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Abstract

In this action research study of a fifth grade mathematics classroom, I investigate how a daily small-group learning environment influenced students' ability to communicate their mathematical thinking in verbal and written form. I discovered that a small-group atmosphere provided more opportunities for direct instruction. I also found that the emphasis on communication helped students to articulate their thinking more clearly when they wrote and spoke of their mathematical ideas. Most students preferred to work in small groups because they appreciated the support of their peers. Students felt they were more likely to ask teammates for help when they had questions as compared to a traditional classroom setting. The research supports the value of small-group settings in the mathematics classroom where students work with their homework teams on a daily basis and where presentations are used as a communication tool for students to share their thinking related to specific problems assigned.

Introduction

The topic of my research project is centered upon mathematical communication in both written and oral forms. I have been interested in helping my fifth grade students improve their ability to communicate their mathematical thinking in order to strengthen their own learning and the learning of their peers.

When I first started teaching 17 years ago, I began to familiarize myself with the work of Marilyn Burns. Over the years, I have incorporated many of her ideas and mathematics problems into my mathematics teaching. I appreciate that Burns encourages an atmosphere of student inquiry, collaboration, and conversation and that many of her problems are applicable to various ability levels. For instance, my favorite first problem of the school year is her rectangular arrays problem. All of my students find success at being able to construct various rectangular arrays for the numbers one through fifty. A few of the students stick to the hands-on tiles to build the different arrays, some begin to see patterns in factors that help them as they build, and others are able to gain new understanding about characteristics of numbers that are prime or composite or square. These latter concepts also lead us to revisit this problem over and over again throughout the year; the arrays are visual reminders of our work and what the work represents.

It has been common practice for me to engage my students in problem solving activities and require them to work and think together like this. However, my Math in the Middle experience has brought a very important layer of instruction to my teaching, and that is the communication layer. Mathematical conversations have existed in my classroom from the beginning, particularly during problem solving sessions which occur at least two class periods a week. However, I never emphasized communication as a tool for learning, and I never asked my students to write about their mathematical thinking. I was afraid students would not like mathematics if I made them write in mathematics class. When I created lesson plans, my primary focus was getting students actively engaged with hands-on problems that connected to our curriculum in a significant way. I still strive to incorporate those rich types of activities, but through my work in Math in the Middle, I have come to understand what it is like to be a learner in an environment that honors mathematical discussion both among students and among teachers and students.

I have spent a great deal of time reflecting on my role within the classroom. I remember when I watched my initial application video of my teaching for Math in the Middle a couple of years ago I was shocked to see myself rush from group to group, trying to get to everyone with raised hands as quickly as I could. I was a question answerer. That is what I did. Whatever students asked, I answered, because I did not want anything to stop my students from doing their important work.

Now I try to model my behavior in the classroom after those individuals who have led our courses in Math in the Middle. Rather than flitting around the room as I did before, I like to think that I stroll around the room tuning in to listen to important student conversation. If I am asked a question, I rarely answer it directly. I ask students questions to help refocus their work, I ask about strategies they have tried, and sometimes I refer them to others in the room whom I know are working on a similar part of the problem and are headed in an appropriate direction. I have come to realize that it is often the questions themselves that lead to students' learning. It is a good thing to allow students to grapple with ideas, talk them out, and listen to each other. I make a conscious effort to allow time in our classroom for these conversations.

I also appreciate the place that writing has in mathematics instruction. It now seems quite natural to ask students to communicate their thinking in written form, and it provides valuable insight into a student's thinking. Last school year I began to incorporate writing into my mathematics classroom as a direct result of my own experiences in Math in the Middle. I chose to provide more opportunities for students to become deeply engaged in problem solving, and for the first time, I was asking students to write in mathematics class. The results were wonderful. I became much stronger in my role as facilitator, and I learned how to make communication a purposeful tool in my problem solving instruction.

For this research project, I knew I wanted to incorporate problem solving, writing, and communication, but I did not know exactly what my approach should be. I also knew that I wanted the project to have an impact in my classroom on a daily basis. My previous instructional format had always been to begin with problem solving work on Tuesdays and Thursdays, and begin with Daily Cumulative Review from our Scott Foresman series on the other days of the week. The second half of each class period was usually for the regular lesson from the textbook, and an assignment was almost always given at the end of the period. I felt my problem solving days were very inquiry based, allowing for lots of great student interaction, but the other class periods during the week were pretty traditional. There was not a lot of communication that occurred on those days, other than teacher to student.

As I began to think of topic ideas for this project, I turned to Marilyn Burns for advice. In her book, *About Teaching Mathematics* (2007), she shares her thoughts about small-group work and homework. I was intrigued by her ideas, because they were things I could implement into our daily mathematics schedule.

Burns (2007) suggests that students be allowed to work together in small groups as they complete their homework assignments, and to also meet again the following day to discuss completed homework problems. She speaks about students presenting their ideas before the whole class and how those ideas can be used to strengthen important concepts in the curriculum. She encourages a student-centered classroom, rather than teacher-centered. Burns believes that small-group work is “invaluable for supporting student learning, maximizing student involvement, reducing isolation among individual students, and establishing a classroom environment that values student thinking, reasoning, and participation. (NEED A PAGE NUMBER) This is exactly the kind of classroom I wish to create for my students, so I chose to implement my own version of her idea.

I began my project by creating five homework teams with four students per team. My students are used to group work and are consistently placed in a seating arrangement of groups of three or four. They were not accustomed to working on homework together, though. Initially, this felt like cheating to them. The homework teams met almost daily to collaborate on their homework assignments, and then table groups met to share and compare their solutions the next day.

With each homework assignment, I asked one student from each group to prepare a presentation of a problem from their work for the following class period. I determined which problem they were to share, and I tried to choose those that I thought would prompt productive discussion. The students were given an envelope with an overhead transparency and pen, and ahead of time they were to prepare a written explanation of the solution for the problem they were assigned.

The ideal classroom is a community of learners where students feel safe and secure and are able to share their ideas in a supportive environment. Students should feel that their voices are honored by all and understand that when they communicate their mathematical thinking they are taking responsibility for their own learning. They also should be provided with consistent opportunities to practice these communication skills, whether it is in verbal or written form. The discussions that took place in this new learning environment became a vital component of our learning.

The Pedagogical Problem

My principal conducted his official observation of me this semester at the same time I was conducting my research study. The day he came in the students were working on proving or disproving the following conjecture: Every square number greater than one can be written as the sum of two prime numbers. The students were working in groups to try to solve the problem. There was much activity in the room. I walked around the classroom and listened to student conversation. I prompted and encouraged as necessary. A couple of times I paused the work of the class to discuss some important points as a whole group, and then they went right back to their work.

When I went to discuss my evaluation, my principal said that he had never seen a classroom operate in that way before. He commended my work and said he was impressed with the way the students were actively engaged in the problem. He talked about how in most classrooms the teacher teaches the lesson, usually in steps or procedures, the students follow the given format, the assignment is given, and students complete their seatwork independently. In our classroom, the students were figuring out how to approach and organize the problem themselves, and they used each other as a resource. He noticed that I did not answer questions directly. Instead, I replied with comments or questions that helped focus the students or help them see their work in a different way. He also noted how some of the students were able to make connections to previous problems we had done. Then he said that not everyone could teach this way. I struggle with that statement. Can that be true?

This experience with my principal is evidence to me that others can be persuaded to see the value of students' communication of mathematical thinking. He recognized the power of it in just a single class period. That is exactly why this idea of mathematical communication is worth exploring. There is a richness that exists when students feel empowered to think and talk about their own learning in their own way. Every classroom should allow for this kind of special interaction, yet the reality is that many classrooms do not. We teach what we know. Most teachers did not learn in this kind of atmosphere where talking about learning was honored. Usually it was the opposite; the message was sit down, be quiet, and do your own work.

A collaborative environment provides students with opportunities to use language as a way to organize their thinking and clearly communicate their ideas. It also encourages them to connect their ideas with the thinking of others, which helps to broaden their own mathematical understandings. This is the kind of setting that most teachers wish for, but it will take some adjustments in methodology to accomplish it. This research project is an opportunity for me to investigate practices and procedures that support a collaborative and communication rich environment for my own classroom, but it will hopefully inspire others to reflect upon their current teaching practices and consider ways they can work towards a more student-centered learning environment.

Literature Review

My goal with this research project was to help students strengthen their ability to communicate their mathematical ideas. The daily small group structure was designed to be a support for whole class discussions, as well as for the students as individual learners. The research articles I found support inquiry and discourse-oriented classrooms. Themes that emerged from my reading include situated perspective learning theory, social versus sociomathematical norms, scaffolding, teacher and student roles within a mathematical community, and methods of communication.

Situated Perspective

The situated perspective is a view of learning and thinking which supports the idea that students must be actively engaged in their own learning in order to strengthen their conceptual understanding. Greeno (1997) has made a significant contribution to Stanford University's Middle-School Mathematics through Applications Project. Greeno emphasizes the need for a collaborative environment where students communicate their mathematical thinking through questioning, explaining, and reasoning. He states, "current studies are finding that students' engagement and effectiveness are enhanced when the participation structures of classrooms are changed to encourage their more active participation in practices of inquiry and sense-making" (1997, p. 99). Greeno relates the situated perspective to the work of John Dewey who believed in developing students' habits of mind through inquiry and reflection.

In an ethnographic case study, Boaler (1999) conducted a study of two different schools in the United Kingdom. In this case study, Boaler explored the differences that often exist between classroom communities. Students at Amber Hill School were in a traditional setting with textbooks, individual work, and homogeneous groups. In contrast, Phoenix Park School provided a relaxed setting for students where mathematical work was centered on projects and open-ended problems. Group discussions were encouraged and students were grouped heterogeneously. Boaler found that a teacher's beliefs and practices have a definite impact on students' conceptual understanding and depth of mathematical communication. Boaler relates "that the behaviours and practices of students in mathematical situations are not solely mathematical, nor individual, but are emergent as part of the relationships formed between learners and the people and systems of their environments" (1999, p. 260). The results of her study show the importance of these relationships. The students at Phoenix Park School, learning in a situated perspective environment, consistently outperformed the students from Amber Hill on various types of assessments. They also saw their mathematics knowledge as relevant to real world situations, while Amber Hill students viewed their mathematics learning at school as separate from real life experience.

It is necessary for all members of a learning community to collaborate in such a way that meaning is created and understood by all participants. Staples (2007) used the work of Greeno and Boaler to provide support for her research study of a ninth-grade prealgebra class that was organized around collaborative inquiry. Staples closely analyzed the various types of collaboration that took place and studied how the classroom community developed over time. She recognizes the situated perspective as a learning theory that focuses upon social processes, yet it is more than just group work and cooperation. Staples repeatedly emphasizes the importance of collaboration where "students must listen to, comprehend, and respond to other students" (2007, p. 205). An environment such as this requires more from the teacher than simply a role of facilitator. It is critical that the teacher is secure in their own mathematical knowledge, as well as how to develop the social norms within a classroom to support the work of a situated perspective learning theory.

Teacher's Role in a Mathematical Community

The authors of several of the research articles I read discuss the importance of the teacher within a community of learners. Teachers are often viewed through two very different lenses depending on the particular philosophy of the teacher – he or she is the primary dispenser of knowledge or the facilitator who believes in a student-centered approach. Williams and Baxter

(1996) conducted a three-year study of a middle school mathematics classroom. Their case study featured a teacher who was a participant in a professional development program known as the QUASAR project. Williams and Baxter examined patterns of discourse that occurred between students, as well as between the teacher and students. Their findings provide a cautionary response to discourse-oriented teaching. It is not enough to have students talk about their work. Communication alone does not guarantee conceptual understanding. Discourse must be meaningful. They state the following: “If students discuss what they are doing, and that discussion is meaningful to them, so too will be the mathematics they discuss” (1996, p. 36). Williams and Baxter make the point that while mathematical communication is a fundamental component of an effective mathematics classroom, the teacher has the responsibility of making sure the talk is pushing students towards greater conceptual understanding.

Boaler (1999) provides similar evidence in her work with the students from Amber Hill and Phoenix Park. The role of the teacher was critical to the students’ success. Students at Amber Hill were accustomed to classroom discourse, however they often received too much guidance from their teacher who, when questioned, would often provide them with correct problem solving methods and procedures. Students at Phoenix Park were supported differently. Their teachers were open to various approaches and would not provide any type of structured help to the students.

Yackel and Cobb (1996) published a research article on an investigation of how elementary students learn mathematics. They also were interested in learning how teachers can be best supported in creating environments that work to increase students’ conceptual understanding. Yackel and Cobb believe that teachers must be careful listeners of their students in order to provide optimal learning opportunities that are in response to students’ explanations, reasoning, and solutions. They see the teacher "as a representative of the mathematical community in classrooms where students develop their own personally meaningful ways of knowing" (1996, p. 461). I appreciate how Yackel and Cobb recognize the teacher as a facilitator of mathematical discussions, but also as a participant of the classroom community. The teacher is a learner, too.

Researchers Zack and Graves (2001) share this view of the teacher's role as listener and learner in their qualitative study of a fifth grade inquiry classroom. These researchers focused upon three different students and observed the ways they participated in problem solving activities. Zack and Graves discovered that the interaction that takes place in an inquiry setting results in both students and teachers being learners.

Another important role of the teacher is to provide meaningful tasks that are mathematically rigorous and highly engaging. In a paper examining classroom-based factors that affect student engagement, authors Henningsen and Stein studied ways to improve students' mathematical understanding and how to "help them become better mathematical doers and thinkers" (1997, p. 524). In their article, Henningsen and Stein strongly emphasized the need for teachers to select appropriate instructional tasks. They found five important factors that positively influenced how well students remained engaged with various mathematical tasks: tasks were built on students' prior knowledge, teacher employed scaffolding strategies, an appropriate amount of time was provided for the work, a high-level of performance was modeled, and the teacher provided the proper amount of press for explanations and meaning from students.

This idea of "press" was mentioned in two other articles I read. Staples (2007) referred to it in her work with a ninth-grade prealgebra class. One of the points she made in her article was the importance of the teacher supporting students' thinking through comments and questioning. The teacher she observed was successful at eliciting ideas from her students. Staples comments that a "noticeable feature of Ms. Nelson's practice was her *press* and persistence in eliciting students' thinking, even when they were reluctant or showed signs of hesitancy" (2007, p. 174). Staples goes on to say that her success was also attributed to the fact that her students knew she truly valued their ideas and they understood that she expected them to clearly explain their thinking.

Kazemi and Stipek (2001) conducted a study about how classroom practices create varying degrees of press for students' conceptual understanding. These authors selected four upper elementary teachers for their research and used "press for learning" as a variable in selecting the four teachers. Kazemi and Stipek chose two teachers who showed a higher degree of press with their students and two teachers who showed a lower degree of press. The high press teachers directly worked with their students on how to talk about mathematics. Students worked in small groups and were expected to come to a consensus. The groups were also accountable for each individual, making sure that everyone understood the strategies and procedures that resulted in the group's solution. In addition, groups were required to provide a written explanation of their work. The high press teachers asked questions about the students' work and placed the responsibility of explanation on the students. All were expected to participate.

The low press teachers did not provide direct instruction on how to communicate mathematically. Kazemi and Stipek state that students in low press classrooms "did not question each other or make sure that each person understood the mathematical relations involved" (2001, p. 78). Often one or two students would complete the task for the entire group. Students worked in small groups, however there was not an equal distribution of work.

Scaffolding

As mentioned in the last section when discussing the work of Henningsen and Stein (1997), scaffolding has been shown to be an effective strategy used by teachers to support mathematical communication. They state,

> Scaffolding occurs when a student cannot work through a task on his or her own, and a teacher or more capable peer provides assistance that enables the student to complete the task alone, but that does not reduce the overall complexity or cognitive demands of the task. (1997, p. 527)

Just as a building can require scaffolding that enables workers to physically rise up to complete a job such as washing windows or painting the second story, scaffolding is a tool for the teacher to help students “rise up” to the next level of new understanding that builds on their prior knowledge.

An important attribute of Ms. Nelson, from the article by Staples (2007), was her ability to scaffold students’ sharing of ideas. Ms. Nelson made use of various visual representations to help students construct their mathematical thinking. She asked important questions and provided appropriate prompts in response to student conversation. Her participation and input was part of her classroom structure that allowed her to guide the mathematical work of her students so that the goals of the task could be accomplished.

Williams and Baxter (1996) elaborate on the idea of scaffolding in their research article by describing both analytic and social scaffolding. Analytic scaffolding involves the structuring of mathematical ideas, while social scaffolding refers to the establishment of classroom norms for social behavior. In their study of a discourse-oriented classroom, they found that social norms were often considered a higher priority than the mathematical content of the problems students solved. The teacher had provided a safe and secure learning environment for students to share their thinking and had worked extensively to model positive social interaction. However, this teacher seemed to “expect analytic scaffolding to arise from tasks she selects and from discourse among students” (1996, p. 37). Communication and task selection does not guarantee conceptual understanding. Teachers must make thoughtful decisions as to the amount of guidance and structure they provide their students. There is a fine line between too much and too little, therefore a balance between analytic and social scaffolding is key.

Social and Sociomathematical Norms

One of the strengths of a discourse-oriented classroom is the wide array of sociomathematical norms. Yackel and Cobb (1996) attempt to provide clarification as to the difference between social and sociomathematical norms, and Kazemi and Stipek (2001) often make reference to that work within their own research article. Social norms include explaining mathematical thinking, discussing different strategies, and working in small groups. Sociomathematical norms consider the specific mathematical task at hand and “what counts as an acceptable mathematical explanation and justification” (Yackel & Cobb, 1996, p. 461). These norms also require students to have an understanding of the various mathematical concepts underlying different strategies and solutions and rely on using mathematical arguments to achieve consensus within small groups. Sociomathematical norms are important in developing a more sophisticated understanding of the mathematical work teachers do with students.

Kazemi and Stipek (2001) documented how the presence of sociomathematical norms was a determining factor of success in the four classrooms they studied. They specifically discuss how collaboration was strengthened when all group participants were held accountable for their mathematical thinking. They noted that full participation was fundamental to the success of the group and "consensus should be reached through mathematical argumentation" (2001, p. 76). It is not enough to communicate procedures and strategies within a social practice. Sociomathematical norms raise the level of expectation for everyone. Teachers need to understand this and work to provide an environment where these norms can be consistently modeled and applied.

Sociomathematical norms require students to be active participants in their own learning. Pape, Bell, and Yetkin (2003) conducted a study on the collaborative instruction of a seventh grade mathematics teacher and a university researcher. Their purpose was to organize and analyze their findings in such a way that they might determine instructional factors that support students' sense of mathematical accomplishment and strategies they employ to control their own learning. They call these learners "self-regulated learners" and believe that students develop reasoning through argument and controversy, which are natural elements of mathematical discourse.

This is all about teaching students to think about their thinking, and that is not an easy task. I need to be mindful about not allowing the discourse within my classroom setting to become shallow or routine. I must continually reflect on instructional practices, as well as the substance of the mathematics. Metacognitive processes are essential to the mathematical communication that takes place in classrooms, and teachers should consciously articulate those processes to students in order that they may adopt similar ways of thinking about their own learning.

Methods of Communication

Effective teachers use several methods of communication to help students develop mathematical knowledge. Williams and Baxter (1996) observed many important techniques within the classroom of Ms. H. She provided as many opportunities as she could for students to talk about mathematics. She emphasized the importance of verbalization, being able to put your thinking into your own words. Students often worked in pairs or small groups. She invited her students to ask questions of each other during class discussions and even awarded points to those students that asked the questions. Ms. H. typically began her class periods with a brief introduction to the problem, and then students worked in small group settings to solve the problems. She encouraged them to discuss strategies and explain their thinking to each other. Whole class discussion was used at the end of most lessons to present solutions, and students often used the overhead projector as tool for communicating their problem solving processes and results. The authors noted that Ms. H. frequently asked students to discuss their thinking with her. She asked specific questions and would share her reasons for knowing with the students. This is consistent with discourse-oriented teaching.

Greeno poses the common question, “Does group work produce unequal participation?” (1997, p. 106). He goes on to offer three different situations where inequality was a factor among groups. However, his conclusion is that it is difficult to determine why different students participate in different ways. He believes more research must be done to inform educators on the varying degrees of participation within small groups. In a socially interactive environment, Greeno states that the “actions of several individuals mutually influence and reinforce or interfere with each other” (p. 113). It is important for teachers to stay alert to individual actions within small groups in order to maximize positive influences and behavior and concept reinforcement, while minimizing interference.

Written explanations are also an important method of communication. Zack and Graves (2001) discussed how one elementary teacher engaged her students in problem solving activities.

Her students were required to complete a “Problem of the Week.” Rather than working in collaborative groups at the start, she assigned them to work independently on them first. They recorded their thinking about the problem in their Math Logs.

> This permits the teacher to see what the child can do independently, via the child’s detailed written explanation, and the child’s own representations by means of drawing, tables, algorithms, arithmetic expressions, and so on. The teacher then has an indication of each child’s individual understanding before engaging with other children to explore the problem further. (2001, p. 237)

This idea of independent think time within a small-group setting is important. Many students need that quiet opportunity to organize their own thinking before they share with others.

Albert (2000) conducted a fourteen-week study of the mathematical thought processes of students in a seventh grade classroom. Within the context of a socially interactive setting, Albert explored the relationship between students’ oral and written thought processes. In her research article, Albert provides a discussion of Vygotsky’s “zone of proximal development” (ZPD) where students “are involved with tasks or problems that go beyond their immediate individual capabilities in which teachers (or other adults) assist their performance, or in collaboration with more knowledgeable peers” (2000, p. 109). This is directly related to the concept of scaffolding. Albert goes on to propose the “zone of proximal practice” (ZPP) which she names as a “new zone of self-regulation” where “self-scaffolding occurs” (p. 110). What an individual learns in collaboration with others is constructed through ZPD, while written language helps the learner internalize his or her understandings, increasing cognitive development through ZPP.

Albert (2000) recognizes both social interaction and reflective writing as important methods of mathematical communication. She has observed how the collaborative, or interpersonal, work of students helps to shape their intrapersonal thinking, providing them a way to make sense of their own learning and understanding. The nature of the writing process helps students to organize information, incorporate visual representations, discover patterns, and reflect on the reasonableness of a solution. It also provides an excellent tool for teachers for assessing students' understanding, thereby more clearly informing future instructional choices made by the teacher.

Students' Roles in a Mathematical Community

While the teacher is responsible for guiding the mathematics work in the classroom and employing various instructional strategies, learning is ultimately the responsibility of the student. Henningsen and Stein (1997) and Yackel and Cobb (1996) both address the importance of students acquiring a mathematical disposition. This means that in addition to conceptual understanding of mathematics, students are also demonstrating persistence in solving problems, exploring patterns and relationships, making connections, flexibly using a variety of strategies, conjecturing, generalizing, and communicating their thinking in various forms.

A discourse-oriented classroom creates different expectations for students than a traditional setting. Students must not only be active participants, but active listeners, as well. In order to develop a deeper understanding of mathematical concepts, "students need to analyze mathematical situations, critically examine their mathematical thinking and that of their classmates, and explain and justify their mathematical reasoning" (Pape et al., 2003, p. 183). Students should be encouraged to continually reflect on their own learning and understanding, and question and comment on the work of their peers.

Zack and Graves (2001) also discuss the importance of listening to others and explaining and justifying mathematical reasoning. They specify social norms such as being respectful and kind as key attitudes to have in order to make collaborative discussions as effective as possible. In the classroom they studied, students were required to credit peers for their ideas that helped to facilitate their own individual learning. These social norms must be communicated and modeled to students.

One discussion shared by Zack and Graves (2001) took place between three students – Hosni, Micky, and Jeff. These three students understood their roles within that discussion. Hosni accepted the role of leader, asked permission from his group to speak, and shared his solution. Both Micky and Jeff took on the role of listener and responded appropriately with comments and questions throughout Hosni's explanation. They were actively engaged in their own dialogue, helping each other to construct meaning from their collective mathematical work. "In a classroom where such learner initiatives are valued and the environment is one of a supportive community of learners, inquiry may follow but the characteristics are dependent on the dynamics of the interactions" (2001, p. 256). This type of discourse cannot be forced, but it can be cultivated and nurtured throughout the school year by each and every member of the classroom learning community.

Conclusion

When I began studying my research topic, I was primarily concerned with the role of students pertaining to mathematical communication. The research has shown me, however, that understanding the role of the teacher is a prerequisite to establishing the roles of the students. I also have a new understanding and appreciation of social and sociomathematical norms. I view myself as a reflective learner and teacher and consciously try to model this for my students. The research reminded me of the value of metacognitive thinking. I have a renewed goal to provide adequate time for students to question, justify, argue their mathematical positions, and reflect upon their own learning.

Research Purposes

The purpose of my project is to understand the consequences of daily small-group learning on student engagement and mathematical communication. This is important to my own teaching because it will provide me with an opportunity to explore the effects of daily discussion groups on the students' verbal and written communication skills. It will also be a time to discover new methods of instruction that require different forms of student collaboration than I have tried in the past. The data gathered during this research project will help me determine if a small-group structure is productive on a daily basis in my mathematics classroom.

One example of a new method used during this project is the homework presentations I implemented which were modeled after what we have done in our Math in the Middle courses. I am interested in seeing how well the students value the process of writing solutions for homework presentations and what they believe are the benefits and drawbacks of listening to others as they present their solutions. I would also like to learn if the successful small-group structure that I have had in place for problem solving lessons is appropriate for daily mathematics activities. The addition of homework teams has taken quite a bit of extra class time. One of the things I am seeking through this project is whether or not the homework teaming is a meaningful use of instructional time.

I have four main research questions that guided me throughout this process. The first relates to my understanding of my own teaching, while the other three focus upon student attitudes and conceptual understanding in the area of mathematics.

1. What does my teaching look like when students participate in a small-group learning environment and specifically how does my role change within that environment?
2. What will happen to students' understanding of daily homework concepts when a small group structure is used?
3. What changes can I observe in students' attitudes towards mathematics and learning mathematics as a result of small group implementation?
4. How will students' capacities to communicate mathematical thinking verbally or in written form change when the classroom is organized for daily small group work?

Action Research Methods

I began my data collection during the first week of February, 2008. All of my students had returned their consent forms before this time. My initial data that I collected was an attitude scale (see Appendix A) to determine individual beliefs about conceptual understanding and student attitudes towards group work. This baseline data was necessary to collect early in the study before students had had experience with working in homework teams. I was able to administer this to the whole class at once, and I did this at the beginning, middle, and end of my project.

I also began keeping a journal of observations and reflections during the first week of my study. I made notes in the journal at least once each week. These notes sometimes captured exact words from students' verbal communication or written communication. Sometimes they were observations about students' work and interactions, and sometimes they were reflections about the work I was doing related to classroom management and mathematical instruction.

Interviews were a significant source of data throughout this study. I conducted these in two different formats (see Appendix B). One objective of the interviews was to determine the relationship of small groups to student understanding. To help determine this, I conducted individual interviews of five different students at three points during the research project. I also conducted two small group interviews. I used the same group of four students both times. My purpose was to learn if changes occur in students' attitudes towards mathematics as a result of the small-group structure. Each time I conducted interviews I used a digital voice recorder and then transcribed the recordings within a couple of days.

Each week I saved copies of various forms of student work. The students composed their homework presentations on overhead transparencies. I gathered these at least once each week and made photocopies of their work. The rubrics that I used to score these were copied, too (see Appendix C). I also kept all of the students' work on their written problem solving solutions, which was about once each month.

My teacher grade book is another source of data. It contains a record of students' grades for homework and test scores. Lastly, a questionnaire (see Appendix D) was given to students at two different times during the study, once at the middle and again at the end. This questionnaire asked student to reflect upon their own ability to communicate mathematical ideas.

Much of my data was stored on my home computer. That is where I did my journaling each week, kept track of my interview recordings, and entered my data from surveys. The only data source that remained in my classroom until the end of the study were the copies of student work. Once my project was complete, I reread the pieces of work I had collected and pulled those that seemed most suitable for my final analysis.

I had survey data from two sources – the attitude scale and the questionnaire on mathematical communication (see Appendix C). I tallied the results and entered those numbers into a table. Then, following the Likert Scale, I assigned the values to each answer. For instance, on the attitude scale “strongly disagree” was a 5, “agree” was a 4, “undecided” was a 3, and so on. I plugged the data into a program to compute the mean, median, and standard deviation for each of the survey questions.

Findings

My fifth grade mathematics class is held in the morning on Tuesdays, Wednesdays, and Thursdays. Since I am an elementary teacher, I have some scheduling flexibility, so often these class periods are at least one hour in length. On the other two days of the week, mathematics is held in the afternoon and those class periods are just 35 minutes. I have a total of 24 students, but I have 20 students for mathematics as four of my students attend special education classes during that time.

For the past several years, I have had two different types of “average” teaching days. Two days a week, we begin class with a problem solving activity. Typically, the problem is introduced on Tuesday, and we finish it on Thursday. Many of these are hands-on where students may work with tiles, cubes, tangrams, or geoboards. We spend a significant amount of time discussing effective problem solving strategies. Students work together in their table groups, which are created within our seating arrangement.

The other three days of the week we start mathematics class with Daily Cumulative Review (DCR). Each student has his or her own workbook published by Scott Foresman, our district’s current curriculum series. There is a page of problems for each lesson that serves as a spiral review to help students retain computational and problem solving skills. On these days, students work independently for 5-10 minutes on the assigned problems. Then they meet with their DCR partner to discuss their work. Students are with the same partner for an entire chapter and then new partners are assigned. After partner work, we discuss the problems as a whole group.

At least four days a week I provide instruction on a lesson from our textbook. Students keep track of notes and practice problems within their own spiral notebooks. The homework assignments are generally worksheets from the series, but I often type up my own. This year I did more of that, because I wanted to provide more thought-provoking problems for their small groups to discuss.

As a part of my research project, we also made time each day for homework presentations where five students presented their work and solutions for selected problems from the homework. I also tried to allow at least ten minutes each day for homework teams to sit together so that they would have a supportive environment in which to work. From the beginning, we had three guiding rules for our homework team time. These are suggestions from Marilyn Burns's book *About Teaching Mathematics* (2007):

1. You are responsible for your own work and behavior.
2. You must be willing to help any group member who asks.
3. You may ask the teacher for help only when everyone in your group has the same question. (p. 401)

This weekly routine seems to work best for me. The students know the basic schedule for the week, yet they also experience much variety from day to day. Whether we are working with problem solving activities or computational practice, small group interaction and classroom discussion is constant. Communication is a valued component of students' learning. Students always know they can ask questions, bring up new ideas, comment on each other's thinking, and clarify their own mathematical understanding.

What my teaching looks like when students participate in a small-group learning environment: Owning the conversation

I explored four different questions during my research project. The first question helped me to consider what my own teaching looked like when students participated in a small-group learning environment. Through the increased communication that took place within our classroom, I found that I was able to gain a much clearer sense of each student's conceptual understanding.

I consistently emphasized mathematical discussions and the work of the teams. This transferred the focus away from me and on to my students. My role changed significantly. I became part of the audience of learners. This allowed me the opportunity to hear each student's distinct voice while they shared their thinking within their small groups. The small-group setting invited every student to participate in the dialogue, and they took ownership of their own mathematical conversations. Instead of answering questions, I was asking questions – questions that pushed the students to clarify their thinking and see their work in different ways. I also found that I was able to weave direct instruction into student-centered conversations more naturally than ever. These conversations were often associated with students' preparations for their homework presentations and provided great insight into their mathematical thinking.

In February, Elizabeth[1] and I had a remarkable conversation about a problem from her assignment on polygon angles. She was working to complete it and was also preparing a written explanation to present before the class the next day. The problem was this: Two angles of a

[1] All names are pseudonyms.

hexagon have equal measure. The other angles have measures of 83, 97, 126, and 98 degrees. How many degrees are each of the two remaining angles?

Elizabeth knew to add the given numbers and then subtract from the sum of all the angles in the hexagon, 720 degrees. Her initial step was 720 – 404 = 316. She realized, though, that this difference would give her the total measurement of two angles, not each angle. She looked at me with this amused look and said, “Then what do you do – magically split that in half?!?” She started talking about counting up from 1 to find the middle. I let her talk about this for a while, and then I had to interrupt. Finding the middle of 316 by counting up would work, but it would be time consuming. Elizabeth is one of the top students in the class, and I was surprised she did not see that she could just divide 316 degrees by two to get the measure of each angle. Finding the middle was an interesting way to look at it. I asked her to explain this idea of “the middle” for me. She said counting up would tell you the middle and that would give you the amount of each angle. I decided to refer her back to her own words from her original question.

“What operation do we use when we want to split something in half?” I asked.

“Division?” she replied, with some uncertainty.

“Yes!” I cheered. “And what do you want to divide by if you are splitting something in half?”

“Two?”

“Yes!” I cheered again.

“Ohhhh.” She took a few seconds to work that out on her paper. “So they each must be 158 degrees!”

I nodded.

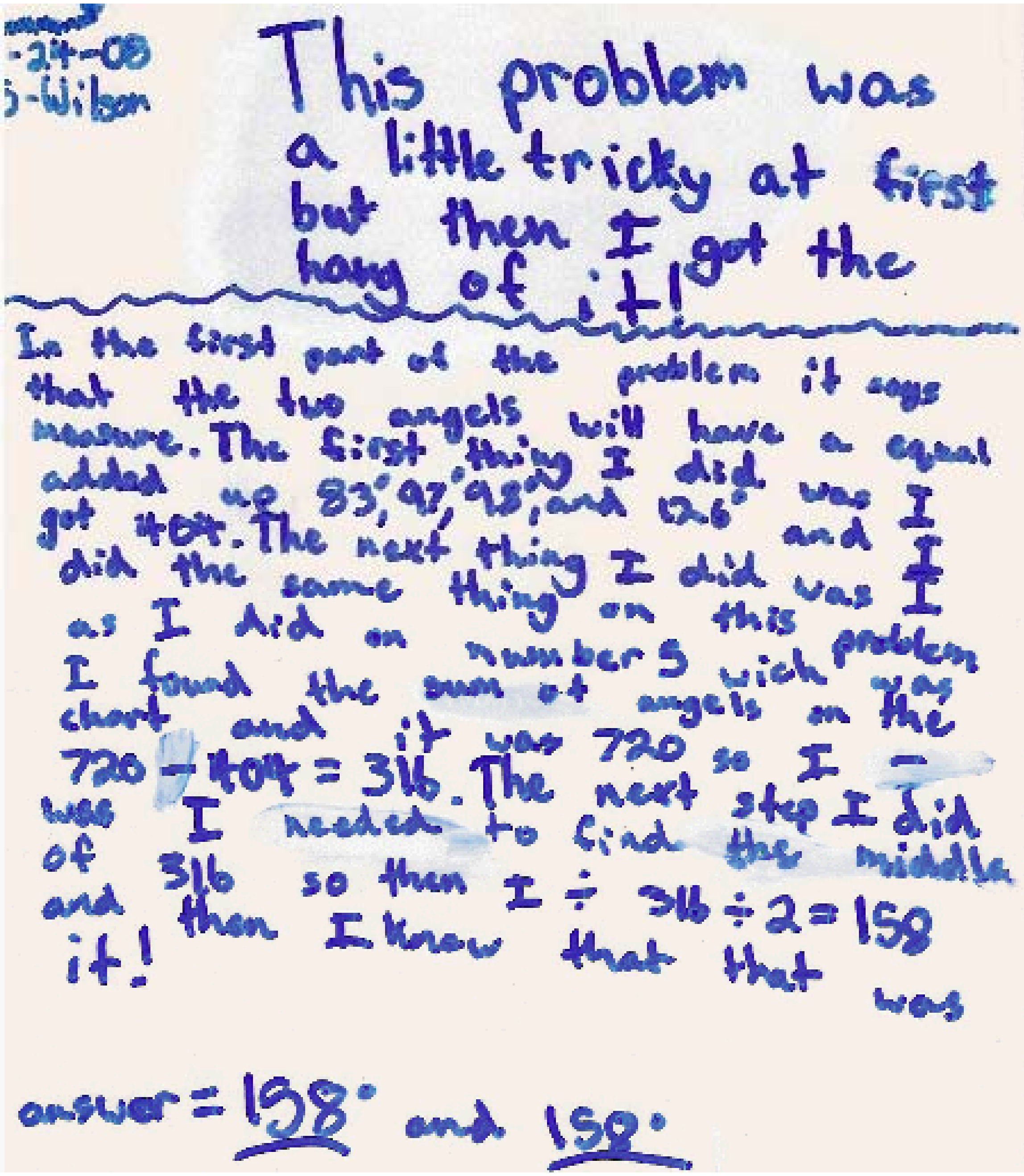
“Whoa! I’m on fire!”

I had to laugh. This was a typical Elizabeth comment. I admire her quick wit and carefree spirit, and I appreciate her creativity. Her unique style made her a significant contributor to our class this year, because she often presented a different way of thinking about problems and situations. She was also someone that I knew was an asset to any group. She has many friends and is always positive and helpful to others.

Ten minutes later Elizabeth came to me to show me her write-up.

“May I read it?” I asked.

“Can I read it to you?” she said right back. “Because that would be even better!” Here is what she read to me, and she was right, it was so much better in her own voice.



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This problem was a little tricky at first but then I got the hang of it!

In the first part of the problem it says that the two angels will have a equal measure. The first thing I did was I added up 83°, 97°, 98°, and 126° and I got 404. The next thing I did was I did the same thing as I did on number 5 on this problem I found the sum of angels on the chart wich was 720 and it was 720 so I − 720 − 404 = 316. The next step I did was I needed to find the middle of 316 so then I ÷ 316 ÷ 2 = 158 and then I know that that was it!

answer = 158° and 158°

Our conversation here is an amazing example of why mathematical communication is so important. It provided me with wonderful insight into her thinking that I would have never gained in a traditional setting. Her written solution gave her an opportunity to digest the work of the problem again reinforcing for her the process she followed to attain her solution. Her explanation is evidence for me that she understands what she did because she successfully put it in words.

I had an opportunity to provide direct instruction to Elizabeth, because she chose to communicate her mathematical thinking to me. This was not a conversation I had anticipated. There were several times, though, throughout this project that I did try to think ahead to the next day's conversations and purposefully plant concepts and questions that I believed would generate rich discussion. One example of this occurred during our geometry unit when we had a lesson on types of triangles. I had assigned a worksheet from our Scott Foresman series. This worksheet contains the following problem: If a triangle has a 45-degree angle, what kind of triangle is it?

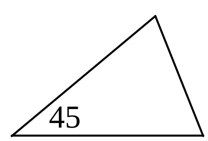
I like this question, because there is more than one right answer to it. In past years, the students have completed the homework assignment and turned it in, but we have never taken the time to discuss this particular problem at all. I chose to have Ryler present this problem to the class during homework presentations, and I was eager to see the level of discussion that would take place around it. Here is his write-up:

> When we started, my group and I did #1 and then skipped to #5, thanks!! We got acute for #5 because Mrs. Wilson told us that, all angles smaller than 90 degrees are acute, all angles that are 90 degrees are right, and all angles over 90 degrees are obtuse, and Tanisha had a 45 degree angle so it would be acute because it is less than 90 degrees.

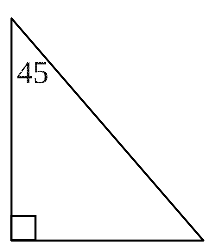
Ryler's explanation was accurate, but I asked the class if anyone had a different solution. No one did. I expressed my appreciation to Ryler for his work, but I also said that we would revisit this problem after the presentations were done. Unfortunately, Ryler is not accustomed to positive attention, although it is what he craves most. He is a very good mathematics student, but too often his poor social skills get in the way of his success. I could tell that my interest in his problem was satisfying to him and made him feel like a valued contributor to our class.

Several minutes later, when we reread Ryler's problem, I realized we were not all reading the problem the same way. Many of us had interpreted the problem in different ways. We had to talk about what the word "it" was referring to in the wording of the question. Some thought it was the triangle; some thought it was the angle. We reread the question again and confirmed that the question was asking about the triangle.

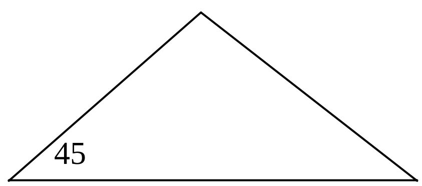
I asked each student to draw a 45-degree angle on the back of his or her paper and complete the shape to create a triangle. Most drew something like this:



Then Zach shared how he had created a right triangle with a 45-degree angle. I asked him to come up to the board to draw his example.



Gail said she had a different way to draw it. Hers was an obtuse triangle.



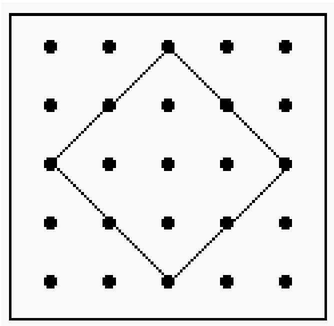
This discussion showed us that Ryler's problem actually had three different solutions to it. A triangle that has a 45-degree angle can be acute, right, or obtuse. It just depends on how it is drawn. Direct instruction was used within student-led conversations to highlight the important ideas behind this problem. I provided the opportunity and asked a key question or two, but the students generated their own thinking and evidence to support their thinking on their own. That was exactly was I was hoping for with this problem.

Towards the end of our geometry unit, I provided the students with a "Geoboard Challenge" activity. Their task was to create various shapes that matched specific criteria, and the students knew from the directions that three of the fourteen challenges would be impossible to solve. This activity really forced the students to understand and apply the mathematical language we had been using, such as acute, obtuse, isosceles, congruent, parallel, perimeter, and area. I was impressed to see several students using their math spirals as resources for many of the terms and definitions.

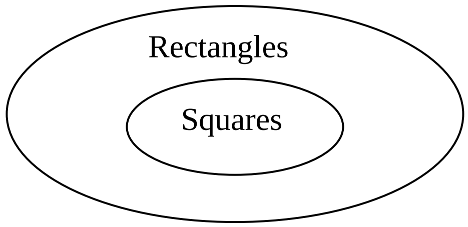
We worked on this activity for two class periods, and at the end of the first period my plan was to have students share their findings for the first seven challenges. The discussion was excellent, particularly around challenge #6 that asked the students to find an isosceles triangle with no congruent angles. Through our conversation, we realized that our definition of isosceles had perhaps been too limited by just referring to a triangle's sides, not its angles. So I drew a picture on the board and showed the students that if a triangle has two congruent sides, it will also have two congruent angles.

However, the most noteworthy portion of this day occurred when the students insisted they were not finished with their discussion at the end of the seventh challenge. The students felt that #7, find a rectangle with three interior pegs, led right into #8, find a rectangle with five interior pegs. This was a beautiful moment, because it was new evidence for me that the students truly felt that they owned the mathematical conversations of our classroom. I chose to be flexible, follow their lead, and the discussion continued.

I knew from my observations during the work time that only a couple of groups had completed the eighth challenge, and their success was largely due to my prompting them to consider thinking of a rectangle in a slightly different way. Most students referred to the solution below as a square or a diamond, so this problem led to an amazing discussion about the relationship between rectangles and squares. We had addressed this the previous week while defining quadrilaterals, yet several students did not believe the shape on the geoboard was indeed a rectangle.



I reminded them of the Venn diagram that we had recorded in our mathematics spirals during Lesson 6-4. We had discussed this same diagram style many times in science class within the context of mixtures and solutions. I asked the students if they could recall the true statements communicated by this diagram when we used those terms. Right away they knew. All solutions are mixtures. Not all mixtures are solutions. We were then able to create similar statements that related to squares and rectangles. All squares are rectangles. Not all rectangles are squares.



The solution for challenge #8 on the geoboard is a rectangle that also happens to be a square with five interior pegs. It is possible. This experience clearly shows how direct instruction was an essential element of the mathematical discourse that occurred, but it developed naturally in a student-centered environment where the students were key players in their own learning and were even allowed to be a part of the decision making related to the instructional pacing of the lesson.

On March 28, Cody presented his written explanation for the following problem: Name a fraction between 1/2 and 1. Our class had completed several introductory lessons on understanding fractions, and this lesson focused upon comparing fractions. Cody's approach to his problem was unique, and I chose to emphasize how his strategy could be transferred to other situations when fractions with larger numerators and denominators are considered. Here is Cody's written explanation.

I think it is 3/4 because 2/4 is a half and for the numerator it is 3 and 3 is larger than 2.

It is clear that Cody values brevity with words, but his idea is excellent. Several students shared other correct answers to this problem, as well. Most of them had thought about this problem with fraction bars or similar pictures. What I really appreciated about Cody's thinking was that he thought of something he knew: 2/4 is half. Then he built off of that idea to create a fraction he knew would work between 1/2 and 1. I complimented his thinking on this in front of the class.

Considering the relationship that exists between the numerator and denominator of a fraction is very helpful when comparing, and using halves as benchmarks for comparison provides a good starting point. Also, drawing visuals is not always practical, particularly when numbers get bigger. I used Cody's idea to talk about fractions with larger numbers. One student suggested we try looking at 750,000/1,000,000. How does that fraction compare to 1/2? The students were able to quickly determine what half of one million is and they realized that this fraction is larger than 1/2. It was wonderful for Cody to see how his strategy could be used for a fraction like that.

This is another example of how direct instruction can be precisely linked to the mathematical communication of students. It is important for teachers to be alert to these moments that provide opportunities for elaboration of students' ideas. These thoughtful decisions help a teacher to guide his or her students' mathematical thinking and the classroom becomes a safe and meaningful place for students as they participate in their own learning that is rich with collaboration, inquiry, and reflection.

What happens to students' understanding of daily homework concepts when a small group structure is used: Constructing meaning together

I found that a small-group learning environment makes students more willing to ask other students for help when they do not understand a concept. The role of the student is always a learner, but in a small-group setting each student is also a teacher. They are encouraged to support each other in their learning through asking questions, sharing strategies, and clarifying ideas. In an interview with Jim, I asked him why he thought I had decided to place students in small groups for mathematics work. This was his reply:

> *I don't really know. I think it's so we can see what other people are thinking. Just seeing what they know. Like Marie, Elizabeth, Mitch and I…one of us will be, 'I got this,' and then we'll kind of explain it a little bit and if somebody else doesn't have it or has something different we'll go back through and ask, 'How did you get this?' and "Why do you think it's that?' It's worked pretty good.*

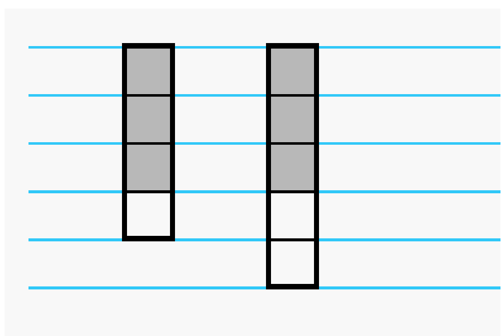
The questionnaire on mathematical communication provides evidence to this assertion as well. Here is question #3 from that survey: When I have a question about my work, I ask other members of my group for help. Twelve out of 20 students said they ask for help "most of the time," while eight students circled "sometimes."

An excerpt from my journal on February 20, 2008, shows how students are relying on each other for help.

> *Students worked in homework teams today. I tried to pay more attention to the interactions of each group. Usually my focus is on classroom management and student understanding. It's hard to step back and just watch and listen. Today I walked around with my sticky notes, so I could jot down observations and comments…It appeared that the groups were doing really well. As I walked past Carrie's group, I heard her say, 'You guys, I need help.' Then she immediately worded it as a question, 'Can you guys help me?' Right away the other three members of the team leaned over to help Carrie with her question. I'm glad that Carrie felt comfortable enough to speak up and ask her group for help.*

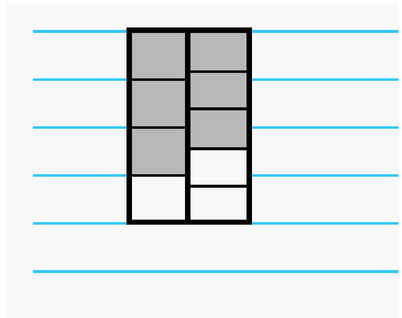
It seems that students are able to take more responsibility for their own learning in a small group structure. They have a supportive environment in which to work and are able to rely on their team to help them increase their own mathematical understanding. This support system was also apparent within the large group setting.

One day at the end of March, while we were studying fraction comparison, I began a lesson where I asked students to draw pictures in their mathematics notebooks to represent 3/4 and 3/5. As I walked around the room, I noticed that a few students were drawing their representations in different sizes. Gail, for instance, drew a vertical fraction bar for 3/4. She divided it into four equal parts and each part was a line of her notebook paper. Then she drew 3/5 and she used the lines of her notebook paper as a guide again. She left the section size the same, so she had to make her 3/5 bar one line longer.



The issue with this illustration is that the shaded amounts appear to be equivalent. I noticed that Craig, right next to Gail, was doing the same kind of thing. I continued around the room, though, and observed many students drawing visual representations that were the same size. I shared with the class that I was glad to see so many people drawing shapes of the same size and how that was extremely important when comparing fractional amounts. A few people started to erase and redraw.

A few minutes later, I began our comparison conversation by inviting Elizabeth up to the board to demonstrate how she had drawn her fraction illustrations. I had seen her work on her notebook paper (shown below) and I liked how she used the blue lines as a guide, yet understood that the bars needed to be equivalent in size. She drew a large rectangle first and then split it in half vertically to create two fraction bars. Then she divided the left side into fourths, the right side into fifths, and shaded in the parts.



We could see that 3/4 was larger so I placed the > symbol between the two fractions I had written on the board above the illustration. Immediately Elizabeth said, “Sometimes I look at the bottom number. That helps me.” With one problem, in just a few minutes, she had jumped to the exact point I was hoping to make in the day’s lesson. I asked her to explain. She said that since four is a smaller number, then 3/4 is bigger, and since five is a bigger number, then 3/5 is smaller.

Morgan said she was thinking it should be the other way around. Since five is bigger, then 3/5 is bigger. I did not say a word, and Elizabeth went on to explain that even though four is a smaller number, the pieces within the fraction bar are larger. The fifths are smaller pieces. This made sense to Morgan. We did several additional examples where the numerator was the same, but the denominators were different. The students seemed to quickly catch on to Elizabeth’s idea.

Then Morgan asked about fractions that have different numerators but same denominators, so we looked at few examples like that. Using my computer and projector, we crafted together two sentences that captured the mathematical thinking that had just taken place.

1. To compare two fractions with same numerators, we look at which denominator is smaller. The one that has the smaller denominator is the bigger fraction.

2. To compare two fractions with same denominators, we look at the numerator and see which is bigger or smaller. The bigger the numerator, the bigger the fraction.

Putting the ideas from our conversation into words was a powerful element of our learning that day. The process of transferring mathematical ideas into written language is usually not an easy task. This concept of comparing fractions with like denominators or like numerators seems like a simple thing, but this experience taught me that all students do not find this to be simple. Our work with visual representations and then connecting verbal and written language to match our mathematical thinking was a sophisticated process for each learner in our classroom. I know that too often I have glossed over this lesson, assuming students understand how to compare fractions like these.

It seems that when students can talk about a problem and write about a problem then they are truly taking responsibility for their own learning. The small-group structure is a means of support for students, and when they are provided consistent opportunities to work together in this kind of environment, they will understand the importance of verbalizing their thinking, they will feel comfortable asking for help, and more confident in asking questions of each other.

Changes observed in students' attitudes towards math and learning math as a result of small group implementation: Practicing positive interaction

I found that the large majority of students liked the idea of working in small groups for both homework and problem solving activities. There was some frustration, however, with uncooperative team members. This dampened some students' attitudes towards learning mathematics at times, and caused some to prefer working independently.

The group interviews I conducted provided a mix of opinions about small groups. When I asked the students to share the positives of small groups, they were quick to share how they appreciated that they could ask their peers for help when they did not understand a concept.

Craig: Well, our group always helps other people out, like always stays on track, and helps us out if we're stuck on a question.

Carrie: If someone gets stuck, then some other people that get that question can help with that question.

Paige: Mine is like what they said, too. I get stuck on some questions sometimes, because my brain doesn't want to think right and so I usually need help and they help me.

Elizabeth: It's really nice to work together in our group. For me I just need five more minutes. That way you have time to write down problems instead of just talking about it. Sometimes Doug and I talk about a problem and it takes us almost the whole time.

Craig: Yeah, I wish we had more time, too. It's kind of like our group gets going and then it is time for us to quit.

I know from casual conversations throughout the research project that several students were dissatisfied with the amount of time they had to work in their homework teams. Many students wanted more than the usual ten minutes to work together. The attitude scale results show this as well. In April, 11 out of 20 students disagreed or strongly disagreed with the following statement: We have an adequate amount of time in class to discuss our work in our small groups. Time is always an issue, and I gave them as much time as I could to work in their homework teams.

However, some of the students also stated that they were not satisfied overall with the individual contributions of some group members. Their responses to my question, “How flexible and open minded are the members of your group towards each other?” show how their frustration with a few affected their views about the success of the small groups.

Elizabeth: Some people get mad, because one time somebody in our group, we were comparing answers, and we had a different answer that was right and that person got really mad and just said, “No. Mine is right and you guys’s is wrong.”

Doug: Some people don’t really pay attention. They make noises and ask for answers.

Craig: That happens in our group, too. Usually it’s just like one person that bothers us, but it’s hard for the rest of us to concentrate because he’s so annoying.

Carrie: Our group is pretty good. If you’re stuck, they will help you. I think we all do a good job of listening to each other.

The students were echoing my own thoughts here. It was difficult to hear their unhappiness with some of their peers, but it did confirm what I had been feeling lately as I observed several of the groups. Just the day before, on March 18, I had written in my teaching journal about the tensions I was feeling.

> *Homework groups are frustrating!! Kids announcing the answers. Some people writing them down without thinking them through for themselves. Others are frustrated because they have to wait for the dawdlers. The kids that don’t understand think the rest of the group is going too fast. UGH!*

Even though the students seemed to become irritated with other group members, my hope was that they believed the small groups were worth it and were increasing their conceptual understanding. So I asked, "What kind of difference are the small groups making on your own understanding of the mathematics we are learning?"

Paige: Not really making a difference. That stuff…I just understand it, but sometimes I don't. So kind of they do and kind of they don't. Sometimes it can be harder because we have to wait on people.

Carrie: Sort of. Some people can help me and some people kind of go through it really fast and don't really care.

Craig: One person in our group speeds ahead and then tells us later what the answers are and we don't understand them.

Paige: I don't really like them [small groups]. I usually can get my work done on my own. And I usually get my work done and I'm ready to hand it in, but I feel like I can't because I'm supposed to help others. I don't really want to help them. I feel like they don't want to listen.

I decided that as a class we needed to re-envision or re-clarify what homework teams were supposed to look like and sound like. I felt like the whole idea needed to be tweaked. The small groups were not turning out how I had envisioned. I wanted the students to support each other, not frustrate each other. Over the next few days, I recorded behaviors that were characteristic of a successful small group and behaviors characteristic of a not-so-successful small group.

After we came back from spring break, on March 25, we spent a good portion of a class period revisiting out homework team rules and discussing their meaning. Then I asked Zach, Elizabeth, Doug, and Cody to role-play the attitudes and actions of a homework team. I gave them a pretend mathematics assignment and specific roles. Zach played the loud one, blurting out answers and telling people what to do and then scoffing at them for not knowing what to do or not keeping up. Elizabeth was the complainer—wanting people to slow down and help her with many of the problems. She kept whining, “I don’t get it.” Cody was the quiet one. He worked on his assignment without engaging with his fellow team members at all. Doug was the distracted one—looking around the room and playing with his pencil. They performed in this manner for a minute or two while the rest of the class stood around them and observed. We talked as a class about the behaviors that we saw. The students thought it was funny, but they also recognized themselves and others in the behaviors that they saw.

Then we did a second role-play. Mitch, Vicki, Paige, and Tyler helped with this one. They quietly worked on their own papers for the first 30 seconds or so. Then Vicki asked how everyone was doing. Did anyone need help? No one did, so they kept working. Then Paige asked if anyone had done #3, because she did not understand it. Tyler and Mitch both said they understood it and then Tyler offered to help her get through it.

I told the class that the homework teams are a work in progress, but that this was the way I wanted them to approach homework teams right now. I communicated to them that it is <u>not</u> important that all members are on the same problem at the same time. They do <u>not</u> have to discuss every single problem either. Rather, they are to work at their own pace and if they have a question about something, then they should feel free to ask someone in their group, and one of the homework team rules is that if a person asks for help, you must help them.

This one day made a huge difference in the way our small groups operated for the rest of the year. The students were much more aware of the specific behaviors that were expected, and I became more conscientious about reminding them of the positive attitudes and behaviors that successful groups display. We often reviewed the homework teams rules and the role-play scenarios. Each time an assignment was given, I reminded them that it was important for them to begin their work quietly so everyone can think and work at their own speed. I also reminded the students that every group needs an encourager just like Vicki was during the role-play -- someone who will periodically check with the other team members to see how everyone is doing.

On April 4, I walked around the classroom during homework team time and recorded various behaviors that I observed:

- *Keagan is usually off task, but better today. Tyler is leaning over him and helping him with a problem.*
- *Gretchen is leaning over helping Craig.*
- *Dillon got up out of his chair to stand by Martin and help him. I stayed close to hear the conversation. Dillon spoke fast and then immediately sat down after his explanation. Martin looked pretty confused, so I asked Dillon to come back over beside Martin and work through a problem together. He did and that helped Martin. He got it.*
- *Craig and Martin are using their math spirals as a resource.*
- *Cody asked Ryler, "Need some more help? You sure?"*
- *Marie said, "Carrie, how are you doing?"*
- *Tyler asked, "Keagan, need any help?"*

This is what I pictured homework teams to be. The students were helping each other, they were helping themselves, and there was no frustration over pacing, because they understood that it was okay if different people were in different places on the assignment. Seeing these positive behaviors reinforced the idea that students need to be taught how to appropriately interact with each other in an academic setting. Consistent reminders should be provided as the students practice the necessary social skills that are required for a team to be successful.

The attitude scale provided further evidence that showed students' attitudes towards learning mathematics and small group implementation. This survey data was collected using a Likert scale. A "strongly agree" response has a value of 5, while a "strongly disagree" response has a value of 1. I collected this data at the beginning, the middle, and at the end of my research project.

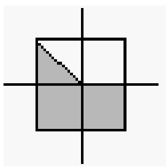
The students consistently showed that they liked to work in small groups for their problem solving work, yet when asked if they understand the mathematics of a problem more when their group works together, the scores fell during the course of the project. Initially, the mean score was 4.5, then in March it was 4.25, and at the end of April the mean score was 4.05.

Survey scores for similar questions related to homework teams fluctuated a bit. The mean scores were higher after the March survey, but then lowered again at the April survey. Students were asked if they liked doing homework assignments in small groups. The mean score in February was 3.55, in March it was 4.3, and in April it was 4.05 – still quite a bit higher than the initial survey. The next question on the survey asked students if the small groups had helped them better understand the mathematical concepts they were learning. The first survey had a mean score of 3.45. In March it went up to 3.95, and then in April the mean score dropped to 3.25. It seems the students like working in groups, yet they are not as certain that these small groups really help their conceptual understanding of the material they are learning.

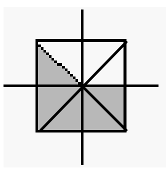
How students' ability to communicate mathematical thinking verbally or in written form changes when the classroom is organized for daily small group work: Developing individual voices through collaboration

The small group structure supports students in clearly communicating their mathematical ideas for their own learning and for the learning of their peers. Here is a homework presentation prepared by Zach. He is a thoughtful young man that prefers animals to people and speaks only when he has something significant to say. I appreciate how he used proper vocabulary and helpful diagrams to illustrate his thinking.

Our group had number five. We thought it was easy. The picture looked like this:



Then we made each little square into two triangles, so it would look like this:



Then you count the triangles in all. There are 8. So then you count the shaded triangles. There are 5. Then that means that the 5 is the numerator and 8 is the denominator, so your answer is 5/8.

Another data source for this research question came from the questionnaire on mathematical communication. The questionnaire showed that 75% of the students agreed with the following statement: “Our small group learning environment helped me to communicate my mathematical ideas more clearly.” Here are a few specific comments:

- If you don’t get it, they will help you.
- It helps to hear and think about someone else’s ideas.
- Your group can help you explain it if it gets confusing on some things.

- We can share our work and if it is different, we can work it out.

Gail's story must be told at this point. Even though the comments above were anonymously provided, in my mind I can hear Gail saying every one of them. Most people would call her a pistol. Her gymnastics training has helped her to be fiercely competitive in P.E. and on the playground, yet in the classroom Gail is quiet and shy. School is not easy for her. We have a microphone system that we use regularly, and Gail's small frame would shrink even smaller every time she held the microphone. She got nervous and giggly whenever it was her turn to speak.

During the first homework team time in early February, Gail had been assigned to present problem #27 for the next day. The lesson was on algebraic expressions. That would be the first time the students had participated in homework presentations. I knew this might be tough for her, so I watched Gail carefully to see how she would react. As I walked around the room while teams began to work, I was very pleased to see that her team had jumped ahead to #27. When I asked them about this, they said they wanted to make sure that they discussed it together in order to help Gail be more prepared in writing up her solution later at home.

When Gail presented the next day, she readily went to the overhead, laid down her transparency, and all of us waited patiently for her to begin. She was quiet for a bit, and then she asked me in a timid voice, "Would it be okay if I reread what I wrote before I say it out loud?" I told her it was completely okay. While she turned around, composed herself, and silently reread her work shining on the screen, the class was silent. Then she shared. It was wonderful. Through her words and the certainty with which she spoke, we could see that Gail understood her problem: Evaluate $n \times n + 7$ for n=3. What would be the value of n, if $n \times n + 7 = 88$?

> My group helped me with this problem. First, we made *n* be 3 and we got 16 for an answer since 3 x 3 + 7 = 16. Then we figured what *n* could be to get 88. I knew it had to be 9, since 9 x 9 + 7 = 81.

Within two weeks, all of the students had taken their turn at presenting a homework solution. It was time to cycle through the groups again. Gail must have been aware of this because on February 22 she came up to me early in the morning, before mathematics class had started, and asked me if it was her turn to share a homework problem again for the next day. I told her it was, and her quick response was, “YES!” This time I assigned her problem #7 from a worksheet on polygon angles that had four parts to it. The presentation she prepared for it was outstanding, and her confidence had increased. She showed how to calculate the sum of the angles in a hexagon. Gail used two transparencies to explain her work, and she illustrated the solution in two different ways. She explained how one could take a measure of one angle and multiply it by six if one had a regular hexagon. For an irregular hexagon, she demonstrated how one could triangulate it into four triangles and then multiply 180 and 4 to get 720 degrees.

This problem was much more complex than the first problem she had presented, but she accepted the challenge and her tremendous effort and courage were admired by all of us. She seemed to genuinely enjoy the opportunity to share her thinking in front of her classmates. Gail successfully articulated her mathematical ideas within an environment she knew to be supportive and kind, and for her that made all the difference.

The last piece of evidence that supports a small-group setting for strengthening mathematical communication is about Doug. Doug was the shortest boy in our class this year and was painfully aware of that fact. You would never know that it bothered him, though. His personality is a wonderful mix of hilarity and serious attention to detail. From his hair to his homework, he likes to be precise.

At the beginning of April, Doug was working on his write-up for his homework presentation. He had to explain how to change 14/10 into a mixed number. He was being careful to write with specific vocabulary. As I walked by, he pointed to 1 4/10 on his paper and asked me, “Should I call it ‘10’ or ‘the denominator’?” I told him either was fine. Then he asked as he pointed to the 1, “What do you call that thing?” I told him it was called the whole number. A few minutes later Doug came up to me and asked about drawing a picture. I told him he sure could if he wanted but that it was not a required part of the problem. He responded with a grin, “Well, I’ve been listening to you and you say it’s cool to show it with a picture, too.”

“Absolutely,” I told him. “That will really help to show the picture, too.”

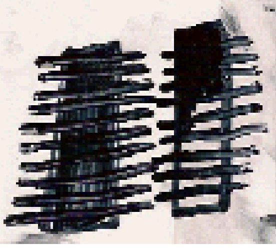
The write-up from Doug’s presentation is below. He carried over into his written work what we had talked about the previous day. This is another great example of how a classroom that invites conversation helps students to value language and want to get it right. Doug took his responsibility seriously and understood that the words he chose to convey his solution mattered. They mattered to his classmates and to me, but most importantly his words mattered to him.

#3
PS-5
4-4-08

The question was $\frac{14}{10}$ and I got $1\frac{4}{10}$, and I can reduce $\frac{4}{10}$ and make it $1\frac{2}{5}$.

How I got my answer was how many times does 10 go into 14, so it is 1 time so the 1 would be the whole number. Carry over the 10, which is the denominator. How I got my numerator is that I said 10 goes into 14, 1 time. 10 times 1 is 10, so to get to 14 you have to add 4 more and that is how I got my answer and numerator.

I drew a picture to represent $1\frac{4}{10}$.



Conclusions

This research project provided daily opportunities for me to understand better how students can collaborate with their peers during problem solving and homework activities. Overall, it seems that students appreciated these experiences and many wished that they had more time to discuss their work in small groups. The communication that took place seemed to strengthen their written communication, particularly connected to their homework presentations. It also allowed for varied ways of thinking to be shared within our classroom. When a classroom is set up with a more teacher-centered approach, then the teacher teaches the process, the students practice it, and then they independently complete their homework assignment using those same processes that they were taught. When do they take ownership of those concepts, though? It seems to me that when students can talk about it, write about it, and see the problem in more than one way, then that is when true ownership of their learning occurs.

It is evident that some teams may work better than others, and some students may not prefer this group approach. The teacher must constantly monitor, support, and reflect upon the work of small groups. Flexibility is important and instructional time is required in order that students are taught the social skills necessary for a successful small group. Students also need to routinely revisit the homework team rules and renew their commitment to the small-group structure that they appreciate.

This study has the potential to inform my colleagues' efforts within my teaching community. Our school district is examining writing instruction quite closely. This problem of practice that focuses on communication in a specific content area is a timely topic that could greatly benefit other teachers and provide them with ideas to strengthen their students' work in similar ways.

The new knowledge discovered through this research project will be of significant interest to other teachers, as well as parents and administrators. Small group discussions, homework teams, and written solutions are unique characteristics in most math classrooms. Many educators would agree that these are exciting methods that we can employ to authentically measure students' conceptual understanding of content.

Implications

Next year our school district will be implementing a new mathematics textbook series. Due to our district's low computational scores on our achievement tests, we have chosen to adopt the Saxon program. I have thought a lot about which instructional practices related to this research project I would choose to implement next year and how I might go about incorporating them.

I know that my table groups will naturally become homework teams, and I do want to have these teams support each other with their mathematical thinking on a daily basis. However, I now recognize that the students really need lessons in small group behavior. The lessons need to be as specific as possible. The two role-playing situations that students acted out during this project were very effective, but they helped for just a few days. I think it is well worth the time to take it slow in setting up the small group routine. The homework rules should be revisited on a regular basis, and the role-playing scenarios can be acted out several times. The teacher cannot assume that students know how to properly communicate with each other. Specific behaviors must be practiced. What does a student look like or say if they have a question? How does a student go about explaining a problem to another student? What are some kinds of nonverbal communication that a group member might display that the other group members should notice?

I also need to consider the personalities that are combined for a team. This year we reorganized the homework teams one time. Do the teams need to be altered more often than that? I also tried to split up the students who tend to cause difficulties within a small group. I think the result was that several of the groups became frustrated because of these people. Would it have been better to put those individuals together in their own group?

The other practice that I will definitely keep for next year is homework presentations. At the end of the year, the students shared with me that they liked preparing for those, and they also enjoyed listening to their peers present. Some said it made them nervous to get up in front of the class. They were afraid they might have a wrong answer, but they also knew it was good experience for them, and they knew what they were sharing came from a group effort, not just their own. One student said the presentations helped her "because it showed me a different way of doing things and it usually helped me understand the problem more."

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Appendix A

Attitude Scale

Please respond to the following items by drawing a circle around the response that most closely reflects your opinion.

SA	**A**	**U**	**D**	**SD**
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

1. I am confident with my ability to solve mathematical problems.

SA **A** **U** **D** **SD**

2. I stick with my problem solving work until I'm done and I don't give up.

SA **A** **U** **D** **SD**

3. I like to look at more than one way to solve a problem.

SA **A** **U** **D** **SD**

4. I enjoy our problem solving work more when I can work with others.

SA **A** **U** **D** **SD**

5. I understand the mathematics of a problem more when our group works together.

SA **A** **U** **D** **SD**

6. I like doing homework assignments in small groups.

SA **A** **U** **D** **SD**

7. Our small groups have helped me better understand the math concepts we are learning.

SA **A** **U** **D** **SD**

8. The members of my group are good listeners and are considerate of everyone's ideas.

SA **A** **U** **D** **SD**

9. We have an adequate amount of time in class to discuss our work in our small groups.

SA **A** **U** **D** **SD**

10. The members of our group encourage each other to do our best.

SA **A** **U** **D** **SD**

11. Our small group stays on task.

SA **A** **U** **D** **SD**

12. Our group takes the time to make sure all members understand the work we're doing.

SA **A** **U** **D** **SD**

Appendix B

Individual Interview Questions Pertaining to Conceptual Understanding of Daily Homework

1. How confident do you usually feel about successfully completing your math homework each day?

2. When you get stuck on a problem, how do you get through it?

3. Do you feel like you can usually explain how to do the homework to someone else? Why or why not?

4. Who do you most often go to when you need help understanding a problem?

5. How do you know when you have made an error in your homework? Can you give a specific example?

6. Which math concepts are your strongest areas? Why do you think you find these concepts easier than others?

7. Which math concepts are you still working on understanding? What do you think makes them hard?

8. How do you check over your work?

9. Why do you think I have been focusing in class on small groups, homework, and having you explain more?

10. What advice would you give me for planning my math classes for next year in the area of small group work? Homework? Explaining?

Small Group Interview Questions Pertaining to Students' Attitudes About Math as a Result of Small Groups

1. Share your opinions about the positive things you see happening during our daily small group work.

2. Is there anything you would change about the small group structure? If so, what would that be?

3. How flexible and open-minded are group members towards each other?

4. Do you believe the small group time is time well spent? Why or why not?

5. What kind of difference are the small groups making on your own understanding of the math we are learning?

6. How has your attitude about math changed since we've started our daily small groups?

7. What type of learning environment works best for you?

8. Think about our goals in math this year (located on the inside cover of each student's math notebook). Which goal is your strongest? Can you think of a time when this was evident to you? Which goal is one you'd like to improve? Have our small group learning environment impacted your growth on any of these goals?

Appendix C

Rubric for Homework Solutions

Criteria	4	3	2	1
Explanation	Clear explanation of solution is provided	Explanation is mostly clear with little confusion	Explanation is somewhat clear but some incomplete parts exist	Explanation is unclear and work is incomplete
Written Solution	Very well written and organized; attention to detail is evident	Mostly well written and organized; reasonably easy to follow	Somewhat clear and organized; some parts are difficult to understand	Poorly written; difficult to understand

Appendix D

Student Questionnaire on Mathematical Communication

Please circle the response that most closely reflects your opinion for questions 1-5.

1. I raise my hand and am willing to share my ideas during whole class discussions.

Most of the time Sometimes Not usually

2. I contribute ideas in my small group.

Most of the time Sometimes Not usually

3. When I have a question about my work, I ask other members of my group for help.

Most of the time Sometimes Not usually

4. Talking about a problem helps me understand it better.

Most of the time Sometimes Not usually

5. Writing about a problem helps me understand it better.

Most of the time Sometimes Not usually

Please answer each of the questions below completely and honestly.

6. What do you find most challenging about communicating your mathematical ideas verbally?

7. What do you find most challenging about communicating your mathematical ideas in writing?

8. Has our small group learning environment helped you to communicate your mathematical ideas more clearly? If so, how?

9. Do you prefer working independently on problems or with a group? Please explain your reasoning.