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Calculus-Based General Physics

Instructional Materials in Physics and Astronomy

1975

Sound

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SOUND

INTRODUCTION

We, much more than our ancestors, are constantly being bombarded by sound. We hear, and are more or less aware of, music, air hammers, television sets, jet planes, conversation, engines, sirens, etc., throughout the day. A sophisticated audio industry tries to improve the quality of musical sound. At the other end of the quality scale, noise pollution is a serious concern, which probably affects our lives more than we realize.

In order to deal with sound, we should have some idea of what it is. What factors determine whether a sound is pleasing or grating? How is sound transmitted? How much power do our ear drums actually receive when we hear a bird, or a rock band? What determines whether our ears detect a sound; what is the meaning of sound beyond the range audible to a human? These are just a sample of a multitude of physical and physiological questions that one might ask about sound. Not all of these questions will be answered in this module, but you will learn enough to begin finding answers to several of them.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Describe the motion of a transverse wave on a string (needed for Objective 1 of this module)	Traveling Waves Module
*Find the partial derivative of a function of two variables (needed for Objective 2 of this module)	Partial Derivatives Review
*Apply Newton's second law to problems in one dimension (needed for Objective 2 of this module)	Newton's Laws Module
*Write the meaning and give an example of the following terms: frequency, angular frequency, amplitude, and phase as they apply to simple harmonic motion (needed for Objectives 1 to 4 of this module)	Simple Harmonic Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. <u>Definitions and relations</u> - Explain what sound is, describe the physical significance of displacement amplitude, pressure amplitude, and intensity, and describe the relationship between pressure and particle displacement.

2. <u>Mathematical description</u> - Given an expression for pressure or displacement, determine whether it represents a possible traveling sound wave, and if so, determine the characteristics of the wave.

3. <u>Superposition</u> - Apply the principle of superposition to determine the characteristics of the sound at a point produced by two sources.

4. <u>Resonance</u> - Calculate the conditions for resonance in a given open or closed air column.

5. <u>Doppler shift</u> - Calculate the effects on the frequency of a sound wave produced by motion of the source and/or the receiver with respect to the medium.

GENERAL COMMENTS

We are fortunate that we don't need to learn entirely new physics, and entirely new methods of analysis, for each of the many different types of waves. Although there are important differences to be found among electromagnetic, elastic, hydrodynamic, and gravitational waves, it is the striking similarities among them that make the subject of wave motion more comprehensible.

You have already studied transverse waves on a string in the module <u>Traveling Waves</u>. Your study of other kinds of wave phenomena will be simplified if you concentrate on the similarities between the waves you study and waves on a string. We mention here some of the important similarities.

1. Wave Equation

All waves are mathematically similar. The mathematical expression for the displaced quantity is a solution of a particular partial-differential equation, the <u>wave equation</u>. The solutions of that equation describe the wave at any position and at any instant of time. In one dimension, the solutions of the wave equation are functions of both variables, x and t. In simple traveling waves, however, x and t always appear in a particular combination:

$$f(x, t) = f(x \mp vt),$$

where v is the wave speed. The minus sign is associated with waves moving in the positive x direction, and the plus sign is associated with waves moving in

the negative x direction. To understand this a little better, consider a wave in the shape of a pulse, as shown in Figure 1. The first graph (a) shows the pressure at a particular instant, t = 0. We could describe that wave form by some function, p = p(x).

The wave moves essentially without distortion to the right, at the wave velocity v. The second graph (b) shows the pulse at the later time, $t = t_1$, when the pulse has moved a distance vt_1. Since the shape of the wave is unchanged, the functional form describing the wave must be the same, but translated to the right a distance vt_1. We can write p = p(x´), where the origin of the x´ reference frame is translated a distance vt_1 to the right,

$$x´ = x - vt_1.$$

Therefore, for any time, the pressure is described by a function p = p(x - vt). If the pulse had been moving to the left, we would have changed the sign of vt in the argument.
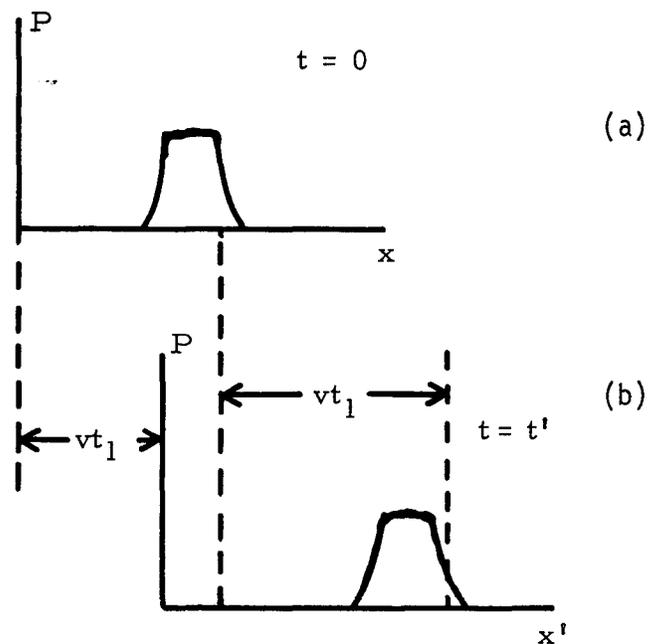

Figure 1

2. Principle of Superposition

If $f_1(x - vt)$ and $f_2(x - vt)$ are two different solutions of the wave equation, corresponding to different waves, the sum of these two functions is also a solution, if the amplitudes are not too large. Physically, this means that complicated wave disturbances can be broken down into a superposition of simpler disturbances. This fact is called the principle of superposition. You will be studying a few of the consequences of that principle.

One important application of the principle of superposition is the addition of two waves of slightly different frequencies. Let us assume that two sources emit sinusoidal waves with angular frequencies ω_A and ω_B. If the sound arriving at a particular point from each source has the same amplitude, the individual disturbances can be written as

$$y_A = y_0 \sin \omega_A t \quad \text{and} \quad y_B = y_0 \sin \omega_B t.$$

The sum of these two waves is then

$$y = y_0(\sin \omega_A t + \sin \omega_B t).$$

If we now apply the trigonometric identity

$$\sin a + \sin b = 2 \cos[(a - b)/2] \sin[(a + b)/2],$$

we get

$$y = 2y_0 \sin[(\omega_A + \omega_B)t/2] \cos[(\omega_A - \omega_B)t/2].$$

If ω_A and ω_B are nearly the same, the disturbance is the product of a rapidly varying term with frequency $(\omega_A + \omega_B)/2$, and a slowly varying term with frequency $(\omega_A - \omega_B)/2$. Figure 2 shows two waves with slightly different frequencies in (a), and in (b) the sum of the two waves. The slowly varying term acts as an envelope that determines the amplitude of the rapidly oscillating part. The maximum amplitude occurs when

$$\cos[(\omega_A - \omega_B)t/2] = \pm 1.$$

There are, therefore, two beats in each cycle of the amplitude term, thus the beat frequency is $\omega_A - \omega_B$.

Figure 2
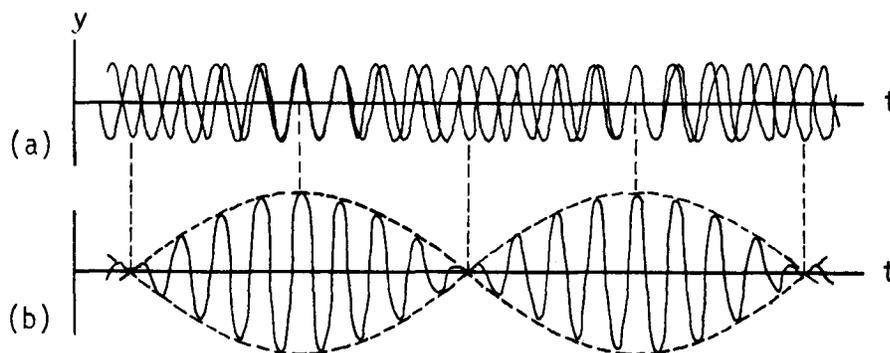
The beat phenomenon is true for all kinds of waves (electromagnetic, sound, water, etc.) at all but the highest intensities. In sound, musicians will frequently tune to each other by playing the same note and tuning out the beats. The human ear can detect beats between two tones up to a frequency of about 7.0 Hz. Highly skilled piano tuners, by tuning out beats, can tune a piano more accurately than all but the most sophisticated electronic tuners.

3. Pressure

In a sound wave, the dependent variable may be chosen to be either the displacement of a particle in the medium, or the deviation of the pressure from its equilibrium value. In either case the variable is a function of both x and t, and satisfies the wave equation. The solution to the wave equation can always be expressed in the form of a function of (x + vt), depending on whether the wave is moving in the direction of decreasing or increasing x, or as a sum of functions of (x + vt). The physical process is that a change in pressure produces a change in the volume of an element of mass of the medium. Since the change in volume is related to the particle displacement, the pressure and displacement are related. Either can be chosen as the dependent variable in describing a sound wave. Usually the pressure is more convenient to work with.
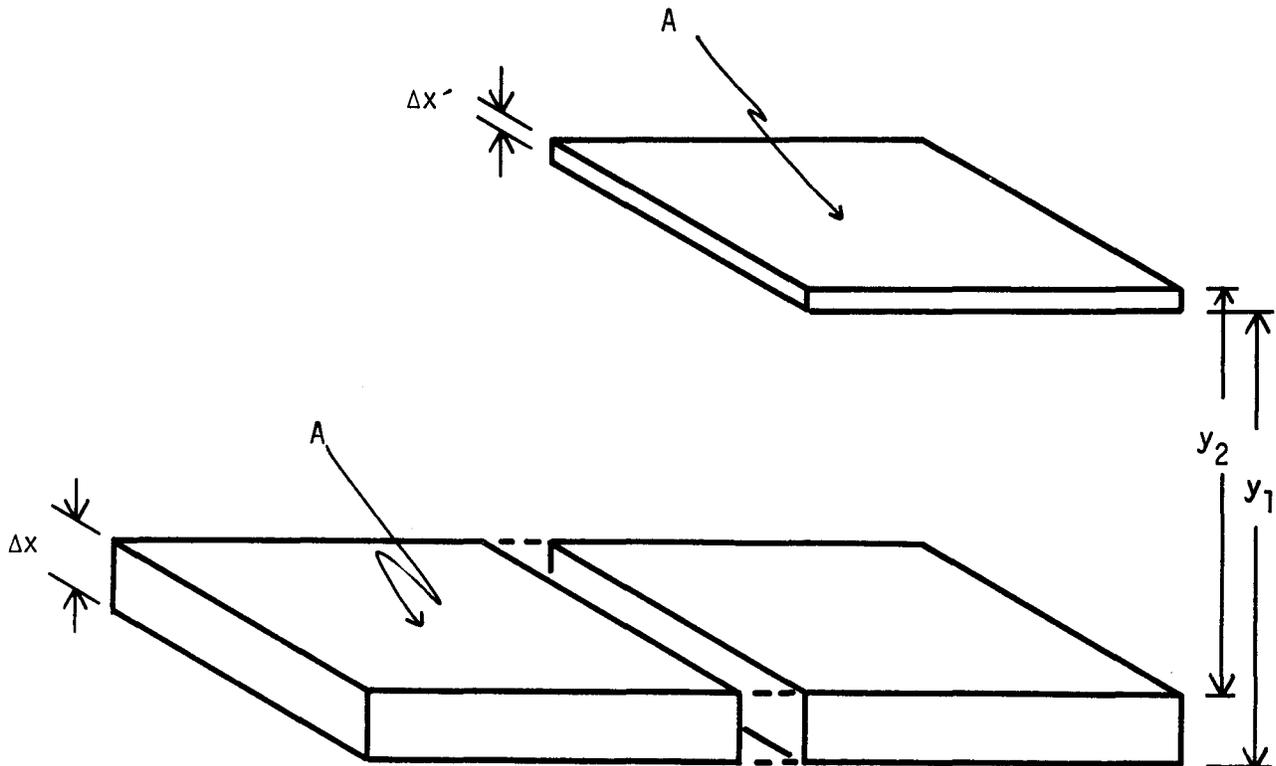

Figure 3

Figure 3 shows a thin slab of medium, containing mass M of the material at equilibrium with pressure p_0. Since the element of the medium is at equilibrium, there is no net force on the element and no acceleration. If a plane sound wave now passes through the region of space that includes the element, the volume containing M oscillates about its equilibrium position. Since the pressure changes, the volume of the element will also change, dependent on position and time.

Suppose that at a particular instant of time the element has been displaced to the position shown in Figure 3. Notice that the element has also been compressed. A particle at the left-hand end of the element has been displaced at distance y_1, and a particle at the right-hand end has moven y_2. The pressure at the left side of the element is $p + p_0$. If we call the displacement of the center of mass from its equilibrium position y, then Newton's second law for this element is

$$\Sigma F_i = Ma = (\rho A \, \Delta x')(d^2y/dt^2), \tag{1}$$

where ρ is the density of the medium. Since all forces are in one direction, we can omit the vector notation. The forces on the element in its displaced position are

$$\Sigma F_i = (p_0 + p)A - (p_0 + p + \Delta p)A = -\Delta pA,$$

and Eq. (1) becomes

$$-\Delta pA = \rho A \, \Delta x'(\partial^2 y/\partial t^2), \qquad \Delta p = -\rho \, \Delta x'(\partial^2 y/\partial t^2). \tag{2}$$

To get an expression in terms of one variable instead of both pressure and displacement, we employ the bulk modulus:

$$B = -\frac{\Delta p}{(\Delta V'/V)},$$

where $V = A \, \Delta x$ and $\Delta V = A(\Delta x' - \Delta x) = -A(y_2 - y_1) = -A \, \Delta y$. Thus

$$\Delta V/V = -\Delta y/\Delta x \qquad \text{and} \qquad \Delta p = -B(\Delta y/\Delta x).$$

Substituting for Δp in Eq. (2) we find

$$\Delta p = -B\frac{\Delta y}{\Delta x} = -\rho \, \Delta x'\frac{\partial^2 y}{\partial t^2} \xrightarrow[\Delta x, \ \Delta x' \to 0]{} \frac{\partial^2 y}{\partial t^2} = \frac{B}{\rho} \frac{\partial^2 y}{\partial x^2}.$$

This is once again the wave equation for displacement. Traveling wave solutions of this equation look like $f(x \pm vt)$, where $v = (B/\rho)^{1/2}$. The main physical idea here is that a change in pressure produces a change in volume of the element. Since the change in volume is related to the displacement, the pressure and displacement are related through the bulk modulus.

4. Sinusoidal Waves

For all sinusoidal waves, there is a relationship between speed v, wavelength λ, frequency ν, angular frequency ω, and wave number k:

$$v = \lambda\nu = (\lambda/2\pi)(2\pi\nu) = \omega/k.$$

For nonsinusoidal waves it is not always possible to define a single, precise wavelength or frequency. You will be concerned in this module with sinusoidal waves.

5. Wave Movement

As a wave progresses through space, the quantity that varies (the "thing that's waving") does not move along with the wave. For example, in a traveling wave on a string, a particle on the string moves perpendicular to the string, while the wave moves along the string. In a sound wave, the individual molecules of the medium execute microscopic, longitudinal oscillations about an equilibrium position, while the wave moves on through the medium.

6. Energy and Momentum

Mechanical and electromagnetic waves therefore do not carry matter, but they do carry energy and linear momentum. (They may also carry angular momentum.) The intensity of a wave is the energy carried across unit area in a unit of time. The units of intensity are joules per second per square meter or watts per square meter. The brightness of a light, the loudness of a sound, the destructiveness of a "tidal wave" or tsunami, are all related to the intensity of the wave. The intensity is proportional to the square of the amplitude of a wave. For example, in Traveling Waves, you learned that the intensity of a wave on a string is proportional to the square of the transverse displacement. The intensity of a sound wave is proportional to the square of the pressure excursion, or to the square of the longitudinal displacement from the equilibrium position. Normally the intensity, or the average intensity, is the quantity that is measured, rather than the pressure amplitude, or the particle displacement.

ADDITIONAL LEARNING MATERIALS

Films

Superposition of Pulses in a Spring, S-81293, Encyclopaedia Britannica Educational Corporation.

Longitudinal Standing Waves in a Spring, S-81298, Encyclopaedia Educational Corporation.

Standing Sound Waves, S-81299, Encyclopaedia Educational Corporation.

Experiments in Physics (24): Standing Acoustic Waves, de Gruyter.

TEXT: Frederick J. Bueche, <u>Introduction to Physics for Scientists and Engineers</u>
       (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Keep this study guide readily available as you study. The comments on the material in the text are intended to direct your attention as you read, and thereby make your reading easier. Complete the entire study procedure before attempting the Practice Test.

For Objective 1, study Section 29.7 in Chapter 29. Recall from the module <u>Traveling Waves</u> that the <u>amplitude</u> of any wave is the maximum value of the <u>oscillating</u> quantity. Thus, ξ_0 in Eq. (29.12) is the <u>displacement amplitude</u> of the sound wave. Study Section 29.8 up to the last paragraph on p. 576, and General Comment 3.

Equation (29.14) is the <u>wave equation</u>. Every kind of wave (displacement of strings, light, water, sound) is described by a differential equation that is mathematically similar to Eq. (29.14). You need not remember the equation. It is enough, for now, if you can accept the fact that the solutions of Eq. (29.14) are superpositions of functions like

$$\xi(x, t) = \xi(x - vt) \quad \text{and} \quad \xi(x, t) = \xi(x + vt),$$

BUECHE

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text[a]	
1	Secs. 29.7, 29.8				Chap. 29, Quest. 8, 9
2	General Comments 1, 3, 4, 5, 6, Sec. 29.7	A	D	Chap. 29, Probs. 2, 6	Chap. 29, Probs. 3, 13, 20
3	Secs. 31.1 to 31.3, General Comment 2	B	E	Chap. 31, Probs. 1, 2, 7	Chap. 31, Probs. 3, 4, 8
4	Secs. 34.5, 34.6	C		Chap. 34, Quest. 6, 7, 11, Prob. 6	
5	Sec. 29.9		E	Chap. 29, Prob. 17	

[a]Quest. = Question(s).

where, from Eq. (29.15), $v = \sqrt{E/\rho}$. Particularly useful solutions of this type
are of the form in Eq. (29.12). Solve Problems A and D in this study guide,
and Problems 2 and 6 in Chapter 29 for Objective 2.

Read Section 31.1 in Chapter 31 for Objective 3, concentrating on the statement
of the superposition principle, and General Comment 2. Then study Sections 31.2
and 31.3. For your own satisfaction, prove the trigonometric identity in the
middle of p. 612. The easiest procedure is to begin by applying the sum and
difference formulas to the right-hand side. Notice that the second equation
from the bottom of p. 612 should, strictly speaking, be a proportionality in-
stead of an equality. The constant of proportionality will, of course, have
appropriate dimensions so that both sides have the units of intensity (joules
per square meter per second)! The important point of these sections is that
the principle of superposition tells us to add the <u>amplitudes</u> of interfering
waves. However, if you were to detect the sound with a microphone, for example,
the signal you received would be proportional to the average <u>intensity</u> rather
than to the amplitude. Solve Problem B in this study guide, and Problems 1,
2, and 7 of Chapter 31.

Study Section 29.9 in Chapter 29. Notice that a source that is moving with
respect to the medium produces sound with a wavelength that differs from
the wavelength due to a source at rest with respect to the medium. If the
source is at rest, but the observer is moving, the wavelength remains unchanged,
of course, but the observer intercepts a different number of wavelengths per
unit time, and therefore perceives a different frequency. Solve Problem E in
this study guide, and Problems 16 and 17 in Chapter 29.

Study Sections 34.5 and 34.6 in Chapter 34. Bueche introduces this material
on resonance late in his text. He is therefore able to use words like "quantum"
and "eigenfrequencies," which you may not have learned. Don't worry about it,
since in the context of Section 34.6, you may cross out the words "quantum" and
the prefix "eigen-," and not change the meaning at all. Solve Problem C in
this study guide. Answer Questions 6 and 7 and solve Problem 6 in Chapter 34.

Text: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New
       York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Keep this study guide open and readily available as you read. The comments
on material in the textbook are intended to direct your attention as you read,
and thereby make your reading easier. Complete the entire study procedure be-
fore attempting the Practice Test.

Read quickly Section 17-1 in Chapter 17. Study Section 17-2 (except for the
final two paragraphs), and General Comment 3. Study Section 17-3. In Eq.
(17-2), note that if displacement (pressure) is represented by such an expression,
pressure (displacement) is represented by a similar expression, 90° out of phase with
displacement (pressure). Solve Problems A and D in this study guide, and Problem 11.

Study (or review) Section 16-7 and General Comment 2. The principle of super-
position is applied to a very large variety of situations. Solve Problem B
in this study guide, and Problems 27 and 29 in Chapter 16. Then study Section
17-4 and solve Problem C in this study guide, and Problems 25, 30, and 31 in
Chapter 17. Study Sections 17-5 and 17-6 and solve Problem E in this study guide,
and Problems 42, 43, and 44 in Chapter 17.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions — Study Guide	Assigned Problems — Study Guide	Assigned Problems — Text	Additional Problems
1	Secs. 17-1 to 17-3				
2	Sec. 17-3, General Comments	A	D	Chap. 17, Prob. 11	
3	Secs. 16-7, 17-5, General Comment 2	B	E	Chap. 16, Probs. 27, 29	
4	Sec. 17-4	C			Chap 17, Probs. 37, 38
5	Sec. 17-6		E	Chap. 17, Probs. 42, 43, 44	Chap. 17, Probs. 41, 45, 46

TEXT: Francis Weston Sears and Mark W. Zemansky, <u>University Physics</u> (Addison-
       Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Keep this study guide open and readily available as you read. The comments on
material in the text are intended to direct your attention as you read, and
thereby make your reading easier. Complete the entire study procedure before
attempting the Practice Test.

Review Sections 21-1 and 21-2 in Chapter 21. Concentrate especially on the
nature of a longitudinal wave, and the mathematical form of a traveling wave,
Eq. (21-3). The wave equation is at the end of Section 21-2. Read the
General Comments.

Solve Problems A and D in this study guide, and Problem 21-3 in the text.
This deals with a transverse wave, but you may pretend it is a longitudinal
displacement. Study Section 21-4, then read Sections 23-1 through 23-4 for
a qualitative discussion of some aspects of sound and music.

Study Sections 22-6 through 22-8, and General Comment 2. Your text distin-
guishes between reinforcement, when waves add constructively, and interference,

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 21-1, 21-4, 23-1, 23-2				
2	Sec. 21-2, General Comments	A	D	21-3	
3	Secs. 22-2, 22-6, 22-7, 23-8, General Comment 2	B	E	23-7(a)	23-15
4	Sec. 22-8	C			
5	Sec. 23-9		E	23-14, 23-16, 23-17	23-11, 23-12, 23-13, 23-15

when waves add destructively. More common usage of the words is to refer to
both of these effects as <u>interference</u>. Solve Problems B and C in this study
guide, and Problem 23-7(a) in your text.

Study Section 23-8, which describes another important application of
the principle of superposition. For your own satisfaction, prove the trigono-
metric identity just before Eq. (23-6). The easiest procedure is to apply the
well-known formulas for the cosine of the sum or difference of two angles to
the right-hand side. Study Section 23-9, and solve Problem E in this study
guide, and Problems 23-14, 23-16, and 23-17 in the text.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics
        (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Keep this study guide open and readily available as you study. The comments on
the material in the text are intended to direct your attention as you read, and
thereby make your studying easier. Complete the entire study procedure before
attempting the Practice Test.

Study carefully Sections 17-1 and 17-2 in Chapter 17. If you have not studied
thermodynamics, omit the final four paragraphs of Section 17-2. Study Section
17-3 and the General Comments. Solve Problems A and D in this study guide, and
Problems 17-3, 17-8, and 17-9 in the text. Study Section 17-4. It is charac-
teristic of virtually all waves that the intensity is proportional to the square
of the amplitude. The intensity, averaged over many oscillations, is usually
the quantity that is measured.

Study Section 17-5. You may also wish to review Sections 16-6 and 16-7. Al-
though these sections are concerned with transverse waves instead of longitudi-
nal waves, the analysis of superposition phenomena is identical to that for
longitudinal waves. Solve Problems B and C in this study guide, and Problems
17-12, 17-13, and 17-14 in the text. Study General Comment 2. Read Section
17-7, and study Section 17-11. Solve Example 17-5 and Problem E in this
study guide. Then solve Problems 17-22 and 17-25.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 17-1 to 17-4, 17-7				
2	Secs. 17-1, 17-3, General Comments	A	D	17-3, 17-8, 17-9	17-4, 17-5
3	Secs. 16-6, 16-7, 17-5, General Comment 2	B	E		
4	Sec. 17-5	C		17-12, 17-13, 17-14	
5	Sec. 17-11		E	17-22, 17-25	17-21, 17-24

PROBLEM SET WITH SOLUTIONS

A(2). The deviation of the air pressure from its equilibrium value in a certain region is given by the expression $\mu(x, t) = 20 \sin(6x - 6000t)$ N m^2. (All units are SI.) Does this expression represent a traveling wave in air?

Solution

Solutions of the wave equation can be expressed as a function of $(x \mp vt)$. If we can put the given expression in that form, we shall be able to determine the answer to the question:

$$p(x, t) = 20 \sin(6x - 6000t) = 20 \sin 6(x - 1000t).$$

By comparing with $p(x - vt)$, we immediately see that the given expression represents a wave traveling in the direction of increasing x, with a speed of 1000 m/s, and a pressure amplitude of 20.0 N/m^2. It may represent a sound wave, but not a sound wave in air at STP, where v = 330 m/s.

Although it was not part of the original question, it's good practice to investigate the properties of the wave a little further. For example, the wave number k is 6 m^{-1}, thus the wavelength is

$$\lambda = 2\pi/k = 6.28/6 = 1.05 \text{ m}.$$

Since ω is 6000 rad/s, the frequency is

$$\nu = \omega/2\pi = 6000/6.28 = 955 \text{ Hz},$$

a value in the audible range.

B(3). Two sources of plane waves, S_1 and S_2, are located distances ℓ_1 and ℓ_2 from an observer P, as shown in Figure 4. The sources broadcast in phase with each other. The pressure at either source is

$$p_s = p_0 \cos(\omega t).$$

Calculate the intensity of sound measured by the observer at P relative to that observed if only one of the sources is working.
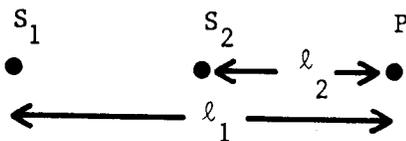

Figure 4

Solution

The principle of superposition tells us to add pressures:

$$P_p = p_0 \cos(k\ell_1 - \omega t) + p_0 \cos(k\ell_2 - \omega t).$$

From the trigonometric identity,

$$\cos a + \cos b = 2 \cos[(a - b)/2] \cos[(a + b)/2],$$

thus

$$P_p = 2p_0 \cos[(k\ell_1 - \omega t - k\ell_2 + \omega t)/2] \cos[(k\ell_1 - \omega t + k\ell_2 - \omega t)/2]$$
$$= 2p_0 \cos[(k\ell_1 - k\ell_2)/2] \cos[k\ell_1 + k\ell_2 - 2\omega t)/2],$$

where $k(\ell_1 - \ell_2) = (2\pi/\lambda)\Delta\ell$ is the <u>relative phase</u> of the sound from the two sources.   Average intensity is proportional to square amplitude, thus

$$I_p \propto 4p_0^2 \cos^2[(k/2)(\ell_1 - \ell_2)].$$

Thus the relative intensity is

$$I_p/I_1 = 4 \cos^2[(k/2)(\Delta\ell)].$$

If $k \Delta\ell/2 = (2n + 1)(\pi/2)$, where n = integer, the intensity is always zero. If $k \Delta\ell/2 = n\pi$, the intensity will be maximum and will be four times as large as I_1.

C(4).   A tuning fork is placed over a vertical tube that is open at the top and can be filled with water.   It produces strong resonances when the water is 0.080 m and 0.280 m from the top of the tube, and under no other conditions.   What is the frequency of the tuning fork?   The speed of sound in air is 330 m/s.
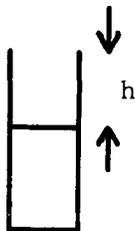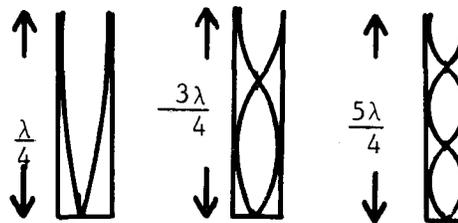

Figure 5

Figure 6

Solution

The water surface closes off the lower end of the tube. See Figure 5. The water presents an immovable barrier to the displacement of the air molecules, thus there is a displacement node at the water surface. Near the top of the tube, on the other hand, there is a displacement antinode. See Figure 6. The top of the tube and the water must therefore be (2n - 1)/4 wavelengths apart.

We have

$$h_1 = [(2n_1 - 1)/4]\lambda \quad \text{and} \quad h_2 = [(2n_2 - 1)/4]\lambda,$$

where h_1 is the minimum length to produce a resonance, thus $n_1 = 1$. The next highest must be $n_2 = 2$:

$$h_2 - h_1 = \left(\frac{2n_2 - 1}{4} - \frac{2n_1 - 1}{4}\right)\lambda = \frac{n_2 - n_1}{2}\lambda = \left(\frac{n_2 - n_1}{2}\right)\left(\frac{v}{\nu}\right),$$

$$\nu = \left(\frac{n_2 - n_1}{2}\right)\left(\frac{v}{h_2 - h_1}\right) = \left(\frac{1}{2}\right)\left(\frac{300}{0.200}\right) = 825 \text{ Hz}.$$

Problems

D(2). The displacement of a particle in a medium is given by the expression

$$y = \{(1.00 \times 10^{-10}) \exp[-(5x)^2] \exp[-(1650t)^2] \exp[-15(1650)xt]\} \text{ m}.$$

Does this represent a traveling wave?

E(3, 5). Maurice André stands beside a railroad track with his trumpet. Dizzy Gillespie stands on a flatcar with his trumpet. The flat car rolls toward André at a speed $v_G = 3.30$ m/s. Each man plays a concert A (440 Hz). How many beats per second do you hear if you are standing on the track between them? The speed of sound in the still air is 330 m/s.

Solutions

D(2). Let us change the product of exponentials to a sum of exponents, and see if $y = y(x \mp vt)$:

$$y = 10^{-10} \exp[-(5x)^2 - (1650t)^2 - 15(1650)xt].$$

Does the exponent factor?

$$(5x)^2 + (1650t)^2 + 15(1650)xt = (5)^2\left[x^2 + 3\left(\frac{1650}{5}\right)xt + \left(\frac{1650}{5}\right)^2 t^2\right].$$

The quantity in brackets is <u>almost</u> a perfect square, but not quite. <u>If</u> the 3 had been a 2, the exponent would have been

$$-(5)^2\left[x^2 + 2\left(\frac{1650}{5}\right)xt + \left(\frac{1650t}{5}\right)^2\right] = -(5)^2\left(x + \frac{1650}{5}t\right)^2.$$

This expression represents a wave, moving in the direction of <u>decreasing</u> x (why?), with

$$v = 1650/5 = 330 \text{ m/s}.$$

This is close to the speed of sound in air. However, the problem $\underline{\text{as given}}$ cannot be put in the form $y(x, t) = y(x + vt)$, and therefore does $\underline{\text{not}}$ represent a simple traveling wave.

E(3, 5). Because the flatcar is moving toward the observer the wavelength of Gillespie's tone will be shortened. At rest, the distance between crests of the sound wave would be $\lambda_0 = v/\nu_0$. However, in one period of the sound, a time $1/\nu_0$ s, Diz advances a distance $v_G(1/\nu_0)$, thus the new wavelength is

$$\lambda = \lambda_0 - \frac{v_G}{\nu_0} = \frac{v - v_G}{\nu_0}, \qquad \frac{1}{\lambda} = \frac{\nu_0}{v - v_G},$$

$$\nu = \frac{v}{\lambda} = \nu_0\left(\frac{v}{v - v_G}\right) = \nu_0\left(\frac{1}{1 - v_G/v}\right).$$

The frequency from Diz's trumpet is

$$\nu = 440(1/1 - 0.0100) \simeq 444 \text{ Hz}.$$

If you remember that the number of beats between two sounds is equal to the difference in frequencies, you can immediately say that the number of beats is

$$\Delta\nu = 444 - 440 = 4 \text{ beats/s}.$$

(On a test, you might be asked to give at least a qualitative explanation of that answer.)

Practice Test Answers

1. Sound is a longitudinal mechanical wave, propagated through a medium. The independent variable can be considered either the deviation of the pressure from its equilibrium value, or the displacement of particles of the medium from their equilibrium positions. The amplitude of the wave is then the maximum excursion of the pressure from its equilibrium value, or the maximum displacement. An increase in pressure will produce a decrease in the volume of an element of the medium. For a plane wave, the volume of an element with sides parallel to the direction of propagation is $A \Delta x$, where A is the cross-sectional area and Δx is the thickness of the element. Thus, the displacement is directly related to the pressure excursion. This answer is adequate. A more quantitative description of the relation between pressure and displacement involves the bulk modulus B:
$p = -Bky_0 \cos(kx - \omega t)$.

2. The expression represents the superposition of two waves traveling at $v = 500$ m/s. Notice that if the two speeds were different, the expression would not represent a wave, since the speed of sound is independent of the frequency.

3. (a) $\ell = v/4\nu = 0.187$ m. (b) $v_1 = v(\nu´/\nu - 1) = 19.8$ m/s^2. Toward.

4. $\Delta\nu = 128$ Hz. This frequency is too high to be heard as "beats." It is the frequency of C an octave below middle C. If you play middle C and the G above it loudly enough, you will hear the lower C.

PRACTICE TEST

Useful information: the speed of sound in air at STP is $v = 330$ m/s,
$$\cos a + \cos b = 2 \cos\left(\frac{a - b}{2}\right) \cos\left(\frac{a + b}{2}\right); \quad \sin a + \sin b = 2 \sin\left(\frac{a + b}{2}\right) \cos\left(\frac{a - b}{2}\right).$$
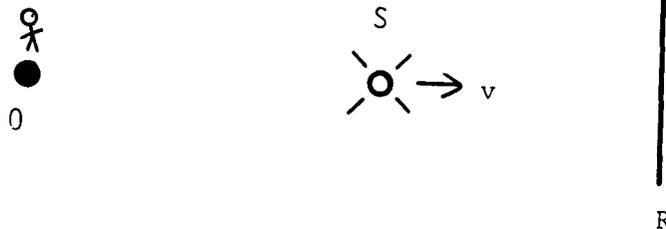
1. Explain, in your own words, what sound is. Define the displacement amplitude, pressure amplitude, and intensity. How are they related? (On the Master Test, your instructor will tell you whether to answer this question in writing or orally.)

2. Does the following expression represent a possible sound wave? If so, determine the amplitude of the wave, and the velocity of sound in the medium. If not, state how you know that it is not a sound wave:
$$p(x, t) = 2.00 \times 10^{-3} \sin(0.50x - 250t) + 3.50 \times 10^{-4} \cos(2.50x + 1250t).$$

3. The fundamental tone of a pipe that is closed on one end is a concert A (440 Hz) when the tone is excited in air at STP. (a) Determine the length of the pipe. (b) How fast must I move if I wish to raise the tone I hear from the pipe to a B-flat (466 Hz)? Should I move toward or away from the pipe?

4. Two adjacent sources emit sound of equal intensity, and of frequencies 256 and 384 Hz, respectively. Calculate the beat frequency when these sources sound simultaneously. Can you interpret your answer physically?

SOUND

pass   recycle

Mastery Test   Form A

1   2   3   4   5

Name _____   Tutor _____

For the speed of sound in air at STP, use v = 330 m/s.

$$\cos a + \cos b = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2}); \quad \sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2}).$$

1. Explain, in your own words, what sound is. Define the displacement ampli-
    tude, pressure amplitude, and intensity. How are they related?

2. State whether the expression

    $$p = 20.0 \sin(337t + 0.876x) \text{ N/m}^2$$

    represents a traveling wave. If it does, what is the speed of sound? Name
    a point on the x axis where the excess pressure is zero at t = 0. Calculate
    the distance to the nearest position at which $p = p_{max}$ = 20.0 N/m^2 at t = 0.

3. A sound source S, emitting a frequency of 500 Hz, moves toward a reflector R
    at 30.0 m/s as in Figure 1. An observer O hears sound from S and from R.
    What beat frequency does he hear?

4. A pipe open on each end has a fundamental frequency of 330 Hz. The first
    overtone of the pipe has the same frequency as the first overtone of another
    pipe closed on one end. Calculate the length of each pipe.

Figure 1

SOUND

Mastery Test   Form B

Name _____   Tutor _____

For the speed of sound in air at STP, use v = 330 m/s.

$$\cos a + \cos b = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2}); \quad \sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2}).$$

1. Explain, in your own words, what sound is. Define the displacement amplitude, pressure amplitude, and intensity. How are they related?

2. If the expression

    $$\xi = \xi_0 \exp(-3x^2) \exp(+1980xt) \exp(-3.267 \times 10^5 t^2) \sin(\sqrt{3}x - \sqrt{3}(330)t)$$

    can represent the displacement in a traveling sound wave, sketch the wave as a function of x at t = 0. If it cannot represent a wave, explain why.

3. A man moves between two identical whistles. When his speed is 20.0 m/s, he hears a beat frequency of 30.0 Hz. What is the frequency of the sound emit ε by the whistles?

4. Two pipes, a and b, have the same fundamental frequency. Pipe a is open on each end, and pipe b is closed on one end.

    (a) Which pipe is longer?

    (b) Compare the frequencies of overtones from the two pipes.

SOUND

Mastery Test   Form C

Name _____   Tutor _____

For the speed of sound in air at STP, use $v = 330$ m/s.

$$\cos a + \cos B = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2}); \quad \sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2}).$$

1. Explain, in your own words, what sound is. Define the displacement amplitude, pressure amplitude, and intensity. How are they related?

2. A purported wave is represented by the expression

    $$p = 0.150 \cos(2.00x + 1000t) + 0.0300 \sin(500t - 1.00x).$$

    Is this expression, in fact, a possible sound wave? If so, calculate the speed of sound. If not, explain why not.

3. Two identical sources of sound, S_1 and S_2, are located distances ℓ_1 and ℓ_2 from an observer P, as shown in Figure 1. The sources broadcast plane waves in phase with each other. The displacement amplitude at each source is

    $\xi = \xi_0 \cos \omega t.$

    (a) Calculate the intensity of sound measured by the observer, relative to the intensity when only S_2 is operating.

    (b) Choose some "reasonable" numbers that will maximize the intensity at P.

    Figure 1

4. A tuning fork is held over a tube partially filled with water. Resonance occurs when the water is 0.100 and 0.260 m below the fork. Calculate the frequency of the tuning fork.

5. A jet aircraft moves at a speed three-fourths that of sound. It overtakes another aircraft moving at only one-fourth that of sound. The jet is emitting a sound with the frequency 280 Hz. What frequency does a passenger on the slower aircraft hear before the jet passes?
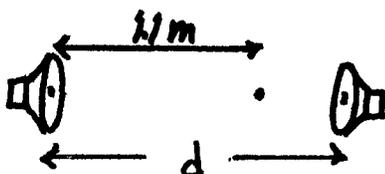
SOUND

Mastery Test   Form **D**

Name _____

Tutor _____

For the speed of sound in air at STP, use v = 330 m/s.

$$\cos a + \cos B = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2}); \qquad \sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2}).$$

1) A train is moving toward a listener with a speed of 20 m/s.
    a) If the whistle of the train has a frequency of 330 Hz
        what frequency does the listener hear?
    b) After the train passes the listener what frequency does he hear?

2) A man makes dinner music with glasses with varying amounts of
    water in them. How far from the top of the glass would the water
    have to be to reach "A" (220 Hz).

3) a) State the principle of superposition.
    b) Two loud speakers are oscillating at 300 Hz in phase.
        (P = $P_0 \cos \omega t$)
    How close could they be located if the point on the line between
    the speakers at 1.1 m from the test speaker is "dead", i.e. the
    intensity is zero?

Courtesy of University of MIssouri-Rolla

Mastery Test   Form E

Name _____

For the speed of sound in air at STP, use v = 330 m/s.

$$\cos a + \cos B = 2 \cos\left(\frac{a - b}{2}\right) \cos\left(\frac{a + b}{2}\right); \qquad \sin a + \sin b = 2 \cos\left(\frac{a - b}{2}\right) \sin\left(\frac{a + b}{2}\right).$$

1) A referee is moving toward the players with a velocity of 5 m/s who are running with a velocity of 5 m/s toward him. If the referees blows his whistle, f = 300 Hz, what frequency do the players hear?

2) An open pipe resonates at 200 Hz, 400 Hz, 600 Hz and 800 Hz. After one end is plugged what are the first four resonant frequencies?

3) A piano tuner strikes his tuning fork (220 Hz). What is the beat frequency he hears if the piano string is oscillating with a frequency f = 230 Hz?

Date _____

Mastery Test   Form F

pass   recycle

1   2   3   4   5

Name _____

Tutor _____

For the speed of sound in air at STP, use v = 330 m/s.

$$\cos a + \cos B = 2 \cos(\frac{a-b}{2}) \cos(\frac{a+b}{2}); \qquad \sin a + \sin b = 2 \cos(\frac{a-b}{2}) \sin(\frac{a+b}{2}).$$

1) A thief is running away from a policeman. If the thief is running with a speed v = 7 m/s and the policeman is running with a speed 5 m/s, what frequency will the thief hear when the policeman blows his whistle whose frequency at rest is 300 Hz?

2) An open pipe resonates at 400 Hz. (fundamental) At what frequency will it resonate when one end is plugged?

3) Two sources of plane waves are located at distances ℓ_1 = 2.2 m and ℓ_2 from a listener. If the pressure at either source is

$P_s = P_o \cos 2\pi(300)t$ find ℓ_2 such that the intensity at the listener

i four times as high as he would receive from a single speaker.

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Longitudinal wave in pressure or displacement from equili-
    brium value. Physical argument relating pressure and displacement. Analytic
    treatment, using bulk modulus, is OK, but not required. Proportionality
    between I and (pressure)^2 or (displacement)^2.

    Solution: Sound is a longitudinal mechanical wave, propagated through a
    medium. The independent variable can be considered either the deviation of
    the pressure from its equilibrium value, or the displacement of particles of
    the medium from their equilibrium positions. The amplitude of the wave is
    then the maximum excursion of the pressure from its equilibrium value, or
    the maximum displacement. An increase in pressure will produce a decrease
    in the volume of an element of the medium. For a plane wave, the volume of
    an element with sides parallel to the direction of propagation is $A \, \Delta x$,
    where A is the cross-sectional area and Δx is the thickness of the element.
    Thus, the displacement is directly related to the pressure excursion. This
    answer is adequate. A more quantitative description of the relation between
    pressure and displacement involves the bulk modulus B: $p = -Bky_0 \cos(kx - \omega t)$.

2. What To Look For: Wave must be expressed as $f(x \mp vt)$. Student should show
    ability to pick a particular time and evaluate what happens.

    Solution:

    $p = 20.0 \sin(337t + 0.876x) = 20.0 \sin 0.876[x + (337/0.876)t]$.

    This is a function of $(x + vt)$, thus it is a sound wave, with speed 337/0.876
    = 385 m/s.

    $p(x, 0) = 0 = 20.0 \sin(0.876x)$, $0.876x = n\pi$, $n = 1, 2, \ldots$.

    $x = n\pi/0.876 = n(3.59)$ m.   Nearest maximum is $\lambda/4$ away:

    $\lambda/4 = 2\pi/4k = \pi/2(0.876) = 1.793$ m.

3. What To Look For: Direction (+) of frequency shift should be correct. Use
    of correct equations, and ability to justify physically the choice of equa-
    tions. Can he justify physically the fact that beat frequency is ν?
    (Modulation of amplitude is $\Delta\nu/2$, but modulation of intensity is $\Delta\nu$.)

    Solution: Source emits ν wavelengths. Source moves $(v_s/\nu) \, m_1$ in one period.
    Wavelength of sound is $\lambda' = \lambda_0 \pm v_s/\nu$ for waves emitted to [right, left].
    The corresponding frequencies are

    $$\nu = \frac{c}{\lambda'} = \frac{c}{\lambda_0 \pm v_s/\nu} = \frac{c\nu}{c \pm v} \cdot$$
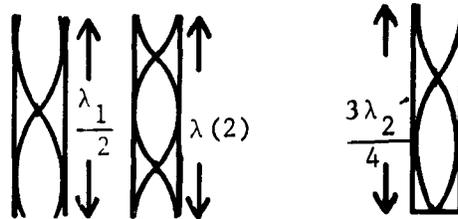
    Observer receives both frequencies, so the beat frequency is

    $$\Delta\nu = \nu c \frac{1}{c - v} - \frac{1}{c + v} = \frac{2\nu\nu c}{c^2 - v^2} = \frac{2\nu\nu c}{(c - v)(c + v)} = \frac{2(500)(30)(330)}{(300)(360)} = \frac{11(10^3)}{10(12)}$$

$\Delta \nu$ = 91.7 Hz.

4. <u>What To Look For</u>: How displacement and pressure depend on configuration
   of pipe. Ability to connect conditions. Ask about ν_2.

Figure 9

<u>Solution</u>: See Figure 9, in which the first two sketches show displacement
amplitude for fundamental (λ_1) and first overtone (λ_2) in open pipe. Third
sketch shows first overtone (λ_2') of second pipe. Let $\ell + \ell'$ be length of
open and closed pipes:

$$\ell = \lambda/2 = c/2\nu_1,$$

where ν_1 = 330, $\ell = \lambda_2 = c/\nu_2$, $\ell' = 3\lambda_2'/4 = 3c/4\nu_2'$. From the first equation,
ℓ = 330/2(330) = (1/2) m. From the second equation, ν_2 = c/ℓ = 330(2) = 660.
That makes sense! From the third equation, ℓ'= 3(330)/4(660) = (3/8) m.

MASTERY TEST GRADING KEY - Form B

1. Solution: Sound is a longitudinal mechanical wave, propagated through a medium. The independent variable can be considered either the deviation of the pressure from its equilibrium value, or the displacement of particles of the medium from their equilibrium positions. The amplitude of the wave is then the maximum excursion of the pressure from its equilibrium value, or the maximum displacement. An increase in pressure will produce a decrease in the volume of an element of the medium. For a plane wave, the volume of an element with sides parallel to the direction of propagation is A Δx, where A is the cross-sectional area and Δx is the thickness of the element. Thus, the displacement is directly related to the pressure excursion. This answer is adequate. A more quantitative description of the relation between pressure and displacement involves the bulk modulus B: $p = -Bky_0 \cos(kx - \omega t)$.

2. What To Look For: Demonstrate that student should look for $f(x \mp vt)$.

   Solution:

   $\xi = \xi_0 \exp(-3x^2) \exp(1980xt) \exp(-3.267 \times 10^5 t^2) \sin[\sqrt{3}x - \sqrt{3}(330)t]$.

   Exponent is $-3[x^2 - 660xt + 1.089 \times 10^5 t^2] = -3(x - 330t)^2$.

   $\xi = \xi_0 \exp[-(kx - \omega t)^2 \sin(kx - \omega t)$,

   where $k = \sqrt{3}$ m^{-1}; $\omega = vk = \sqrt{3}(330)$ rad/s,   and V = 330 m/s. It is a wave.

3. What To Look For: Does wavelength change? (No.) Student should know which has highest frequency.

   Solution: See Figure 10. In 1 s, man moves v_0 m, or v_0/λ wavelengths. Number of wavelengths he intercepts is $\nu \pm v_0/\lambda$, depending on whether he moves toward or away from the source. The difference in frequency is $2(v_0/\lambda)$, which is the beat frequency:

   $2(\frac{v_0}{\lambda}) = 2(\frac{v_0 v_0}{c}) = 30$, $v_0 = \frac{30(330)}{2(20)} = \frac{3}{4}(330) = 247.5$ Hz.

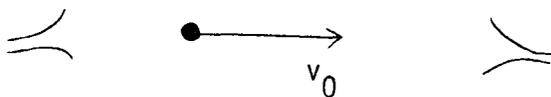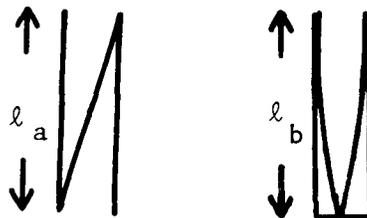   Figure 10   Figure 11

4. <u>What To Look For</u>: Know conditions for resonance. Be able to relate conditions in the two pipes.

<u>Solution</u>: See Figure 11. (a) Displacement distribution is shown in Figure 11:

$$\ell_a = \lambda/2, \qquad \ell_b = \lambda/4,$$
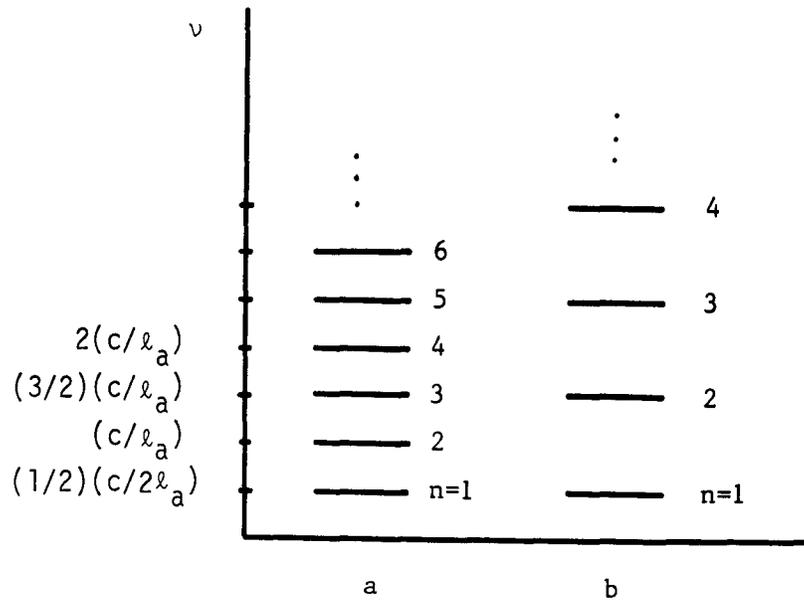
(b) For pipe a,

$$\ell_a = \frac{n\lambda_n^{(a)}}{2} = \frac{nc}{2\nu_n^{(a)}}, \qquad \nu_n^{(a)} = \frac{nc}{2\ell_a}, \qquad n = 1, 2, \ldots.$$

For pipe b,

$$\ell_b = (2n - 1)\frac{\lambda_n^{(b)}}{4} = \frac{(2n - 1)c}{4\nu_n^{(b)}}, \qquad \nu_n^{(b)} = \frac{(2n - 1)c}{4\ell_b}.$$

For n = 1, we know the two frequencies are the same: $\nu_1^{(a)} = \nu_1^{(b)}$. See Figure 12.

Figure 12

MASTERY TEST GRADING KEY - Form C

1. Solution: Sound is a longitudinal mechanical wave, propagated through a
   medium. The independent variable can be considered either the deviation of
   the pressure from its equilibrium value, or the displacement of particles
   of the medium from their equilibrium positions. The amplitude of the wave
   is then the maximum excursion of the pressure from its equilibrium value, or
   the maximum displacement. An increase in pressure will produce a decrease
   in the volume of an element of the medium. For a plane wave, the volume of
   an element with sides parallel to the direction of propagation is $A \Delta x$,
   where A is the cross-sectional area and Δx is the thickness of the element.
   Thus, the displacement is directly related to the pressure excursion. This
   answer is adequate. A more quantitative description of the relation between
   pressure and displacement involves the bulk modulus B: $p = -Bky_0 \cos(kx - \omega t)$.

2. What To Look For: Student should know that wave is of the form $f(x + vt)$.
   Ask student whether it would be a sound wave if one of the ω's were changed.

   Solution:

   $p = 0.150 \cos(2.00x + 1000t) + 0.0300 \sin(500t - 1.00x)$

   $\quad = 0.150 \cos[2(x + 500t)] + 0.0300 \sin[-(x - 500t)]$.

   It is a sound wave, with $v = 500$ m/s.

3. Solution: Choose origin of coordinate system at S_1 as in Figure 13. Then

   $\xi_1(x, t) = \xi_0 \cos(kx - \omega t), \quad \xi_2(x, t) = \xi_0 \cos[k(x - \ell_1 - \ell_2) - \omega t]$.

   At point P,
   $\xi(\ell_1, t) = \xi_0 \cos(k\ell_1 - \omega t) + \xi_0 \cos(-k\ell_2 - \omega t)$
   $\quad = 2\xi_0 \cos\{[k(\ell_1 - \ell_2) + 2\omega t]/2\} \cos[k(\ell_1 + \ell_2)/2]$
   $\quad = 2\xi_0 \cos[k(\ell_1 + \ell_2)/2]\{\cos[k(\ell_1 - \ell_2)/2] \cos \omega t$
   $\quad\quad - \sin[k(\ell_1 - \ell_2)/2] \sin \omega t\}$

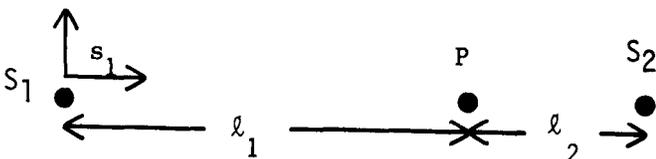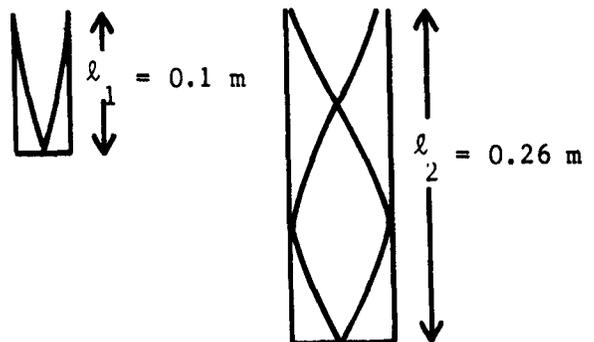   Figure 13   Figure 14

For a maximum, we clearly need

$$\frac{k(\ell_1 + \ell_2)}{2} = n_1\pi, \qquad n = 0, 1, 2, \ldots \text{ and } \frac{k(\ell_1 - \ell_2)}{2} = n_2\pi,$$

$$\underline{\text{or}} \quad \frac{k(\ell_1 - \ell_2)}{2} = (2n_2 + 1)\frac{\pi}{2} \, .$$

Let us choose

$$k(\ell_1 + \ell_2)/2 = n_1\pi, \qquad k(\ell_1 - \ell_2)/2 = n_2\pi.$$

Then $k\ell_1 = (n_1 + n_2)\pi$ and $k\ell_2 = (n_1 - n_2)\pi$. Let $\ell_1 = \ell_2$. Then $n_2 = 0$, and choose

$$k_2\left(\frac{2\ell}{2}\right) = n_1\pi, \qquad \ell = \frac{n_1\pi}{k} = \frac{n_1\lambda}{2} = \frac{n_1 c}{2\nu} \, . \quad \text{If we choose } \nu \text{ as, say, 330 Hz, then}$$

$$\ell = (1/2) \text{ m.}$$

4. <u>What To Look For</u>: Student should know and apply condition for resonance.

    <u>Solution</u>: See Figure 14.

    $$\ell_1 = \frac{\lambda}{4} = \frac{c}{4\nu} \, , \qquad \ell_2 = \frac{3\lambda}{4} = \frac{3c}{4\nu} \, , \qquad \ell_2 - \ell_1 = \frac{c}{4\nu}(3 - 1) = \frac{c}{2\nu} \, ,$$

    $$\nu = \frac{c}{2(\ell_1 - \ell_2)} = \frac{330}{2(0.320)} = 516 \text{ Hz.}$$

5. <u>What To Look For</u>: Student should distinguish between motion of <u>source</u> and <u>receiver</u> and use appropriate formulation.

    <u>Solution</u>: In one period $T = 1/\nu_0$, the fast jet advances

    $$\Delta\ell = v_1 T = v_1/\nu_0 = (3/4)(c/\nu_0).$$

    Wavelength is shortened by $\lambda_1 = \lambda_0 - 3c/4\nu_0 = \lambda_0/4$. The slow craft, in one second, receives $v_1 - v_2/\lambda_1$ wave crests so

    $$\nu_2 = \nu_1 - v_2/\lambda_1 = 4\nu_0 - (c/4)(v_1/c) = 4\nu_0 - (4\nu_0/4) = 3\nu_0.$$

    Thus $\nu_2 = 3(280) = 840$ Hz.