2011

Analysis of Turbulent Hydraulic Jump over a Transitional Rough Bed of a Rectangular Channel: Universal Relations

Noor Afzal

Aligarh Muslim University, India

A. Bushra

University of Nebraska-Lincoln

Abu Seena

Korea Advanced Institute of Science and Technology, Daejeon, Korea

Follow this and additional works at: <http://digitalcommons.unl.edu/civilengfacpub>

Part of the [Hydraulic Engineering Commons](#)

Afzal, Noor; Bushra, A.; and Seena, Abu, "Analysis of Turbulent Hydraulic Jump over a Transitional Rough Bed of a Rectangular Channel: Universal Relations" (2011). *Civil Engineering Faculty Publications*. 54.

<http://digitalcommons.unl.edu/civilengfacpub/54>

This Article is brought to you for free and open access by the Civil Engineering at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Civil Engineering Faculty Publications by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.

Analysis of Turbulent Hydraulic Jump over a Transitional Rough Bed of a Rectangular Channel: Universal Relations

Noor Afzal,¹ A. Bushra,² and Abu Seena³

1. Visiting Professor, Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daejeon, Korea, and Faculty of Engineering, Aligarh Muslim University, Aligarh, India

2. Department of Civil Engineering, University of Nebraska

3. Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daejeon, Korea.

Corresponding author – Noor Afzal, email prof_noor_afzal@yahoo.com

Abstract

The streamwise flow structure of a turbulent hydraulic jump over a rough bed rectangular channel has been investigated. The flow is divided into inner and outer layers, where upstream supercritical flow changes to downstream subcritical flow. The analysis is based on depth averaged Reynolds momentum equations. The molecular viscosity on the rough bed imposes the no slip boundary condition, but close to the wall the turbulent process in inner layer provides certain matching conditions with the outer layer, where molecular viscosity has no dominant role. It is shown that the bed roughness in the inner layer has a passive role in imposing wall shear stress during formation of hydraulic jump in the outer layer. The Belanger's jump condition of rectangular channel has been extended to account for the implications of the drag attributable to channel bed roughness, kinetic energy correction factor, and coefficient of the Reynolds normal stresses. For depth averaged Reynolds normal stress, an eddy viscosity model containing gradient of depth averaged axial velocity is considered. Analytical solutions for sequent depth ratio, jump length, roller length, and profiles of jump depth and velocity were found to depend upon the upstream Froude number, drag owing to bed roughness, and kinetic energy correction factor. On the basis of dynamical similarity, the roller length and aeration length were proposed to be of the same order as the jump length. An effective upstream Froude number, introduced in the present work, yields universal predictions for sequent depth ratio, jump length, roller length, jump profile, and other hydraulic jump characteristics that are explicitly independent of bed roughness drag. Thus, results for hydraulic jump over a rough bed channel can be directly deduced from classical smooth bed hydraulic jump theory, provided the upstream Froude number is replaced by the effective upstream Froude number. These findings of universality have been supported by experimental data over a rough bed rectangular channel.

Keywords: Hydraulic jump analysis, Universal jump relations for rough bed, Effective upstream Froude number, Rectangular channel, Turbulent flow, Closure model

CE Database subject headings: Hydraulic jump, Bed roughness, Froude number, Channels, Turbulent flow

Introduction

The hydraulic jump in an open channel is formed when upstream supercritical flow changes into downstream subcritical flow. Hydraulic jumps have been extensively studied owing to their frequent occurrence in nature and have been widely used as energy dissipaters for hydraulic structures (Rajaratnam 1968; Hager 1992). The primary concern with jumps on rough beds is that the roughness elements located near upstream might be subjected to cavitation and possible erosion. In such cases, the end of the jump moves downstream, thereby causing erosion and possibly damage to the structure itself. The hydraulic jump over a natural rough bed is made up of rocks consisting of various ranges of relative roughness, where the relative roughness t is the ratio of the equivalent grain roughness k_s to the effective flow depth.

A sketch of a hydraulic jump over a rough bed is shown in Figure 1, where k is the bed roughness, F_τ is the rough bed shear stress imposed on the fluid, h_1 is the fluid depth at toe of the jump, h_2 is fluid depth at the end of the jump, L_j is the jump length, and L_R is the roller length. The application of the momentum conservation in a control volume bounded by upstream supercritical flow (marked 1) and downstream subcritical flow (marked 2) yields the equation $\Pi_1 + M_1 + N_1 = \Pi_2 +$

$M_2 + N_2 + F_\tau$ where Π_1 and Π_2 are hydrostatic forces; M_1 and M_2 are the mean momentum fluxes; N_1 and N_2 are turbulent Reynolds normal momentum fluxes; and h_1 and h_2 are sequent depths, respectively, at the toe (Section 1) and exit (Section 2) of the hydraulic jump. The integrated bed shear stress is generally adopted as $F_\tau = \lambda(M_1 - M_2)$ where λ is bed shear force coefficient.

For a hydraulic jump over a smooth bed rectangular channel, Belanger (1840) proposed a sequent depth ratio $h_2/h_1 = \alpha^{-1}$ versus upstream Froude number F_1 , which may be extended to a rough bed:

$$\frac{h_2}{h_1} = \frac{1}{2} \left[-1 + \sqrt{1 + aF_1^2} \right], \quad a = 8(1 + e) \quad (1)$$

where $a = 8$ corresponds to the channel of the smooth bed. Based on the experimental data, Govindarao and Ramaprasad (1966) proposed $a > 8$ for rough bed channels. The work of Leutheusser and Kartha (1972) adopted bed shear stress as $F_\tau = \lambda M_1$ and their prediction

$$e = 0.0296 - \frac{0.0122}{\alpha(1 - \alpha)} \quad (2)$$

Figure 1. Sketch of a hydraulic jump over a rough bed: k_s is the bed roughness, F_r is the rough bed shear stress imposed on the fluid, h_1 is the fluid depth at toe of the jump, h_2 is fluid depth at the end of the jump, L_j is the jump length, and L_R is the roller length

was supported by experimental data. Leutheusser and Schiller (1975) investigated the characteristics of mean turbulent motion in an artificially roughened channel in which bed roughness consisted of spheres (acrylic plastic balls on acrylic plastic base plates) and strips (small sheet-metal angles attached to a red wood base plate). For a rough bed rectangular channel, Rajaratnam (1967) presented interesting data for the sequent depth and jump length in terms of parameter ke/h_1 , where ke is rough bed equivalent roughness and h_1 is supercritical stream depth (Figure 1). Hughes and Flack (1984) represented the relative roughness of the bed in terms of d_{xx} (the material size of the bed mixture for which xx of material is finer) and equivalent grain roughness height k_s (assumed to be a function of d_{65}). Leutheusser and Schiller (1975) and Hughes and Flack (1984) analyzed their experimental data for a sequent depth ratio that supported Equation (2). Recently, Pagliara et al. (2008) proposed another correlation:

$$e = -0.05 \frac{k}{h_1} [1 - 1.256\alpha^{-1} + 0.256\alpha^{-2}] + 1.1\chi^{-4.5},$$

$$\chi = \frac{\sigma^2 \delta}{\sigma^2 - 1} \quad (3)$$

where k/h_1 = roughness ratio; and parameter χ = function of geometric standard deviations $\delta = d_{90}/d_{84}$ and $\sigma = d_{84}/d_{16}$ of the bed roughness. Gill (1980) predicted the effects of bed roughness upon sequent depth h_2/h_1 as a first perturbation over smooth surface sequent depth $\alpha_*^{-1} = (h_2/h_1)_{e=0}$ in a rectangular channel as

$$\frac{h_2}{h_1} = \alpha_*^{-1} \left[1 - \frac{0.0082(F_1^2 - 1)^{1.535}}{3\alpha_*^{-2} - (1 + 2F_1^2)} \right] \quad (4)$$

which compares with experimental data for $F_1 \leq 10$. Carollo et al. (2007) considered $M_2 \neq 0$ and proposed sequent depth Equation (1), where $-e = -\lambda$ was bed roughness effects correlated with bed roughness k/h_1 . Later, Carollo et al. (2009) adopted k/h_c as bed roughness and proposed an empirical expression:

$$\frac{h_2}{h_1} - 1 = \sqrt{2} \exp\left(\frac{k}{h_c}\right) (F_1 - 1)^{0.963} \quad (5)$$

where h_c = critical water depth, instead of h_1 used earlier by Carollo et al. (2007).

The hydraulic jump on a corrugated bed has been studied by Ead and Rajaratnam (2002), Tokay (2005), Yadav and Ahmad (2007), and Abbaspour et al. (2009), in which the height of corrugation from crest to trough and wave length of

corrugation play significant roles in the corrugated beds. Mohamed-Ali (1991), Negm (2002), Izadjoo and Bajestan (2005), and Bejestan and Neisi (2009) have considered the effects of roughened-bed stilling basin on the length of a hydraulic jump in a rectangular channel.

The objective of the present work is to analyze the axial flow structure of a turbulent hydraulic jump over a rough bed in a rectangular channel by using depth averaged Reynolds equations at large Reynolds numbers. The flow in the domain of the turbulent hydraulic jump is divided into two layers: inner and outer. In inner layer near the wall, molecular viscosity, bed roughness, and turbulent process play dominant roles, which satisfies the no slip boundary condition over the rough bed of the rectangular channel.

Because of three-dimensional transitional roughness in streamwise and cross streamwise directions, the roughness sublayer in the immediate neighborhood of the channel bed would produce a complicated three-dimensional mean flow pattern, but slightly above this roughness sublayer the mean turbulent flow would be two-dimensional and dominated by the oncoming stream velocity. The skin friction force is no greater and likely smaller than form drag owing to irregular random bed roughness. In a typical rough bed, the separation attributable to the irregular transitional bed surface is primarily confined in the roughness sublayer. The flow separation in the roughness sublayer does not directly affect the outer layer of flow, but implicitly imposes drag force owing to skin friction and foam drag refereed as drag force. In fact, the foam drag that arises owing to separation is also caused by implications of molecular kinematic viscosity effects. In the roughness sublayer, a traditional no slip condition has been satisfied, implying small changes in velocity (compared to velocity of outer stream) and consequently the Froude number based on sublayer velocity and sublayer depth in the roughness sublayer would be much less than unity. The analogy of a hydraulic jump with a shock wave (Duncan et al. 1967) and analysis of the shock wave structure becomes relevant, which for laminar flow may be found in the work of Thompson (1972). Thus, in the outer layer, the turbulence, inertia of fluid, and imposed drag owing to bed roughness play a major role and the molecular viscosity has a negligible effect. The matching of the outer layer to the inner layer imposes the drag owing to bed roughness, which has a passive role in imposing the wall shear stress owing to transitional bed roughness in the formation of the hydraulic jump. Thus, supercritical flow $F_1 > 1$ at the toe of the jump changes to subcritical Froude number $F_2 < 1$ at the exit of the jump. In the present work, implications of the upstream Froude number, bed roughness drag, energy correction

factor, and effective Reynolds normal stresses have been analyzed. The sequent depth ratio h_2/h_1 proposed expression leads to a rational choice of parameter e in Equation (1). The analytical solutions of the depth averaged Reynolds equation predicted velocity and jump depth profiles in the turbulent hydraulic jump. The dynamic similarity shows that the roller length L_R and aeration length L_A are of the order of the jump length L_j , i.e., $L_R = n_1 L_j$ and $L_A = n_2 L_j$, where n_1 and n_2 , the universal numbers for roller and aeration lengths, are explicitly independent of the channel shape. The analytical expressions for jump length, roller length, and aeration length have also been proposed. The proposed predictions compare well with the experimental data over rough bed rectangular channels.

Analysis of a Rough Bed Channel

The depth averaged equations of continuity and momentum for a hydraulic jump over a smooth or rough bed in a rectangular channel are (Afzal and Bushra 2002)

$$\frac{\partial}{\partial x} \int u dz = 0 \quad (6)$$

$$\int \left(u^2 + \frac{p}{\rho} - \frac{1}{\rho} \tau_{nn} - \nu \frac{\partial u}{\partial x} \right) dz - \frac{1}{\rho} \int \tau_w dx = C \quad (7)$$

where $u = u(x, z)$ = velocity at a point in the streamwise direction; $p = \rho g(h - z)$ = hydrostatic pressure distribution; $\tau_{nn} = \tau_{xx} - \tau_{zz}$ = effective normal Reynolds stress in the streamwise x direction; h = depth of flow in the z direction over a transitional rough bed channel; g = acceleration attributable to gravity; τ_w = bed roughness drag force owing to friction and foam drags per unit flow depth at the bottom surface; $A = bh$ = cross-sectional area of the flow; b = width of the channel; and C = constant of integration. In Equation (7), the first term is the mean momentum flux, the second term is the hydrostatic pressure, the third term is the turbulent momentum flux, the fourth term is the molecular viscous stress, and the fifth term is the bottom bed shear stress. In terms of drag force coefficient, $\lambda = C_d/2 = \int \tau_w dx / \rho u^2 b dz$ the momentum Equation (7) becomes

$$\int \left[(1 - \lambda) u^2 + \frac{p}{\rho} - \frac{1}{\rho} \tau_{nn} \right] dz = C \quad (8)$$

where $p = \rho g(h - z)$ hydrostatic pressure distribution. The upstream and downstream boundary conditions are $x \rightarrow -\infty$, $h \rightarrow h_1$, $u \rightarrow u_1$, $\tau_{nn} \rightarrow \tau_{nn1}$, $x \rightarrow +\infty$, $h \rightarrow h_2$, $u \rightarrow u_2$, and $\tau_{nn} \rightarrow \tau_{nn2}$, respectively. The continuity Equation (6) and the momentum Equation (8) have been integrated to yield

$$Uh = U_1 h_1 \quad (9)$$

$$\begin{aligned} & h \left[(1 - \lambda)(1 + \beta) U^2 + \frac{1}{2} g h - \frac{1}{\rho} T_{nn} \right] \\ & = h_1 \left[(1 - \lambda_1)(1 + \beta_1) U_1^2 + \frac{1}{2} g h_1 - \frac{1}{\rho} T_{nn1} \right] \end{aligned} \quad (10)$$

where $U(x)$ = depth averaged velocity; and T_{nn} = depth averaged effective Reynolds normal stress. The upstream and downstream boundary conditions are $x \rightarrow -\infty$, $h \rightarrow h_1$, $U \rightarrow U_1$, $T_{nn} \rightarrow T_{nn1}$, $x \rightarrow +\infty$, $h \rightarrow h_2$, $U \rightarrow U_2$, and $T_{nn} \rightarrow T_{nn2}$, respectively.

Jump Conditions

The continuity Equation (9) and momentum Equation (10) have been simplified, upstream and downstream of jump, as

$$U_1 h_1 = U_2 h_2 \quad (11)$$

$$\begin{aligned} & h_2 \left[(1 - \lambda_2)(1 + \beta_2) U_2^2 + \frac{1}{2} g h_2 - \frac{1}{\rho} T_{nn2} \right] \\ & = h_1 \left[(1 - \lambda_1)(1 + \beta_1) U_1^2 + \frac{1}{2} g h_1 - \frac{1}{\rho} T_{nn1} \right] \end{aligned} \quad (12)$$

The momentum Equation (12) may also be expressed as

$$\begin{aligned} & h_2 \left[\{(1 - \lambda_2)(1 + \beta_2) - \Gamma_2\} U_2^2 + \frac{1}{2} g h_2 \right] \\ & = h_1 \left[\{(1 - \lambda_1)(1 + \beta_1) - \Gamma_1\} U_1^2 + \frac{1}{2} g h_1 \right] \end{aligned} \quad (13)$$

where $\Gamma_1 = T_{nn1}(\rho U_1^2)^{-1}$ and $\Gamma_2 = T_{nn2}(\rho U_2^2)^{-1}$ = effective Reynolds normal stresses coefficients upstream and downstream of jump, respectively.

The first invariant of the jump is the sequent depth ratio relation, obtained after elimination of velocity between Equations (11) and (12) as

$$\begin{aligned} & \left(\frac{h_2}{h_1} \right)^3 - \left(\frac{h_2}{h_1} \right) [1 + 2\{(1 - \lambda_1)(1 + \beta_1) - \Gamma_1\} F_1^2] \\ & + 2\{(1 - \lambda_2)(1 + \beta_2) - \Gamma_2\} F_1^2 = 0. \end{aligned} \quad (14)$$

where $F_1 = U_1 / (gh_1)^{1/2}$ = upstream Froude number. The first two terms in the jump Equation (14) are expressed in factors and the remaining third and fourth terms from the left-hand side are moved on the right-hand side to obtain

$$\begin{aligned} & \frac{h_2}{h_1} \left(\frac{h_2}{h_1} + 1 \right) \left(\frac{h_2}{h_1} - 1 \right) = \left[\{(1 - \lambda_1)(1 + \beta_1) - \Gamma_1\} \frac{h_2}{h_1} \right. \\ & \left. - \{(1 - \lambda_2)(1 + \beta_2) - \Gamma_2\} \right] 2F_1^2 \end{aligned} \quad (15)$$

The sequent depth Equation (15) may be represented as

$$\frac{h_2}{h_1} \left(\frac{h_2}{h_1} + 1 \right) = 2(1 + e) F_1^2 \quad (16)$$

where function e is related to the sequent depth, drag owing to transitional bed roughness, energy correction factor, and coefficient of Reynolds normal stress as

$$e = \frac{[(1 - \lambda_1)(1 + \beta_1) - \Gamma_1] \frac{h_2}{h_1} - [(1 - \lambda_2)(1 + \beta_2) - \Gamma_2]}{\frac{h_2}{h_1} - 1} - 1 \quad (17)$$

The solution to sequent depth Equation (16) yields

$$\frac{h_2}{h_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8(1 + e) F_1^2} \right] \quad (18)$$

where e predicted here by Equation (17), which differs from Equation (2) after Leutheusser and Kartha (1972) and Leutheusser and Schiller (1975) and Equation (3) after Pagliara et al. (2008). The solution to sequent velocity ratio from Equation (11) yields

$$\frac{U_1}{U_2} = \frac{1}{2} \left[-1 + \sqrt{1 + 8(1 + e) F_1^2} \right] \quad (19)$$

$$\frac{U_2}{U_1} = \frac{1}{4(1+e)F_1^2} \left[1 + \sqrt{1 + 8(1+e)F_1^2} \right] \quad (20)$$

The second invariant of the jump is the product of the upstream and downstream velocities, which provides

$$U_1 U_2 = U_c^2 = gh_c^2, \quad h_c = \frac{h_2 + h_1}{2(1+e)} \quad (21)$$

where $U_c = (gh_c)^{1/2}$ = critical velocity; and h_c = critical depth ratio. Equation (21) is analogous to the Prandtl relation for a shock wave.

The momentum Equation (10) may be expressed as

$$gh^2(2F_s^2 + 1) = gh_1^2(2F_{s1}^2 + 1) = gh_2^2(2F_{s2}^2 + 1) \quad (22)$$

where $F_s = F[(1-\lambda)(1+\beta) - \Gamma]^{1/2}$ is defined here as an effective Froude number, a function of conventional Froude number $F = U/\sqrt{gh}$, bed drag force coefficient λ , kinetic energy correction factor β and Reynolds normal stress coefficient $\Gamma = T_{nm}/(\rho U^2)^{-1}$. The suffixes 1 and 2 denote the upstream and downstream values.

Jump Profile Equation

The depth averaged Reynolds Equation (10) in the hydraulic jump becomes

$$\frac{T_{xx}}{\rho} = (1-\lambda)(1+\beta)U^2 + \frac{1}{2}gh - \frac{h_1}{h} \left((1-\lambda_1)(1+\beta_1)U_1^2 + \frac{1}{2}gh_1 - \frac{T_{xx1}}{\rho} \right) \quad (23)$$

whose solution describes the turbulent flow structure in the hydraulic jump of a rectangular rough bed channel. The hydraulic jump is analogous to a shock wave (Duncan et al. 1967; Afzal and Bushra 1999). In both situations, the upstream and downstream flows approach certain limiting conditions: in shock waves, these are the well known Rankine-Hugoniot relations (Thompson 1972), whereas for the hydraulic jump, these are the Belanger relations for a rectangular channel. The shock wave structure in laminar flow, reported by Thompson (1972), show that the molecular viscous normal stress plays the dominant role. From the measurement of Resch et al. (1976) in a turbulent hydraulic jump, Afzal and Bushra (1999) proposed that the Reynolds normal stress plays a major role, as considered in Equation (23) in terms of depth average flow. It is well known that the study of all turbulent flows is handicapped by the problem of closure. Despite numerous attempts, a closure hypothesis that describes the essential physics in a reasonably general fashion has yet to be constructed, and therefore analyses based upon closure hypotheses that are not fully satisfactory are the subject of some uncertainty. The Reynolds normal stress T_{nm} may be expressed by a simple closure eddy viscosity model in terms of gradient of depth averaged axial velocity with respect to axial distance as

$$T_{nm} = \rho \nu_\tau \frac{\partial U}{\partial x} \quad (24)$$

where ν_τ = kinematic eddy viscosity of flow. In the present work, the jump is associated with the outer layer, where the bed roughness boundary condition imposed by the inner layer has a passive role in the formation of the hydraulic jump. In the outer layer, the eddy viscosity ν_τ from the dimensional argument is proportional to a velocity scale ΔU and length scale Δh , and a number of proportionality. In a traditional work, Clauser (1956), in the outer layer of a turbulent boundary layer over a smooth surface, adopted the velocity scale $\Delta U = U_c$ and $\Delta h = \delta^*$, where U_c is the velocity at the edge of

turbulent boundary layer and δ^* is the displacement thickness of turbulent boundary layer displacement thickness, thus $\nu_\tau = \epsilon U_c \delta^*$. From extensive experimental data, Clauser proposed $\epsilon = 0.18$, a universal number, and Townsend (1976) proposed $\epsilon = 0.2$. The dimensional eddy viscosity ν_τ in a hydraulic jump formed in the streamwise x-direction, was proposed by Afzal and Bushra (2002) on a smooth channel bed in a proper analogy with Clauser (1956) for a turbulent boundary layer formed in the normal y-direction. In a hydraulic jump, kinematic eddy viscosity ν_τ depends on drag owing to transitional bed roughness, kinetic energy correction factor, overall velocity jump $\Delta U = U_1 - U_2$, jump depth $\Delta h = h_2 - h_1$, and a universal number ϵ independent of channel geometry and bed roughness. Thus kinematic eddy viscosity ν_τ may be expressed as

$$\nu_\tau = \epsilon(1-\lambda)(1+\beta)(U_1 - U_2)(h_2 - h_1) \quad (25)$$

The governing Equation (23) based on eddy viscosity models Eqs. (24) and (25) becomes

$$\begin{aligned} & \epsilon(1-\lambda)(1+\beta)(U_1 - U_2)(h_2 - h_1)h \frac{\partial U}{\partial x} \\ & = (1-\lambda)(1+\beta)hU^2 - (1-\lambda_1)(1+\beta_1)h_1U_1^2 \\ & \quad + \frac{1}{2}g(h^2 - h_1^2) - \frac{h_1}{\rho}T_{nm1} \end{aligned} \quad (26)$$

Closed Form Solution

If the function $(1-\lambda)(1+\beta) - \Gamma$ remains invariant at two ends of the jump, then Equation (17) is simplified and the sequent depth ratio Equation (16) may be expressed as

$$\frac{h_2}{h_1} \left(\frac{h_2}{h_1} + 1 \right) = 2[(1-\lambda)(1+\beta) - \Gamma]F_1^2 \quad (27)$$

and solution becomes

$$\frac{h_2}{h_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8[(1-\lambda)(1+\beta) - \Gamma]F_1^2} \right] \quad (28)$$

In the present work, the variation of bed drag coefficient λ , effective Reynolds normal stress coefficient Γ , and kinetic energy correction factor β across the jump have also been neglected, i.e., $\lambda \approx \lambda_1 \approx \lambda_2$, $\beta \approx \beta_1 \approx \beta_2$, and $\Gamma \approx \Gamma_1 \approx \Gamma_2$, first owing to the invariance of e at two ends (upstream and downstream of the jump), and second, because an additional condition across the jump was adopted for analytical integration of the hydraulic jump equation across the jump. In terms of effective upstream Froude number $F_{s1} = [(1-\lambda)(1+\beta) - \Gamma]^{1/2}F_1$, the sequent depth ratio Equation (28) becomes

$$\frac{h_2}{h_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8F_{s1}^2} \right] \quad (29)$$

which is explicitly independent of bed roughness, analogous to the Belanger (1840) relation. The second invariant is the product of upstream and downstream velocities across the hydraulic jump:

$$\begin{aligned} U_1 U_2 &= U_c^2 = gh_c, \quad h_c = \frac{h_m}{(1-\lambda)(1+\beta) - \Gamma}, \\ h_m &= \frac{1}{2}(h_1 + h_2) \end{aligned} \quad (30)$$

The turbulence level attributable to Reynolds normal stresses Γ may be neglected, and the sequent depth Equation (28) becomes

$$\frac{h_2}{h_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8(1 - \lambda)(1 + \beta)F_1^2} \right] \quad (31)$$

and the effective upstream Froude number becomes $F_{S1} = [(1 - \lambda)(1 + \beta)]^{1/2} F_1$. In this situation, the governing Equation (26) of the hydraulic jump in the axial direction velocity and depth profiles becomes

$$\epsilon(U_1 - U_2)(h_2 - h_1)h \frac{\partial U}{\partial x} = hU^2 - h_1U_1^2 + \frac{g}{2} \frac{h^2 - h_1^2}{(1 - \lambda)(1 + \beta)} \quad (32)$$

where $Uh = U_1h_1$. The flow at the toe of the jump is supercritical (upstream Froude number $F_1 > 1$), and at the exit of jump is subcritical (downstream Froude number $F_2 < 1$). The length of a hydraulic jump is often an important factor to know when considering the design of structures like settling basins. The length of a hydraulic jump is often hard to measure in the field and during laboratory investigations owing to the sudden changes in surface turbulence level and because of the formation of roller eddies. The jump length may be defined as the distance measured from the front face of the jump to the point on the surface immediately downstream of the rollers. The most common definition is by passing a tangent to the depth (of velocity) at the center and finding out where the theoretical upstream and downstream depth conditions are met. Thus, hydraulic jump length $L_j = (h_2 - h_1)/(\partial h/\partial x)_m$ is shown in Figure 2(a), and is defined as the ratio of the jump in fluid depth $h_2 - h_1$ to the slope of the jump depth profile $(\partial h/\partial x)_m$ at mean depth profile $h = h_m = (h_2 + h_1)/2$. Likewise, the jump thickness based on the fluid velocity profile $L_u = (U_1 - U_2)/(\partial U/\partial x)_m$ shown in Figure 2(b) if the ratio of the velocity jump $U_1 - U_2$ to the slope of the jump velocity profile $(\partial U/\partial x)_m$ at mean velocity $U = U_m = (U_1 + U_2)/2$.

Velocity Distribution in the Jump

The jump Equation (32) is simplified in terms of the nondimensional axial velocity profile $V = (U - U_1)/(U_2 - U_1)$ and the nondimensional streamwise coordinate $X = x/h_2$ to provide

$$\epsilon[1 + V(\alpha - 1)V] \frac{\partial V}{\partial X} = V(1 - V) \left(\frac{1 + 2\alpha}{1 - \alpha^2} - V \right) \quad (33)$$

subjected to the upstream and downstream boundary conditions $X \rightarrow -\infty$, $V \rightarrow 0$, $X \rightarrow \infty$, and $V \rightarrow 1$, respectively. The closed form solution yields

$$V^{-(2+\alpha)}(1 - V)^{1+2\alpha} \left(\frac{1 + 2\alpha}{1 - \alpha^2} - V \right)^{1-\alpha} = \exp \left[-\frac{(1 + 2\alpha)(2 + \alpha)X + L_v}{\epsilon} \right] \quad (34)$$

The constant L_v in the jump profile Equation (34) is related to the origin of the hydraulic jump, and is left underdetermined, which may be estimated from experimental data. The length of the hydraulic jump in terms of velocity U is shown in Figure 2(b). In terms of nondimensional V , the jump length may be expressed as $L_u = (V_2 - V_1)/(\partial V/\partial x)_m$, where $(\partial V/\partial x)_m$ is the slope of surface profile at mean velocity $V_m = (V_2 + V_1)/2$. The length of jump L_u from Equation (33) becomes

$$\frac{L_u}{h_2} = 4\epsilon(1 - \alpha) \frac{(1 + \alpha)^2}{1 + 4\alpha + \alpha^2} \quad (35)$$

Figure 2. The turbulent hydraulic jump length over a rough bed rectangular channel: (a) $L_j = (h_2 - h_1)/(\partial h/\partial x)_m$, ratio of jump in axial depth $h_2 - h_1$ with slope $(dh/dx)_m$ at mean depth of fluid $h_m = (h_2 + h_1)/2$; (b) $L_u = (U_1 - U_2)/(\partial U/\partial x)_m$, the ratio of axial velocity jump $U_1 - U_2$ to slope $(dU/dx)_m$ at mean velocity $U_m = (U_2 + U_1)/2$

Depth Distribution in the Jump

Based on the nondimensional depth profile $\eta = (h - h_1)/(h_2 - h_1)$, the jump Equation (32) may be expressed as

$$\epsilon(1 + \alpha) \frac{\partial \eta}{\partial X} = \eta(1 - \eta) \left(\eta + \frac{1 + 2\alpha}{1 - \alpha} \right) \quad (36)$$

subjected to upstream and downstream boundary conditions $X \rightarrow -\infty$, $\eta \rightarrow 0$, $X \rightarrow +\infty$, and $\eta \rightarrow 1$, respectively. The solution of the jump profile Equation (36) becomes

$$\eta^{-(2+\alpha)}(1 - \eta)^{1+2\alpha} \left(\eta + \frac{1 + 2\alpha}{1 - \alpha} \right)^{1-\alpha} = \exp \left[-\frac{(1 + 2\alpha)(2 + \alpha)X + L_\eta}{\epsilon} \right] \quad (37)$$

For a large effective upstream Froude number F_{S1} , the parameter $\alpha \rightarrow 0$ and the asymptotic Equation (37) yield $X + L_\eta = 0.5 \ln[\eta^2/(1 - \eta^2)]$, which is independent of α or F_{S1} (as in a classical hydraulic jump on the smooth bed of a rectangular channel). The constant L_η in the solution is related to the origin of the hydraulic jump and left underdetermined in the analysis, but may be estimated from experimental data of the initial condition of the jump.

The length of the hydraulic jump $L_j = (h_2 - h_1)/(\partial h/\partial x)_m L_j$ (analogous to shock wave thickness) shown in Figure 2(a) estimated from jump profile Equation (36) becomes

$$\frac{L_j}{h_2} = \epsilon \Lambda (1 - \alpha), \quad \Lambda = \frac{8}{3} \quad (38)$$

Substitution of $\alpha = h_1/h_2$ from sequent depth ratio Equation (27) yields

$$\begin{aligned} \frac{L_j}{h_2} &= \frac{8}{3} \epsilon \left[1 - \frac{1 + \sqrt{1 + 8(1 - \lambda)(1 + \beta)F_1^2}}{4(1 - \lambda)(1 + \beta)F_1^2} \right] \\ &= \frac{8}{3} \epsilon \left(1 - \frac{1 + \sqrt{1 + 8F_{S1}^2}}{4F_{S1}^2} \right) \end{aligned} \quad (39)$$

where $F_{S1} = [(1 - \lambda)(1 + \beta)]^{1/2} F_1$ = effective upstream Froude number.

The roller length, L_R , is the horizontal distance between the toe section with the flow depth h_1 and the roller end. The analysis of the depth averaged Reynolds equations of mean turbulent flow in a channel is sufficiently general. From the dynamic similarity, it is postulated that the length scale of the roller L_R is of the order of the jump length scale L_j i.e., $L_R = n_1 L_j$ and $\epsilon_R = n_1 \epsilon$, where n_1 is a universal number explicitly independent of channel shape. The roller length in view of this postulate yields

$$\begin{aligned} \frac{L_R}{h_2} &= \epsilon_R \Lambda (1 - \alpha) \\ \frac{L_R}{h_2} &= \frac{8}{3} \epsilon_R \left[1 - \frac{1 + \sqrt{1 + 8(1 - \lambda)(1 + \beta)F_1^2}}{4(1 - \lambda)(1 + \beta)F_1^2} \right] \\ &= \frac{8}{3} \epsilon_R \left(1 - \frac{1 + \sqrt{1 + 8F_{S1}^2}}{4F_{S1}^2} \right) \end{aligned} \quad (40)$$

The aeration length L_A is defined as the reach between the upstream end of the longer wing and the location at which air clouds have left the flow, according to the analysis of depth averaged Reynolds equations of mean turbulent flow in a channel of arbitrary cross section. From the dynamic similarity, it is also postulated that aeration length L_A is of the order of the length scale of the jump L_j i.e., $L_A = m_2 L_j$ and $\epsilon_A = m_2 \epsilon$ where m_2 for roller length is a universal number explicitly independent of channel shape. The expression for aeration length becomes

$$\begin{aligned} \frac{L_A}{h_2} &= \epsilon_A \Lambda (1 - \alpha) \\ \frac{L_A}{h_2} &= \frac{8}{3} \epsilon_A \left[1 - \frac{1 + \sqrt{1 + 8(1 - \lambda)(1 + \beta)F_1^2}}{4(1 - \lambda)(1 + \beta)F_1^2} \right] \\ &= \frac{8}{3} \epsilon_A \left(1 - \frac{1 + \sqrt{1 + 8F_{S1}^2}}{4F_{S1}^2} \right) \end{aligned} \quad (41)$$

Regarding the experimental data in a rectangular smooth channel for $2 < F_1 < 15$, Afzal and Bushra (2002) proposed $\Lambda = 8/3$, $\epsilon \Lambda = 6.9$, and $\epsilon = 2.58$ for jump length, $\epsilon_R \Lambda = 5.2$ and $\epsilon_R = 1.95$ for roller length, and $\Lambda \epsilon_A = 3.90$ and $\epsilon_A = 2.58/0.66$ for aeration length. The universal constants ϵ , ϵ_R and ϵ_A are related as

$$\epsilon = 1.32 \epsilon_R = 0.66 \epsilon_A = 2.58 = \frac{6.90}{\Lambda} \quad (42)$$

which are independent of the shape of the channel cross section (Bushra and Afzal 2006). The sequent depth ratio Equation (31),

jump length Equation (38), roller length Equation (40), and aeration length Equation (42) may also be expressed as

$$\frac{h_2}{h_1} - 1 = \phi, \quad \frac{L_j}{h_1} = \frac{8}{3} \epsilon \phi, \quad \frac{L_R}{h_1} = \frac{8}{3} \epsilon_R \phi, \quad \frac{L_A}{h_1} = \frac{8}{3} \epsilon_A \phi \quad (43)$$

The function ϕ sequent depth Equation (31) is expanded in the powers of $F_{S1} - 1$ by a Taylor series as

$$\phi = \frac{4}{3} (F_{S1} - 1) + \frac{2}{27} (F_{S1} - 1)^2 - \frac{32}{739} (F_{S1} - 1)^3 + \dots \quad (44)$$

Based on the leading order term in Equation (46), the sequent depth ratio becomes

$$\frac{h_2}{h_1} - 1 = \frac{4}{3} (F_{S1} - 1) \quad (45)$$

and jump length, roller length, and aeration length become

$$\frac{L_j}{h_1} = \epsilon \frac{32}{9} (F_{S1} - 1) \quad (46)$$

$$\frac{L_R}{h_1} = \frac{32}{9} \epsilon_R (F_{S1} - 1) \quad (47)$$

$$\frac{L_A}{h_1} = \frac{32}{9} \epsilon_A (F_{S1} - 1) \quad (48)$$

Equation (47) may be compared with empirical Equation (5) by Carollo et al. (2009) for sequent depth ratio. The roller length L_R empirical relations proposed by Carollo et al. (2007) are

$$\begin{aligned} \frac{L_R}{h_1} &= 6.525 \left[\exp \left(-0.60 \frac{k}{h_1} \right) \right] (F_1 - 1), \\ \frac{L_R}{h_1} &= 2.244 \left(\frac{h_2}{h_1} \right)^{-1.272} \end{aligned} \quad (49)$$

The work of Pagliara et al. (2008) proposed the sequent depth relation Equation (1), subjected to Equation (3), and the empirical relations for jump length L_j and roller length L_R are

$$\frac{L_j}{h_1} = 5,000(1 - F_1^m), \quad m = -0.0086 \exp \left(-\frac{0.466}{\chi} \right) \quad (50)$$

$$\frac{L_R}{h_1} = \left(27.457 \frac{k}{h_1} - 73.517 + \frac{20}{\chi^{2.1}} \right) \left(\frac{1}{F_1} - \frac{1}{2} \right) \quad (51)$$

The empirical Equation (5) by Carollo et al. (2009), Equation (51) by Carollo et al. (2007), and Eqs. (52) and (53) by Pagliara et al. (2008) are valid for small values on $F_1 - 1$. On other hand, the writers' first order analytical predictions in Eqs. (47)–(50) in terms of parameter $F_{S1} - 1$ are explicitly independent of the bed roughness of the channel.

Results and Discussion

The analysis of the hydraulic jump over a rough bed rectangular channel has been presented. The solution Equation (31) obtained for sequent depth ratio is described as follows:

$$\frac{h_2}{h_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8(1 - \lambda)(1 + \beta)F_1^2} \right] \quad (54)$$

and has been compared with the experimental data of the hydraulic jump. In terms of $F_{S1} = [(1 - \lambda)(1 + \beta)]^{1/2} F_1$, the effective upstream Froude number, the sequent depth ratio becomes

$$\frac{h_2}{h_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8F_{S1}^2} \right] \quad (55)$$

The relative roughness is connected to bed drag force coefficient λ through sequent depth ratio for each Froude number. Thus, bed roughness drag coefficient λ is estimated from sequent depth Equation (54) for each set $(h_2/h_1, F_1)$ of experimental data for prescribed roughness, after neglecting the β effect. The effects of relative roughness drag are considered in terms of the roughness parameter k/h_1 and alternate parameter k/h_c , where $h_c = (q^2/g)^{1/3} = h_1 F_1^{-1/3}$ is the critical depth. The data of bed roughness drag coefficient λ is shown in Figure 3(a) against relative roughness k/h_1 . Therefore, the current work proposes the following prediction for bed drag force coefficient:

$$\lambda = 1 - \exp \left[-0.55 \left(\frac{k}{h_1} \right)^{0.75} \right] \quad (56)$$

Figure 3. The prediction of bed roughness drag coefficient λ versus relative roughness from experimental data for transitional bed roughness in a rectangular channel: (a) k/h_1 ; (b) k/h_c

which is also shown Figure 3(a), and compares well with the experimental data. Furthermore, a simple linear relation

$$\lambda = 0.45 \frac{k}{h_1} \quad (57)$$

which is also shown in Figure 3(a) compares slightly better with the experimental data for $k/h_1 \leq 1$. The relation proposed by Carollo et al. (2007)

$$\lambda = \frac{2}{\pi} \arctan \left[0.8 \left(\frac{k}{h_1} \right)^{0.75} \right] \quad (58)$$

is also shown in Figure 3(a), and compares well with Equation (56). The bed roughness drag coefficient λ against alternate roughness parameter k/h_c is shown in Figure 2(b) along with the proposed relations

$$\lambda = 1 - \exp \left[-2.05 \left(\frac{k}{h_c} \right)^{0.75} \right] \quad (59)$$

$$\lambda = \frac{2}{\pi} \arctan \left[1.4 \left(\frac{k}{h_c} \right)^{0.75} \right] \quad (60)$$

that also compare well with the experimental data. Furthermore, a simple linear prediction

$$\lambda = 1.5 \frac{k}{h_c} \quad (61)$$

which is also shown in Figure 3(b) agrees with the data for $k/h_c \leq 0.3$. The experimental data shown in Figures 3(a) and 3(b) have appreciable scatter, but these predictions nearly represent the mean of the data. The roughness drag coefficient λ in terms of roughness scale k/h_1 from the experimental data in Figure 3(a) show that Eqs. (56) and (57) describe the data roughly of the same order. Furthermore, the roughness drag coefficient λ in terms of the roughness scale k/h_c from experimental data in Figure 3(b) shows that the Eqs. (59) and (60) also describe the data, roughly to the same order. In the present work, the bed roughness drag Equation (55) is adopted for comparison of the writers' prediction with the experimental data on jump characteristics over transitional rough beds. The sequent depth ratio h_2/h_1 with the upstream Froude number F_1

Figure 4. Comparison of experimental data with sequent depth ratio h_2/h_1 versus upstream Froude number F_1 Equation (51) with roughness drag coefficient Equation (52) attributable to transitional bed roughness in a rectangular channel

Figure 5. Comparison of experimental data with this study's universal prediction of sequent depth ratio h_2/h_1 versus effective upstream Froude number F_{S1} , Equation (28), for all bed roughness in a rectangular channel

Figure 6. Comparison of Equation (39) for jump length versus upstream Froude number F_1 with transitional rough bed experimental data in a rectangular channel: (a) L_j/h_2 ; (b) L_j/h_1

shown in Figure 4 from the experimental data for relative bed roughness $0 \leq k/h_1 < 2.5$ and bed drag coefficient $0 \leq \lambda < 0.7$ has been compared with Equation (54) on the sequent depth ratio involving bed roughness drag coefficient from Equation (56). The comparison of this study's prediction for the universal sequent depth ratio h_2/h_1 Equation (55) based on the upstream effective Froude number F_{S1} , shown in Figure 5, compares with the experimental data for all types of bed roughness.

In a rectangular channel, the nondimensional jump length L_j/h_2 and L_j/h_1 versus upstream Froude number F_1 from experimental data are shown in Figures 6(a) and 6(b), which reveals the dependence on drag owing to transitional bed roughness. For large values of upstream Froude numbers $F_1 \rightarrow \infty$, the jump length L_j/h_2 approaches a constant value that depends on bed roughness. This study's prediction of jump length from Equation (38) with $8\epsilon/3 = 6.9$ is also shown in the same figure for relative roughness $k/h_1 = 0, 1/4, 1/2, 3/4, 1, 2, 2.5$, and 3. Accurate data is needed, particularly in terms of certain fixed value of k/h_1 , rather than particular values of k . The non-dimensional jump length Equation (39) in terms of effective Froude number F_{S1} is a universal relation that explicitly does

Figure 7. (a) Comparison of universal Equation (37) for jump length L_j/h_1 versus sequent depth ratio h_2/h_1 with experimental data for all bed roughness in a rectangular channel; (b) comparison of this study's universal predictions with experimental data for jump length L_j/h_1 versus effective upstream Froude number $F_{S1} = F_1 \sqrt{(1 - \lambda)(1 + \beta) - \Gamma}$ for all bed roughness

not depend on the bed roughness. The jump length L_j/h_1 versus h_2/h_1 and F_{S1} from Equation (39) is also a universal relation. To test this universal proposition, the same experimental data are also shown in Figure 7(a), where the data scatter fits well with coefficient $8\epsilon/3 = 6.3$. Subramanya (1998) suggested the jump length $L_j/h_2 = 6.1$ over a smooth bed rectangular channel, which practically remains constant for $F_1 > 5$, whereas Elevatorski (1959) proposed $L_j/h_2 = 6.9$. Clearly, better experiments are needed with respect to bed roughness for particular fixed values of relative roughness k/h_1 for moderate and higher Froude numbers. The role of the effective upstream Froude number F_{S1} based on bed roughness is investigated for jump length L_j/h_1 . The experimental data of Hughes and Flack (1984) and Ead and Rajaratnam (2002) for jump length L_j/h_1 versus F_{S1} are shown in Figure 7(b); which also provides strong support for the writers' universal relations, explicitly independent of bed roughness. The leading term approximations in ($F_{S1} \rightarrow 1$) in Equation (43) for jump length L_j/h_1 is also shown in Figure 7(b) and is in good agreement with the experimental data.

In rectangular channels the nondimensional roller length L_R/h_2 and L_R/h_1 against the upstream Froude number F_1 are shown in Figures 8(a) and 8(b), respectively, from the rough

Figure 8. Comparison of Equation (41) versus upstream Froude number F_1 with a transitional rough bed experimental data in rectangular channel for two jump lengths: (a) L_R/h_2 ; (b) L_R/h_1

bed data of Hughes and Flack (1984), Ead and Rajaratnam (2002), and Carollo et al. (2007). For large values of upstream Froude numbers $F_1 \rightarrow \infty$, the roller length L_R/h_2 approaches a constant universal value that depends on the bed roughness. This study's prediction of roller length from Equation (41) is also shown for $k/h_1 = 0, 1/4, 1/2, 3/4, 1, 2, 2.5$, and 3. Accurate data is needed, particularly in terms of a certain fixed value of k/h_1 , rather than particular values of k . The nondimensional roller length Equation (40) is a universal relation that does not depend on bed roughness. The roller length L_R/h_1 versus h_2/h_1 from Equation (36) is also a universal relation. To test this linear proposition, the same experimental data is shown in Figure 9(a); within the scatter, the data fit well with coefficient $8\epsilon_R/3 = 4.2$. The role of the effective upstream Froude number F_{S1} on roller length L_R/h_1 from the experimental data of Carollo et al. (2007) and Ead and Rajaratnam (2002) is shown in Figure 9(b), which also provides strong support for this study's universal relations, explicitly independent of bed roughness. The leading term perturbation solution in parameter $F_{S1} \rightarrow 1$ from roller length Equation (44) in terms of L_R/h_1 is also shown in Figure 9(b) and is in good agreement with the experimental data.

Figure 9. Comparison of this study's universal predictions versus effective upstream Froude number $F_{S1} = F_1 \sqrt{(1 - \lambda)(1 + \beta) - \Gamma}$ with experimental data for various bed roughness of a rectangular channel: (a) jump length L_R/h_1 versus sequent depth ratio h_2/h_1 ; (b) roller length L_R/h_1

Conclusions

1. The turbulent hydraulic jump theory over a rough bed rectangular channel has been proposed from depth averaged analysis of the Reynolds momentum equation. The bed shear stress attributable to the transitionally rough bed surface of the channel is considered while integrating the depth averaged Reynolds equations. The flow at the toe of the jump is supercritical (upstream Froude number $F_1 > 1$), which at the exit of jump is subcritical (downstream Froude number $F_2 < 1$).
2. The skin friction force is not greater and likely smaller than form drag owing to irregular random bed roughness. In a typical bed roughness, the separation attributable to irregular transitional bed surface is primarily confined in the roughness sublayer. The flow separation in the roughness sublayer does not directly affect the flow in the outer layer, but implicitly imposes drag force owing to the bed friction and foam drag owing to bed roughness. In fact, foam drag that arises because of separation flow on a transitional bed is also caused by the fluid molecular kinematic viscosity effects.
3. The flow invariant relations in the jump are attributable to the upstream and downstream fluxes where sequent depth Equation (18) and velocity jump Equation (19) are based on parameter e defined by Equation (17). For a particular case if $(1 - \lambda)(1 + \beta) - \Gamma$ is invariant at two ends of the jump, then sequent depth Equation (25) and velocity jump ratios are very much simplified. The sequent depth ratio and critical depth depend on bed roughness drag coefficient λ , in addition to the upstream Froude number F_1 for a particular channel shape. In terms of effective Froude number $F_s = [(1 - \lambda)(1 + \beta)]^{1/2} F$, the sequent depth ratio, and other hydraulic jump characteristics over the rough bed can be deduced from the classical hydraulic jump over smooth beds, provided that the upstream Froude number F_1 is replaced by the upstream friction Froude number F_{s1} . The Belanger's jump condition in turbulent flow is extended for transitional bed roughness, kinetic energy correction factor, and turbulent normal stress fluctuations of the momentum transfer.
4. The bed roughness drag coefficient λ as a function of bed roughness scale k/h_1 has been predicted by Eqs. (55) and (57), which describe the data to the same level of accuracy. Furthermore, λ as a function of alternate bed roughness scale k/h_c is predicted by Eqs. (55) and (56), which also describe the data to the same order. In the present work, Equation (55) is adopted for the prediction of jump characteristics.
5. The depth averaged Equation (23) over a rough channel bed is closed by a simple eddy viscosity model $T_{mn} = \rho \nu_t \partial U / \partial x$. The eddy viscosity expression $\nu_t = \epsilon(1 - \lambda)(1 + \beta)(U_1 - U_2)(h_2 - h_1)$ incorporates the effects of transitional roughness attributable to bed roughness drag coefficient λ , kinetic energy correction factor β , overall jump velocity scale $\Delta U = U_1 - U_2$, and jump length scale $\Delta h = h_2 - h_1$. Here ϵ is a universal constant, independent of channel geometry and bed roughness.
6. The length of a hydraulic jump is often hard to measure in the field and during laboratory investigations because of the sudden changes in surface turbulence, in addition to the formation of rollers and eddies. The most common definition is by passing a tangent to the depth (of velocity) at the center and finding out where the theoretical upstream and downstream depth conditions are met. The jump thickness based on axial velocity profile $L_u = (U_1 - U_2)/(\partial U / \partial x)_m$ is the ratio of the jump in velocity U_1 / U_2 to the velocity gradient $(\partial U / \partial x)_m$ at mean velocity $U = U_m = (U_1 + U_2)/2$ in the jump. Likewise, the hydraulic jump length $L_j = (h_2 - h_1)/(\partial h / \partial x)_m$ is the ratio of the rise of fluid depth $h_2 - h_1$ to depth gradient $(\partial h / \partial x)_m$ at mean depth $h = h_m = (h_2 + h_1)/2$. The theory predicts $L_j = \epsilon \Lambda (1 - \alpha)$, where $\Lambda = 8/3$ and bed roughness data agree with $\epsilon = 2.58$, the smooth bed value (Afzal and Bushra 2002). The data show that the validity of the dynamic similarity that the roller length L_R and aeration length L_A are of the order of the jump length L_j , and that the constant of proportionality is explicitly independent of channel shape and bed roughness. The roller length, $L_R = \epsilon_R \Lambda (1 - \alpha)$, and aeration length $L_A = \epsilon_A \Lambda (1 - \alpha)$ are of the same order as L_j , which leads to eddy viscosity universal number $\epsilon = 1.32 \epsilon_R = 0.66 \epsilon_A = 2.58 = 6.9 / \Lambda$, the same as the smooth bed channel.
7. The solution of the jump profile η versus X for a rough bed rectangular channel has been proposed. The jump length L_j relation is obtained in analogy with the shock wave thickness, leading to $L_j/h_2 = \epsilon \Lambda (1 - \alpha)$, which is explicitly independent of λ the drag of bed roughness, but depends on channel geometric shape factor, which for a rectangular channel is $\Lambda = 8/3$. The jump length L_j/h_2 versus α and L_j/h_2 versus F_{s1} are universal relationships that are explicitly independent of λ the drag attributable to bed roughness and β the energy correction parameter. However, the jump length L_j/h_2 versus F_1 is a nonuniversal relation that depends on drag owing to bed roughness and energy correction factors in the channel. The expressions for the roller length L_R/h_2 and the aeration length L_A/h_2 turn out to be analogous with the jump length L_j/h_2 , which in terms of F_1 depend on bed roughness but are universal in terms of F_{s1} as explicitly independent of bed roughness and energy correction factor.
8. The proposed theory for rough bed rectangular channels predicted universal solution depth profile, sequent depth ratio, jump length, and roller lengths in terms of effective upstream Froude number F_{s1} and are explicitly independent of bed roughness. The experimental data of Hughes and Flack (1984), Carollo et al. (2007, 2009), and Ead and Rajaratnam (2002) support the proposed universal theory. Thus, the results for rough bed channels can be directly deduced from the classical smooth bed hydraulic jump theory, provided the upstream Froude number F_1 may be replaced by the effective upstream Froude number F_{s1} .

Notation

The following symbols are used in this paper:

$A(x) = bh(x)$ area of flow in rectangular channel;

b = width of the rectangular channel;

$C_d = 2 \int \tau_w dx / \int \rho u^2 b dz$ = coefficient of drag attributable to channel bed;

e = Equation (17) in sequent depth Equation (16) of the hydraulic jump;

$e = (1 - \lambda)(1 + \beta) - \Gamma - 1$ = assumed invariant function across the jump, which includes special case $\lambda \approx \lambda_1 \approx \lambda_2$, $\beta \approx \beta_1 \approx \beta_2$, and $\Gamma \approx \Gamma_1 \approx \Gamma_2$;

$F = U / \sqrt{gh}$ = Froude number;

$F_s = F \sqrt{(1 - \lambda)(1 + \beta) - \Gamma}$ = effective Froude number;

g = gravitational acceleration;

$h(x)$ = depth of fluid layer in the channel;

$h_c = (q^2/g)^{1/3} = h_1 F_{s1}^{2/3}$ = critical depth of flow;

$h_m = (h_1 + h_2)/2$ = mean of upstream and downstream depths in a hydraulic jump;

k_s = bed roughness height;

L = arbitrary constant representing the streamwise location of

the jump origin;
 L_A = aeration length of the jump;
 L_j = length of the hydraulic jump;
 L_R = roller length in the formation of the hydraulic jump;
 L_V = velocity length attributable to velocity profile in the jump;
 $p = \rho g(h - z)$ = hydrostatic pressure distribution;
 $q = Q/b$ = discharge per unit width of the channel flow;
 $T_{nn} = T_{xx} - T_{zz}$ = effective depth averaged Reynolds normal stress in the hydraulic jump;
 $T_{nn} = \rho \nu_t \partial U / \partial x$ = eddy viscosity closure model in the hydraulic jump;
 $T_{xx} = (1/h) \int \tau_{xx} dz$ = depth averaged Reynolds normal stress $\tau_{xx} = -\rho u'u'$ in the streamwise x-direction;
 $T_{zz} = (1/h) \int \tau_{zz} dz$ = depth averaged Reynolds normal stress $\tau_{zz} = -\rho w'w'$ in the z-direction;
 $U(x) = (1/h) \int u dh$ = cross-sectional averaged velocity in the x-direction;
 $U_h(x)$ = free surface velocity at $z = h(x)$;
 $u(x, z)$ = local velocity at a point in the streamwise x-direction;
 $V(x) = (U(x) - U_1)/(U_2 - U_1)$ = nondimensional axial velocity profile in the hydraulic jump;
 $w(x, z)$ = local velocity at a point in the normal z-direction;
 $X = x/h_2$ = nondimensional streamwise variable;
 x = streamwise horizontal coordinate of the flow;
 z = vertical coordinate measured above the bottom wall;
 $\alpha = h_2/h_1$ = sequent depth ratio;
 β_1, β_2 = upstream and downstream kinetic energy correction factors;
 $\Gamma = T_{nn}(\rho U_2)^{-1}$ = normal Reynolds stress turbulence level coefficient in the jump;
 $\Gamma_1 = T_{nn1}(\rho U_1)^{-1}$ = upstream normal turbulence level coefficient;
 $\Gamma_2 = T_{nn2}(\rho U_2)^{-1}$ = downstream normal turbulence level coefficient;
 ϵ = universal number for eddy viscosity constant independent of channel section;
 ϵ_A = universal number for aeration length independent of channel section;
 ϵ_R = universal number for roller length independent of channel section;
 $\eta(x) = (h(x) - h_1)/(h_2 - h_1)$ = nondimensional depth profile in the jump;
 Λ = constant in the jump based on shape of the channel;
 $\lambda = C_d/2$ = drag coefficient of the channel in the hydraulic jump;
 ν = molecular kinematic viscosity of fluid;
 ν_t = eddy viscosity of flow in the jump;
 ρ = fluid density;
 τ_w = bed roughness drag force per unit flow depth at the bottom;
 τ_{xx}, τ_{zz} = Reynolds normal stresses in streamwise and normal directions; and
 τ_{xz} = Reynolds shear stress in x-z plane.

Subscripts

1 = upstream of jump; and
 2 = downstream of jump.

References

- Abbaspour, A., Dalir, A. H., Farsadizadeh, D., and Sadraddini, A. A. (2009). "Effect of sinusoidal corrugated bed on hydraulic jump characteristics." *J. Hydro-Environ. Res.*, 3(2), 109–117.
- Afzal, N., and Bushra, A. (1999). "Turbulent hydraulic jump in a channel of arbitrary cross-section: Analogy with shock wave." *26th National Conf. of Fluid Mechanics and Fluid Power*, B. Mati, S. Som, S. K. Dash, and S. K. Som, eds., Allied Publishers, New Delhi, India, 18–33.
- Afzal, N., and Bushra, A. (2002). "Structure of the turbulent hydraulic jump in a trapezoidal channel." *J. Hydraul. Res.*, 40(2), 205–214.
- Bejestan, M. S., and Neisi, K. (2009). "A new roughened bed hydraulic jump stilling basin." *Asian J. Appl. Sci.*, 2(5), 436–445.
- Belanger, J. B. (1840). *Essai sur la solution nume rique de quelques problemes relatifs au mouvement permanent des eaux courantes* (Essay on the numerical solution of some problems relative to steady flow of water), Carilian-Goeury, Paris (in French).
- Bushra, A., and Afzal, N. (2006). "Turbulent hydraulic jump in circular and U-shaped." *J. Hydraul. Res.*, 44(4), 567–576.
- Carollo, F. G., Ferro, V., and Pampalone, V. (2007). "Hydraulic jump on rough beds." *J. Hydraul. Eng.*, 133(9), 989–999.
- Carollo, F. G., Ferro, V., and Pampalone, V. (2009). "New solution of classical hydraulic jump." *J. Hydraul. Eng.*, 135(6), 527–531.
- Clauser, F. H. (1956). "The turbulent boundary layers." *Advances in applied mechanics*, Vol. 4, Academic Press, New York.
- Duncan, W. J., Thom, A. S., and Young, A. D. (1967). *Mechanics of fluids*, Edward English Language Book Society, UK, 437.
- Ead, S. A., and Rajaratnam, N. (2002). "Hydraulic jumps on corrugated beds." *J. Hydraul. Eng.*, 128(7), 656–663.
- Elevatorski, E. A. (1959). *Hydraulic energy dissipators*, McGraw-Hill, New York.
- Gill, A. E. (1980). "Effect of boundary roughness in hydraulic jump." *Water Power Dam Constr.*, 32, 22–24.
- Govindarao, N. S., and Ramaprasad, N. (1966). "Application of momentum equation in the hydraulic jump." *La Houille Blanche*, 24(4), 451–453.
- Hager, W. H. (1992). *Energy dissipater and hydraulic jump*, Kluwer Academic Publishers, Dordrecht, Netherlands.
- Hughes, W. C., and Flack, J. E. (1984). "Hydraulic jump properties over a rough bed." *J. Hydraul. Eng.*, 110(12), 1755–1771.
- Izadjoo, F., and Bajestan, M. (2005). "Effects of trapezoidal shape corrugated bed on the characteristics of hydraulic jump." *17th Canadian Hydrotechnical Conf.: Cornerstone of a Sustainable Environment*.
- Leutheusser, H. J., and Kartha, V. C. (1972). "Effects of inflow condition on hydraulic jump." *J. Hydraul. Div.*, 98(8), 1367–1383.
- Leutheusser, H. J., and Schiller, E. J. (1975). "Hydraulic jump in a rough channel." *Water Power Dam Constr.*, 27(5), 186–191.
- Mohamed-Ali, H. S. (1991). "Effect of roughened-bed stilling basin on length of rectangular hydraulic jump." *J. Hydraul. Eng.*, 117(1), 83–93.
- Negm, A. A. (2002). "Optimal roughened length of prismatic stilling basins." *Proc., 5th Int. Conf. on Hydro-Science and Engineering*. (<http://kfki.baw.de/conference/ICHE/2002-Warshaw/author.htm>).
- Pagliara, S., Lotti, I., and Palermo, M. (2008). "Hydraulic jump on rough bed of stream rehabilitation structures." *J. Hydro-Environ. Res.*, 2(1), 29–38.
- Rajaratnam, N. (1967). "Hydraulic jumps." *Advances in hydrosiences*, Vol. 4, Academic Press, New York, 197–280.
- Rajaratnam, N. (1968). "Hydraulic jump on rough beds, *Trans. Eng. Inst. Canada*, 11(A2), 1–8.
- Resch, F. I., and Leutheusser, H. J., and Coantic, M. (1976). "Study of kinematic and dynamic structure of the hydraulic jump." *J. Hydraul. Res.*, 14(4), 293–319.
- Subramanya, K. (1998). *Flow in open channel*, Tata McGraw-Hill, New Delhi, India.
- Thompson, P. G. (1972). *Compressible fluid dynamics*, McGraw-Hill, New York, 517.
- Tokuyay, N. D. (2005). "Effects of channel bed roughness on hydraulic jumps." *Proc., ASCE World Water and Environmental Resources Congress*, ASCE, Reston, VA.
- Townsend, A. A. (1976). *Structure of turbulent shear flows*, Cambridge University Press, Cambridge, UK.
- Yadav, P. K., and Ahmad, Z. (2007). "Parameters of hydraulic jumps on corrugated beds." *J. Hydraul. Eng.*, 13(7), 93–105.