The Canoe Curve

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I met my favorite equation in a canoe. We had just bought it and were trying it out (the canoe, not the equation). Since my wife, who is also my business manager, had some concern whether we would use it enough to justify the purchase, I set to ruminating as we paddled around in Lincoln's Holmes Lake Park.

If we only use this canoe just this once, the cost-per-use is the full price, $300. If we use it twice, the cost per use is $150. If we use it three times, the cost is $100. Four times, $75. Five times, $60. Six times, $50. Seven times, $43...

At home, I programmed what I called the "Canoe Curve" on our Apple IIe. (This was some time ago!) The program was a loop to divide an incrementing number successively into 300. I eventually noticed that the numbers were symmetrical! I mean, just as four uses brought the cost-per-use to $75, 75 uses brought the cost-per-use to $4! Five times, $60, 60 times, $5, and so on, everywhere. This was wonderful. So I used the same loop to plot a curve. It had a big bend. Obviously, the efficient thing to do would be to get around the bend.

It seemed a very useful curve. You could use it to guide any business decision whether to rent or buy any piece of equipment. Eventually, after a "College Algebra" course, I realized how very efficently both programs, the dividing loop and the plotting loop, could be expressed. All that successive dividing could be expressed with just $y = n/x$! Talk about tight programming. No computer code could be tighter than that.

I eventually realized that every such equation is not only symmetrical, it is symmetrical about the square root of the cost of the canoe! That's the big bend. The business decision comes to this: would you use the equipment the square root of the dollar cost and thus get round the big bend to the shallow end of the curve?

If not, rent. If yes, buy. How very practical a square root can be.

Since then, I've seen $y = n/x$ arise everywhere.

Years later, I helped organize a flying club, ten guys to purchase one airplane, and then to share, and thus afford, the high costs of flying. We needed to make several decisions. When it came time to make the particular decision whether to charge high monthly dues and low hourly rental or to get our costs more from high hourly rentals, I was ready with the Cost per Use Curve.

The annual dues were n. It came to $y = (n/x) + a$, a being the low hourly rental. It didn't change the shape, it merely moved the entire curve upwards on the y-axis. Frequent flying would bring down the total cost to approximate the low club rate.

The members looked it over and saw that significant dues plus low hourly rental would encourage high usage. Without high usage, we'd go broke, so that is how we set up the club dues and rates. Arguably, since the flying club is still going, a curve from algebra was the foundation of a successful flying club.

Being an old-fashioned professor of Latin and Greek, I require my Homer students in Greek and my Vergil Students in Latin to memorize 100 lines of Epic by the end of the term. If they started 100 days in advance, they only had to learn 1 line a day; if they started the night before, they had to memorize 100 lines in one day. If they waited until there were 10 days to go, they had to memorize 10 lines a day, and from there on out, the stack of memorization became daunting. Hmm. This seemed very familiar.

How to help them see the efficiency of an early start and regular work at this assignment? Aha! It's $y = n/(n - x)$, specifically, with the assignment and the days, $y = 100/(100 - x)$. It shows how any cumulative task can be easy with an early start, and how a late start will require a hyperbolic labor of Sisyphus!

Then I started an in-line skating club. We are all interested in going fast. Lincoln, home of miles and miles of paved hiker-biker trails, also has mile-markers on them. This called for a table to convert elapsed time per mile into miles per hour. I did many conversions, solving each as an individual problem, thinking it through each time. Finally, the light-bulb. One single equation. Mph = 60/elapsed time in minutes: $y = n/x$ all over again. Just plug in 60 for n. Find your elapsed time on the x-axis, go up and across to read your speed on the y-axis.

GROUND-SPEED TABLE	25	144	43	84
	26	139	44	82
1. Time flight from	27	133	45	80
section line to section	28	129	46	78
line.	29	124	47	77
	30	120	48	75
2. Find seconds.	31	116	49	73
	32	113	50	72
3. Groundspeed is	33	109	51	70
other number in next	34	105	52	69
column.	35	103	53	69
	36	100	54	67
Example: 144 seconds	37	97	55	65
means 25 mph; 25	38	95	56	64
seconds means 144	39	92	57	63
mph.	40	90	58	62
	41	88	59	61
Math & Table by	42	86	60	60
Tom Winter				

And two weeks ago, having at my advanced age gotten a private pilot license, and bought a Cessna 150, I was flying to Harvard, Nebraska. (Yes, Harvard is in Nebraska. Or at least, there IS a Harvard, NE!) I had a horrible headwind. In such a fix, knowing your groundspeed could be life or death: stay up longer than your fuel endurance, and you risk doing an emergency landing. Fortunately, everywhere below me in the heart of the midwest, there are section lines. They are all a mile on a side. I needed my groundspeed. I clocked my flight from one section line to the next, and started thinking. It took 51 seconds. Okay, so my groundspeed is… 70 mph. Oh my, there it is again. Mph = 3600/elapsed time in seconds, $y = n/x$. Plug in 3600 for n.

Once home, I used the graphing calculator in my Mac and produced a table, so any time I — or any pilot favored with the section-lines of the midwest for reference — need to figure out groundspeed, I can simply find the elapsed seconds in either of the two columns (remember, this graph is always symmetrical!) and read the groundspeed in the other column. Example from the top of the columns: if the elapsed time from section-line to section-line is 25 seconds, you're making good 144 mph over the ground. Correspondingly, if it takes 144 seconds to hit the second section-line, you're going 25! I've got it on a handy plastic-laminated card that fits in my shirt pocket.

I give these groundspeed tables away to fellow pilots, like party favors. They wonder why it works. And I wonder if I should show them the graph of $y=n/x$ and give it away! ∎