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Norman R. Simon
University of Nebraska - Lincoln, nsimon@unl.edu

Shashi M. Kanbur
University of Nebraska-Lincoln, kanbur@oswego.edu

Dimitri Mihalas
University of Illinois at Urbana-Champaign

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NORMAN R. SIMON AND SHASHI M. KANBUR
Department of Physics and Astronomy, University of Nebraska, Lincoln, NE 68588-0111
AND
DIMITRI MIHALAS
Department of Astronomy, University of Illinois, Urbana, IL 61801
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ABSTRACT

It has been known for many decades that the spectral type of Cepheids at light maximum is constant with period. We use hydrodynamic pulsation models to explain this result in terms of the outward reach of the hydrogen ionization front. On the other hand, we show that, at minimum light, the Cepheid photospheric temperature is mainly a function of amplitude. A number of observed Cepheids have published temperatures that seem too hot at both maximum and minimum. We attribute this to an overestimation of the reddening for these stars. A list is given.

Subject headings: Cepheids — stars: oscillations

1. INTRODUCTION

Many years ago, Code (1947) pointed out a surprising characteristic of the light curves of classical Cepheids: Whereas the spectral type at minimum light becomes later with increasing pulsation period, that at maximum light shows no variation with period, remaining in the rather narrow range, F5–F8.

In Figure 1 we reproduce a plot of Code's data. Filled squares indicate maximum light, open squares minimum light. Straight lines have been drawn in to mark crude envelopes enclosing the observed points. At minimum light the spectral type ranges from F7 at 2 days to K1 at 27 days, while at maximum the spectral type remains roughly constant throughout. To our knowledge, this behavior has never been explained.

2. HYDRODYNAMIC MODELS

We have used the code TGRID (Simon & Aikawa 1986) to construct eight hydrodynamic pulsation models with parameters appropriate to the classical Cepheids. The opacity employed was that of Rogers & Iglesias (1992). Convection was neglected. Table 1 gives the model parameters: mass and luminosity in solar units, effective temperature (kelvins), period (days), and limiting amplitude (magnitudes). Models B and B′ have the same parameters, but the latter was integrated with larger artificial viscosity, causing its amplitude to be smaller. Model A was converged in the first overtone mode to mimic certain stars on Code's list, such as SU Cas and DT Cyg.

For each of the hydrodynamic models we extracted the photospheric temperatures at maximum and minimum light and converted them to $B-V$ colors using the scale of Teays & Schmidt (1987). These colors were then transformed to spectral types by linear interpolation in the supergiant table of Johnson (1966). Figure 2 shows a plot of spectral type versus log period for the models, with notation as in Figure 1, including the observational envelopes. One sees that the model calculations reproduce the observed properties quite well, in particular the flat slope with period at maximum light. The theoretical points for minimum light also lie generally within the observational envelope, although a flatter slope seems to be suggested by the models. We shall argue below that this is probably due to a well-known deficiency of current hydrodynamic calculations: the limiting amplitudes they produce are, to a certain extent, arbitrary.

3. THE CEPHEID AT MAXIMUM AND MINIMUM

It is well known from observations that maximum light corresponds to the phase of maximum velocity of expansion in the photosphere. Models show that at this phase the star's radius assumes a value close to that of the average radius over the cycle. Observed colors and spectral types indicate, and models confirm, that when the star is brightest its surface is also hottest. Finally, the phase of maximum light is known to correspond to maximum outward reach in mass of the hydrogen ionization front (HIF) (Castor 1968). At minimum light, the star once again passes through its average radius, while all the other properties are reversed—i.e., one has maximum *contraction* velocity, minimum photospheric temperature, and minimum reach of the HIF.

Figure 3 shows temperature profiles in the outer layers of a typical model (model F) at a number of phases at and prior to maximum light. The time sequence is from right to left. The ordinate gives temperature in units of 10^4 K, and the abscissa mass in the form $\log(1-q)$, where $q = M_r/M$. The steep (nearly vertical) temperature rise in each profile shows the location of the HIF, which moves outward in mass as light maximum is approached. The leftmost profile corresponds to maximum light. In the time following maximum (not shown) the HIF begins to move inward. The filled squares in each profile denote the location of the photosphere ($\tau = \frac{2}{3}$).

In Figure 4 we plot opacity profiles for the same model. As is well known, a steep rise in opacity, the hydrogen opacity bump, occurs at the HIF. Figures 3 and 4 together explain in a simple manner why all of Code's (1947) Cepheids display (roughly) the same spectral type at maximum light. At this phase, the HIF is so shallow in all these stars that one sees in as far as the opacity bump, but then no further. Since the bump always begins in earnest at a temperature with some range around 6000 K, this is the temperature that characterizes the photosphere at maximum light. It is interesting to note that an

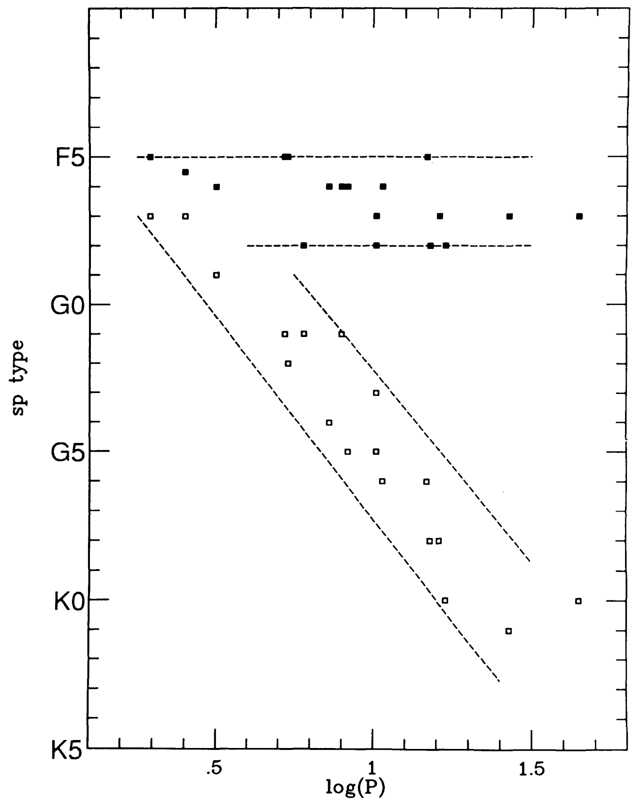

FIG. 1.—Spectral type at maximum light (*filled squares*) and at minimum light (*open squares*) vs. log period, reproduced from Code (1947). Dashed lines give crude envelopes for the observed points.

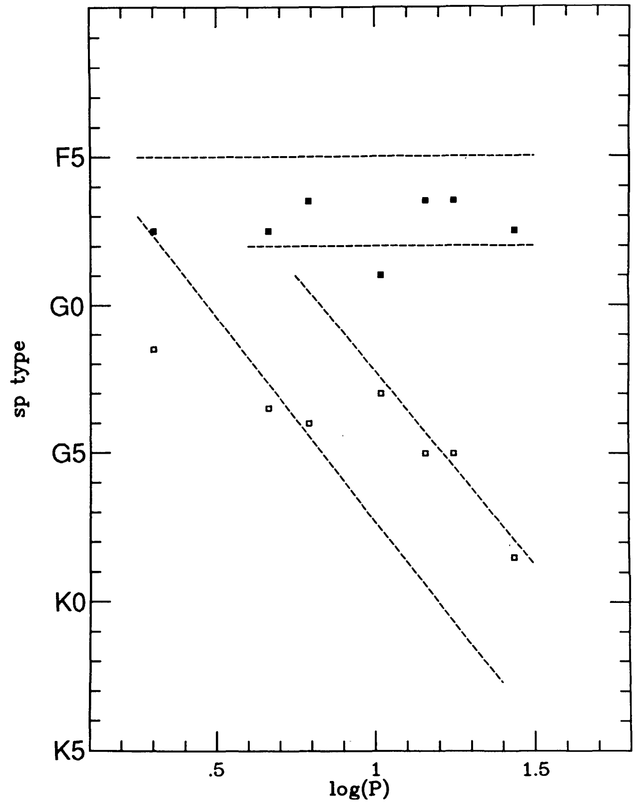

FIG. 2.—Same as Fig. 1, but symbols now refer to hydrodynamic models

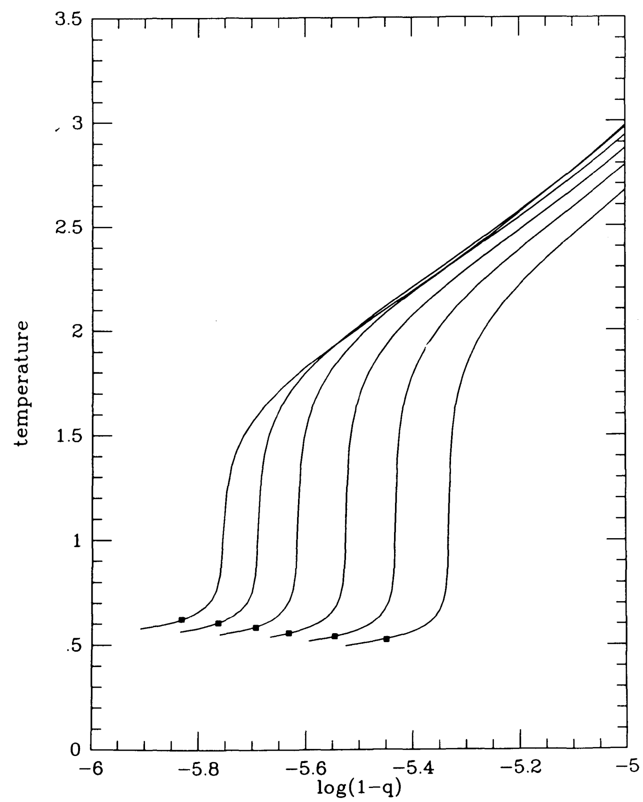

FIG. 3.—Temperature profiles (in 10^4 K) for model F as maximum light is approached. Filled squares indicate position of photosphere. Time sequence goes from right to left; leftmost curve corresponds to light maximum; $q = M_r/M$.

entirely analogous effect explains why nuclear fireballs all have the same maximum surface temperature (Zel'dovich & Raizer 1967).

At minimum light, the situation is quite different. Figure 5 compares the temperature profiles, and Figure 6 the opacity profiles, at maximum and minimum. At minimum light (right curve on each figure), the HIF is deep enough that the photosphere is located above the hydrogen opacity bump (i.e., at smaller $1 - q$). In this case the photospheric temperature is dictated by different considerations and will vary according to other properties of the pulsation—in particular, as we shall see, the amplitude.

4. ADDITIONAL DATA

Let us now extend Code's (1947) Cepheid sample with two additional data sets—first, that due to Moffett & Barnes (1980, 1984). Figure 7 shows photospheric temperatures at maximum

TABLE 1

HYDRODYNAMIC MODELS

Model	M	log L	T_e	Period	Amplitude
A......	3.45	2.865	5900	1.99	0.62
B......	3.98	3.089	5640	4.61	1.09
B'......	3.98	3.089	5640	4.61	0.93
C......	4.57	3.306	5707	6.20	1.12
D......	5.44	3.578	5550	10.4	0.70
E......	6.03	3.741	5461	14.3	1.19
F......	6.46	3.850	5404	17.7	1.19
G......	7.42	4.067	5293	27.1	1.12

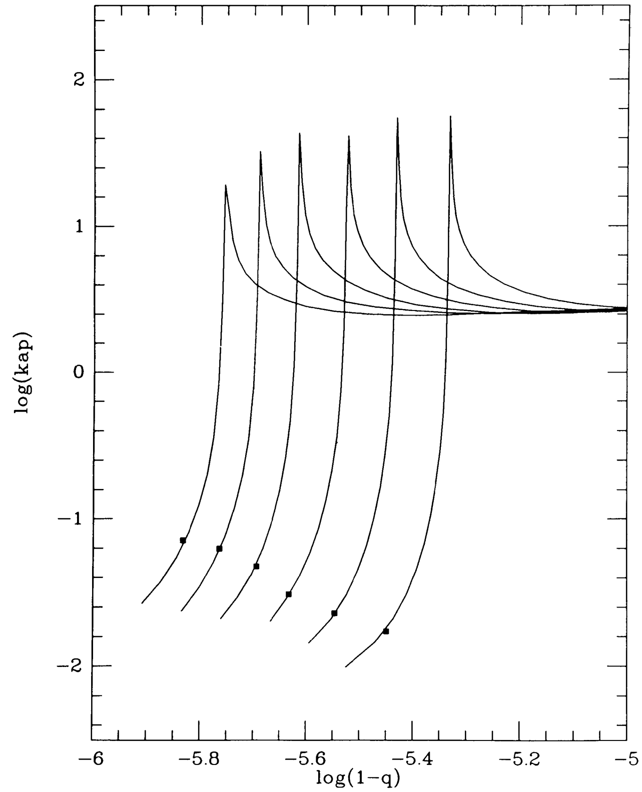

FIG. 4.—Same as Fig. 3, but opacity profiles ($\log \kappa$ in cm^2 g^{-1}) are presented.

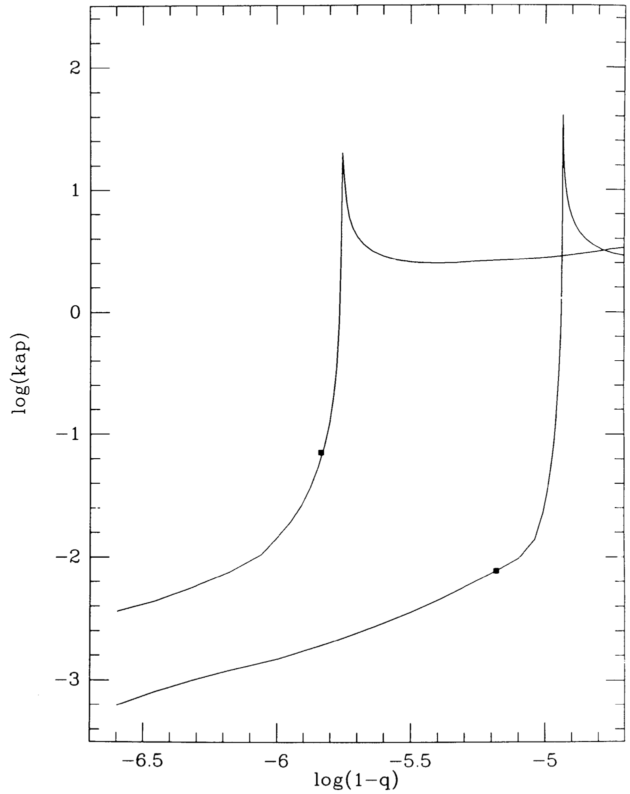

FIG. 6.—Same as Fig. 5, but opacity profiles are presented. Notation as in Fig. 4.

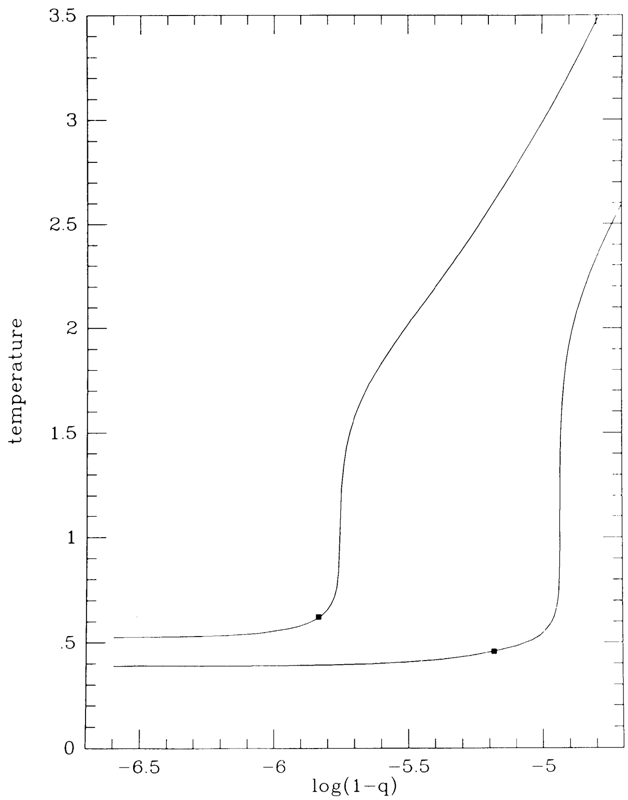

FIG. 5.—Temperature profiles (in 10^4 K) for model F at maximum light (*left curve*) and minimum light (*right curve*). Notation as in Fig. 3.

light (*filled squares*) and minimum light (*open squares*) for 105 Cepheids in the Moffett-Barnes (MB) study. Temperatures were obtained from MB's $B-V$ colors using the transformation of Teays & Schmidt (1987), viz.,

$$\log T = 3.904 - 0.237\,(B-V)_0\,, \quad (1)$$

and reddenings from Fernie (1990). The horizontal lines reproduce Code's upper envelope from Figure 1, bounded by spectral types F5 and F8. The lower curves are the minimum light envelope from Figure 1, transformed from spectral type into temperature. The ragged shape of these curves stems from the discrete and nonlinear nature of the supergiant table of Johnson (1966).

From Figure 7, one sees that most of the MB Cepheids conform to Code's strip at maximum light. A few of the long-period stars are too cool. These stars are perhaps so distended that even at maximum light the hydrogen opacity bump lies well below the photosphere. In addition, there are seven stars which seem too hot. A number of possible explanations exist for these, but we shall defer their discussion to a later section. At minimum light most of the stars (*open squares*) also conform to the Code envelopes. We shall see that the stars lying above this envelope are perfectly normal, their high minimum temperatures having been dictated by their modest amplitudes.

In Figure 8 we plot further data, these from the large Cepheid sample of Pel (1976). The notation is as in Figure 7. For stars with period less than 11 days, the temperatures at maximum and minimum light were given explicitly by Pel and are merely reproduced here. Pel did not publish these temperatures for Cepheids with periods greater than 11 days since at minimum light the stars' observed parameters broached the

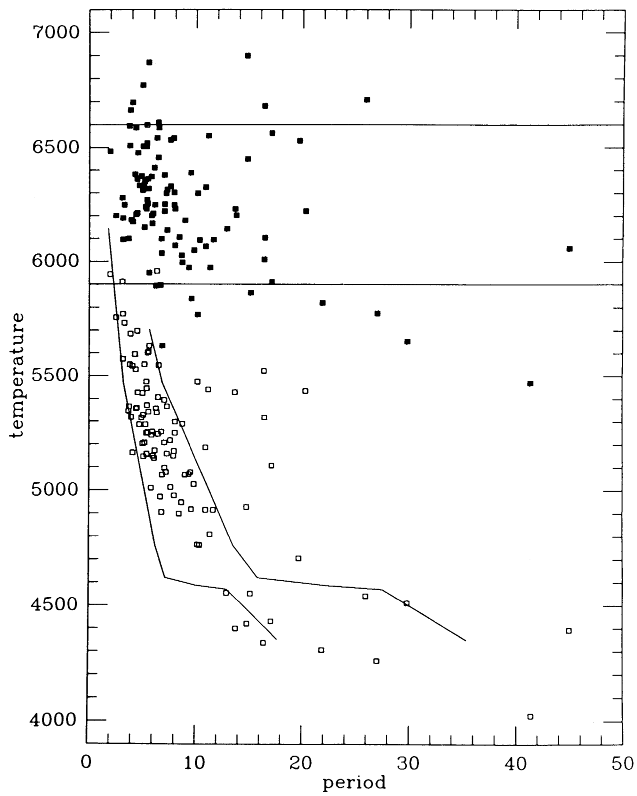

FIG. 7.—Temperature vs. period for MB Cepheids at maximum light (*filled squares*) and minimum light (*open squares*). Solid lines represent the dashed loci mapped from Fig. 1.

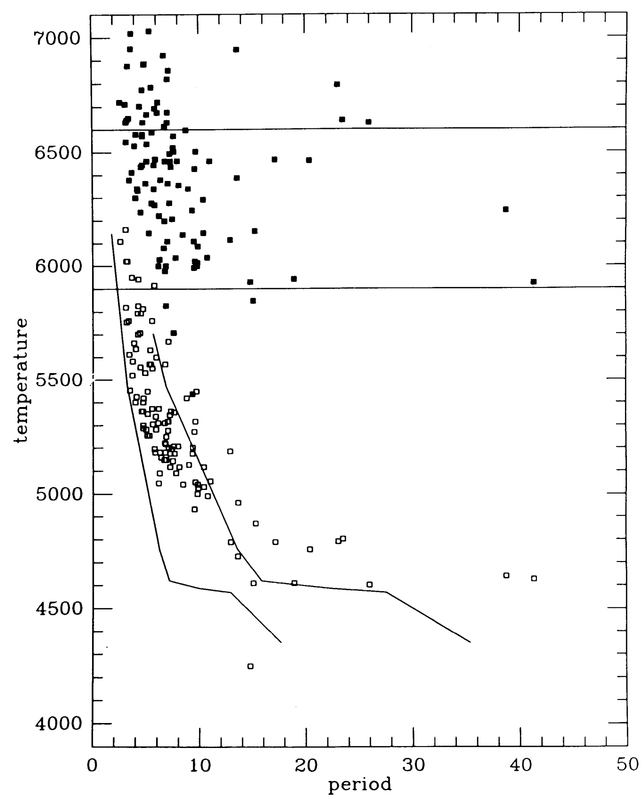

FIG. 8.—Same as Fig. 7, but symbols represent the Cepheids of Pel (1976)

boundaries of his model atmosphere grid. However, at maximum light, these parameters fall safely within the grid and we have calculated maximum temperatures for Pel's long-period sample using his prescription on the published (Pel 1976) Walraven colors, gravities and reddenings.[1] Figure 8 shows that the majority of Pel's Cepheids at maximum light also lie inside the Code envelope, although here the number of stars that appear too hot at maximum is rather larger.

5. AMPLITUDE AND MINIMUM LIGHT

Consider the radius of a pulsating star at two arbitrary phases, call them 1 and 2. We have

$$\frac{R_1^2}{R_2^2} = \frac{L_1 T_2^4}{L_2 T_1^4} . \tag{2}$$

If we take phase 1 to be that of light maximum, and phase 2 light minimum, then $R_1 \cong R_2$ and equation (2) may be written

$$\log T_{\min} \cong \log T_{\max} - \tfrac{1}{10}(V_{\min} - V_{\max}) , \tag{3}$$

where we ignore any differential bolometric corrections.

Figure 9 shows a plot of log $T_{\min}$ versus amplitude for the MB Cepheids of Figure 7. The open squares denote the objects which seem too hot at maximum, that is, those stars which fall above the upper horizontal line in Figure 7. From Figure 9, it is clear that these stars also tend to be hotter at minimum. Most of the remaining points in Figure 9 (*filled squares*) corre-

[1] For completeness, we have also extrapolated Pel's method somewhat beyond the grid boundaries to calculate *minimum* temperatures for the stars with periods longer than 11 days. These temperatures should be considered approximate.

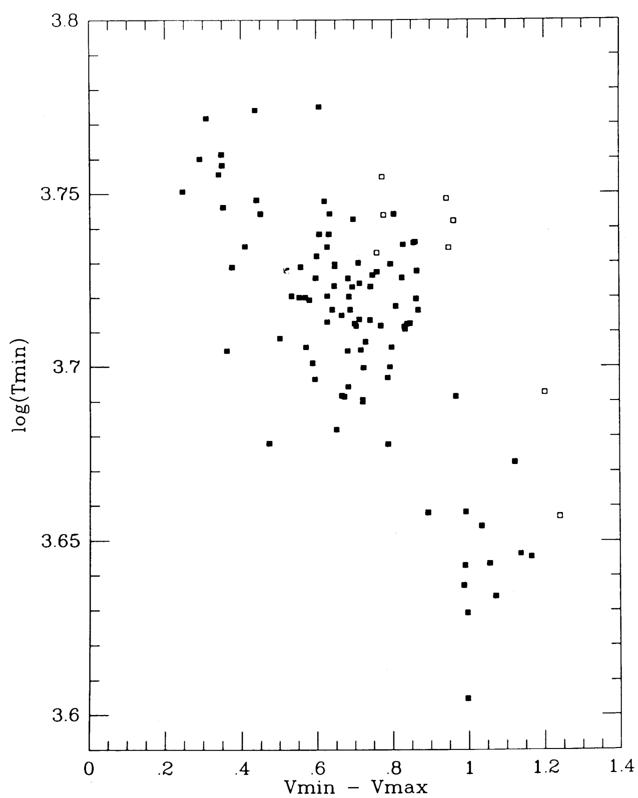

FIG. 9.—Temperature at minimum light vs. amplitude for the MB Cepheids. Open squares indicate the stars which lie above the upper horizontal line in Fig. 7.

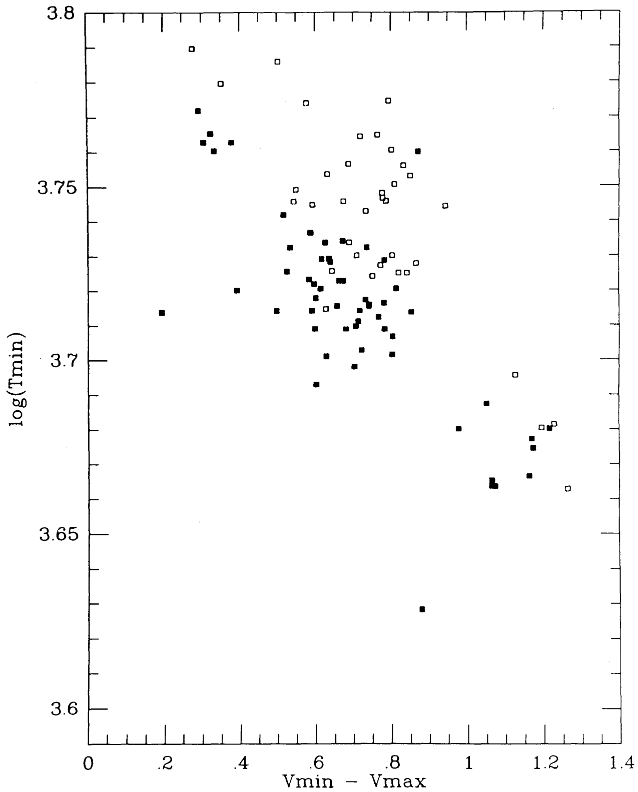

FIG. 10.—Temperature at minimum light vs. amplitude for the Cepheids of Pel (1976). Open squares indicate the stars which lie above the upper horizontal line in Fig. 8.

spond to stars whose maximum temperature falls within Code's envelope (i.e., between the two horizontal lines in Fig. 7); these objects satisfy, in the mean, the condition log $T_{max} \sim$ constant. Comparing equation (3) with Figure 9, one can see by eye that the filled squares do indeed crudely display the predicted slope. Thus the minimum temperature is, in the mean, a function of amplitude.

In Figure 10, we plot minimum temperature against amplitude for the Cepheid sample of Pel (1976). Notation is the same as in Figure 9. Once again, the stars which are too hot at maximum are also hot at minimum, while the rest of the stars in Figure 10 (*filled squares*) define a sequence whose slope crudely agrees with that in equation (3). One final confirmation of the effect of amplitude on minimum temperature may be seen from Table 1, where the slightly smaller amplitude of model B′ (vs. model B, with the same parameters) gives rise to a slightly higher temperature at minimum light.

Referring to Figure 2, it now seems logical to attribute the shallowish slope at minimum light (*open squares*) to an amplitude effect. The short-period models were converged with too large an amplitude (e.g., model A vs. SU Cas or DT Cyg), and the long-period models with too small an amplitude (e.g., model F vs. X Cyg or CD Cyg) with respect to observed stars (Code 1947) of similar period.

6. REDDENING AND MAXIMUM TEMPERATURE

Comparing Figure 1 to, say, Figure 7, one notes that the domain of Cepheids at maximum light (region between the two horizontal lines) seems much broader in temperature than it does in spectral type. This is due merely to the relative insensitivity of spectral type to temperature, over the range in question, in Johnson's (1966) table. On the other hand, the flat slope of the spectral type (or temperature) versus period at maximum light is explained by the location of the HIF as described above.

Let us now attempt to treat the relatively small number of stars in Figure 7, and larger number in Figure 8, whose temperature at light maximum exceeds the upper envelope of Code's domain. Three possible explanations come to mind for these stars: (1) the Teays-Schmidt temperature scale (used for Fig. 7), and the Pel (1976) scale (used for Fig. 8) are both too hot; (2) the stars in question are binaries with hot companions; or (3) the stars in question have published reddenings which are too large.

The first explanation seems unattractive. While it is true that reducing the temperatures across the board could move the "too hot" stars below the upper bound of Code's strip, it would push an equal number of stars below the lower bound, making them "too cool." Could the larger number of "too hot" stars in the Pel versus the MB sample be attributed to the use of a hotter temperature scale for the former? The answer seems to be no. While the Pel scale is indeed hotter than Teays-Schmidt at low temperatures, in the domain near the upper boundary of Code's strip ($T \sim 6600$ K), the two temperature scales seem virtually indistinguishable (Teays 1986).

To explore the second and third alternatives, we present in Table 2 a list of the "too hot" Cepheids from both the MB and Pel samples. Column (2) shows the value of T_{max} for each star—the higher T_{max}, the more the star exceeds the upper envelope of Code's domain. The reddenings are given in column (3): for

TABLE 2

CEPHEIDS WHICH APPEAR TOO HOT AT MAXIMUM

Star (1)	T_{max} (2)	E^a (3)	ΔE^a (4)
Moffett & Barnes (1980, 1984)			
SU Cyg	6662	0.069	0.017
TX Cyg	6899	1.193	0.081
RZ Gem	6869	0.554	0.073
V Lac..................	6772	0.337	0.047
RR Lac.................	6608	0.341	0.002
SZ Mon	6681	0.310	0.022
X Pup	6706	0.472	0.029
ST Tau	6695	0.330	0.026
Pel (1976)			
FM Aql	6720	0.294	0.014
Y Car..................	6951	0.050	0.040
SX Car	6885	0.138	0.032
UX Car	7019	0.037	0.047
WZ Car	6793	0.170	0.022
V Cen	6783	0.117	0.021
AZ Cen	6640	0.065	0.005
UZ Cen	6875	0.095	0.031
V381 Cen	6666	0.083	0.008
R Cru	6693	0.064	0.011
S Cru...................	6631	0.066	0.004
AG Cru	7304	0.097	0.077
SU Cru................	8119	1.008	0.158
RT Mus	7455	0.151	0.093
X Pup	6627	0.183	0.003
AD Pup	6947	0.144	0.039
AT Pup	6923	0.084	0.036
RV Sco................	6675	0.149	0.009
X Sgr	6820	0.086	0.025
R TrA	6649	0.060	0.006
V Vel	7400	0.117	0.087

[a] For the MB sample, $E(B-V)$; for the Pel sample, $E(V-B)$ in the Walraven system.

the MB sample, the color excess is $E(B-V)$ taken from Fernie (1990); for the Pel sample, the color excess is $E(V-B)$ in the Walraven system, obtained from Pel (1976). The final column of Table 2 estimates possible overkill in published reddenings, as explained below.

The first star given in Table 2, SU Cyg, is a known binary, and appears on the Evans (1992) list of Cepheids with hot companions. According to N. Evans (private communication), the measured $B-V$ color of SU Cyg must be increased by 0.06 to correct for the contribution of the companion. Using equation (1), we infer an overestimation of the temperature of SU Cyg by about 200 K due to the companion's light. This is more than enough to account for the temperature excess at maximum for SU Cyg or indeed for any of the MB stars in Table 2. However, of the 31 stars included in the table, SU Cyg is the only one with a confirmed hot companion according to the Evans (1992) study. On the contrary, there are nine stars in Table 2, which Evans (1992) finds *not* to have a hot companion. These objects are V Cen, V381 Cen, R Cru, S Cru, AT Pup, RV Sco, X Sgr, R TrA, and V Vel. In our opinion, the existence of these stars argues strongly against binarity as a general explanation for the temperature excess, although a minority of the "too hot" stars might perhaps be accounted for in this way.

Let us turn then to the reddenings. The last column in Table 2 indicates how much the color excess given in column (3) would need to be reduced for each star, in order to drop its maximum temperature to the upper boundary of Code's domain, i.e., to 6600 K. For the MB sample, the entries in column (4) were obtained directly from equation (1). In the case of the Pel sample, the temperature scale is more complex, but we have made a temperature-color plot for the stars in Figure 8, and fitting it with a straight line, have obtained

$$\log T_e = 3.919 - 0.569(V-B)\,, \qquad (4)$$

where $V-B$ is, once more, the Walraven color. While equation (4) provides only a crude representation of the Pel (1976) temperature scale, it suffices for our purpose of obtaining the estimates of excess reddening given in column (4) of Table 2.

Comparing columns (3) and (4), one sees that the explanation of excess reddening to account for the "too hot" stars seems plausible for all of the MB and most of the Pel entries in Table 2. For a few stars, e.g., Y car, AG Cru, RT Mus, and V Vel, the estimate of excess reddening (col. [4]), large in itself, is also such a large fraction of the published reddening (col. [3]), that the explanation becomes questionable. In another star, UX Car, column (4) actually exceeds column (3) by a small amount. For some of these objects, another explanation should perhaps be sought, although excess reddening may still contribute.

7. DISCUSSION

In the previous section we have argued that an overestimation of the reddening seems the most plausible explanation for most of the Cepheids whose temperatures at light maximum stand out as too high. If this is true, then we may present Table 2 as a list of stars whose published values of $E(B-V)$ or $E(V-B)$ are suspect in the sense that they are probably too large. Indeed, it was Code (1947) himself who suggested that reddenings be determined by studying Cepheids at maximum light. Thus, Table 2 constitutes at least a partial fulfillment of Code's original plan.

However, perhaps the most interesting result of the present work is simply the success of the hydrodynamic models in reproducing the observational data at both maximum and minimum light. These models employ dynamic zoning in the hydrogen ionization region, but neglect convection and treat shocks very crudely, if at all. Their atmospheres are extremely simple: they are gray, and calculated in the Eddington approximation. Nonetheless, the pulsation calculations seem to be getting the Cepheids right, at least as regards the progress of the HIF at minimum and (particularly) at maximum light. This success is a plus for the models and a positive indication for the near future, when a new generation of hydrodynamic codes will emerge, built upon the relatively unsophisticated calculations that are currently employed.

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