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Power of Models in Longitudinal Study: Findings From a Full-Crossed Simulation Design

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Abstract

Because the power properties of traditional repeated measures and hierarchical multivariate linear models have not been clearly determined in the balanced design for longitudinal studies in the literature, the authors present a power comparison study of traditional repeated measures and hierarchical multivariate linear models under 3 variance-covariance structures. The results from a full-crossed simulation design suggest that traditional repeated measures have significantly higher power than do hierarchical multivariate linear models for main effects, but they have significantly lower power for interaction effects in most situations. Significant power differences are also exhibited when power is compared across different covariance structures.

Keywords

covariance structure; hierarchical multivariate linear models; longitudinal study; power analysis; traditional repeated measures

In longitudinal studies, both traditional repeated measures (TRM) and hierarchical multivariate linear models (HMLM) can be applied for a balanced design when the focus is testing fixed main effects. The balanced design assumes an equal number and spacing of measurements over time for each subject. TRM can be used for this design with univariate or multivariate approaches. When sphericity is met, the univariate tests are appropriate; when sphericity is not met, we can use adjusted univariate tests or traditional multivariate tests, which do not assume a given variance-covariance (VC) structure (cf. Greenhouse & Geisser, 1959; Huynh & Feldt, 1976; Jennrich & Schluchter, 1986; Wolfinger & Chang, 1995). For the same longitudinal design, HMLM treats the repeated observations nested within the subjects; that is, repeated measures at Level 1 and subjects at Level 2. A third or higher level of HMLM can be introduced to represent the contextual effects on the subjects' growth (Raudenbush & Bryk, 2002).

HMLM and TRM are essentially interrelated in their theoretical development, especially after advanced computational methods were developed to handle missing values and estimate the VC structures (Dempster, Laird, & Rubin, 1977; Dempster, Rubin, & Tsutakawa, 1981; Goldstein 1995; Jennrich & Schluchter, 1986; Littell, Milliken, Stroup, & Wolfinger, 2006; Little, 1995; Little & Rubin, 2002; Maas & Snijders, 2003; McCulloch & Searle, 2001; Raudenbush & Bryk, 2002; Van der Leeden, Vrijburg, & de Leeuw, 1996). Jennrich and Schluchter were to first model specific VC structures directly through maximum likelihood estimation on the basis of traditional multivariate repeated measures approach, whereas HMLM incorporates Jennrich and Schluchter's multivariate repeated measures approach to longitudinal data analysis (Raudenbush & Bryk; Jennrich & Schluchter; Schluchter, 1988; Van der Leeden, 1998). In the literature, HMLM is simply called *hierarchical linear models*, *multilevel models*, or more generally known as *generalized latent variable models* (e.g., Goldstein, 1994, 1995; Hox, 2002; Maas & Snijders, 2003; B. Muthén, 2002, 2004; L. Muthén & Muthén, 2006; Raudenbush & Bryk; Singer & Willett, 2003; Skrondal & Rabe-Hesketh, 2004).

The power analysis in longitudinal studies has been an active area but a uniform and standard criterion has not been established, especially based on the VC structures (cf. Hedeker, Gibbons, & Waternaux, 1999; Littell et al., 2006; Raudenbush, Spybrook, Liu, & Congdon, 2005; Snijders, 2005). Because the current analytical power approximations are not comprehensive or necessarily accurate (Littell et al.) and the power properties of TRM and HMLM have not yet been clearly compared in the balanced design, using a Monte Carlo (MC) simulation approach is efficient for examining their power properties simultaneously.

For parsimonious and exploratory purposes, TRM and three common VC structures were examined with the longitudinal data generated from a 2-level HMLM in this study. The three VC structures were (a) random slope with homogeneous Level 1 variance (RC), (b) unstructured (UN), and (c) first-order autoregressive (AR(1)).

Two-Level HMLM Model

The hypotheses tested in this simulation assumed the hypothesis of no fixed effects on the individuals' scores over time. The fixed effects were the two-group treatment effect (β_{01}), time effect (β_{10}), and interaction (β_{11}). The underlying mathematical model for this simulation is as follows:

$$\text{Level 1: } y_{it} = \pi_{0i} + \pi_{1i} * \text{TIME} + e_{it} \quad (1)$$

$$\begin{aligned} \text{Level 2: } \pi_{0i} &= \beta_{00} + \beta_{01} * \text{TREATMENT} + u_{0i} \\ \pi_{1i} &= \beta_{10} + \beta_{11} * \text{TREATMENT} + u_{1i}, \end{aligned} \quad (2)$$

where y_{it} represents the score of person i at time t ; π_{0i} is the score of person i at time 0; π_{1i} refers to the slope of person i (i.e., rate of change with respect to time); β_{00} is the average overall initial score at time 0; β_{01} stands for the hypothesized difference in average status from the effect of treatment; β_{10} is the average overall annual rate of change at Level 2; β_{11} represents the hypothesized difference in average annual rate of change from the effect of treatment; u_{0i} is the random effect for intercepts (i.e., random error of intercepts at Level 2); u_{1i} is the random effect for slopes (i.e., random error of slopes at Level 2); e_{it} refers to the random error at the t^{th} time point of the i^{th} person at Level 1.

The aforementioned two-level model can be reduced to a single-level model by substituting Equation (2) into Equation (1):

$$y_{it} = (\beta_{00} + \beta_{01} \times \text{TREATMENT} + \beta_{10} \times \text{TIME} + \beta_{11} \times \text{TREATMENT} \times \text{TIME}) + r_{it}, \quad (3)$$

where the residual term, $r_{it} = u_{0i} + u_{1i} \times \text{TIME} + e_{it}$, includes the Level-1 random error (e_{it}) and Level-2 random effects (u_{0i} and u_{1i}); β_{00} , β_{01} , β_{10} , β_{11} , u_{0i} , u_{1i} , and e_{it} are the same as those in Equation (1) and Equation (2). Hence, the HMLM model is also expressed as the mixed effect model with a mix of fixed effects in the parentheses and random effects embodied in the residual term r_{it} .

TRM and Three Covariance Structures Under Study

The TRM approach to Equation (3) can be simply expressed in a matrix form:

$$Y = X\beta + r, \quad (4)$$

where Y is a $t_i \times 1$ response vector for subject i , t represents the number of time points, and $i = 1, \dots, n$; X is a $t_i \times a$ design matrix for fixed effect β , where a is the number of fixed effects (i.e., the three parameters, β_{01} , β_{10} , and β_{11} in this study), and β is an $a \times 1$ vector; residual r is independently and normally distributed with a mean vector of 0 and variance of Σ , $r \sim N(0, \Sigma)$. The parameter estimates in the traditional approach are obtained using the method of moments (McCulloch & Searle, 2001; Montgomery, 2005; Wolfinger & Chang, 1995).

Random Slope With Homogeneous Level-1 Variance (RC)

Random slope with homogeneous Level-1 variance is often described as the covariance structure for standard MLM, also known as *standard hierarchical linear model* (HLM) or *random coefficient model* (RC; Raudenbush & Bryk, 2002, p. 191; Raudenbush, Bryk, & Congdon, 2004; Singer & Willett, 2003, p. 244–245, 251–265; Kreft, 1996). For convenience, RC is used for this covariance structure hereafter. The RC covariance structure of Model (3) residual r_{it} , Σ_r , is expressed as two components:

$$e_{it} \sim N(0, \sigma^2), \text{ and } \begin{pmatrix} u_{0i} \\ u_{1i} \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix} \right].$$

The variance of Level-1 error term (e_{it}) is homogeneous and the covariance structure of Level-2 random errors (u_{0i} and u_{1i}) is arbitrary. For Model (3), only 4 VC parameters need to be estimated; that is, σ^2 , τ_{00} , τ_{11} , and τ_{01} . Level-1 variance, σ^2 , is independent of Level-2 variance, τ .

Unstructured Covariance Matrix (UN)

The unstructured covariance matrix (also called *unrestricted structure* in the literature) places no restrictions on the structure of covariance matrix, Σ_r , and there is redundancy in mathematical formulation of this covariance structure (Littell, Henry, & Ammerman, 1998, pp. 1229–1230; Raudenbush et al., 2004). If the covariance structure of Σ_r is assumed to be unknown, one could fit a UN covariance matrix. The UN matrix for each Level-2 subject with 3 time points can be expressed as

$$\begin{pmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 \\ \sigma_{31}^2 & \sigma_{32}^2 & \sigma_{33}^2 \end{pmatrix} \quad (5)$$

and requires the estimation of three variance parameters and three covariance parameters. In general, the number of parameters to estimate in the UN approach is of order d^2 for a d by d covariance matrix, which may be computationally excessive when a large number of time points are involved.

First-Order Auto-Regressive (AR(1))

For Model (3), AR(1) can be written as follows:

$$\begin{aligned} \text{Var}(r_{it}) &= \tau + \sigma^2 \\ \text{Cov}(r_{it}, r_{it'}) &= \tau + \sigma^2 \rho^{|t-t'|}, \end{aligned} \quad (6)$$

where τ stands for the Level-2 variance and $|t-t'|$ is the lag between two time points; ρ is the auto-correlation and σ^2 is the Level-1 variance at each time point. AR(1) allows the Level-1 errors to be correlated under Markov assumptions and Level-1 covariance structure is expressed as

$$\sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{pmatrix} \quad (7)$$

As the redundancy is not in the mathematical formulation of AR(1), the covariance structure of Level-2 random effects (u_{0i} and u_{1i}) must be specified to estimate the Level-2 variance τ (τ_{00} , τ_{11} , and τ_{01}), which is usually assumed unstructured (cf. Littell et al., 1998, pp. 1229–1230; McCulloch & Searle, 2001; Raudenbush & Bryk, 2002; Singer & Willett, 2003; Wolfinger, 1993). Thus, five VC parameters need to be estimated for AR(1) of Model (3).

Method

MC Design

This research used a MC study to compare the empirical power of RC, UN, and AR(1). To make the results applicable over many possible situations (Raudenbush & Liu, 2000), a standardized model, Model (4), was employed in this simulation where the grand mean in Model (3) was set to zero (i.e., $\beta_{00} = 0$),

$$\begin{aligned} y_{it} &= (\beta_{01} \times \text{TREATMENT} + \beta_{10} \times \text{TIME} + \beta_{11} \\ &\times \text{TREATMENT} \times \text{TIME}) + r_{it}, \end{aligned} \quad (8)$$

A stacked SAS macro was written by Fang (2006) to generate the two-level repeated measures data with the RC covariance structure and to calculate the power for RC, UN, and AR(1). The number of iterations for this MC study was 10,000 and the nominal alpha for each sample test was .05. RC was selected as the underlying covariance structure mainly in view of the efficiency and complexity of this MC design and future expansion of the data generator. For example,

more parameters would be required to specify for UN and AR(1) than RC for the same data generation. Also, the redundancy problem of UN mentioned earlier would increase the complexity for the design of the data generator. In fact, choosing either of the three covariance structures would mimic the real application situation in which researchers would usually not know the real underlying covariance structure in advance but would need to fit one for their hierarchical data during analyses. Thus the choice of an appropriate covariance structure is at issue, and this power comparison study naturally becomes necessary.

Data Generation

The data generation procedure based on Model (3) was carried out as follows:

Level-1 Data—The error term at Level 1 (i.e., e_{it}) was assumed to be independent of the Level-2 random effects (i.e., u_{0i} and u_{1i}), that is, $cov(u_i, e_{it}) = 0$. The Level-1 error term followed a normal distribution, $e_{it} \sim N(0, \sigma^2)$.

Level-2 Data—The random intercepts u_{0i} ($X_{intercepts}$) and slopes u_{1i} (X_{slopes}) assumed a standard bivariate normal distribution. A standardized G matrix for $X_{intercepts}$ and X_{slopes} ,

$$G = \begin{pmatrix} \sigma_{00}^2 & \sigma_{01}^2 \\ \sigma_{10}^2 & \sigma_{11}^2 \end{pmatrix}, \text{ and random mean vector, } \begin{pmatrix} \mu_0 \\ \mu_1 \end{pmatrix},$$

were specified to simulate correlated bivariate normal data for $X_{intercepts}$ and X_{slopes} . The Cholesky decomposition method was utilized to generate the correlated Level-2 normal data. This simulation was accomplished by multiplying the normal data by L , which is the Cholesky decomposition of G . The estimated variables were $\hat{X}_{intercepts}$ and \hat{X}_{slopes} .

Complete Data—Data were generated in the appropriate format required by PROC MIXED (SAS Institute, 2003). An index matrix was created for time, treatment, and individuals (IDs). Two treatment groups were coded by 0 and 1, respectively. IDs were considered nested within each treatment group; for instance, IDs ranged from 1 to 25 for Group 1, and 26 to 50 for Group 2. Time started from 0 and extended to the maximum specified for each study condition. On the basis of Model (4), a univariate response vector of y_{it} was created. For example, each subject might have had three time points and each treatment group 25 subjects.

The data generator (Fang, 2006; Fang, Brooks, Rizzo, & Barcikowski, 2006) was validated with parameter estimates from Potthoff and Roy's (1964) data. The results (see the Appendix) show that the estimates from simulated data are slightly different in the third or fourth decimal from the true parameters. Also, as suggested by Littell et al. (2006), under the null hypothesis of no treatment effect, the p values based on the simulated data have an approximate uniform distribution, which further validates the data generator.

Power Comparison

When the time points are fixed, this study implemented a full-crossed design for power comparison; that is, the power was compared across the levels of the four factors: (a) correlation in G matrix (G), (b) reliability of Level-1 coefficients (λ), (c) effect size (β), and (d) sample size per treatment group (n). The decision was made to fix the time points at 3 because of three main facets. First, three time points are practically the base requirement for longitudinal studies, although theoretically two time points would suffice (McCulloch & Searle, 2001). Second, a full-crossed design based on three time points lays the foundation for the second step where we fixed the total sample size while varying the sample size per treatment group and the number

of time points (i.e., the ratio of n to t). Third, in many real-world studies, it is simply not feasible to collect data at more than three time points due to costs, particularly in applied settings such as educational studies, clinical trials, and epidemiological research found in other social and physical science areas.

As shown in Figure 1, we used Cohen's indexes (1988) for correlation, $\rho \in \{.1 .3 .5\}$; correspondingly, G matrix (G) for random intercepts (u_{0i}) and slopes (u_{1i}) was specified as

$$\begin{pmatrix} 1 & .1 \\ .1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & .3 \\ .3 & 1 \end{pmatrix}, \text{ or } \begin{pmatrix} 1 & .5 \\ .5 & 1 \end{pmatrix};$$

the averaged reliability of Level-1 coefficients was set to vary from .01 to 1 by .25, $\lambda \in \{.01 .25 .5 .75 1\}$ (Raudenbush & Bryk, 2002; Raudenbush et al., 2004); the effect sizes also used Cohen's indexes for the two-group treatment effect (β_{01}), time effect (β_{10}), and interaction (β_{11}), $\beta \in \{.2 .5 .8\}$; the sample size per treatment group changed from 25 to 200 by 25 ($n \in \{25 75 100 125 150 175 200\}$). On the basis of this design, a general power pattern of TRM, RC, UN, and AR(1) were presented at each level of the four factors for the three tests (i.e., $4 \times 3 \times 5 \times 3 \times 7 \times 3 = 3,780$ cells).

For exploratory purpose, we also compared the power of the four models by holding the total sample size (N) at 800 and varying the ratio of the treatment group size to the number of time points under a specified condition in which the reliability was fixed at .50. The sample size ratio was set at 100/4, 80/5, and 40/10, respectively. The power of the four models was examined across different levels of the correlation in G matrix and effect sizes for the three tests (i.e., $4 \times 3 \times 3 \times 3 \times 3 = 324$ cells; see Figure 1).

MC Analysis

We used the following function to calculate the upper bound of standard errors for pairwise empirical power (i.e., the standard error for the difference in proportions).

$$SE_{upper} = \sqrt{2 \times \frac{p \times (1-p)}{n}}, \quad (9)$$

where $p = .5$ and $n = 10,000$. If the pairwise differences are twice the upper bound of SE (i.e., $SE_{upper} \cong .007$), then the differences are labeled as significant. The power patterns are illustrated in tables and graphs.

Results

The results shown in Tables 1–3 indicate a general pattern when varying the G matrix in all three tests, treatment effect (β_{01}), time effect (β_{10}) and interaction (β_{11}). The pattern showed that as the correlation in G matrix increases, the power of TRM, RC, UN, and AR(1) decreased slightly, which may imply that the lower the correlation between intercepts and slopes, the higher the power we can obtain. But it should be noted that the power change across G matrixes seems to be visually minimal. Because of this small difference among the four models, Figures 2–4 merely present the power pattern at $G = .1$, whereas Tables 1–3 provide the entire power information by varying G matrixes.

Empirical Power in Treatment Test

By fixing the number of time points at 3, this power study indicated that in the treatment test, the TRM power was significantly higher than the three HMLM models across different levels of sample sizes, G matrixes, reliability, and effect sizes, except when the reliability is very low, around .01, or when the reliability is perfect at 1 and effect size at .8 (see Table 1a). As displayed in Figure 2, the power of all four models is below .20 at the reliability of .01 and the power difference among the four is not statistically different based on the criteria discussed earlier (i.e., twice the upper bound of SE , $2 \times SE = .014$), even when the sample size changes to 200 and the effect size becomes .8 (see Table 1b).

Although the pairwise power difference among the four models seems to be visually indistinguishable at the reliability of 1, the power of four models is significantly different from each other across the sample sizes; correlation in the G matrix and effect sizes, except the difference between TRM and UN vanishes at the reliability of 1 (see Figure 2 and Table 1b). The pairwise power difference also displays that the power of UN, AR(1), and RC generally keeps the same descending order but is not significantly different across all levels of the four factors, especially at the reliability of .01 (see Figure 2a and Table 1b).

When the total sample size is fixed at 800 and reliability at .50, the results show that the power of the four models decreases as the ratio of treatment group size to the number of time points decreases (see Figure 2b). In other words, it seems that when the total sample size is held constant, the higher power is easily achieved by increasing the treatment group size rather than the number of time points in detecting the treatment effect (i.e., increasing the sample size at a higher level rather than the lower level, using the terminology of HMLM). Table 3 indicates that in the treatment test, the power at 100/4 is at least 1.5 times the power at 80/5 and at least twice the power at 40/10 for all four models. Under this setting, TRM power is still significantly higher than the HMLM models across the sample sizes, effect sizes, and correlation in the G matrix, except that TRM and UN power seem to be similar at the sample size ratio of 40/10 or when the effect size is small. Among the three HMLM models, UN power is still higher than the other two, and AR(1) is higher than RC, except at the ratio of 40/10.

Empirical Power in Interaction Test

The interaction test with three fixed time points indicates that the TRM power is generally significantly lower than HMLM models at the medium or large effect sizes with the reliability between .25 and .75 (see Table 2). At the reliability of 1, RC power is not always significantly higher than TRM across the samples sizes, effect sizes, or the correlation in the G matrix, whereas UN and AR(1) power are generally significantly higher than TRM. At a very low reliability of .01, the power of all four models is below .20 across the power factors and the pairwise power differences seem not to be identifiable (see Figure 3a).

Among the three HMLM models, UN power ranks the highest, AR(1) the second, and RC the lowest across the factors. At the medium or large effect sizes, the pairwise power difference among the three VC structures is always significant, except when the reliability is at or below .25. At the small effect size, the pairwise differences among the three VC structures are significant, except when the reliability is at or below .50.

Holding the total sample size at 800 and the reliability at .50, the power pattern between TRM and HMLM models in the interaction test seems to be magnified (see Figure 3b). The power of UN, AR(1), and RC is still higher than TRM across the sample sizes, effect sizes, and the correlation in the G matrix. The pairwise differences among the four models are generally significant except with the power difference between RC and TRM at the small effect size and with the pairwise differences among the three HMLM models at a large effect size of .8 and

small correlation of .1 in the G matrix, as their power is all close to the asymptote of 1 (see Table 3).

Empirical Power in Time Test

As shown in Figure 4a, TRM and the three HMLM models are still not comparable at the reliability of .01. At the small or medium effect size, TRM power is significantly higher than the three HMLM models across sample sizes and the correlation indexes in G matrix when the reliability is at or above .25; among the three VC structures, UN is significantly higher than the other two, and AR(1) is higher than RC when the reliability is at or above .50. At the large effect size, as the power of all four models are close to the asymptote of 1, TRM power seems to be higher than the three HMLM models only when the sample size is below 75 and the reliability above .25.

Figure 4b indicates that while holding the total sample size at 800 and the reliability of .50, TRM power is higher than the other three models in the time test across the sample size ratios at the small or medium effect size, except at the ratio of 40/10 where UN is higher than TRM. At the large effect size, the power of all four models is close to 1 across the specified sample size ratios, and no significant pairwise power difference is found.

Discussion and Conclusions

In the present MC study, we were primarily concerned with the empirical power of TRM and HMLM under three VC structures in the longitudinal study. Specifically, we compared the power of TRM, RC, AR(1), and UN in three tests, two-group treatment effect (β_{01}), time effect (β_{10}), and time-by-treatment interaction (β_{11}), under the balanced design in longitudinal studies, assuming the underlying covariance structure is RC. The four factors in this power study are the G matrix (G), reliability (λ), effect size (β), and ratio of sample size per treatment group (n) to the number of time points (t).

Generally, the findings from this full-crossed design based on the fixed time points or the fixed total sample size show that TRM, in the treatment or time test, is more powerful than the HMLM under the three VC structures except for the situation in which the reliability is close to 0 or perfect at 1. Yet, in reality, the two extreme conditions are rare. This study also indicates that in the treatment test, it is easier to achieve the higher power by increasing the treatment group size rather than the number of time points, which corresponds to the discussion in the literature (e.g., Kreft, 1996; Raudenbush & Bryk, 2002). For example, the increase of time points typically would comparatively incur more research cost by following up on the subjects, and it would probably confront more participants' mobility, dropout, and other attrition problems and likely result in greater missing data, which would further complicate the research.

As for the interaction test, researchers have questioned what the power is to detect the interactions when they exist in the HMLM data. They expected HMLM to perform better than traditional models, but they did not find such proof (Davison, Kwak, Seo, & Choi, 2002; Kreft, 1996; Raudenbush, 1995). This study provided empirical power estimates in the interaction test for both TRM and HMLM. One of the interesting findings in this power study indicates that TRM has significantly lower power than the other three HMLM models—RC, AR(1), and UN—in the interaction test, except in the two extreme reliability conditions (close to 0 or 1).

From this study, we noticed that the power can be significantly different among different VC structures when using the HMLM models in the longitudinal study. In addition to referring to the model fit statistics (Akaike, 1973; Littell et al., 2006; Pinheiro & Bates, 2000; Schwarz, 1978; Singer & Willett, 2003), the empirical power results from this study could be a reference source when applying HMLM models. Also from these empirical results, practitioners may

estimate the sample sizes, the reliability, effect size, or the correlation in the G matrix for their studies if scenarios are similar to this study. On the basis of this study, TRM could be the choice if researchers are further interested in main effect tests and the practical situation is most similar to this research in which the balanced design is assumed and fixed effects are primarily the concern. If researchers are more concerned with interaction tests (i.e., the group difference in the change rate over time using the HMLM terminology), this study recommends that UN, AR (1), or RC be the method of choice. UN could be the best among the three HMLM models if the practical situation is most similar to this research and if there is a need to try an exploratory analysis when the VC structure is assumed unknown. In practice, if the candidate models are appropriate for a given analysis, practitioners tend to choose the more powerful model, which is reasonable. However, we encourage the thoughtful and responsible application of TRM and HMLM in recommended tests similar to this balanced design in order to properly contextualize the value of our findings.

Because no uniform agreement was made regarding the typical values for the settings, such as the effect sizes and the correlation among random effects in different applied research areas (e.g., education, psychology, clinical trials, and biology, etc.), we used the traditional Cohen's indexes, which are at least widely accepted in social science research. On the basis of our post hoc demonstration, the change of the sign of these indexes would not affect the power pattern among the four models compared in this study, although the magnitude of the power of these models changed slightly. However, we would recommend that practitioners closely collaborate with their project colleagues to make full sense of the results generated from this study. If there is a strong rationale that the expected indexes differ from the present study, the values should be changed to generate corresponding output using our macro.

Future studies may consider extending this MC study by comparing power at different total sample sizes across more variable ratios of sample size per group to the number of time points to examine whether the power differences could diminish or increase at the low ratio. Also, the data generator could be modified to further investigate the power differentiation under other underlying covariance structures. Instead of reliability, interclass correlation could be considered in the power analysis. Although the magnitude of power difference and power decreasing or increasing rates can vary, the general power patterns among TRM and the three VC structures are expected to be similar to this study. The HMLM data generator and power comparison macro (Fang, 2006; Fang et al., 2006) could be expanded to generate missing data or nonnormal longitudinal data in order to be more practical and to examine the statistical properties and power of more complex growth models.

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Factors ^a	Conditions (Cells) ^b
I	
Correlation in <i>G</i> Matrix (<i>G</i>):	{.1 .3 .5}
Reliability (λ):	{.01 .25 .5 .75 1}
Effect Size (β):	{.2 .5 .8}
Sample Size per Treatment Group (<i>n</i>):	{25 50 75 100 125 150 175 200}
II	
Correlation in <i>G</i> Matrix (<i>G</i>):	{.1 .3 .5}
Effect Size (β):	{.2 .5 .8}
Sample Size Ratio (<i>n/t</i>):	{100/4 80/5 40/10}

FIGURE 1.

Monte Carlo (MC) design for power analysis of TRM, RC, AR(1), and UN by *G* matrix, effect size, and sample size of 10,000 MC samples at $\alpha = .05$. TRM = traditional repeated measures; RC = random slope with homogeneous level-1 variance; AR(1) = first-order autoregressive; UN = unstructured. ^aThe factors are crossed. ^bConditions are specified for testing each fixed effect (β_{01} , β_{10} , and β_{11}) across levels of factors. ^c“Fixed” indicates the fixed parameters in the design. ^dSettings for testing the three fixed effects.

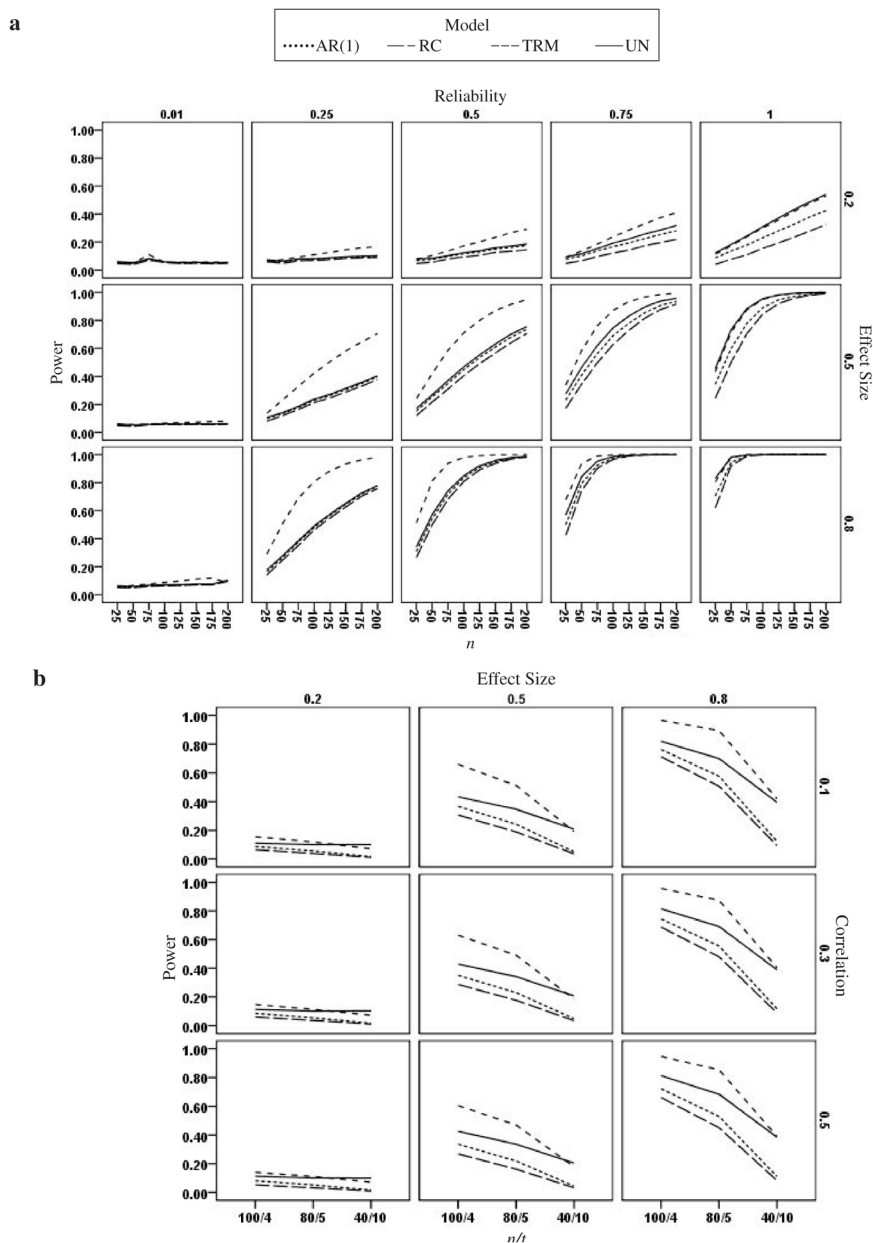


FIGURE 2. Power patterns of variance-covariance structures, random slope with homogeneous Level-1 variance (RC), first-order autoregressive (AR(1)), unstructured (UN), and traditional repeated measures (TRM) at $\alpha = .05$ of 10,000 MC samples in treatment test. For Figure 2a, $t = 3.0$ and $G = 0.1$ in treatment test. For Figure 2b, $N = 800$ and $\lambda = .50$ in treatment test.

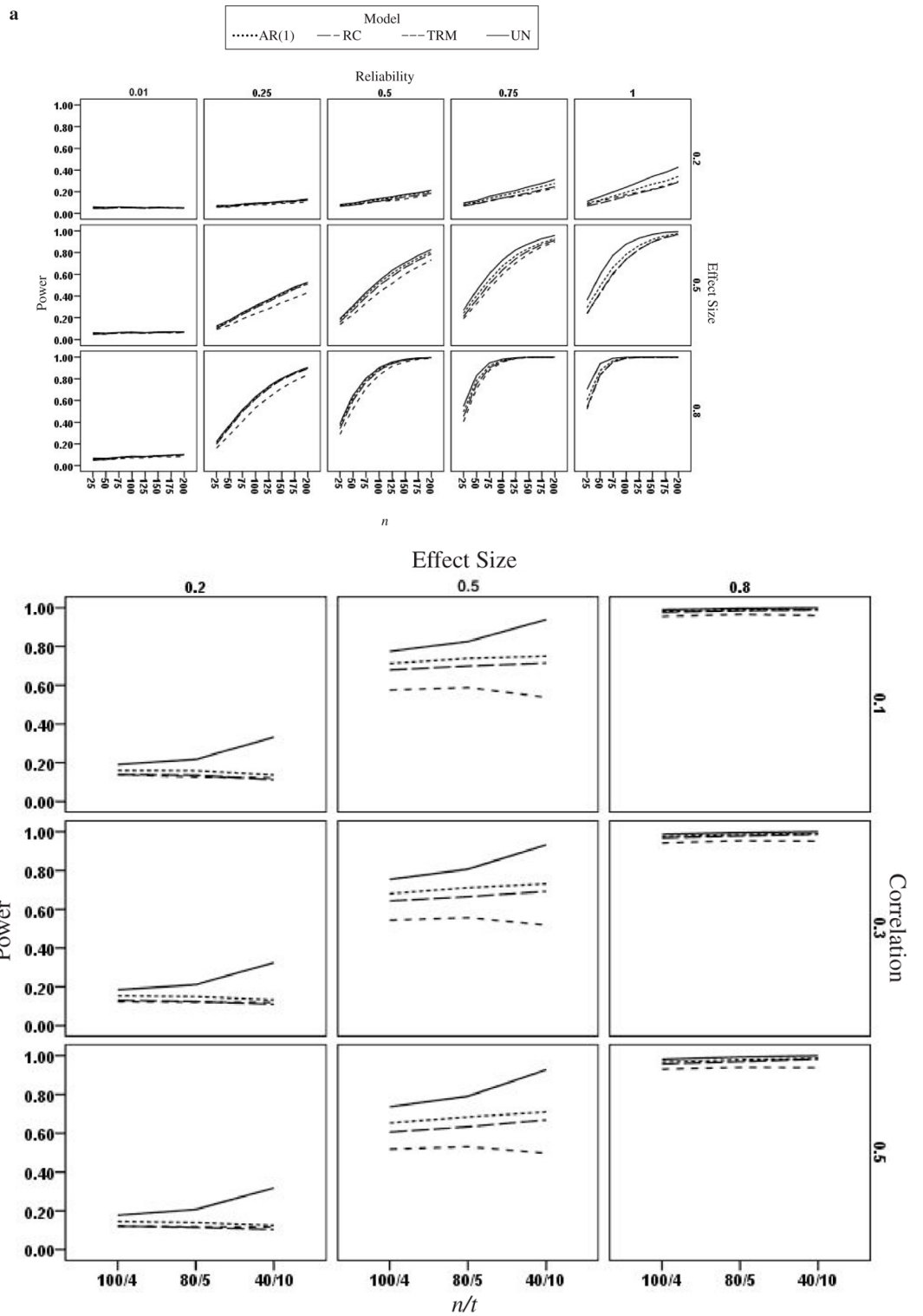


FIGURE 3. Power patterns of variance-covariance structures, random slope with homogeneous Level-1 variance (RC), first-order autoregressive (AR(1)), unstructured (UN), and traditional repeated

measures (TRM) at $\alpha = .05$ of 10,000 MC samples in the interaction test. For Figure 3a, $t = 3.0$ and $G = 0.1$ in interaction test. For Figure 3b, $N = 800$ and $\lambda = .50$ in interaction test.

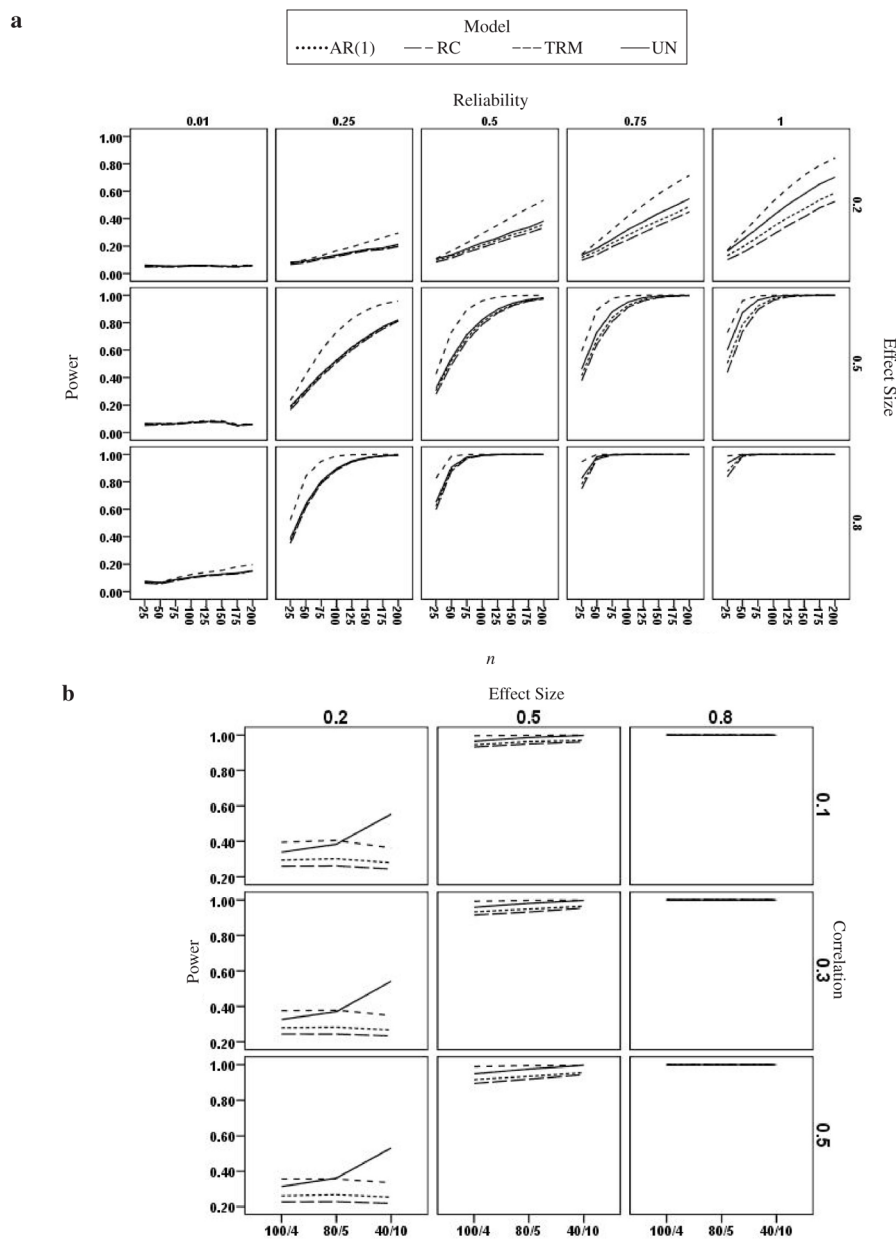


FIGURE 4. Power patterns of variance-covariance structures, random slope with homogeneous Level-1 variance (RC), first-order autoregressive (AR(1)), unstructured (UN), and traditional repeated measures (TRM) at $\alpha = .05$ of 10,000 MC samples in time test. For Figure 4a, $t = 3.0$ and $G = 0.1$ in time test. For Figure 4b, $N = 800$ and $\lambda = .50$ in time test.

Table 1

TABLE 1a. Power Pattern of TRM, RC, AR(I), and UN When $t = 3$ of 10,000 MC Samples at $\alpha = .05$ in Treatment Test

λ	n	$G = 1$					$G = 3$					$G = 5$						
		RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	
1	25	0.0409	0.0866	0.1218	0.1130	0.0331	0.0761	0.1197	0.1042	0.0274	0.0703	0.1197	0.1042	0.0274	0.0703	0.1197	0.1042	
	50	0.0764	0.1365	0.1831	0.1733	0.0637	0.1224	0.1829	0.1585	0.0533	0.1128	0.1815	0.1585	0.0533	0.1128	0.1815	0.1585	
	75	0.1090	0.1787	0.2430	0.2384	0.0915	0.1596	0.2399	0.2132	0.0777	0.1471	0.2381	0.2132	0.0777	0.1471	0.2381	0.2132	
	100	0.1556	0.2308	0.3088	0.3004	0.1297	0.2097	0.3091	0.2897	0.1075	0.1895	0.3057	0.2897	0.1075	0.1895	0.3057	0.2897	
	125	0.1939	0.2805	0.3728	0.3584	0.1628	0.2521	0.3699	0.3206	0.1388	0.2302	0.3687	0.3206	0.1388	0.2302	0.3687	0.3206	
	150	0.2357	0.3280	0.4335	0.4211	0.2000	0.2987	0.4303	0.3781	0.1730	0.2721	0.4289	0.3781	0.1730	0.2721	0.4289	0.3781	
	175	0.2789	0.3812	0.4885	0.4761	0.2383	0.3458	0.4850	0.4274	0.2047	0.3175	0.4831	0.4274	0.2047	0.3175	0.4831	0.4274	
	200	0.3253	0.4248	0.5406	0.5303	0.2788	0.3911	0.5365	0.4762	0.2392	0.3587	0.5343	0.4762	0.2392	0.3587	0.5343	0.4762	
	.75	25	0.0472	0.0775	0.0933	0.0909	0.0396	0.0751	0.0927	0.0871	0.0344	0.0715	0.0934	0.0871	0.0344	0.0715	0.0934	0.0871
		50	0.0662	0.1017	0.1164	0.1161	0.0581	0.1061	0.1162	0.1290	0.0512	0.0913	0.1151	0.1290	0.0512	0.0913	0.1151	0.1290
		75	0.0963	0.1375	0.1536	0.1857	0.0828	0.1249	0.1514	0.1717	0.0748	0.1171	0.1486	0.1717	0.0748	0.1171	0.1486	0.1717
		100	0.1218	0.1677	0.1922	0.2378	0.1075	0.1558	0.1908	0.2199	0.0962	0.1462	0.1904	0.2199	0.0962	0.1462	0.1904	0.2199
		125	0.1434	0.1944	0.2224	0.2858	0.1305	0.1819	0.2195	0.2646	0.1179	0.1739	0.2168	0.2646	0.1179	0.1739	0.2168	0.2646
		150	0.1743	0.2256	0.2578	0.3307	0.1553	0.2132	0.2555	0.3025	0.1414	0.1990	0.2533	0.3025	0.1414	0.1990	0.2533	0.3025
		175	0.1964	0.2544	0.2861	0.3757	0.1762	0.2413	0.2844	0.3459	0.1584	0.2275	0.2828	0.3459	0.1584	0.2275	0.2828	0.3459
200		0.2197	0.2805	0.3189	0.4102	0.1995	0.2661	0.3168	0.3779	0.1775	0.2504	0.3141	0.3779	0.1775	0.2504	0.3141	0.3779	
.50		25	0.0475	0.0696	0.0810	0.0757	0.0459	0.0667	0.0806	0.0746	0.0424	0.0661	0.0804	0.0746	0.0424	0.0661	0.0804	0.0746
		50	0.0556	0.0758	0.0835	0.1043	0.0519	0.0741	0.0833	0.1013	0.0479	0.0712	0.0829	0.1013	0.0479	0.0712	0.0829	0.1013
		75	0.0791	0.0983	0.1035	0.1359	0.0744	0.0966	0.1031	0.1292	0.0705	0.0930	0.1013	0.1292	0.0705	0.0930	0.1013	0.1292
		100	0.0935	0.1161	0.1237	0.1726	0.0869	0.1118	0.1226	0.1651	0.0810	0.1082	0.1222	0.1651	0.0810	0.1082	0.1222	0.1651
		125	0.1059	0.1303	0.1377	0.2016	0.1013	0.1275	0.1365	0.1928	0.0953	0.1247	0.1359	0.1928	0.0953	0.1247	0.1359	0.1928
		150	0.1264	0.1510	0.1600	0.2340	0.1182	0.1473	0.1586	0.2218	0.1120	0.1415	0.1579	0.2218	0.1120	0.1415	0.1579	0.2218
		175	0.1351	0.1624	0.1716	0.2671	0.1267	0.1562	0.1707	0.2545	0.1201	0.1520	0.1705	0.2545	0.1201	0.1520	0.1705	0.2545
	200	0.1444	0.1755	0.1862	0.2917	0.1359	0.1697	0.1853	0.2767	0.1274	0.1654	0.1849	0.2767	0.1274	0.1654	0.1849	0.2767	
	.25	25	0.0598	0.0671	0.0730	0.0593	0.0468	0.0530	0.0699	0.0629	0.0459	0.0611	0.0692	0.0629	0.0459	0.0611	0.0692	0.0629
		50	0.0484	0.0585	0.0632	0.0749	0.0469	0.0580	0.0631	0.0732	0.0460	0.0567	0.0631	0.0732	0.0460	0.0567	0.0631	0.0732
		75	0.0651	0.0753	0.0790	0.0933	0.0643	0.0754	0.0791	0.0912	0.0625	0.0753	0.0785	0.0912	0.0625	0.0753	0.0785	0.0912
		100	0.0685	0.0778	0.0804	0.1102	0.0662	0.0773	0.0805	0.1105	0.0646	0.0776	0.0806	0.1105	0.0646	0.0776	0.0806	0.1105
		125	0.0750	0.0853	0.0870	0.1254	0.0728	0.0841	0.0869	0.1231	0.0716	0.0846	0.0864	0.1231	0.0716	0.0846	0.0864	0.1231
		150	0.0827	0.0926	0.0952	0.1442	0.0810	0.0922	0.0956	0.1425	0.0798	0.0918	0.0960	0.1425	0.0798	0.0918	0.0960	0.1425
		175	0.0862	0.0943	0.0991	0.1596	0.0846	0.0949	0.0988	0.1555	0.0820	0.0937	0.0985	0.1555	0.0820	0.0937	0.0985	0.1555
200		0.0890	0.0986	0.1030	0.1681	0.0869	0.0979	0.1026	0.1656	0.0838	0.0975	0.1029	0.1656	0.0838	0.0975	0.1029	0.1656	
.01		25	0.0475	0.0547	0.0604	0.0489	0.0474	0.0547	0.0604	0.0490	0.0476	0.0545	0.0606	0.0490	0.0476	0.0545	0.0606	0.0490
		50	0.0419	0.0469	0.0515	0.0508	0.0418	0.0470	0.0517	0.0470	0.0416	0.0467	0.0517	0.0470	0.0416	0.0467	0.0517	0.0470
		75	0.0685	0.0778	0.0804	0.1102	0.0508	0.0546	0.0573	0.0519	0.0509	0.0546	0.0571	0.0519	0.0509	0.0546	0.0571	0.0519
		100	0.0538	0.0569	0.0589	0.0540	0.0540	0.0569	0.0590	0.0538	0.0535	0.0567	0.0590	0.0538	0.0535	0.0567	0.0590	0.0538
		125	0.0476	0.0499	0.0523	0.0477	0.0477	0.0500	0.0522	0.0478	0.0478	0.0499	0.0520	0.0478	0.0478	0.0499	0.0520	0.0478
		150	0.0498	0.0520	0.0544	0.0569	0.0499	0.0519	0.0544	0.0567	0.0497	0.0521	0.0545	0.0567	0.0497	0.0521	0.0545	0.0567
		175	0.0459	0.0488	0.0510	0.0567	0.0456	0.0491	0.0509	0.0566	0.0455	0.0495	0.0509	0.0566	0.0455	0.0495	0.0509	0.0566
	200	0.0501	0.0516	0.0533	0.0531	0.0500	0.0520	0.0532	0.0529	0.0500	0.0518	0.0532	0.0529	0.0500	0.0518	0.0532	0.0529	
	1	25	0.2423	0.3432	0.4543	0.4311	0.2046	0.3119	0.4529	0.3862	0.1755	0.2874	0.4511	0.3862	0.1755	0.2874	0.4511	0.3862
		50	0.5003	0.5985	0.7337	0.7147	0.4459	0.5554	0.7300	0.6568	0.3974	0.5183	0.7289	0.6568	0.3974	0.5183	0.7289	0.6568
		75	0.7055	0.7781	0.8839	0.8755	0.6532	0.7406	0.8826	0.8294	0.6012	0.7045	0.8818	0.8294	0.6012	0.7045	0.8818	0.8294
		100	0.8544	0.8907	0.9459	0.9499	0.7979	0.8636	0.9208	0.9208	0.7505	0.8312	0.9208	0.9208	0.7505	0.8312	0.9208	0.9208
		125	0.9199	0.9467	0.9829	0.9801	0.8864	0.9250	0.9818	0.9645	0.8523	0.9012	0.9808	0.9645	0.8523	0.9012	0.9808	0.9645
		150	0.9588	0.9718	0.9933	0.9921	0.9398	0.9602	0.9927	0.9846	0.9170	0.9445	0.9925	0.9846	0.9170	0.9445	0.9925	0.9846
		175	0.9796	0.9864	0.9969	0.9979	0.9694	0.9786	0.9967	0.9952	0.9557	0.9697	0.9966	0.9952	0.9557	0.9697	0.9966	0.9952
EZ = 2		25	0.0761	0.1224	0.1829	0.1585	0.0331	0.0761	0.1197	0.1042	0.0274	0.0703	0.1197	0.1042	0.0274	0.0703	0.1197	0.1042
		50	0.1596	0.2097	0.2399	0.2132	0.0637	0.1224	0.1829	0.1585	0.0533	0.1128	0.1815	0.1585	0.0533	0.1128	0.1815	0.1585
		75	0.2097	0.2521	0.2699	0.2306	0.0915	0.1596	0.2399	0.2132	0.0777	0.1471	0.2381	0.2132	0.0777	0.1471	0.2381	0.2132
		100	0.2521	0.2987	0.3091	0.2897	0.1297	0.2097	0.3091	0.2897	0.1075	0.1895	0.3057	0.2897	0.1075	0.1895	0.3057	0.2897
		125	0.3280	0.3458	0.3458	0.3206	0.1628	0.2521	0.3699	0.3206	0.1388	0.2302	0.3687	0.3206	0.1388	0.2302	0.3687	0.3206
		150	0.4248	0.4248	0.4248	0.4211	0.2000	0.2987	0.4303	0.3781	0.1730	0.2721	0.4289	0.3781	0.1730	0.2721	0.4289	0.3781
		175	0.4248	0.4248	0.4248	0.4211	0.2000	0.2987	0.4303	0.3781	0.1730	0.2721	0.4289	0.3781	0.1730	0.2721	0.4289	0.3781
		200	0.4248	0.4248	0.4248	0.4211	0.2000	0.2987	0.4303	0.3781	0.1730	0.2721	0.4289	0.3781	0.1730	0.2721	0.4289	0.3781
	EZ = 5	25	0.3119	0.4529	0.4529	0.4311	0.2046	0.3119	0.4529	0.3862	0.1755	0.2874	0.4511	0.3862	0.1755	0.2874	0.4511	0.3862
		50	0.5554	0.7406	0.8826	0.8294												

TABLE 1a. Power Pattern of TRM, RC, AR(I), and UN When $r = 3$ of 10,000 MC Samples at $\alpha = .05$ in Treatment Test

λ	n	$G = .1$					$G = .3$					$G = .5$						
		RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	
.75	200	0.9921	0.9950	0.9994	0.9994	0.9867	0.9916	0.9994	0.9980	0.9785	0.9868	0.9993	0.9953	0.9785	0.9868	0.9993	0.9953	
	25	0.1718	0.2301	0.2736	0.3377	0.1529	0.2195	0.2711	0.3112	0.1398	0.2072	0.2686	0.2881	0.1398	0.2072	0.2686	0.2881	
	50	0.3445	0.4096	0.4641	0.5847	0.3170	0.3902	0.4607	0.5424	0.2911	0.3702	0.4579	0.5061	0.2911	0.3702	0.4579	0.5061	
	75	0.4976	0.5633	0.6206	0.7622	0.4631	0.5384	0.6161	0.7211	0.4309	0.5172	0.6117	0.6818	0.4309	0.5172	0.6117	0.6818	
	100	0.6285	0.6921	0.7474	0.8719	0.5898	0.6641	0.7422	0.8367	0.5582	0.6366	0.7401	0.8042	0.5582	0.6366	0.7401	0.8042	
	125	0.7301	0.7804	0.8309	0.9345	0.6969	0.7564	0.8275	0.9114	0.6630	0.7330	0.8239	0.8826	0.6630	0.7330	0.8239	0.8826	
	150	0.8159	0.8550	0.8959	0.9666	0.8328	0.8811	0.9359	0.9768	0.8054	0.8518	0.9325	0.9325	0.8054	0.8518	0.9325	0.9325	
	175	0.8775	0.9067	0.9378	0.9856	0.8539	0.8923	0.9359	0.9768	0.8284	0.8731	0.9338	0.9643	0.8284	0.8731	0.9338	0.9643	
	200	0.9166	0.9360	0.9579	0.9932	0.9005	0.9245	0.9563	0.9867	0.8794	0.9131	0.9553	0.9788	0.8794	0.9131	0.9553	0.9788	
	.50	25	0.1188	0.1507	0.1701	0.2401	0.1116	0.1446	0.1699	0.2262	0.1060	0.1421	0.1689	0.2145	0.1060	0.1421	0.1689	0.2145
50		0.2166	0.2544	0.2685	0.4195	0.2051	0.2468	0.2666	0.3975	0.1936	0.2387	0.2668	0.3750	0.1936	0.2387	0.2668	0.3750	
75		0.3091	0.3493	0.3697	0.5858	0.2926	0.3372	0.3678	0.5595	0.2782	0.3272	0.3655	0.5320	0.2782	0.3272	0.3655	0.5320	
100		0.4051	0.4488	0.4676	0.7109	0.3872	0.4380	0.4656	0.6818	0.3720	0.4280	0.4627	0.6557	0.3720	0.4280	0.4627	0.6557	
125		0.4910	0.5333	0.5526	0.7993	0.4708	0.5193	0.5477	0.7705	0.4515	0.5047	0.5463	0.7442	0.4515	0.5047	0.5463	0.7442	
150		0.5667	0.6099	0.6314	0.8699	0.5460	0.5965	0.6279	0.8474	0.5246	0.5818	0.6242	0.8227	0.5246	0.5818	0.6242	0.8227	
175		0.6414	0.6802	0.7015	0.9148	0.6230	0.6682	0.6976	0.8974	0.6033	0.6548	0.6934	0.8768	0.6033	0.6548	0.6934	0.8768	
200		0.7075	0.7376	0.7561	0.9488	0.6889	0.7287	0.7514	0.9350	0.6678	0.7170	0.7508	0.9188	0.6678	0.7170	0.7508	0.9188	
.25		25	0.0793	0.0976	0.1062	0.1360	0.0771	0.0960	0.1056	0.1334	0.0750	0.0949	0.1069	0.1311	0.0750	0.0949	0.1069	0.1311
		50	0.1198	0.1336	0.1409	0.2320	0.1159	0.1306	0.1401	0.2283	0.1127	0.1296	0.1384	0.2222	0.1127	0.1296	0.1384	0.2222
	75	0.1636	0.1784	0.1857	0.3292	0.1591	0.1763	0.1842	0.3215	0.1551	0.1741	0.1824	0.3106	0.1551	0.1741	0.1824	0.3106	
	100	0.2128	0.2297	0.2378	0.4240	0.2076	0.2256	0.2350	0.4121	0.2004	0.2231	0.2331	0.4034	0.2004	0.2231	0.2331	0.4034	
	125	0.2460	0.2642	0.2712	0.5057	0.2403	0.2612	0.2693	0.4973	0.2354	0.2583	0.2676	0.4853	0.2354	0.2583	0.2676	0.4853	
	150	0.2884	0.3062	0.3159	0.5820	0.2825	0.3047	0.3134	0.5685	0.2755	0.3018	0.3119	0.5547	0.2755	0.3018	0.3119	0.5547	
	175	0.3307	0.3490	0.3574	0.6466	0.3244	0.3476	0.3554	0.6327	0.3171	0.3445	0.3531	0.6212	0.3171	0.3445	0.3531	0.6212	
	200	0.3766	0.3979	0.4044	0.7054	0.3684	0.3925	0.4031	0.6916	0.3612	0.3874	0.4010	0.6792	0.3612	0.3874	0.4010	0.6792	
	.01	25	0.0485	0.0546	0.0618	0.0518	0.0487	0.0549	0.0620	0.0517	0.0482	0.0546	0.0618	0.0522	0.0482	0.0546	0.0618	0.0522
		50	0.0441	0.0484	0.0537	0.0569	0.0443	0.0485	0.0540	0.0569	0.0445	0.0494	0.0545	0.0564	0.0445	0.0494	0.0545	0.0564
75		0.0534	0.0593	0.0571	0.0597	0.0536	0.0597	0.0597	0.0597	0.0533	0.0592	0.0615	0.0656	0.0533	0.0592	0.0615	0.0656	
100		0.0569	0.0592	0.0605	0.0662	0.0567	0.0596	0.0611	0.0657	0.0566	0.0592	0.0615	0.0656	0.0566	0.0592	0.0615	0.0656	
125		0.0551	0.0574	0.0586	0.0688	0.0550	0.0573	0.0589	0.0689	0.0548	0.0568	0.0591	0.0688	0.0548	0.0568	0.0591	0.0688	
150		0.0579	0.0596	0.0610	0.0727	0.0576	0.0594	0.0610	0.0725	0.0577	0.0596	0.0606	0.0721	0.0577	0.0596	0.0606	0.0721	
175		0.0557	0.0588	0.0591	0.0771	0.0556	0.0587	0.0590	0.0761	0.0557	0.0587	0.0593	0.0760	0.0557	0.0587	0.0593	0.0760	
200		0.0587	0.0610	0.0622	0.0772	0.0586	0.0612	0.0623	0.0774	0.0585	0.0612	0.0620	0.0770	0.0585	0.0612	0.0620	0.0770	
1		25	0.6179	0.7048	0.8302	0.8051	0.5611	0.6613	0.8260	0.7546	0.5062	0.6250	0.8228	0.7034	0.5062	0.6250	0.8228	0.7034
		50	0.9195	0.9436	0.9820	0.9807	0.8917	0.9253	0.9810	0.9645	0.8586	0.9013	0.9800	0.9463	0.8586	0.9013	0.9800	0.9463
	75	0.9897	0.9943	0.9994	0.9990	0.9815	0.9894	0.9994	0.9968	0.9706	0.9827	0.9993	0.9926	0.9706	0.9827	0.9993	0.9926	
	100	0.9987	0.9994	1.0000	0.9999	0.9969	0.9985	1.0000	0.9997	0.9938	0.9968	1.0000	0.9993	0.9938	0.9968	1.0000	0.9993	
	125	1.0000	1.0000	1.0000	1.0000	0.9998	0.9999	1.0000	1.0000	0.9998	0.9996	1.0000	0.9999	0.9998	0.9996	1.0000	0.9999	
	150	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9997	0.9998	1.0000	1.0000	0.9997	0.9998	1.0000	1.0000	
	175	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
	200	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
	.75	25	0.4239	0.4995	0.5692	0.6784	0.3928	0.4741	0.5644	0.6381	0.3642	0.4547	0.5596	0.6027	0.3642	0.4547	0.5596	0.6027
		50	0.7420	0.7934	0.8431	0.9373	0.7093	0.7707	0.8384	0.9133	0.6788	0.7488	0.8355	0.8882	0.6788	0.7488	0.8355	0.8882
75		0.9037	0.9253	0.9532	0.9910	0.8820	0.9128	0.9512	0.9837	0.8595	0.8956	0.9492	0.9744	0.8595	0.8956	0.9492	0.9744	
100		0.9664	0.9762	0.9869	0.9992	0.9575	0.9692	0.9858	0.9975	0.9472	0.9632	0.9848	0.9942	0.9472	0.9632	0.9848	0.9942	
125		0.9905	0.9937	0.9975	0.9999	0.9858	0.9917	0.9971	0.9998	0.9817	0.9878	0.9967	0.9991	0.9817	0.9878	0.9967	0.9991	
150		0.9972	0.9989	0.9991	1.0000	0.9960	0.9974	0.9991	1.0000	0.9959	0.9974	0.9992	1.0000	0.9959	0.9974	0.9992	1.0000	
175		0.9990	0.9992	0.9997	1.0000	0.9987	0.9992	0.9997	1.0000	0.9977	0.9986	0.9996	1.0000	0.9977	0.9986	0.9996	1.0000	
200		0.9985	0.9990	0.9996	1.0000	0.9994	0.9997	0.9997	1.0000	0.9992	0.9996	0.9996	1.0000	0.9992	0.9996	0.9996	1.0000	
.50		25	0.2641	0.3092	0.3396	0.5108	0.2503	0.2979	0.3370	0.4861	0.2381	0.2924	0.3354	0.4633	0.2381	0.2924	0.3354	0.4633
		50	0.6179	0.7048	0.8302	0.8051	0.5611	0.6613	0.8260	0.7546	0.5062	0.6250	0.8228	0.7034	0.5062	0.6250	0.8228	0.7034

TABLE Ia. Power Pattern of TRM, RC, AR(I), and UN When $r = 3$ of 10,000 MC Samples at $\alpha = .05$ in Treatment Test

λ	n	$G = .1$					$G = .3$					$G = .5$					
		UN vs. RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM
.25	50	0.4984	0.5422	0.5715	0.8111	0.4790	0.5280	0.5679	0.7837	0.4596	0.5154	0.5632	0.7837	0.4596	0.5154	0.5632	0.7837
	75	0.6814	0.7172	0.7407	0.9380	0.6632	0.7075	0.7379	0.9222	0.6441	0.6950	0.7348	0.9222	0.6441	0.6950	0.7348	0.9222
	100	0.8083	0.8354	0.8494	0.9801	0.7925	0.8250	0.8471	0.9737	0.7771	0.8124	0.8434	0.9737	0.7771	0.8124	0.8434	0.9737
	125	0.8905	0.9075	0.9208	0.9944	0.8801	0.9004	0.9188	0.9915	0.8681	0.8929	0.9173	0.9915	0.8681	0.8929	0.9173	0.9915
	150	0.9377	0.9496	0.9581	0.9987	0.9291	0.9439	0.9555	0.9979	0.9202	0.9374	0.9534	0.9979	0.9202	0.9374	0.9534	0.9979
	175	0.9687	0.9750	0.9791	0.9995	0.9629	0.9715	0.9786	0.9994	0.9579	0.9673	0.9779	0.9994	0.9579	0.9673	0.9779	0.9994
	200	0.9816	0.9851	0.9877	1.0000	0.9783	0.9838	0.9870	0.9998	0.9746	0.9813	0.9866	0.9998	0.9746	0.9813	0.9866	0.9998
	25	0.1408	0.1646	0.1772	0.2892	0.1382	0.1651	0.1771	0.2781	0.1345	0.1623	0.1782	0.2781	0.1345	0.1623	0.1782	0.2781
	50	0.2467	0.2648	0.2780	0.5083	0.2421	0.2622	0.2757	0.4947	0.2372	0.2616	0.2761	0.4947	0.2372	0.2616	0.2761	0.4947
	75	0.3515	0.3772	0.3854	0.6900	0.3463	0.3729	0.3836	0.6736	0.3415	0.3689	0.3816	0.6736	0.3415	0.3689	0.3816	0.6736
100	0.4624	0.4800	0.4916	0.8083	0.4544	0.4779	0.4899	0.7954	0.4461	0.4723	0.4870	0.7954	0.4461	0.4723	0.4870	0.7954	
125	0.5478	0.5660	0.5762	0.8840	0.5381	0.5600	0.5716	0.8737	0.5301	0.5556	0.5684	0.8737	0.5301	0.5556	0.5684	0.8737	
150	0.6268	0.6457	0.6534	0.9341	0.6189	0.6396	0.6506	0.9262	0.6114	0.6345	0.6486	0.9262	0.6114	0.6345	0.6486	0.9262	
175	0.7041	0.7190	0.7270	0.9630	0.6950	0.7152	0.7257	0.9579	0.6856	0.7105	0.7244	0.9579	0.6856	0.7105	0.7244	0.9579	
200	0.7581	0.7729	0.7795	0.9823	0.7499	0.7679	0.7757	0.9789	0.7429	0.7629	0.7742	0.9789	0.7429	0.7629	0.7742	0.9789	
25	0.0494	0.0560	0.0641	0.0563	0.0492	0.0562	0.0592	0.0663	0.0481	0.0537	0.0598	0.0663	0.0481	0.0537	0.0598	0.0663	
50	0.0483	0.0537	0.0599	0.0663	0.0480	0.0534	0.0599	0.0663	0.0481	0.0537	0.0598	0.0663	0.0481	0.0537	0.0598	0.0663	
75	0.0594	0.0646	0.0706	0.0771	0.0595	0.0649	0.0704	0.0768	0.0596	0.0649	0.0703	0.0768	0.0596	0.0649	0.0703	0.0768	
100	0.0647	0.0678	0.0706	0.0882	0.0643	0.0679	0.0704	0.0877	0.0640	0.0679	0.0703	0.0877	0.0640	0.0679	0.0703	0.0877	
125	0.0665	0.0692	0.0721	0.0974	0.0667	0.0694	0.0722	0.0981	0.0669	0.0697	0.0721	0.0981	0.0669	0.0697	0.0721	0.0981	
150	0.0715	0.0750	0.0772	0.1110	0.0716	0.0751	0.0775	0.1105	0.0669	0.0697	0.0721	0.1105	0.0669	0.0697	0.0721	0.1105	
175	0.0715	0.0743	0.0766	0.1189	0.0712	0.0739	0.0764	0.1195	0.0709	0.0736	0.0765	0.1195	0.0709	0.0736	0.0765	0.1195	
200	0.0967	0.0995	0.1019	0.0814	0.0756	0.0778	0.0797	0.1222	0.0760	0.0786	0.0798	0.1222	0.0760	0.0786	0.0798	0.1222	

TABLE Ib. Power Difference in Treatment Test

λ	n	$G = .1$					$G = .3$					$G = .5$								
		UN vs. RC	AR(I)	UN vs. AR(I)	TRM vs. UN	UN vs. RC	AR(I) vs. RC	TRM vs. RC	UN vs. AR(I)	TRM vs. UN	UN vs. RC	AR(I) vs. RC	TRM vs. RC	UN vs. AR(I)	TRM vs. UN	UN vs. RC	AR(I) vs. RC	TRM vs. RC		
1	25	0.081	0.035	0.046	0.072	0.072	0.026	-0.009	0.087	0.044	0.043	0.071	0.028	-0.016	0.092	0.049	0.043	0.077	0.028	-0.021
	50	0.107	0.047	0.060	0.097	0.097	0.037	-0.010	0.119	0.061	0.059	0.095	0.036	-0.024	0.128	0.069	0.060	0.096	0.037	-0.032
	75	0.134	0.064	0.070	0.129	0.129	0.060	-0.005	0.148	0.080	0.068	0.122	0.054	-0.027	0.160	0.097	0.069	0.119	0.050	-0.041
	100	0.153	0.078	0.075	0.145	0.145	0.070	-0.008	0.177	0.097	0.080	0.139	0.059	-0.038	0.198	0.116	0.082	0.138	0.056	-0.061
	125	0.179	0.092	0.087	0.165	0.165	0.078	-0.014	0.207	0.118	0.089	0.158	0.069	-0.049	0.230	0.139	0.091	0.152	0.060	-0.078
	150	0.198	0.106	0.092	0.185	0.185	0.092	-0.012	0.230	0.132	0.099	0.178	0.079	-0.052	0.256	0.157	0.099	0.173	0.060	-0.083
	175	0.210	0.107	0.102	0.197	0.197	0.095	-0.012	0.247	0.139	0.108	0.189	0.082	-0.058	0.278	0.166	0.113	0.187	0.075	-0.091
	200	0.215	0.116	0.100	0.205	0.205	0.106	-0.010	0.258	0.145	0.112	0.197	0.085	-0.060	0.295	0.176	0.120	0.190	0.071	-0.105
	25	0.046	0.016	0.030	0.044	0.044	0.013	-0.002	0.053	0.018	0.036	0.048	0.012	-0.006	0.059	0.022	0.037	0.049	0.012	-0.010
	50	0.050	0.015	0.036	0.070	0.070	0.034	0.020	0.058	0.020	0.038	0.071	0.033	0.013	0.064	0.024	0.040	0.071	0.031	0.007
75	0.057	0.016	0.041	0.089	0.089	0.048	0.032	0.069	0.027	0.042	0.089	0.064	0.029	0.074	0.032	0.042	0.086	0.044	0.012	
100	0.070	0.025	0.046	0.116	0.116	0.070	0.046	0.083	0.035	0.048	0.112	0.064	0.029	0.094	0.044	0.050	0.109	0.059	0.015	
125	0.079	0.028	0.051	0.142	0.142	0.091	0.063	0.089	0.038	0.051	0.134	0.083	0.045	0.099	0.043	0.056	0.126	0.070	0.027	
150	0.084	0.032	0.051	0.156	0.156	0.105	0.073	0.100	0.042	0.058	0.147	0.089	0.047	0.112	0.054	0.058	0.139	0.081	0.027	
175	0.090	0.032	0.058	0.179	0.179	0.121	0.090	0.108	0.043	0.065	0.170	0.105	0.062	0.124	0.055	0.069	0.166	0.097	0.042	
200	0.099	0.038	0.061	0.191	0.191	0.130	0.091	0.117	0.051	0.067	0.178	0.112	0.061	0.137	0.063	0.073	0.173	0.100	0.036	
25	0.034	0.011	0.022	0.028	0.028	0.006	-0.005	0.035	0.014	0.021	0.029	0.008	-0.006	0.038	0.014	0.024	0.031	0.007	-0.007	
50	0.028	0.008	0.020	0.049	0.049	0.029	0.021	0.031	0.009	0.022	0.049	0.027	0.018	0.035	0.012	0.023	0.050	0.027	0.015	
75	0.024	0.005	0.019	0.057	0.057	0.038	0.032	0.029	0.006	0.022	0.055	0.033	0.026	0.031	0.008	0.023	0.052	0.030	0.021	

Note. Italicized number indicates the difference is significant; that is, twice the upper bound of standard error for empirical power ($SE = \sqrt{2 \times \frac{p \times (1-p)}{n}} \approx .007$, where $p = .5$ and $n = 10000$), $2 \times SE = .014$, $\lambda =$ reliability, $n =$ sample size per group, $G =$ Correlation in G matrix, i.e., between growth parameter estimates, $EZ =$ effect size of treatment. TRM = traditional repeated measures; RC = random slope with homogeneous Level-1 variance; AR(1) = first-order autoregressive; UN = unstructured; all are power patterns of variance-covariance structures.

TABLE 2
Power Pattern and Pairwise Power Difference of TRM, RC, AR(1), and UN When $t = 3$ of 10,000 MC Samples at $\alpha = .05$ in the Interaction Test

λ	n	$G = .1$					$G = .3$					$G = .5$						
		RC	AR(1)	UN	TRM	RC	AR(1)	UN	TRM	RC	AR(1)	UN	TRM	RC	AR(1)	UN	TRM	
.75	25	0.0691	0.0949	0.1113	0.0839	0.0606	0.0874	0.1076	0.0802	0.0549	0.0821	0.1067	0.0775	0.0549	0.0821	0.1067	0.0775	
	50	0.0956	0.1229	0.1532	0.1138	0.0848	0.1141	0.1441	0.1090	0.0744	0.1061	0.1360	0.1025	0.0744	0.1061	0.1360	0.1025	
	75	0.1248	0.1575	0.1978	0.1422	0.1097	0.1454	0.1838	0.1298	0.0744	0.1302	0.1721	0.1217	0.0744	0.1302	0.1721	0.1217	
	100	0.1572	0.1944	0.2422	0.1723	0.1354	0.1762	0.2232	0.1549	0.1198	0.1595	0.2096	0.1409	0.1198	0.1595	0.2096	0.1409	
	125	0.1893	0.2322	0.2909	0.1988	0.1606	0.2062	0.2671	0.1782	0.1406	0.1845	0.2451	0.1629	0.1406	0.1845	0.2451	0.1629	
	150	0.2186	0.2717	0.3406	0.2234	0.1869	0.2389	0.3141	0.2000	0.1651	0.2167	0.2915	0.1836	0.1651	0.2167	0.2915	0.1836	
	175	0.2499	0.2984	0.3784	0.2599	0.2185	0.2681	0.3479	0.2315	0.1886	0.2454	0.3256	0.2094	0.1886	0.2454	0.3256	0.2094	
	200	0.2901	0.3416	0.4281	0.2963	0.2508	0.3084	0.3920	0.2643	0.2176	0.2773	0.3656	0.2384	0.2176	0.2773	0.3656	0.2384	
	.50	25	0.0689	0.0858	0.0978	0.0757	0.0641	0.0799	0.0932	0.0718	0.0600	0.0713	0.0877	0.0713	0.0600	0.0713	0.0877	0.0713
		50	0.0889	0.1047	0.1177	0.0937	0.0834	0.1007	0.1130	0.0900	0.0778	0.0967	0.1117	0.0877	0.0778	0.0967	0.1117	0.0877
		75	0.1175	0.1387	0.1570	0.1169	0.1092	0.1304	0.1454	0.1116	0.0984	0.1220	0.1389	0.1084	0.0984	0.1220	0.1389	0.1084
		100	0.1417	0.1645	0.1844	0.1422	0.1295	0.1539	0.1749	0.1367	0.1168	0.1427	0.1669	0.1273	0.1168	0.1427	0.1669	0.1273
125		0.1646	0.1913	0.2114	0.1553	0.1476	0.1763	0.2017	0.1466	0.1337	0.1632	0.1931	0.1384	0.1337	0.1632	0.1931	0.1384	
150		0.1897	0.2199	0.2476	0.1787	0.1700	0.1988	0.2323	0.1667	0.1548	0.1842	0.2201	0.1558	0.1548	0.1842	0.2201	0.1558	
175		0.2198	0.2491	0.2770	0.2089	0.1967	0.2290	0.2605	0.1923	0.1789	0.2123	0.2490	0.1795	0.1789	0.2123	0.2490	0.1795	
200		0.2473	0.2785	0.3151	0.2319	0.2204	0.2589	0.2975	0.2139	0.1993	0.2362	0.2797	0.2005	0.1993	0.2362	0.2797	0.2005	
.25		25	0.0657	0.0766	0.0832	0.0686	0.0631	0.0747	0.0826	0.0675	0.0596	0.0718	0.0798	0.0662	0.0596	0.0718	0.0798	0.0662
		50	0.0783	0.0893	0.0962	0.0784	0.0746	0.0856	0.0926	0.0751	0.0856	0.0902	0.0724	0.0662	0.0751	0.0902	0.0724	0.0662
		75	0.0989	0.1116	0.1181	0.0929	0.0934	0.1080	0.1144	0.0904	0.0885	0.1047	0.1116	0.0881	0.0885	0.1047	0.1116	0.0881
		100	0.1148	0.1284	0.1364	0.1131	0.1090	0.1216	0.1321	0.1082	0.1041	0.1180	0.1291	0.1043	0.1041	0.1180	0.1291	0.1043
	125	0.1306	0.1429	0.1527	0.1179	0.1228	0.1365	0.1469	0.1143	0.1160	0.1322	0.1427	0.1115	0.1160	0.1322	0.1427	0.1115	
	150	0.1524	0.1695	0.1743	0.1333	0.1397	0.1604	0.1674	0.1289	0.1298	0.1498	0.1632	0.1245	0.1298	0.1498	0.1632	0.1245	
	175	0.1667	0.1828	0.1903	0.1531	0.1557	0.1730	0.1853	0.1485	0.1471	0.1676	0.1787	0.1422	0.1471	0.1676	0.1787	0.1422	
	200	0.1857	0.2011	0.2142	0.1684	0.1729	0.1926	0.2051	0.1617	0.1617	0.1851	0.1983	0.1568	0.1617	0.1851	0.1983	0.1568	
	.01	25	0.0598	0.0671	0.0730	0.0583	0.0581	0.0667	0.0724	0.0585	0.0558	0.0646	0.0711	0.0580	0.0558	0.0646	0.0711	0.0580
		50	0.0634	0.0695	0.0738	0.0581	0.0627	0.0685	0.0729	0.0581	0.0622	0.0678	0.0715	0.0597	0.0622	0.0678	0.0715	0.0597
		75	0.0797	0.0861	0.0864	0.0719	0.0770	0.0828	0.0849	0.0712	0.0763	0.0824	0.0839	0.0727	0.0763	0.0824	0.0839	0.0727
		100	0.0881	0.0938	0.0958	0.0811	0.0844	0.0930	0.0961	0.0787	0.0841	0.0921	0.0940	0.0783	0.0841	0.0921	0.0940	0.0783
125		0.0935	0.0980	0.1008	0.0808	0.0893	0.0957	0.0981	0.0823	0.0876	0.0949	0.0983	0.0814	0.0876	0.0949	0.0983	0.0814	
150		0.1041	0.1099	0.1126	0.0944	0.1013	0.1086	0.1112	0.0912	0.0983	0.1067	0.1101	0.0897	0.0983	0.1067	0.1101	0.0897	
175		0.1093	0.1157	0.1166	0.0966	0.1059	0.1128	0.1153	0.0963	0.1035	0.1116	0.1140	0.0957	1.035	0.1116	0.1140	0.0957	
200		0.1244	0.1286	0.1325	0.1079	0.1210	0.1285	0.1301	0.1079	0.1176	0.1262	0.1279	0.1050	0.1176	0.1262	0.1279	0.1050	
1		25	0.0468	0.0529	0.0601	0.0505	0.0469	0.0525	0.0593	0.0509	0.0470	0.0526	0.0578	0.0505	0.0470	0.0526	0.0578	0.0505
		50	0.0462	0.0506	0.0554	0.0456	0.0468	0.0510	0.0551	0.0459	0.0467	0.0508	0.0550	0.0455	0.0467	0.0508	0.0550	0.0455
		75	0.0544	0.0576	0.0599	0.0552	0.0545	0.0583	0.0599	0.0557	0.0548	0.0579	0.0597	0.0556	0.0548	0.0579	0.0597	0.0556
		100	0.0517	0.0536	0.0555	0.0536	0.0513	0.0533	0.0550	0.0534	0.0511	0.0532	0.0550	0.0537	0.0511	0.0532	0.0550	0.0537
	125	0.0478	0.0497	0.0525	0.0502	0.0480	0.0501	0.0520	0.0500	0.0479	0.0500	0.0526	0.0500	0.0479	0.0500	0.0526	0.0500	
	150	0.0520	0.0540	0.0553	0.0563	0.0519	0.0540	0.0557	0.0559	0.0520	0.0537	0.0560	0.0555	0.0520	0.0537	0.0560	0.0555	
	175	0.0493	0.0512	0.0516	0.0511	0.0487	0.0507	0.0517	0.0514	0.0487	0.0510	0.0515	0.0510	0.0487	0.0510	0.0515	0.0510	
	200	0.0480	0.0497	0.0518	0.0522	0.0480	0.0489	0.0514	0.0521	0.0477	0.0489	0.0512	0.0520	0.0477	0.0489	0.0512	0.0520	
	1	25	0.2402	0.2934	0.3609	0.2373	0.2053	0.2675	0.3334	0.2119	0.1846	0.2419	0.3117	0.1903	0.1846	0.2419	0.3117	0.1903
		50	0.4313	0.4921	0.5987	0.4247	0.3825	0.4486	0.5597	0.3778	0.3409	0.4123	0.5226	0.3436	0.3409	0.4123	0.5226	0.3436
		75	0.6115	0.6672	0.7773	0.6033	0.5425	0.6133	0.7368	0.5410	0.4821	0.5567	0.6937	0.4917	0.4821	0.5567	0.6937	0.4917
		100	0.7382	0.7837	0.8787	0.7400	0.6734	0.7337	0.8461	0.6751	0.6132	0.6834	0.8138	0.6162	0.6132	0.6834	0.8138	0.6162
125		0.8318	0.8678	0.9333	0.8326	0.7786	0.8232	0.9134	0.7807	0.7248	0.7810	0.8886	0.7281	0.7248	0.7810	0.8886	0.7281	
150		0.8980	0.9213	0.9669	0.9018	0.8529	0.8880	0.9515	0.8584	0.8055	0.8518	0.9304	0.8129	0.8055	0.8518	0.9304	0.8129	

G = 5

G = 3

G = 1

λ	n	G = 1			G = 3			G = 5						
		RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	RC	AR(I)	UN	TRM	
.75	175	0.9388	0.9571	0.9869	0.9427	0.9011	0.9302	0.9761	0.9082	0.8606	0.8983	0.9651	0.8723	
	200	0.9644	0.9740	0.9931	0.9681	0.9370	0.9555	0.9880	0.9445	0.9051	0.9330	0.9811	0.9163	
	25	0.2121	0.2421	0.2688	0.1908	0.2561	0.1713	0.1911	0.1766	0.1713	0.1766	0.2077	0.1653	
	50	0.3644	0.4037	0.4508	0.3284	0.3290	0.3738	0.4231	0.2981	0.2988	0.3453	0.4047	0.2776	
	75	0.5146	0.5522	0.6077	0.4741	0.4685	0.5108	0.5772	0.4354	0.4305	0.4767	0.5495	0.4048	
	100	0.6409	0.6787	0.7335	0.6020	0.5899	0.6360	0.7047	0.5483	0.5483	0.5929	0.6743	0.5140	
	125	0.7441	0.7738	0.8253	0.7050	0.6931	0.7334	0.7962	0.6568	0.6462	0.6932	0.7710	0.6125	
	150	0.8175	0.8401	0.8828	0.7895	0.7757	0.8076	0.8596	0.7472	0.7330	0.7719	0.8343	0.7090	
	175	0.8741	0.8916	0.9280	0.8545	0.8360	0.8621	0.9082	0.8144	0.7965	0.8289	0.8895	0.7756	
	200	0.9148	0.9302	0.9586	0.8994	0.8837	0.9046	0.9440	0.8679	0.8514	0.8790	0.9285	0.8353	
.50	25	0.1613	0.1808	0.1905	0.1382	0.1510	0.1706	0.1836	0.1317	0.1418	0.1631	0.1789	0.1267	
	50	0.2704	0.2947	0.3101	0.2277	0.2533	0.2770	0.2968	0.2174	0.2390	0.2677	0.2863	0.2082	
	75	0.3890	0.4135	0.4337	0.3363	0.3614	0.3900	0.4162	0.3154	0.3404	0.3719	0.4018	0.2987	
	100	0.4969	0.5230	0.5417	0.4353	0.4675	0.4947	0.5203	0.4074	0.4373	0.4655	0.5001	0.3863	
	125	0.5843	0.6102	0.6375	0.5155	0.5495	0.5821	0.6150	0.4872	0.5220	0.5525	0.5927	0.4624	
	150	0.6645	0.6863	0.7081	0.6049	0.6313	0.6587	0.6882	0.5767	0.6014	0.6314	0.6670	0.5484	
	175	0.7298	0.7506	0.7734	0.6689	0.6973	0.7228	0.7511	0.6379	0.6681	0.6959	0.7328	0.6118	
	200	0.7881	0.8068	0.8292	0.7334	0.7598	0.7811	0.8070	0.7039	0.7306	0.7552	0.7865	0.6768	
	25	0.1040	0.1144	0.1250	0.0926	0.1010	0.1121	0.1220	0.0905	0.0999	0.1121	0.1204	0.0876	
	50	0.1617	0.1733	0.1778	0.1329	0.1571	0.1682	0.1750	0.1303	0.1527	0.1647	0.1715	0.1271	
.25	25	0.2308	0.2424	0.2480	0.1883	0.2254	0.2380	0.2442	0.1844	0.2174	0.2302	0.2385	0.1790	
	50	0.2931	0.3066	0.3101	0.2388	0.2855	0.2969	0.3031	0.2324	0.2747	0.2899	0.2952	0.2263	
	75	0.3480	0.3611	0.3658	0.2824	0.3370	0.3515	0.3598	0.2747	0.3262	0.3410	0.3531	0.2681	
	100	0.4077	0.4199	0.4288	0.3393	0.3965	0.4084	0.4182	0.3838	0.4341	0.4405	0.4500	0.3202	
	125	0.4662	0.4790	0.4830	0.3833	0.4520	0.4672	0.4719	0.3708	0.4341	0.4521	0.4616	0.3641	
	150	0.5110	0.5236	0.5270	0.4327	0.4943	0.5108	0.5161	0.4202	0.4792	0.4965	0.5074	0.4108	
	175	0.5472	0.5539	0.5616	0.0500	0.0473	0.0539	0.0539	0.0498	0.0473	0.0502	0.0502	0.0476	
	200	0.5510	0.5511	0.5576	0.0480	0.0504	0.0552	0.0576	0.0481	0.0502	0.0549	0.0575	0.0476	
	25	0.0586	0.0618	0.0644	0.0592	0.0586	0.0618	0.0647	0.0589	0.0596	0.0614	0.0645	0.0581	
	50	0.0615	0.0639	0.0669	0.0596	0.0611	0.0638	0.0667	0.0645	0.0659	0.0671	0.0689	0.0569	
.01	125	0.0580	0.0605	0.0629	0.0576	0.0576	0.0606	0.0632	0.0573	0.0573	0.0607	0.0624	0.0569	
	150	0.0642	0.0663	0.0672	0.0640	0.0646	0.0668	0.0676	0.0643	0.0643	0.0668	0.0679	0.0641	
	175	0.0668	0.0685	0.0698	0.0600	0.0669	0.0691	0.0696	0.0594	0.0669	0.0685	0.0694	0.0641	
	200	0.0665	0.0685	0.0695	0.0623	0.0660	0.0680	0.0698	0.0620	0.0664	0.0683	0.0700	0.0628	
	1	25	0.5400	0.6033	0.7025	0.5220	0.4802	0.5519	0.6605	0.4668	0.4262	0.5045	0.6202	0.4210
		50	0.8394	0.8768	0.9404	0.8353	0.7813	0.8304	0.9163	0.7824	0.7265	0.7864	0.8882	0.7319
		75	0.9542	0.9680	0.9899	0.9577	0.9291	0.9486	0.9838	0.9333	0.9009	0.9279	0.9744	0.9059
		100	0.9902	0.9933	0.9992	0.9923	0.9780	0.9858	0.9973	0.9825	0.9634	0.9754	0.9941	0.9698
		125	0.9973	0.9981	0.9999	0.9984	0.9928	0.9963	0.9998	0.9955	0.9859	0.9923	0.9993	0.9895
		150	0.9996	0.9997	1.0000	0.9998	0.9993	0.9993	0.9998	0.9991	0.9967	0.9981	0.9997	0.9978
175		1.0000	1.0000	1.0000	1.0000	0.9998	0.9999	1.0000	0.9998	0.9995	0.9995	1.0000	0.9995	
200		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9999	1.0000	0.9999	
25		0.4506	0.4962	0.5457	0.4049	0.4122	0.4611	0.5188	0.3754	0.3786	0.4283	0.4920	0.3490	
50		0.7522	0.7857	0.8318	0.7110	0.7043	0.7432	0.8028	0.6673	0.6596	0.7044	0.7785	0.6233	
.75	75	0.9026	0.9203	0.9460	0.8817	0.8706	0.8921	0.9308	0.8478	0.8354	0.8641	0.9142	0.8123	
	100	0.9623	0.9701	0.9817	0.9547	0.9448	0.9561	0.9753	0.9356	0.9251	0.9396	0.9677	0.9171	
	125	0.9896	0.9921	0.9949	0.9865	0.9789	0.9849	0.9920	0.9758	0.9677	0.9757	0.9891	0.9635	
	150	0.9987	0.9992	0.9997	0.9983	0.9929	0.9952	0.9983	0.9918	0.9872	0.9907	0.9972	0.9857	
	175	0.9995	0.9996	0.9999	0.9993	0.9987	0.9992	0.9996	0.9983	0.9967	0.9980	0.9992	0.9961	
	200	0.9996	0.9997	1.0000	0.9998	0.9991	0.9994	1.0000	0.9992	0.9981	0.9988	0.9998	0.9982	
	25	0.3384	0.3683	0.3872	0.2867	0.3192	0.3512	0.3742	0.2711	0.3009	0.3354	0.3605	0.2545	
	50	0.6017	0.6260	0.6493	0.5259	0.5657	0.5943	0.6290	0.4981	0.5359	0.5701	0.6080	0.4737	

EZ=8

λ	n	G = .1				G = .3				G = .5			
		RC	AR(1)	UN	TRM	RC	AR(1)	UN	TRM	RC	AR(1)	UN	TRM
.25	75	0.7692	0.7884	0.8061	0.7135	0.7380	0.7618	0.7868	0.6867	0.7119	0.7386	0.7698	0.6561
	100	0.8773	0.8892	0.9018	0.8355	0.8528	0.8682	0.8887	0.8126	0.8296	0.8505	0.8764	0.7879
	125	0.9403	0.9462	0.9549	0.9160	0.9248	0.9338	0.9446	0.8957	0.9073	0.9210	0.9352	0.8776
	150	0.9718	0.9747	0.9803	0.9559	0.9594	0.9667	0.9745	0.9429	0.9482	0.9562	0.9680	0.9280
	175	0.9867	0.9892	0.9929	0.9803	0.9800	0.9834	0.9892	0.9730	0.9736	0.9775	0.9857	0.9652
	200	0.9948	0.9956	0.9967	0.9902	0.9915	0.9930	0.9944	0.9867	0.9877	0.9897	0.9938	0.9813
	25	0.1957	0.2098	0.2204	0.1595	0.1900	0.2046	0.2157	0.1561	0.1839	0.2000	0.2093	0.1546
	50	0.3507	0.3640	0.3716	0.2823	0.3435	0.3559	0.3661	0.2734	0.3347	0.3511	0.3595	0.2685
	75	0.4972	0.5090	0.5159	0.4082	0.4809	0.4967	0.5052	0.3984	0.4648	0.4819	0.4943	0.3898
	100	0.6137	0.6274	0.6348	0.5346	0.5973	0.6125	0.6213	0.5208	0.5817	0.5981	0.6089	0.5045
.01	125	0.7126	0.7233	0.7304	0.6271	0.6968	0.7087	0.7187	0.6104	0.6822	0.6972	0.7088	0.5963
	150	0.7910	0.7997	0.8042	0.7148	0.7733	0.7838	0.7936	0.6983	0.7591	0.7710	0.7816	0.6848
	175	0.8494	0.8581	0.8606	0.7812	0.8358	0.8462	0.8520	0.7658	0.8232	0.8342	0.8420	0.7549
	200	0.8937	0.8991	0.9053	0.8385	0.8820	0.8891	0.8953	0.8248	0.8688	0.8771	0.8857	0.8140
	25	0.0501	0.0570	0.0663	0.0529	0.0504	0.0568	0.0660	0.0528	0.0508	0.0568	0.0661	0.0520
	50	0.0571	0.0616	0.0644	0.0540	0.0567	0.0616	0.0645	0.0534	0.0570	0.0618	0.0651	0.0534
	75	0.0691	0.0738	0.0752	0.0639	0.0693	0.0739	0.0752	0.0638	0.0680	0.0731	0.0746	0.0646
	100	0.0780	0.0815	0.0846	0.0716	0.0781	0.0819	0.0844	0.0712	0.0785	0.0817	0.0845	0.0715
	125	0.0785	0.0808	0.0824	0.0705	0.0784	0.0801	0.0830	0.0701	0.0784	0.0801	0.0830	0.0701
	150	0.0863	0.0890	0.0896	0.0802	0.0854	0.0884	0.0895	0.0807	0.0853	0.0880	0.0892	0.0800
175	0.0904	0.0930	0.0952	0.0773	0.0900	0.0925	0.0945	0.0775	0.0901	0.0928	0.0944	0.0775	
200	0.0967	0.0995	0.1019	0.0814	0.0967	0.0990	0.1016	0.0817	0.0966	0.0985	0.1012	0.0826	

Note. The significance test for the pairwise power difference is the same as the treatment test. Referring to Table 1b, if the pairwise differences are twice the upper bound of SE

$$SE = \sqrt{2 \times \frac{p \times (1-p)}{n}} \approx .007, \text{ where } p = .5 \text{ and } n = 10,000, 2 \times SE = .014, \text{ then the differences are labeled as significant. The results for the time test will be provided upon request. } \lambda = \text{reliability, } n = \text{sample size per group, } G = \text{correlation in } G \text{ matrix, i.e., between growth parameter estimates, } EZ = \text{effect size of interaction, TRM} = \text{traditional repeated measures; RC} = \text{random slope with homogeneous Level-1 variance; AR(1)} = \text{first-order autoregressive; UN} = \text{unstructured; all are power patterns of variance-covariance structures.}$$

TABLE 3
Power Pattern and Pairwise Power Difference of TRM, RC, AR(1), and UN When $t = 3$ and $\lambda = .50$ of 10,000 MC Samples at $\alpha = .05$ in Three Tests

EZ	Ratio	G = .1			G = .3			G = .5			
		AR(1)	UN	TRM	RC	AR(1)	UN	TRM	RC	AR(1)	UN
.2	100/4	0.0652	0.0883	0.1113	0.1552	0.1847	0.1466	0.0525	0.0814	0.1123	0.1400
	80/5	0.0391	0.0567	0.1014	0.1203	0.1533	0.1130	0.0317	0.0524	0.1009	0.1110
	40/10	0.0114	0.0167	0.0115	0.0712	0.1095	0.0723	0.0095	0.0165	0.0094	0.0713
	100/4	0.3068	0.3674	0.4338	0.6602	0.6426	0.6303	0.2684	0.3366	0.4263	0.6053
.5	80/5	0.1900	0.2435	0.3481	0.5128	0.5752	0.4908	0.1622	0.2192	0.3361	0.4701
	40/10	0.0362	0.0508	0.0283	0.1930	0.2743	0.2071	0.0323	0.0440	0.1843	0.2048
	100/4	0.7132	0.7631	0.8197	0.9667	0.9548	0.9562	0.6612	0.7223	0.8131	0.9462
	80/5	0.5077	0.5772	0.7000	0.8937	0.9774	0.8740	0.4509	0.5289	0.6841	0.8538
.8	40/10	0.0939	0.1234	0.3958	0.4200	0.9857	0.4038	0.0822	0.1111	0.3845	0.3895
	100/4	0.1382	0.1603	0.1906	0.1382	0.1309	0.1239	0.1208	0.1460	0.1773	0.1204
	80/5	0.1349	0.1593	0.2170	0.1269	0.1243	0.1211	0.1140	0.1403	0.2072	0.1174
	40/10	0.1128	0.1365	0.3322	0.1209	0.1095	0.1199	0.1030	0.1249	0.3179	0.1161
.5	100/4	0.6782	0.7120	0.7743	0.5752	0.6808	0.5441	0.6067	0.6534	0.7367	0.5184
	80/5	0.6992	0.7382	0.8252	0.5882	0.6651	0.5578	0.6339	0.6844	0.7910	0.5310
	40/10	0.7140	0.7495	0.9378	0.5383	0.6922	0.5184	0.6686	0.7106	0.9275	0.4981
	100/4	0.9754	0.9810	0.9889	0.9547	0.9679	0.9418	0.9576	0.9678	0.9809	0.9299
.8	80/5	0.9840	0.9890	0.9965	0.9643	0.9774	0.9535	0.9696	0.9781	0.9930	0.9394
	40/10	0.9880	0.9914	0.9999	0.9583	0.9857	0.9514	0.9833	0.9859	0.9995	0.9378
	100/4	0.2591	0.2938	0.3383	0.3951	0.2427	0.3758	0.2262	0.2612	0.3131	0.3561
	80/5	0.2610	0.3014	0.3835	0.4050	0.2425	0.3782	0.2274	0.2678	0.3612	0.3559
.5	40/10	0.2431	0.2796	0.5529	0.3630	0.2326	0.3487	0.2182	0.2540	0.5317	0.3350
	100/4	0.9326	0.9458	0.9653	0.9960	0.9147	0.9928	0.8948	0.9167	0.9493	0.9898
	80/5	0.9502	0.9636	0.9858	0.9984	0.9322	0.9979	0.9190	0.9355	0.9760	0.9959
	40/10	0.9622	0.9705	0.9979	0.9988	0.9545	0.9981	0.9451	0.9562	0.9976	0.9963
.8	100/4	0.9998	0.9999	0.9999	1.0000	0.9996	1.0000	0.9995	0.9997	0.9999	1.0000
	80/5	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	0.9999	0.9999	1.0000	1.0000
	40/10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	100/4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Note. The significance test for the pairwise power difference is the same as the treatment test. Referring to Table 1b, if the pairwise differences are twice the upper bound of SE

$(SE = \sqrt{2 \times \frac{p \times (1-p)}{n}} \approx .007)$, where $p = .5$ and $n = 10000$, $2 \times SE = .014$, then the differences are labeled as significant. $\lambda =$ reliability, $n =$ sample size per group, $G =$ correlation in G matrix, i.e., between growth parameter estimates, $EZ =$ effect size of treatment, interaction, or time, $Ratio(n/t) =$ sample size per group (n) to the number of time points (t) holding total sample size (N) constant. TRM = traditional repeated measures; RC = random slope with homogeneous Level-1 variance; AR(1) = first-order autoregressive; UN = unstructured; all are power patterns of variance-covariance structures.

APPENDIX

Validation of Data Generator Using R. F. Potthoff and S. N. Roy's (1964) Data

Variable	Potthoff and Roy's data	Simulated data
Intercept	21.2091	21.2063
Gender	1.4065	1.4065
Time	0.9591	0.9587
Gender \times Time	0.6097	0.6115