profit increased. Conversely, when θ = 0.4, too much inventory could satisfy customers even when service level was low. The extra inventories became redundancy and increased inventory purchasing cost and holding cost, resulting in a profit decrease.

For a system without backorder but with RFID, the joint effects of service level as well as α and θ were illustrated by Figures 5.11 to 5.28. The conclusions were as following. First, when θ was smaller than 0, the profit was proportional to the service level at each α level; when θ was larger than 0, the profit was inversely proportional to the service level. This was because when θ was smaller than 0, the smaller amount of on hand inventories could not satisfy the customers. With the increase of service level, the increased reorder point guaranteed the customer satisfactions and then the profit increased. On the contrary, when θ was larger than 0, excessive on hand inventories caused loss in profit due to the increased purchasing and holding costs of the excessive inventories. The higher the service level was, the more excessive inventories were, the more the purchasing and holding costs were, and the lower the profit was. Second, when the value of θ reduced, the influence of α decreased. Third, the value of α reduced, the influence of θ decreased. The last two results were the same with fifth result from the profit model comparison with respect to Discrepancy Ratio and Inaccuracy Ratio, which was that when one of Discrepancy Ratio and Inaccuracy Ratio was large, the effect of the other one on annual profits was large.

For system with backorder but without RFID and system with backorder and RFID, the conclusions were the same as previous two systems separately. This meant that the effect of service level on the profit was not influenced by the existence of backorders.

6.1.3.2 Fixed order cost per order

Figure 5.31 to 5.34 as well as figures in Appendix C gave the trends of profits of different models according to the change of fixed order cost per order. It came to the conclusion that regardless of backorder as well as α and θ, the annual profit reduced when the fixed order increased. This was because the higher the fixed order cost per order, the more the annual fixed order cost was, and the lower the annual profit was.

6.1.3.3 Penalty cost per backorder

System 1 and system 3 were compared because none of them had RFID. The profit of system 1 stayed constant when the penalty cost per backorder changed because it did not include backorders. The profit of system 2 decreased with the increase of penalty cost per backorder when θ = -0.4, and stayed the same with the profit of system 1 when θ = 0.4. This was because when θ = 0.4, there were enough inventories on hand to meet the customer satisfaction and there was no backorder. The effect of backorders disappeared.

System 2 and system 4 were compared because both of them had RFID. In order to determine the joint effects of both penalty cost per backorder and RFID (α and θ), α and θ were divided into different levels, as shown in Table 5.9. Table 5.10 gave the profit collection of both system 2 and 4 according to the change of penalty cost per backorder at each combination of α and θ. Conclusions were made as following. First, the increased penalty cost per backorder reduced the profits regardless of α and θ. This was because the higher the penalty cost per backorder was, the higher the annual penalty cost was, and then the lower the annual profit was. Second, the larger the value of α was, the higher the possible that the profit without RFID was larger than the profit with RFID would be. This was because larger α resulted in more losing inventories during shipment. Thus, the loss due to these missing inventories offset the profit that came from the backorders. As a result, when α was larger, the profit with RFID became even smaller than the profit without RFID. Third, when θ increased from -0.4 to 0, the influence of penalty cost per backorder became smaller. This was because when θ increased from -0.4 to 0, on hand inventories increased. The increased inventories resulted in fewer shortages. As a result, the effect of penalty cost per backorder decreased.

The results came to the following practical conclusions. First, when a retail inventory management system with RFID had a discrepancy ratio equal to 0.7, this system should not have backorders no matter how much the penalty cost per backorder was.

Second, when a retail inventory management system with RFID had a positive Inaccuracy Ratio, this system would make the same profit no matter it had backorders or not.

Third, for a retail inventory management system with high negative Inaccuracy Ratio, for example -0.4, the Discrepancy Ratio should be carefully control under 0.2 or the penalty cost per backorder should be less than average of $25. Otherwise, this retail inventory management system should not have backorders.

Third, for a retail inventory management system with low negative Inaccuracy Ratio, for example -0.1, the system would make more profit if it had backorders when the Discrepancy Ratio could be controlled under 0.5 or the penalty cost per backorder could be controlled under $30. Otherwise, this retail inventory management system should not have backorders.

Fourth, if a retail inventory management system with RFID had high negative Inaccuracy Ratio, reduction of every dollar in penalty cost per backorder resulted in higher profit improvement. The smaller the negative Inaccuracy Ratio was, the smaller profit improvement would be made by reducing penalty cost per backorder.

6.1.3.4 Percentage of shortage that became backorder

System 1 and 3 were compared, neither of which had RFID. The profit of system 1 stayed the same because it had no backorder. The profit of system 2 reduced according to the increase of percentage of backorders when θ = -0.4, with the maximum value equal to the profit of system 1. When θ = 0.4, the profit of system 2 equaled the profit of system 1.

System 2 and 4, both having RFID, were compared. The combination of α and θ in Table 5.9 were used for this comparison. Table 5.11 listed the profit collection of both system 2 and 4 according to the change of percentage of backorders at each combination of α and θ. Three conclusions were made from this table. First, the two profits were the same when the percentage of backorders was zero. Second, according to the increase of percentage of backorders, smaller α made the profit of system 4 increase and larger α made this profit decease. This was because more inventories were lost when α was large. The increased percentage of backorder resulted in more loss due to these missing inventories, which offset the profit caused by the backorders. Third, the effect of percentage of backorders on the profit was reduced when θ was increasing.

The result came to the following practical conclusions. First, when a retail inventory management system with RFID had a high Discrepancy Ratio, for example, more than 0.5, the percentage of backorders over shortages should be reduced to increase the annual profit. If the Discrepancy Ratio of this system was controlled under 0.5, the percentage of backorders over shortages should be increased to improve the annual profit. This was because when Discrepancy Ratio was larger than 0.5, this high Discrepancy Ratio also existed in backorders. A large number of backorders were lost during shipments. As a result, in order to improve the profit, the number of backorders should be reduced. On the other side, when Discrepancy Ratio was smaller than 0.5, less backorders were lost in reordering process. The loss due to the lost backorders could be offset by the improvements of profit caused by the backorders. As a result, the profit increased when the percentage of backorders increased.

Second, for a retail inventory management system with high negative Inaccuracy

Ratio, for example -0.4, the Discrepancy Ratio should be control under 0.4 or the percentage of backorders over shortages should be less than average of 70%. Otherwise, this retail inventory management system should not have backorders.

Third, for a retail inventory management system with low negative Inaccuracy Ratio, for example -0.1, the system would make more profit if it had backorders when the Discrepancy Ratio could be control under 0.6. Otherwise, this retail inventory management system should not have backorders.

Fourth, if a retail inventory management system with RFID had high negative Inaccuracy Ratio, reduction of every percentage in percentage of backorder over shortages resulted in high profit improvement. The smaller the negative Inaccuracy Ratio was, the smaller profit improvement would be made by reducing percentage of backorder. The reduced negative Inaccuracy Ratio resulted in more inventories on hand, which reduced the shortages. As a result, the effect of percentage of backorders over shortages on the profit decreased.

6.2 Research Limitation and Future Work

There were five limitations in this research work.

1) Labor cost was ignored. In reality, labor cost in retail would impact annual profit depending on the amount reduced labor costs associated with implemented RFID systems.
2) The initial cost of RFID system was ignored, which impacted on the profit at the beginning of the implementation a lot.

3) In sensitivity analysis of one parameter, other uncertain parameters were set to be constant to simplify the analysis. As a result, joint effects of these parameters were not studied.
4) Women's clothes were not the only goods that satisfied the assumptions of this research. Other types of goods might result in different outcomes and conclusions.
5) The pattern of buying women's clothes in this research was based upon 2010 US Census data and profit amounts would change if buying patterns change.

The future work could be developed in the following aspects according to the five limitations. First, labor cost and the initial cost of RFID system should be taken into account in the profit model. Second, joint effect of different critical parameters should be analyzed. Third, other types of goods should be discussed to find the effect of RFID on the inventory management system of that type of inventory. Fourth, results should be updated with newest data to model the changed buying patterns and to obtain possible profits according to latest buying patterns.

Reference

[1] Y. Kang, “Information inaccuracy in inventory systems: stock loss and stockout,” *IIE transactions*, 2005.

[2] H. Clampitt, “RFID certification textbook,” 2007.

[3] B. Konsynski, “Developments in practice x: Radio frequency identification (rfid)-an internet for physical objects,” *Communications of the Association for*, 2003.

[4] H. Jung and F. Chen, “Trends in supply chain design and management: technologies and methodologies,” 2007.

[5] S. Jiang, “Economic Evaluation of RFID Implementation at a Retail Store,” 2009.

[6] L. Zhang, “Analytic Hierarchy Process (AHP) Approach and Fuzzy AHP Approach in Multiple-criteria Decision Making for Commercial Vehicle Information Systems and Networks,” 2010.

[7] J. Landt, “The history of RFID,” *Potentials, IEEE*, 2005.

[8] B. Nath and F. Reynolds, “RFID technology and applications,” *Pervasive Computing, IEEE*, 2006.

[9] P. Ranky, “An introduction to radio frequency identification (RFID) methods and solutions,” *Assembly Automation*, 2006.

[10] G. Oh, D. Kim, and S. Kim, “A quality evaluation technique of RFID middleware in ubiquitous computing,” *Technology, 2006. ICHIT’*, 2006.

[11] H. Clampitt, “The RFID certification textbook,” 2007.

[12] A. Durfee, “RFID, MEMS, and their Applications in the Field of Construction,” 2002.

[13] G. Chappell, D. Durdan, G. Gilbert, and L. Ginsburg, “Auto-ID in the box: the value of Auto-ID technology in retail stores,” *Auto-ID Center*, 2003.

[14] G. Chappell, L. Ginsburg, P. Schmidt, and J. Smith, “Auto-ID on demand: the value of auto-ID technology in consumer packaged goods demand planning,” *Retrieved*, 2002.

[15] G. Chappell, D. Durdan, G. Gilbert, and L. Ginsburg, “Auto-ID on delivery: the value of Auto-ID technology in the retail supply chain,” *Auto-ID*, 2002.

[16] K. Alexander, T. Gilliam, K. Gramling, and C. Grubelic, “Applying auto-ID to reduce losses associated with shrink,” *Auto ID Center*, 2002.

[17] A. Bensoussan and M, “Optimality of base-stock and (s, S) policies for inventory problems with information delays,” *Journal of optimization theory*, 2006.

[18] N. DeHoratius and A. Mersereau, “Retail inventory management when records are inaccurate,” *Operations Management*, 2008.

[19] E. Fleisch, “Inventory inaccuracy and supply chain performance: a simulation study of a retail supply chain,” *International journal of production economics*, 2005.

[20] G. Gaukler and R. Seifert, “Item- Level RFID in the Retail Supply Chain,” *Production and*, 2007.

[21] A. D. Kok and K. V. Donselaar, “A break-even analysis of RFID technology for inventory sensitive to shrinkage,” *International Journal of*, 2008.

[22] E. Europe and U. of Leicester, “Shrinkage: Introducing a collaborative approach to reducing stock loss in the supply chain,” 2001.

[23] N. DeHoratius, “Inventory record inaccuracy: an empirical analysis,” *Management Science*, 2008.

[24] A. Niemeyer and M. Pak, “Smart tags for your supply chain,” *The McKinsey Quarterly*, 2003.

[25] C. Booth-Thomas, “The see-it-all chip,” *Time*, 2003.

[26] “The best thing since the bar-code,” *Economist*, 2003.

[27] A. SAP, “SAP Auto-ID Infrastructure,” 2005.

[28] A. Kearney, “RFID/EPC: Managing the Transition,” *Research analysis report*, 2004.

[29] T. Pisello, “The three Rs of RFID: Rewards, risk and ROI,” *Technology-Evaluation. com*, 2004.

[30] A. Atali and H. Lee, “If the inventory manager knew: Value of visibility and RFID under imperfect inventory information,” 2009.

[31] M. Group, “RFID: benefits for retailers and suppliers through effective deployment and rool-out,” 2004.

[32] R. M. Hardgrave, B. C., M. Waller, "Does RFID reduce out of stock? A preliminary analysis." .

[33] R. Clark, "Inventory tops RFID benefits list in web-based pool," *Using RFID. com*, 2004.

[34] J. Johnson, "METRO reduces out of stocks with RFID; Is Wal-Mart next," *DC Velocity*, 2005.

[35] R. J. Tersine, *Material management and inventory system*. North Holland, 1976.

[36] D. Hartman, "Rules on merchandise inventory taxes." [Online]. Available: http://www.ehow.com/info_8587528_rules-merchandise-inventory-taxes.html.

[37] K. M. Clark, "How does inventory affect taxes." [Online]. Available: http://www.ehow.com/how-does_4796980_inventory-affect-taxes.html.

[38] "Inventory tax law & legal definition." [Online]. Available: http://definitions.uslegal.com/i/inventory-tax/.

[39] "How to account for inventory for income tax purposes." [Online]. Available: http://www.googobits.com/articles/2400-how-to-account-for-inventory-for-income-tax-purposes.html.

[40] J. Spiceland and J. Sepe, "Intermediate Accounting," 2004.

[41] H. Lee, "Unlocking the value of RFID," *Production and Operations Management*, 2007.

[42] Y. Rekik and E. Sahin, "Analysis of the impact of the RFID technology on reducing product misplacement errors at retail stores," *International Journal of Production*, 2008.

[43] J. Visich, S. Li, and B. Khumawala, "Empirical evidence of RFID impacts on supply chain performance," *International Journal of*, 2009.

[44] D. Montgomery and M. Bazaraa, "Inventory models with a mixture of backorders and lost sales," *Naval Research*, 1973.

[45] M. Romney and P. Steinbart, "Accounting information systems," 2000.

[46] "Monthly & Annual Retail Trade." [Online]. Available: http://www.census.gov/retail/.

[47] "About the Monthly Retail Trade Survey." [Online]. Available: http://www.census.gov/retail/mrts/about_the_surveys.html.

[48] "Income, Expenditures, Poverty, & Wealth: Consumer Expenditures." [Online]. Available: http://www.census.gov/compendia/statab/cats/income_expenditures_poverty_wealth/consumer_expenditures.html.

[49] L. L. Chao, *Statistics: Methods and Analysis*. McGraw-Hill, 1969.

[50] "Identifying the Distribution of Your Data." [Online]. Available: http://www.minitab.com/en-US/training/tutorials/accessing-the-power.aspx?id=1706.

[51] H. Mattila, "Merchandising strategies and retail performance for seasonal fashion products," *Acta Universitatis Lappeenrantaensis*, 1999.

[52] "Inventory Optimization: Retail Strategies for Eliminating Stock-Outs and Over-Stocks." [Online]. Available: http://apparel.edgl.com/reports/Inventory-Optimization--Retail-Strategies-for-Eliminating-Stock-Outs-and-Over-Stocks64377.

[53] S. Timme, "The real cost of holding inventory," *MANAGEMENT REVIEW, V. 7, NO. 4 (*, 2003.

[54] "Retail Margin And Markup Table." [Online]. Available: http://www.csgnetwork.com/marginmarkuptable.html.

[55] "Strategy and Implementation Summary." [Online]. Available: http://www.bplans.com/womens_clothing_boutique_business_plan/strategy_and_implementation_summary_fc.php.

[56] V. Aguirregabiria, "The dynamics of markups and inventories in retailing firms," *The Review of Economic Studies*, 1999.

[57] J. ZAPPONE, "Inventory Theory," 2006.

[58] "Application of RFID radio frequency in carton production of recognition technology." [Online]. Available: http://printtechskill.com/?action-viewnews-itemid-26382.

[59] Quantria Strategies, "Effective Federal Income Tax Rates Faced by Small Businesses in the United States," *Small Business Research Summary*.

Appendix A

Table A.1 Decomposition of customer demand in thousand persons (year 1992-2009)

SERIES	ORIGINAL	TREN	DETR	SEAS	DESE	FITS	RESI
1	11.63	14.95	0.78	0.75	15.52	11.20	0.43
2	12.37	14.97	0.83	0.79	15.67	11.81	0.55
3	14.92	14.99	1.00	1.01	14.79	15.12	-0.20
4	16.55	15.01	1.10	1.03	16.11	15.42	1.13
5	17.10	15.02	1.14	1.06	16.12	15.94	1.16
6	15.05	15.04	1.00	0.97	15.48	14.63	0.42
7	14.74	15.06	0.98	0.89	16.49	13.46	1.27
8	16.51	15.08	1.09	0.98	16.85	14.77	1.73
9	15.90	15.10	1.05	0.95	16.66	14.41	1.49
10	17.11	15.12	1.13	1.00	17.06	15.16	1.95
11	18.29	15.14	1.21	1.06	17.23	16.07	2.22
12	27.42	15.16	1.81	1.50	18.29	22.72	4.70
13	13.65	15.18	0.90	0.75	18.22	11.37	2.28
14	12.89	15.19	0.85	0.79	16.33	11.99	0.89
15	15.70	15.21	1.03	1.01	15.56	15.35	0.35
16	17.75	15.23	1.17	1.03	17.28	15.65	2.10
17	18.48	15.25	1.21	1.06	17.42	16.18	2.30
18	16.41	15.27	1.07	0.97	16.87	14.85	1.56
19	16.32	15.29	1.07	0.89	18.25	13.67	2.65
20	16.89	15.31	1.10	0.98	17.24	15.00	1.89
21	16.86	15.33	1.10	0.95	17.67	14.63	2.24
22	17.44	15.35	1.14	1.00	17.40	15.39	2.05
23	18.80	15.37	1.22	1.06	17.71	16.31	2.49
24	26.82	15.38	1.74	1.50	17.89	23.07	3.75
25	11.77	15.40	0.76	0.75	15.71	11.54	0.23
26	12.98	15.42	0.84	0.79	16.45	12.17	0.81
27	16.88	15.44	1.09	1.01	16.73	15.58	1.30
28	16.86	15.46	1.09	1.03	16.41	15.88	0.98
29	16.82	15.48	1.09	1.06	15.86	16.42	0.41
30	16.13	15.50	1.04	0.97	16.59	15.07	1.06
31	14.82	15.52	0.96	0.89	16.58	13.87	0.95
32	16.73	15.54	1.08	0.98	17.08	15.22	1.51
33	15.94	15.55	1.02	0.95	16.71	14.84	1.10
34	16.96	15.57	1.09	1.00	16.91	15.62	1.34
35	18.45	15.59	1.18	1.06	17.38	16.55	1.90
36	27.29	15.61	1.75	1.50	18.20	23.41	3.89
37	11.21	15.63	0.72	0.75	14.96	11.71	-0.50
38	11.79	15.65	0.75	0.79	14.93	12.35	-0.56
39	15.58	15.67	0.99	1.01	15.44	15.81	-0.23

SERIES	ORIGINAL	TREN	DETR	SEAS	DESE	FITS	RESI
40	15.87	15.69	1.01	1.03	15.44	16.12	-0.25
41	16.51	15.71	1.05	1.06	15.56	16.66	-0.15
42	15.35	15.73	0.98	0.97	15.78	15.29	0.05
43	14.14	15.74	0.90	0.89	15.82	14.07	0.07
44	15.14	15.76	0.96	0.98	15.46	15.44	-0.30
45	15.43	15.78	0.98	0.95	16.17	15.06	0.37
46	15.06	15.80	0.95	1.00	15.02	15.84	-0.78
47	16.72	15.82	1.06	1.06	15.75	16.80	-0.08
48	24.05	15.84	1.52	1.50	16.04	23.75	0.30
49	9.39	15.86	0.59	0.75	12.53	11.88	-2.49
50	11.00	15.88	0.69	0.79	13.94	12.53	-1.53
51	13.16	15.90	0.83	1.01	13.04	16.04	-2.88
52	14.07	15.91	0.88	1.03	13.70	16.35	-2.28
53	15.31	15.93	0.96	1.06	14.43	16.90	-1.60
54	14.25	15.95	0.89	0.97	14.65	15.51	-1.27
55	12.42	15.97	0.78	0.89	13.90	14.28	-1.85
56	14.11	15.99	0.88	0.98	14.40	15.67	-1.56
57	13.65	16.01	0.85	0.95	14.30	15.28	-1.63
58	14.47	16.03	0.90	1.00	14.43	16.07	-1.60
59	15.54	16.05	0.97	1.06	14.64	17.04	-1.50
60	21.97	16.07	1.37	1.50	14.65	24.09	-2.12
61	10.76	16.09	0.67	0.75	14.35	12.06	-1.30
62	11.48	16.10	0.71	0.79	14.54	12.71	-1.23
63	14.90	16.12	0.92	1.01	14.77	16.27	-1.37
64	14.10	16.14	0.87	1.03	13.73	16.58	-2.48
65	15.38	16.16	0.95	1.06	14.50	17.14	-1.76
66	13.94	16.18	0.86	0.97	14.33	15.73	-1.80
67	13.33	16.20	0.82	0.89	14.91	14.48	-1.15
68	14.81	16.22	0.91	0.98	15.11	15.89	-1.08
69	14.37	16.24	0.89	0.95	15.06	15.49	-1.12
70	14.97	16.26	0.92	1.00	14.93	16.30	-1.33
71	15.80	16.27	0.97	1.06	14.88	17.28	-1.48
72	22.60	16.29	1.39	1.50	15.08	24.43	-1.83
73	11.45	16.31	0.70	0.75	15.27	12.23	-0.78
74	11.75	16.33	0.72	0.79	14.88	12.89	-1.15
75	14.59	16.35	0.89	1.01	14.45	16.50	-1.91
76	16.66	16.37	1.02	1.03	16.21	16.82	-0.16
77	16.70	16.39	1.02	1.06	15.74	17.38	-0.69
78	14.97	16.41	0.91	0.97	15.39	15.95	-0.98
79	14.76	16.43	0.90	0.89	16.51	14.68	0.07
80	15.45	16.45	0.94	0.98	15.77	16.11	-0.66
81	14.90	16.46	0.90	0.95	15.61	15.71	-0.81

SERIES	ORIGINAL	TREN	DETR	SEAS	DESE	FITS	RESI
82	16.08	16.48	0.98	1.00	16.04	16.53	-0.44
83	16.81	16.50	1.02	1.06	15.83	17.52	-0.71
84	24.11	16.52	1.46	1.50	16.08	24.77	-0.66
85	11.59	16.54	0.70	0.75	15.47	12.40	-0.80
86	12.51	16.56	0.76	0.79	15.85	13.07	-0.56
87	16.65	16.58	1.00	1.01	16.50	16.73	-0.08
88	17.38	16.60	1.05	1.03	16.92	17.05	0.33
89	17.85	16.62	1.07	1.06	16.83	17.63	0.22
90	15.97	16.64	0.96	0.97	16.43	16.18	-0.20
91	15.30	16.65	0.92	0.89	17.11	14.89	0.41
92	15.99	16.67	0.96	0.98	16.32	16.33	-0.35
93	15.63	16.69	0.94	0.95	16.38	15.93	-0.29
94	16.21	16.71	0.97	1.00	16.17	16.76	-0.55
95	17.03	16.73	1.02	1.06	16.04	17.76	-0.73
96	24.18	16.75	1.44	1.50	16.13	25.11	-0.93
97	10.08	16.77	0.60	0.75	13.45	12.57	-2.48
98	11.94	16.79	0.71	0.79	15.13	13.25	-1.31
99	16.01	16.81	0.95	1.01	15.87	16.96	-0.95
100	16.23	16.82	0.96	1.03	15.80	17.29	-1.06
101	16.85	16.84	1.00	1.06	15.88	17.87	-1.02
102	15.38	16.86	0.91	0.97	15.81	16.40	-1.02
103	14.29	16.88	0.85	0.89	15.98	15.09	-0.80
104	15.83	16.90	0.94	0.98	16.16	16.56	-0.72
105	15.94	16.92	0.94	0.95	16.70	16.15	-0.21
106	15.88	16.94	0.94	1.00	15.84	16.98	-1.10
107	16.93	16.96	1.00	1.06	15.95	18.00	-1.07
108	23.23	16.98	1.37	1.50	15.50	25.45	-2.22
109	12.60	17.00	0.74	0.75	16.82	12.74	-0.13
110	13.95	17.01	0.82	0.79	17.67	13.43	0.52
111	17.30	17.03	1.02	1.01	17.14	17.19	0.11
112	18.14	17.05	1.06	1.03	17.66	17.52	0.62
113	17.79	17.07	1.04	1.06	16.77	18.11	-0.31
114	16.24	17.09	0.95	0.97	16.70	16.62	-0.38
115	14.97	17.11	0.87	0.89	16.74	15.29	-0.33
116	17.23	17.13	1.01	0.98	17.59	16.78	0.45
117	15.51	17.15	0.90	0.95	16.25	16.36	-0.85
118	16.94	17.17	0.99	1.00	16.89	17.21	-0.27
119	18.05	17.18	1.05	1.06	17.00	18.24	-0.19
120	25.01	17.20	1.45	1.50	16.68	25.79	-0.79
121	12.32	17.22	0.72	0.75	16.44	12.91	-0.59
122	13.39	17.24	0.78	0.79	16.97	13.61	-0.22
123	17.24	17.26	1.00	1.01	17.08	17.42	-0.18

SERIES	ORIGINAL	TREN	DETR	SEAS	DESE	FITS	RESI
124	16.62	17.28	0.96	1.03	16.18	17.75	-1.13
125	17.09	17.30	0.99	1.06	16.11	18.35	-1.26
126	15.38	17.32	0.89	0.97	15.81	16.84	-1.46
127	14.01	17.34	0.81	0.89	15.67	15.50	-1.49
128	15.73	17.36	0.91	0.98	16.06	17.00	-1.27
129	14.71	17.37	0.85	0.95	15.42	16.58	-1.87
130	15.69	17.39	0.90	1.00	15.65	17.44	-1.75
131	16.99	17.41	0.98	1.06	16.00	18.49	-1.50
132	24.59	17.43	1.41	1.50	16.40	26.13	-1.54
133	14.15	17.45	0.81	0.75	18.88	13.08	1.07
134	14.39	17.47	0.82	0.79	18.24	13.79	0.61
135	18.03	17.49	1.03	1.01	17.87	17.65	0.38
136	18.75	17.51	1.07	1.03	18.25	17.99	0.77
137	19.36	17.53	1.10	1.06	18.26	18.59	0.77
138	17.45	17.54	0.99	0.97	17.95	17.06	0.39
139	16.86	17.56	0.96	0.89	18.85	15.70	1.15
140	17.87	17.58	1.02	0.98	18.24	17.23	0.65
141	18.07	17.60	1.03	0.95	18.94	16.80	1.28
142	19.21	17.62	1.09	1.00	19.16	17.67	1.54
143	20.29	17.64	1.15	1.06	19.11	18.73	1.57
144	29.63	17.66	1.68	1.50	19.76	26.48	3.16
145	13.06	17.68	0.74	0.75	17.42	13.25	-0.19
146	13.83	17.70	0.78	0.79	17.52	13.97	-0.14
147	16.78	17.72	0.95	1.01	16.63	17.88	-1.09
148	17.40	17.73	0.98	1.03	16.93	18.22	-0.82
149	17.29	17.75	0.97	1.06	16.30	18.83	-1.54
150	15.97	17.77	0.90	0.97	16.43	17.28	-1.31
151	14.92	17.79	0.84	0.89	16.69	15.90	-0.98
152	15.20	17.81	0.85	0.98	15.52	17.45	-2.25
153	15.95	17.83	0.89	0.95	16.71	17.01	-1.07
154	17.19	17.85	0.96	1.00	17.14	17.90	-0.71
155	17.76	17.87	0.99	1.06	16.73	18.97	-1.21
156	25.69	17.89	1.44	1.50	17.14	26.82	-1.13
157	13.39	17.90	0.75	0.75	17.87	13.42	-0.03
158	14.05	17.92	0.78	0.79	17.80	14.15	-0.10
159	17.79	17.94	0.99	1.01	17.63	18.11	-0.32
160	18.19	17.96	1.01	1.03	17.71	18.45	-0.26
161	18.12	17.98	1.01	1.06	17.09	19.07	-0.95
162	17.45	18.00	0.97	0.97	17.95	17.50	-0.05
163	15.71	18.02	0.87	0.89	17.58	16.11	-0.40
164	16.46	18.04	0.91	0.98	16.80	17.67	-1.21
165	17.00	18.06	0.94	0.95	17.81	17.23	-0.23

SERIES	ORIGINAL	TREN	DETR	SEAS	DESE	FITS	RESI
166	17.91	18.08	0.99	1.00	17.86	18.12	-0.22
167	19.19	18.09	1.06	1.06	18.07	19.21	-0.02
168	27.30	18.11	1.51	1.50	18.21	27.16	0.14
169	14.58	18.13	0.80	0.75	19.45	13.59	0.99
170	14.58	18.15	0.80	0.79	18.47	14.33	0.25
171	18.50	18.17	1.02	1.01	18.33	18.34	0.16
172	19.56	18.19	1.08	1.03	19.04	18.69	0.88
173	19.33	18.21	1.06	1.06	18.22	19.31	0.01
174	18.41	18.23	1.01	0.97	18.93	17.72	0.68
175	16.88	18.25	0.92	0.89	18.88	16.31	0.56
176	17.07	18.26	0.93	0.98	17.42	17.89	-0.83
177	18.22	18.28	1.00	0.95	19.09	17.45	0.77
178	18.38	18.30	1.00	1.00	18.33	18.35	0.03
179	19.58	18.32	1.07	1.06	18.44	19.45	0.13
180	28.75	18.34	1.57	1.50	19.18	27.50	1.25
181	15.01	18.36	0.82	0.75	20.03	13.76	1.25
182	15.28	18.38	0.83	0.79	19.36	14.51	0.78
183	19.82	18.40	1.08	1.01	19.64	18.56	1.25
184	19.94	18.42	1.08	1.03	19.41	18.92	1.02
185	21.19	18.44	1.15	1.06	19.98	19.56	1.64
186	19.17	18.45	1.04	0.97	19.71	17.95	1.22
187	17.29	18.47	0.94	0.89	19.34	16.51	0.77
188	18.74	18.49	1.01	0.98	19.13	18.12	0.63
189	18.56	18.51	1.00	0.95	19.45	17.66	0.89
190	19.36	18.53	1.05	1.00	19.31	18.58	0.78
191	21.22	18.55	1.14	1.06	19.98	19.69	1.52
192	28.35	18.57	1.53	1.50	18.91	27.84	0.51
193	15.49	18.59	0.83	0.75	20.67	13.93	1.56
194	16.77	18.61	0.90	0.79	21.24	14.69	2.08
195	20.48	18.62	1.10	1.01	20.29	18.79	1.68
196	20.74	18.64	1.11	1.03	20.19	19.16	1.58
197	21.85	18.66	1.17	1.06	20.60	19.80	2.05
198	19.37	18.68	1.04	0.97	19.92	18.17	1.20
199	17.93	18.70	0.96	0.89	20.05	16.72	1.21
200	18.92	18.72	1.01	0.98	19.32	18.34	0.59
201	18.40	18.74	0.98	0.95	19.28	17.88	0.52
202	18.51	18.76	0.99	1.00	18.46	18.81	-0.30
203	19.80	18.78	1.05	1.06	18.65	19.93	-0.13
204	25.34	18.80	1.35	1.50	16.90	28.18	-2.84
205	14.71	18.81	0.78	0.75	19.63	14.10	0.61
206	15.45	18.83	0.82	0.79	19.58	14.87	0.59
207	18.78	18.85	1.00	1.01	18.62	19.02	-0.24

SERIES	ORIGINAL	TREN	DETR	SEAS	DESE	FITS	RESI
208	20.78	18.87	1.10	1.03	20.23	19.39	1.39
209	21.06	18.89	1.11	1.06	19.85	20.04	1.02
210	18.78	18.91	0.99	0.97	19.32	18.39	0.40
211	17.38	18.93	0.92	0.89	19.45	16.92	0.46
212	18.74	18.95	0.99	0.98	19.13	18.56	0.18
213	18.66	18.97	0.98	0.95	19.56	18.10	0.56
214	19.72	18.98	1.04	1.00	19.66	19.04	0.68
215	20.80	19.00	1.09	1.06	19.59	20.18	0.63
216	27.04	19.02	1.42	1.50	18.04	28.52	-1.48

Appendix B

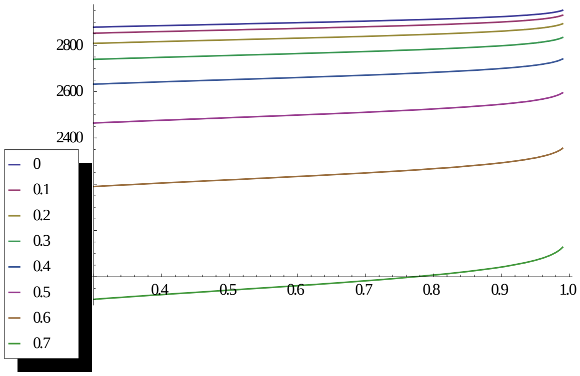


Figure B.1 π_4 **VS SL,** α **when** θ**=-0.4**

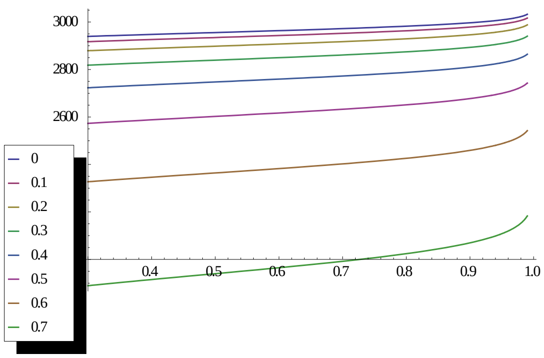


Figure B.2 π_4 **VS SL,** α **when** θ**=-0.3**

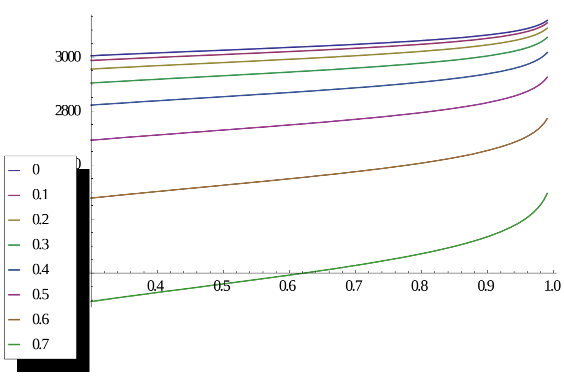


Figure B.3 π_4 **VS SL,** α **when** θ**=-0.2**

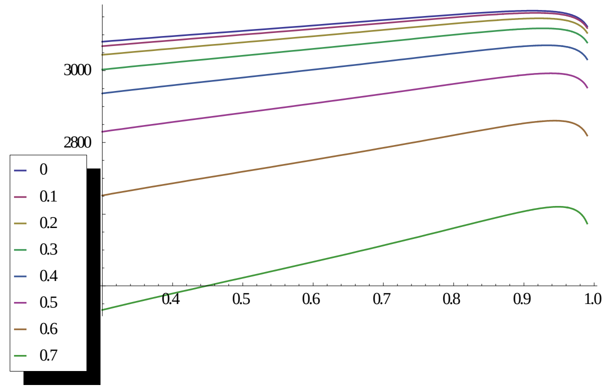


Figure B.4 π_4 **VS SL,** α **when** θ**=-0.1**

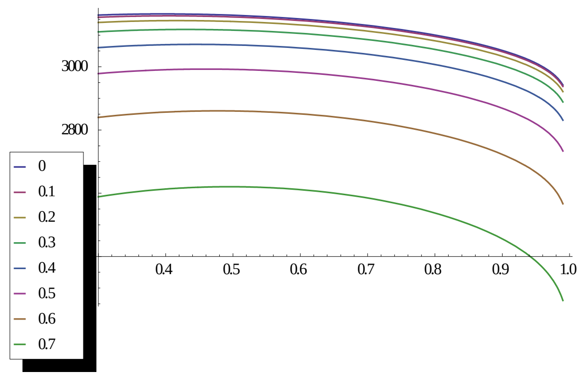


Figure B.5 π_4 **VS SL,** α **when** θ**=0**

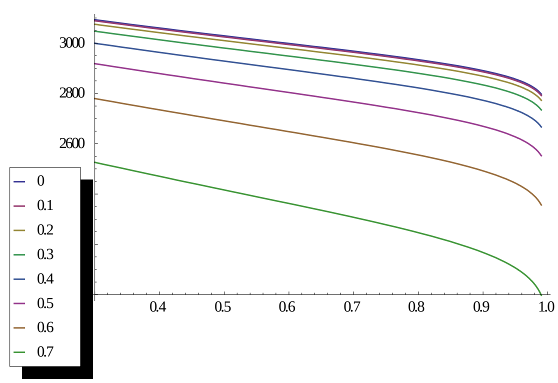


Figure B.6 π_4 **VS SL,** α **when** θ**=0.1**

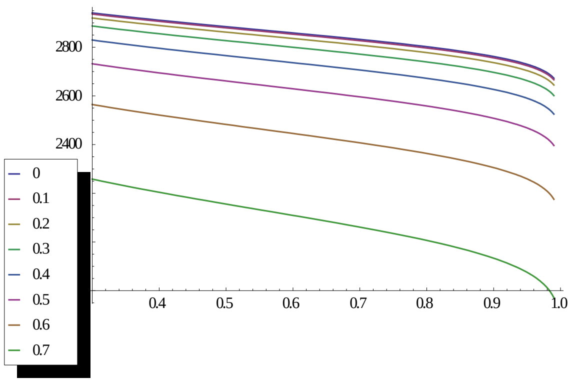


Figure B.7 π_4 **VS SL,** α **when** θ**=0.2**

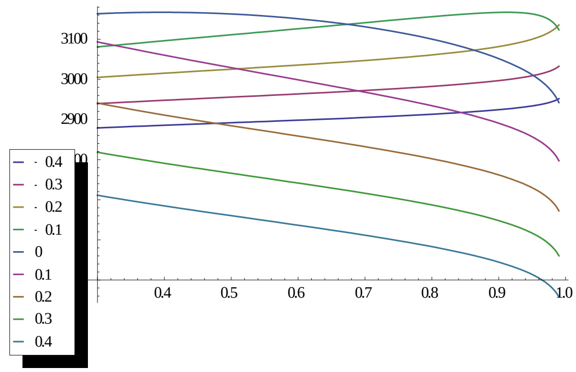


Figure B.10 π_4 **VS SL,** θ **when** α**=0**

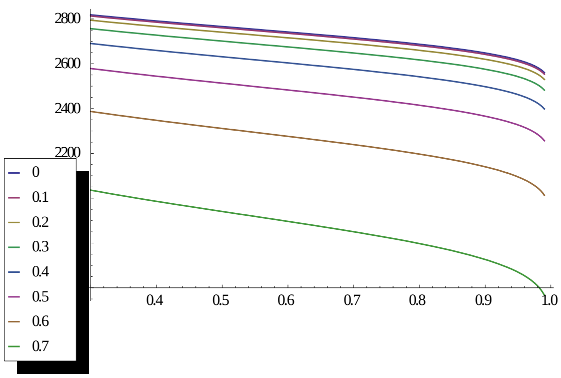


Figure B.8 π_4 **VS SL,** α **when** θ**=0.3**

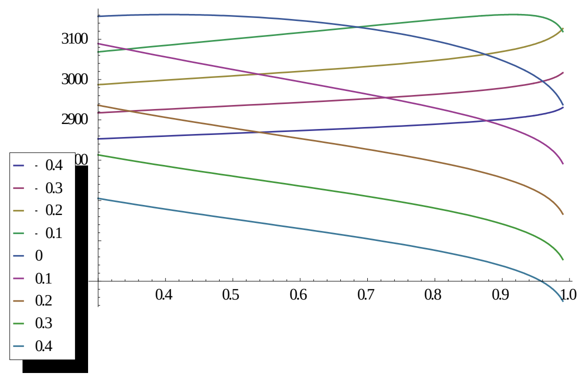


Figure B.11 π_4 **VS SL,** θ **when** α**=0.1**

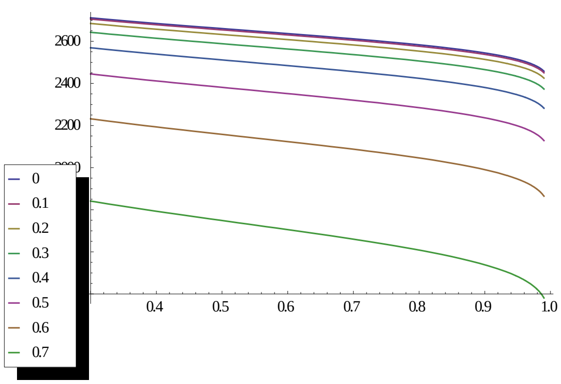


Figure B.9 π_4 **VS SL,** α **when** θ**=0.4**

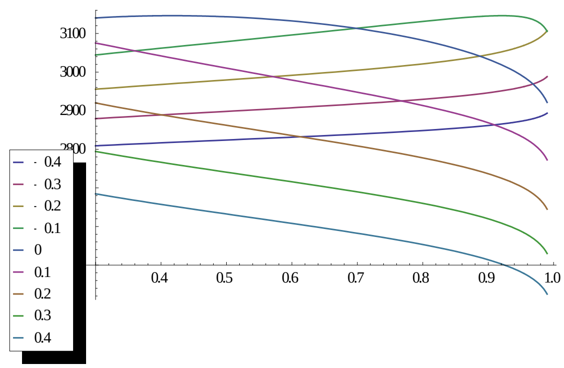


Figure B.12 π_4 **VS SL,** θ **when** α**=0.2**

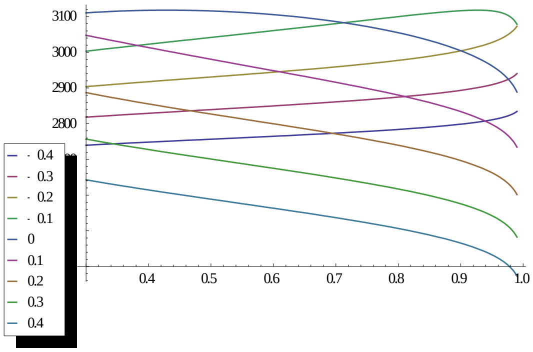


Figure B.13 π_4 **VS SL,** θ **when** α **=0.3**

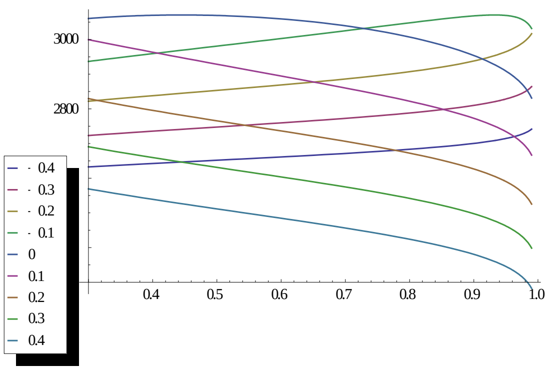


Figure B.14 π_4 **VS SL,** θ **when** α **=0.4**

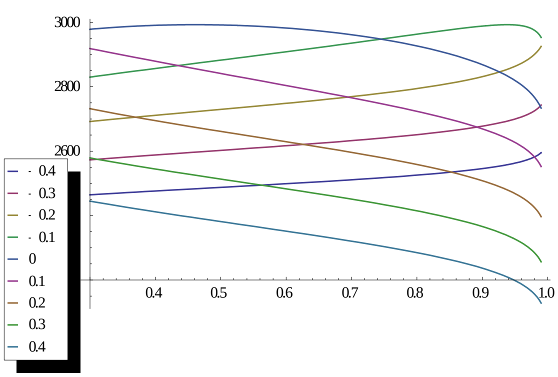


Figure B.15 π_4 **VS SL,** θ **when** α **=0.5**

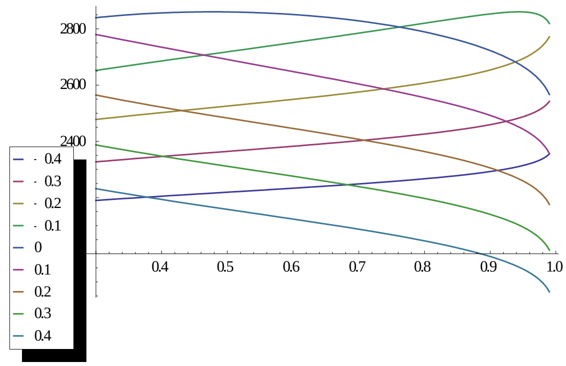


Figure B.16 π_4 **VS SL,** θ **when** α **=0.6**

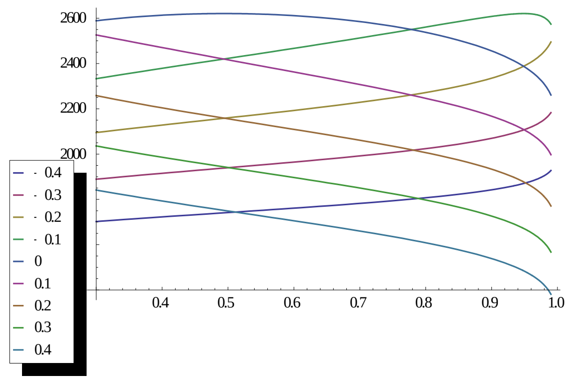


Figure B.17 π_4 **VS SL,** θ **when** α **=0.7**

Appendix C

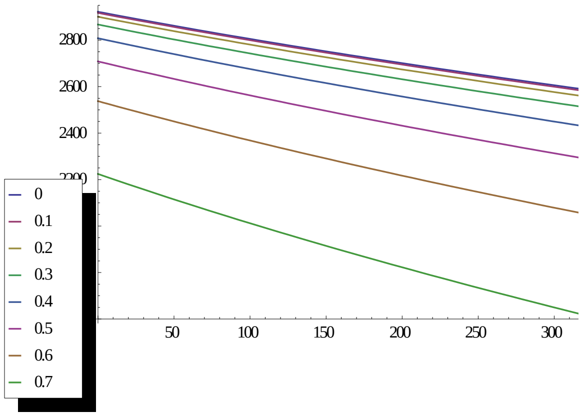


Figure C.1 π_2 VS K, α when θ=-0.4

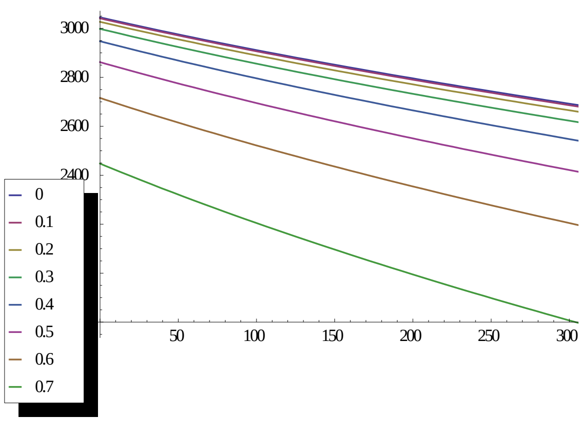


Figure C.2 π_2 VS K, α when θ=-0.3

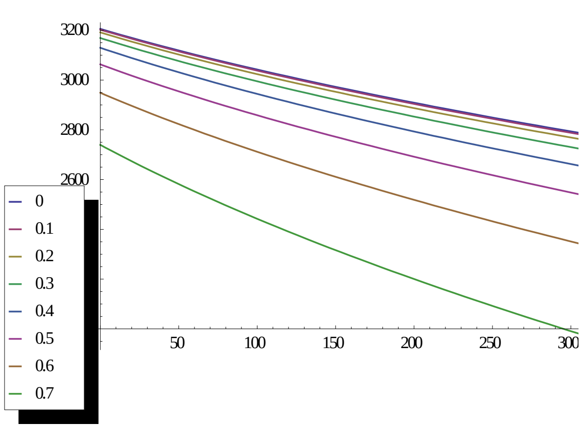


Figure C.3 π_2 VS K, α when θ=-0.2

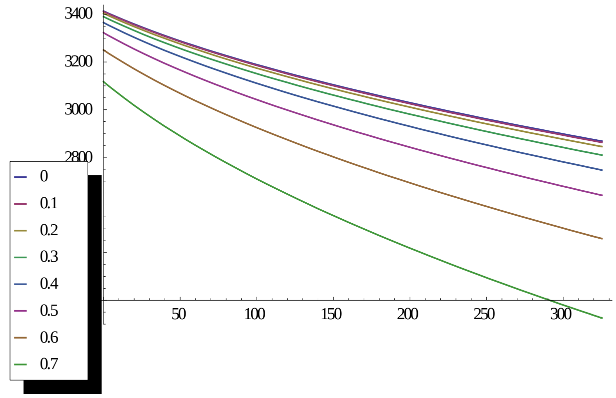


Figure C.4 π_2 VS K, α when θ=-0.1

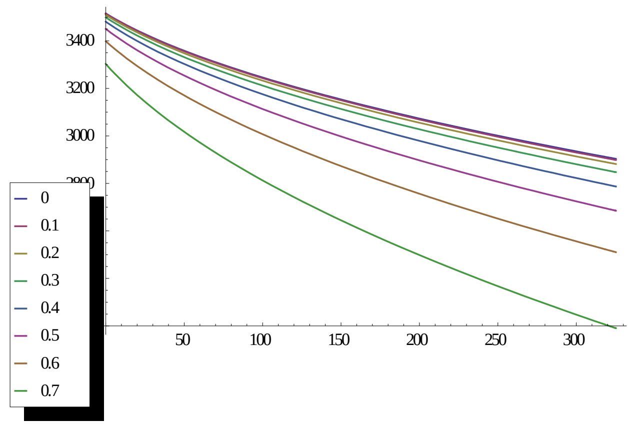


Figure C.5 π_2 VS K, α when θ=0

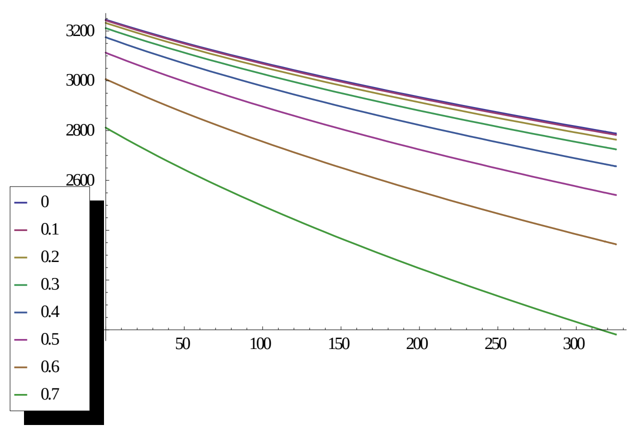


Figure C.6 π_2 VS K, α when θ=0.1

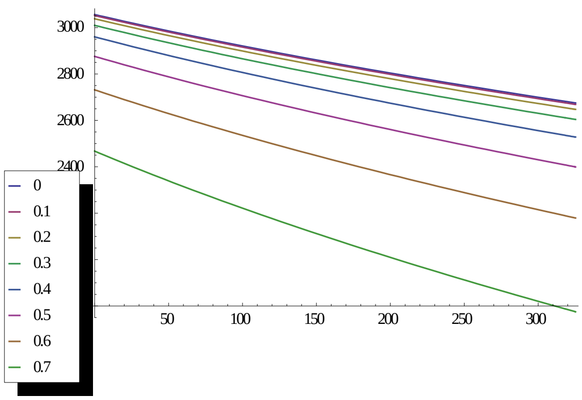


Figure C.7 π_2 **VS K,** α **when** θ**=0.2**

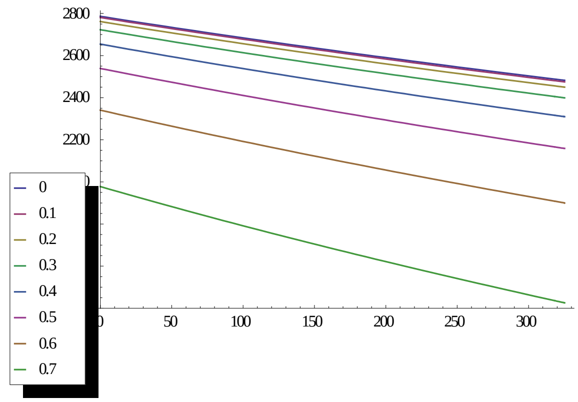


Figure C.9 π_2 **VS K,** α **when** θ**=0.4**

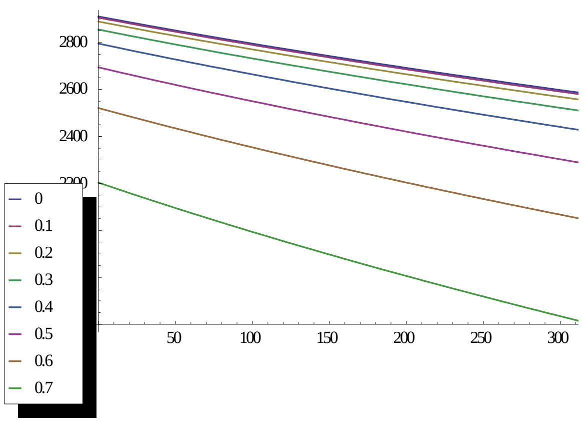


Figure C.8 π_2 **VS K,** α **when** θ**=0.3**

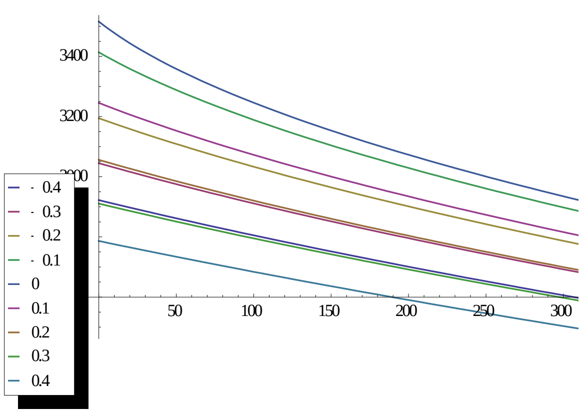


Figure C.10 π_2 **VS K,** θ **when** α**=0**

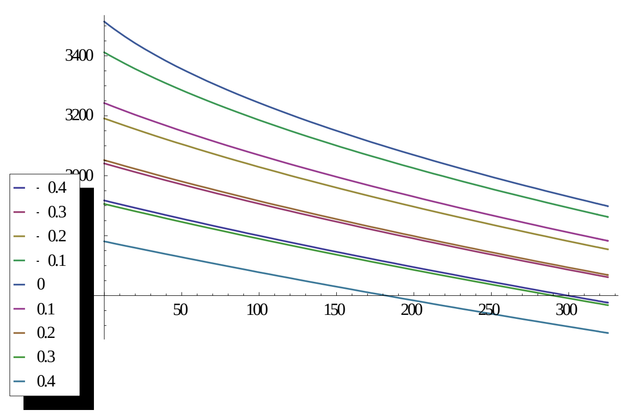


Figure C.11 π_2 **VS K,** θ **when** α**=0.1**

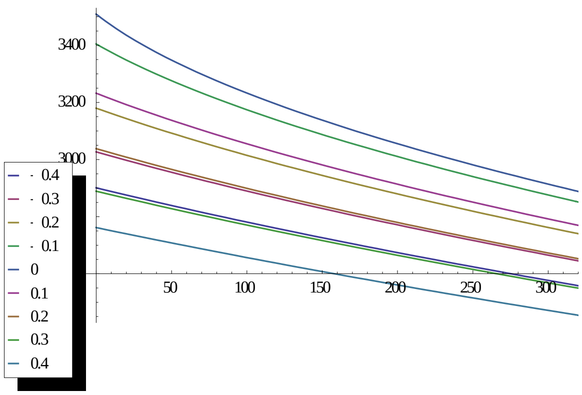


Figure C.12 π_2 VS K, θ when α =0.2

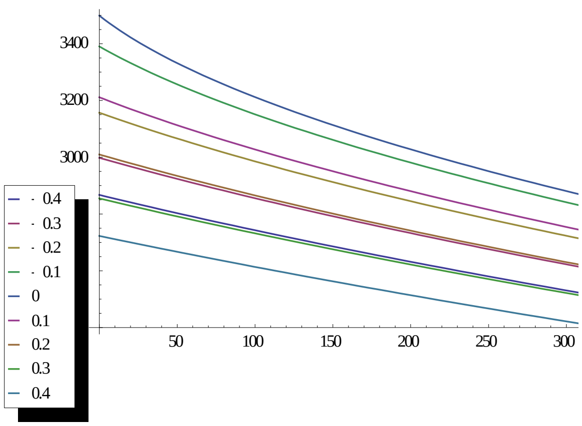


Figure C.13 π_2 VS K, θ when α =0.3

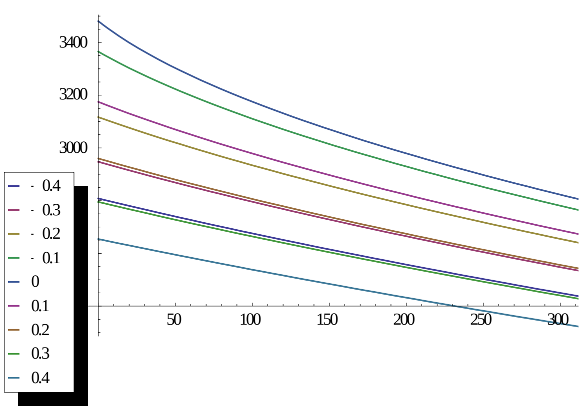


Figure C.14 π_2 VS K, θ when α =0.4

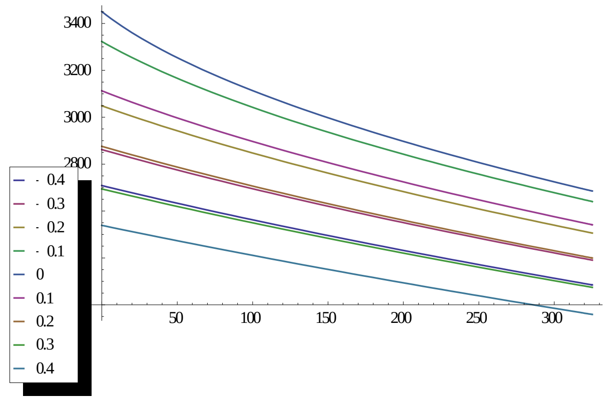


Figure C.15 π_2 VS K, θ when α =0.5

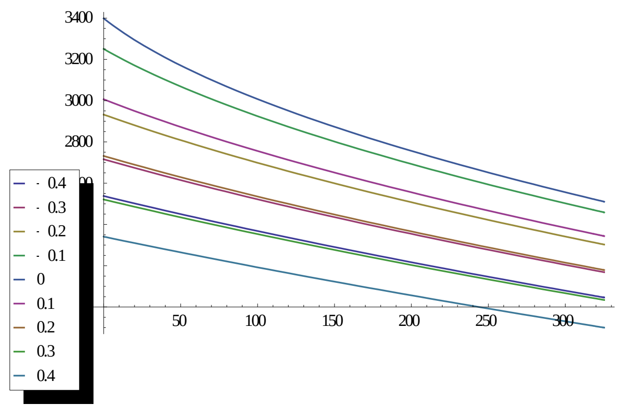


Figure C.16 π_2 VS K, θ when α =0.6

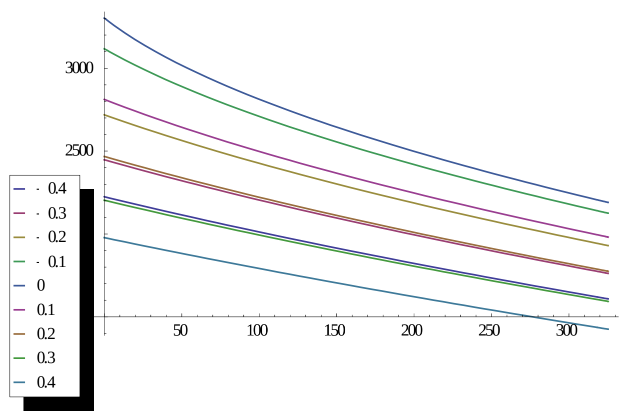


Figure C.17 π_2 VS K, θ when α =0.7

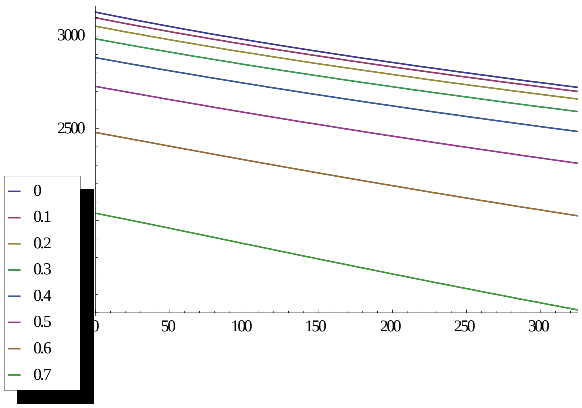


Figure C.18 π_4 VS K, α when θ=-0.4

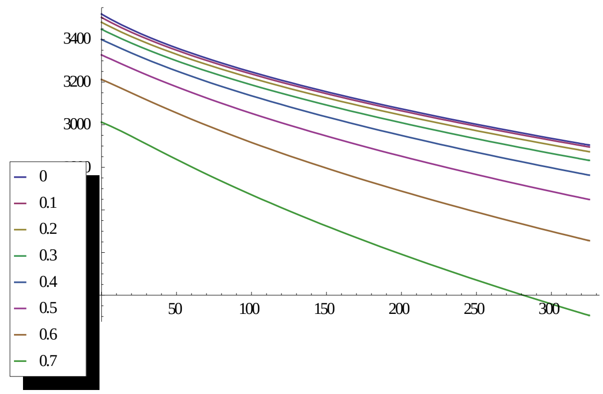


Figure C.21 π_4 VS K, α when θ=-0.1

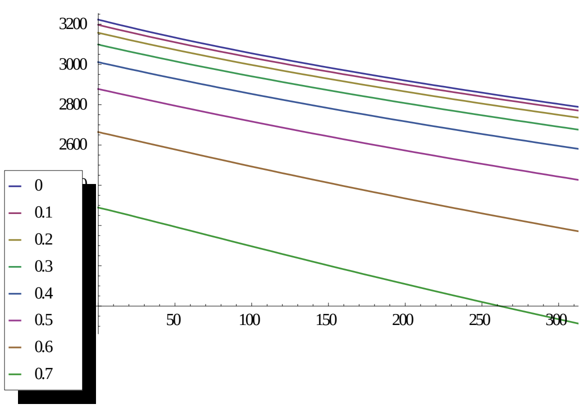


Figure C.19 π_4 VS K, α when θ=-0.3

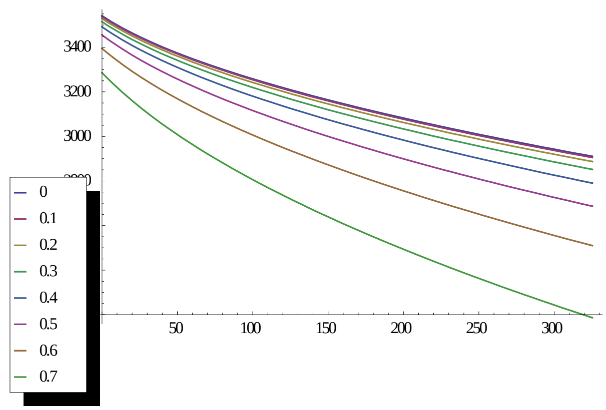


Figure C.22 π_4 VS K, α when θ=0

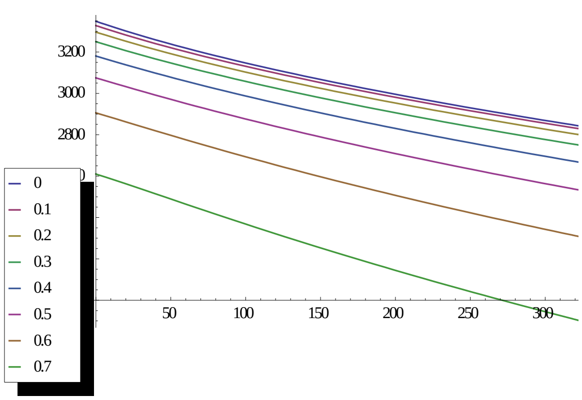


Figure C.20 π_4 VS K, α when θ=-0.2

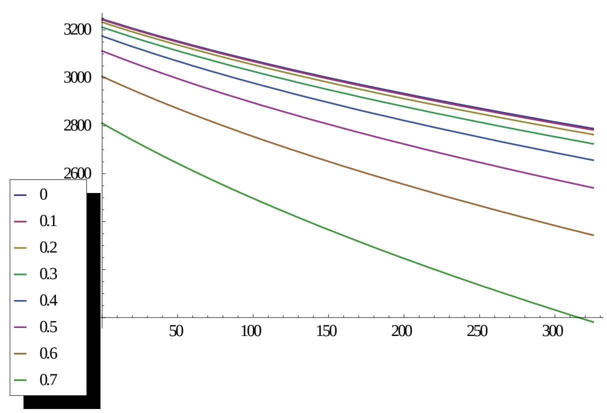


Figure C.23 π_4 VS K, α when θ=0.1

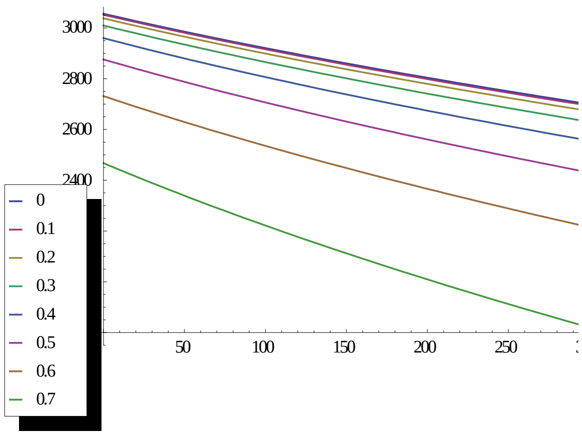


Figure C.24 π_4 **VS K,** α **when** θ**=0.2**

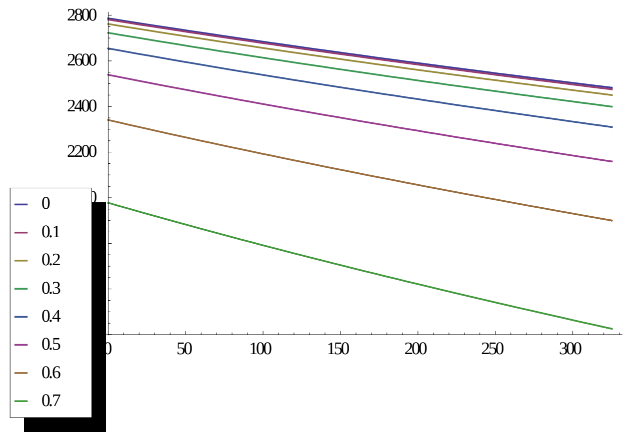


Figure C.26 π_4 **VS K,** α **when** θ**=0.4**

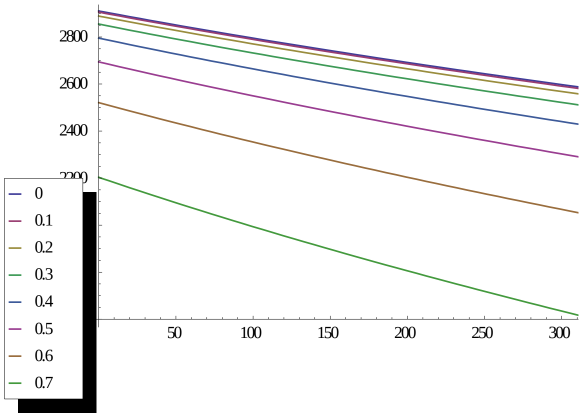


Figure C.25 π_4 **VS K,** α **when** θ**=0.3**

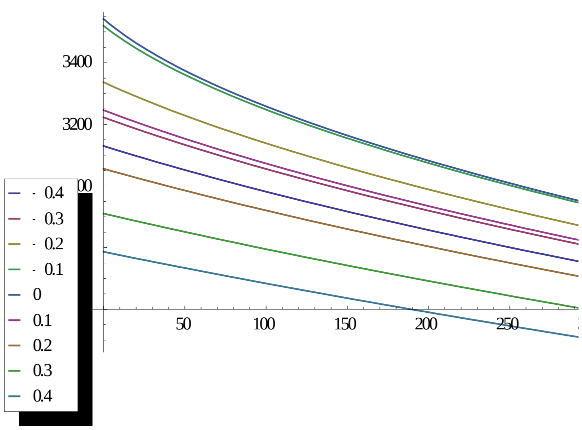


Figure C.27 π_4 **VS K,** θ **when** α**=0**

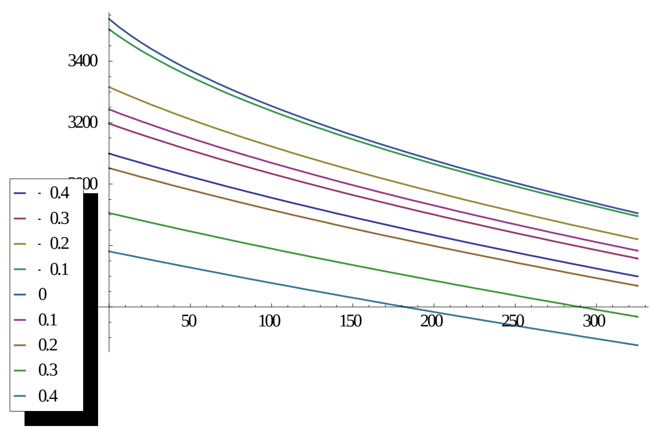


Figure C.28 π_4 **VS K,** θ **when** α**=0.1**

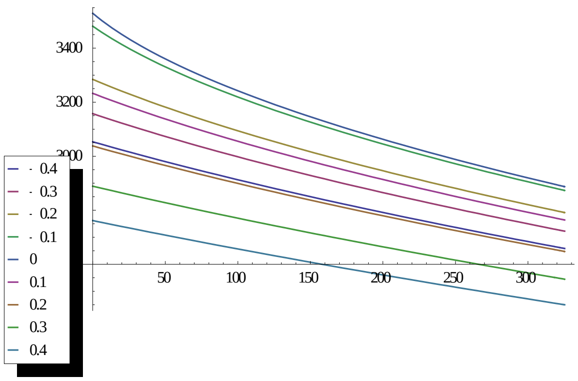


Figure C.29 π_4 VS K, θ when α =0.2

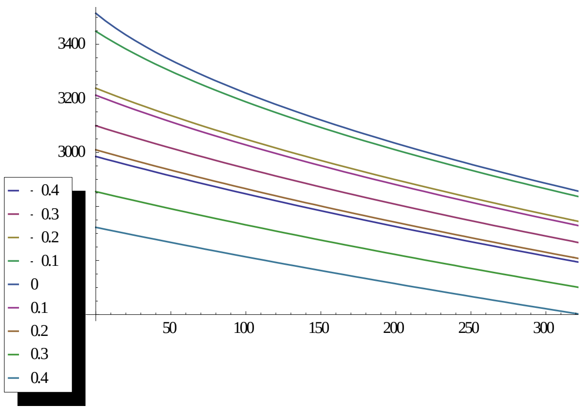


Figure C.30 π_4 VS K, θ when α =0.3

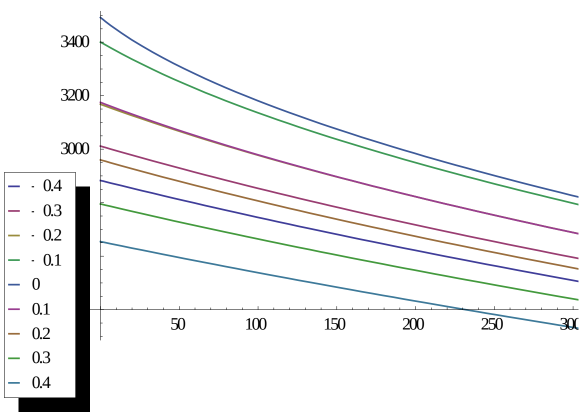


Figure C.31 π_4 VS K, θ when α =0.4

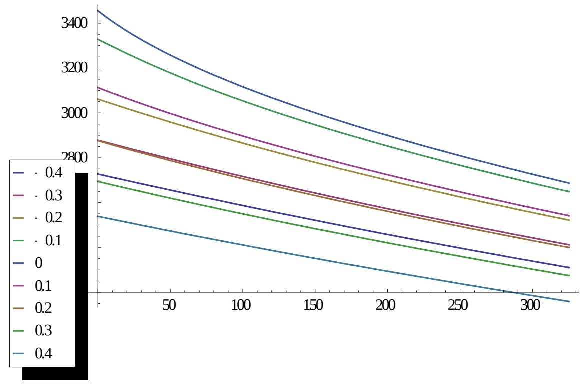


Figure C.32 π_4 VS K, θ when α =0.5

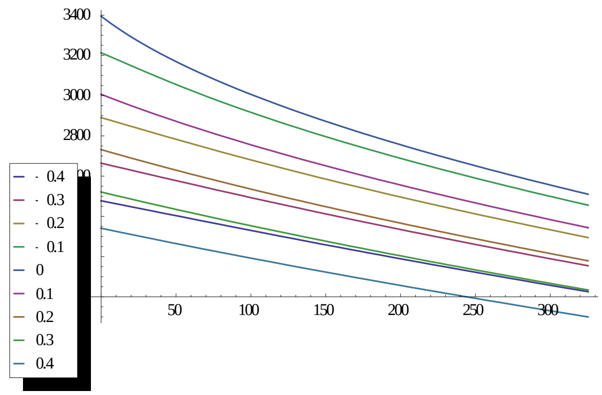


Figure C.33 π_4 VS K, θ when α =0.6

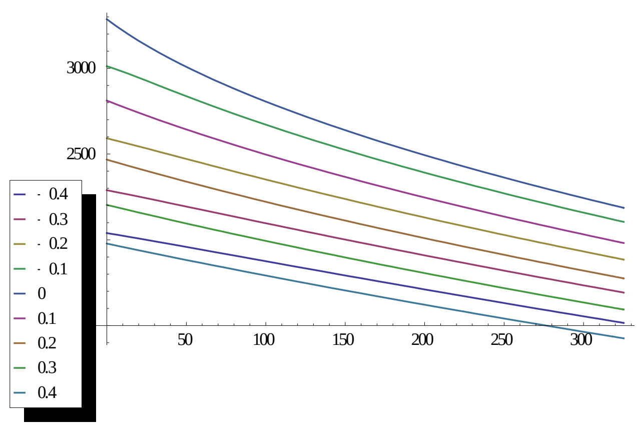


Figure C.34 π_4 VS K, θ when α =0.7