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Suitability of Reduced Pressure Center-Pivots

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SUITABILITY OF REDUCED PRESSURE CENTER-PIVOTS

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ABSTRACT: Selection criteria for reduced pressure center-pivot irrigation sys-
tems are developed. An analysis of the combined effects of the application rate
characteristics of center-pivot irrigation systems and the Soil Conservation Ser-
vice (SCS) soil intake family curves is used to determine the maximum depth
of water which could be applied per irrigation for various types of soils and
sprinkler packages. These irrigation depths are used to determine guidelines
for proper selection of reduced pressure center-pivot systems. The results can
be used as a general guide to determine if a particular system may have a runoff
problem under a given situation.

INTRODUCTION

The energy costs for pumping and distributing water through center-
pivot irrigation systems have increased rapidly in the past few years and
will probably continue to rise in the future. Because of these price in-
creases and the relatively large energy requirements for center-pivot sys-
tems there is considerable interest in reducing the operating pressure of
these systems.

Reduced pressure center-pivot systems, using spray nozzles or low-
pressure impact sprinklers, are becoming increasingly popular and many
existing high pressure systems are being retrofitted with reduced pres-
sure devices. While conventional high-pressure systems require pivot
pressures between 410 and 590 kPa (60–85 psi), reduced pressure sys-
tems can be operated at pivot pressures approaching 130–200 kPa (20–
30 psi), thereby resulting in considerable energy savings. These energy
savings are not without consequences such as runoff of applied water.
In some cases, runoff problems may become so severe that certain types
of reduced pressure devices should probably not be used on some soils.

The analysis presented in this paper does not include the effect of the
sprinkler water droplet impact and the resulting soil sealing on the in-
filtration processes. These highly important components are presently
being investigated by the author and others (4,11,13,14). These authors
have investigated the interaction of a time varying application rate for
certain types of infiltration functions. While these infiltration equations
are more physically based than the SCS soil intake families and they can
be related to different management procedures, their use in a design
fashion similar to that presented in this paper has not been completed.
The use of the infiltration models in an analysis such as the one pre-
sented here is the next logical step to rationalize the design of center-
pivot systems. The procedure presented here was developed because of
the current need for design information for reduced pressure systems
and the widespread use of the SCS intake family soils.

The analysis developed here is similar to that used by Dillon, et al. (2). However, the analysis is extended to include different types of reduced pressure systems and to include additional soil intake families.

ANALYSIS

The design or specification of a center-pivot system for a given site consists primarily of selecting two primary variables: (1) The proper system capacity or flow rate to satisfy the crop water requirements plus leaching fraction, if necessary; and (2) the pivot or end pressure of the system based upon the type of sprinkler package used. The selection of the system capacity can be determined from the peak crop water requirements of the crop or crops to be irrigated or it can be determined using the procedure of Heermann, et al. (6) which includes the contribution of stored soil moisture and the probability of rainfall.

The selection of the pivot or outer end pressure is more difficult, however. Many combinations of sprinkler types or hardware, nozzle sizes, sprinkler spacings and pressures are possible. In general, as the sprinkler pressure is lowered, larger nozzle sizes are required for the same discharge. Depending upon the flow-rate, nozzle-size combination, the calculated sprinkler flow may not be achievable because of a number of factors, including sprinkler performance, thereby requiring a closer spacing. These relationships are sometimes referred to as the nozzling parameters for center-pivots and the equations necessary for determining the required sprinkler discharges based upon the sprinkler spacing are given by Kincaid and Heermann (9).

The required pressure for different sprinkler nozzles can be combined with the equations of Kincaid and Heermann to develop the "sprinkler package" giving the proper sprinkler, nozzle sizes and sprinkler location for the system. Currently, the pivot or end pressure is selected primarily to conserve energy. However, as the pressure is lowered, the application rate of the system will generally rise increasing the possibility of runoff of the applied water. Thus an analysis of the potential runoff of applied water should be used in the selection of proper sprinkler packages for center-pivot irrigation systems. This analysis requires the consideration of both the application rate of the system and the infiltration rate of the soil (3).

Center-Pivot Application Rate.—The rate of water application beneath a center-pivot irrigation system varies continuously with time during the irrigation event. Mathematical expressions for describing the application rate from overlapped individual sprinkler heads on the center-pivot system have been developed. Heermann and Hein (5) presented application rate equations for the center-pivot system assuming both triangular and elliptical distribution patterns. Kincaid, et al. (8) conducted field experiments to test the validity of the theoretical application patterns and concluded that the elliptical pattern was most appropriate. Assuming the water distribution of the sprinklers is elliptical, the application rate $AR(t)$ can be written as

$$AR(t) = \frac{hp}{t_p} (2tt_p - t^2)^{1/2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(1)}$$
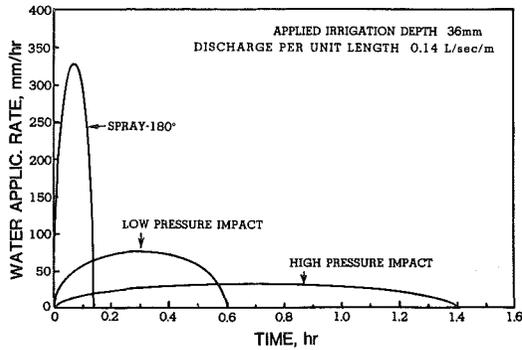
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FIG. 1.—Elliptical Application Rate Characteristics for Three Types of Sprinkler Packages

in which $AR(t)$ = the application rate at a particular point (in mm/h); hp = the peak application rate (mm/h); t_p = the time to the peak rate (hr); and t = time, starting when the application rate begins (hr).

As the sprinkler pressure is reduced, the radii of coverage of the individual sprinklers decreases causing an increase in the application rate of the system. The application rates of the three different types of systems are compared in Fig. 1. While the gross irrigation depths of the three systems are identical, their application rate distributions are vastly different both in magnitude and in time.

The distribution of water along the center-pivot lateral is given by Kincaid and Heermann (9):

$$q_i = \frac{2QR_i}{R_n^2} \dotfill (2)$$

in which q_i = the discharge per unit length of lateral at location R_i, (L/s/m); Q = the total system discharge (L/s); R_i = the distance from the pivot point to the one meter length band (m); and R_n = the distance from the pivot point to the last sprinkler on the system, approximately the system length, R (m). Assuming that the combined distribution from the overlap of the sprinklers is elliptical, the relationship between the unit discharge, q_i, and the peak application rate which occurs directly beneath the lateral is given by

$$q_i = \frac{hp_i r_i}{2,292} \dotfill (3)$$

in which hp_i is the peak application rate at location R_i (mm/h); and r_i = the effective radius of coverage of the sprinklers from the center of the pattern to their wetted edge at location R_i (m). The peak application rate of the system at location R_i can be determined by combining Eqs. 2 and 3:

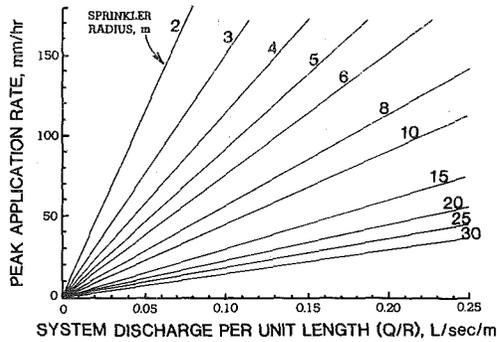$$hp_i = \frac{4,584\,QR_i}{R_n^2 r_i} \dotfill (4)$$

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FIG. 2.—Peak Application Rates for Center-Pivot Systems Assuming Elliptical Pattern Distribution

The greatest potential for runoff occurs at the distal end of the center-pivot lateral where the application rate is the greatest. Thus, Eq. 4 can be reduced to

$$hp = \frac{4{,}584Q}{Rr} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \tag{5}$$

in which hp is the peak application near the distal end of the lateral (mm/h); r = the effective radius of coverage of the sprinklers at the distal end (m); and R is the system length (m).

The solution to Eq. 5 for several values of the effective sprinkler radius, r, is given in Fig. 2. For conventional size systems (52.6 ha, 130 acres), typical values of system flow to system length range between 0.10–0.15 L/s/m. Approximate pivot pressures and sprinkler radii for several types of sprinkler packages for center-pivot systems are given in Table 1.

TABLE 1.—Approximate Pivot Pressure and Sprinkler Radii for Different Types of Center-Pivot Systems

Type of system (1)	Approximate Pivot Pressure		Approximate Sprinkler Radius[a]	
	In kilo-pascals (2)	In pounds force per square inch (3)	In meters (4)	In feet (5)
High pressure impact sprinkler	450–520	65–75	20	66
Medium pressure impact sprinkler	275–350	40–50	14	46
Low pressure impact sprinkler	170–240	25–35	10	33
Low pressure spray, 360°	170–240	25–35	5	16
Low pressure spray, 180°	170–240	25–35	3[b]	10[b]

[a]Sprinkler radius at distal end of lateral.
[b]One direction only.

TABLE 2.—Peak Application Rate Characteristics for Five Types of Center-Pivot Systems at Distal End of Pivot

Type of system (1)	Peak application rate, in milli-meters per hour (2)	Change in peak application rate per meter of sprinkler throw,[a] in millimeters per hour per meter (3)
High pressure impact	29	−1.4
Medium pressure impact	41	−2.9
Low pressure impact	57	−5.7
Spray, 360°	115	−22.9
Spray, 180°	191	−63.7

[a]Assuming a flowrate to pivot length of 0.125 L/s/m. An increase in sprinkler radius results in a decrease in peak application rate.

The effect of the sprinkler radius on the application rate of the system is given in Table 2. Because of their relatively small wetted radius, the application rate of the reduced pressure devices is quite large. Furthermore, as shown in Table 2, the effect of sprinkler radius on the peak application rate becomes more important as the magnitude of the radius decreases. For those devices having a relatively small area of coverage (e.g., spray) a small change in radii will cause a large change in application rate. Conversely, for those devices having a relatively large area of coverage (high pressure sprinkler for example) a small change in effective radius will have little effect on the peak application rate.

Soil Infiltration Rate.—The rate of water entry into the soil profile, the soil intake rate, is a process of great practical importance to irrigation design. The infiltration capacity of the soil determines the maximum rate that water can be applied to the soil surface without runoff. Because of the relatively high application rates of center pivot systems, an understanding of the infiltration process and the factors affecting it are highly important to the design and operation of efficient center-pivot irrigation systems.

Infiltration can be characterized by theoretical methods for most boundary and initial conditions of interest. However, these equations and their solutions are rarely used in practice to describe the infiltration process (12). Attempts to characterize infiltration for field applications have usually involved simplified concepts which permit the infiltration rate or cumulative infiltration depth to be expressed in terms of time and certain soil parameters. One of the simplest equations is the power function proposed by Kostiakov, as referenced by Skaggs, et al. (12):

$$I(t) = kt^n \dotfill (6)$$

in which $I(t)$ = the soil intake rate (mm/h); t = the time after infiltration starts (hr); and k and n = empirical constants.

While Eq. 6 is highly empirical, and the constants (k and n) have no physical interpretation, this function has been used to describe the infiltration under all types of irrigation systems including center-pivot systems (8). The Soil Conservation Service has classified soils into intake families using a modification of Eq. 6 as a basis (13). The empirical con-

Soil intake family	Soil Infiltration Constants	
	k	n
(1)	(2)	(3)
0.1	6.83	−0.485
0.3	15.16	−0.381
0.5	21.77	−0.340
1.0	36.59	−0.305
1.5	47.90	−0.290

[a]Constants used in Eq. 6 with the intake rate in mm/h and time in hours. Modified from the Soil Conservation Service (1).

stants for Eq. 6 determined by a least squares fit to the SCS data for several soil families are given in Table 3.

The soil infiltration function (Eq. 6) and the constants given in Table 3 were developed under the assumption that the intake rate is independent of the application rate during the initial period of application and are valid for flooded infiltration only. Thus the use of these constants for describing the infiltration process under center-pivot systems requires some modification of the infiltration equation.

The technique used to modify the flooded intake functions to account for nonsurface saturated conditions encountered with the application rate of center-pivot systems was that given by Kincaid, et al. (8) with changes to include an elliptical application rate function. A description of the variables used to modify the intake function is shown in Fig. 3.

During the time period before surface saturation (t_s), or when the potential runoff begins, the modified intake rate, I_m is given by

$$I_m(t) = \frac{I(t) \int_0^t I(t)dt}{\int_0^t AR(t)dt} \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (7)$$
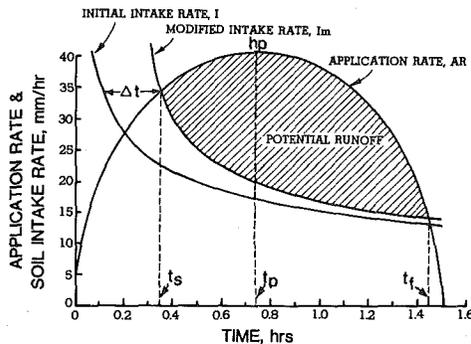

FIG. 3.—Example Application Rate, Soil Intake Rates, and Potential Runoff of Center-Pivot Systems

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in which $I_m(t)$ = the modified intake rate as a function of time (mm/h); $I(t)$ = the flooded intake rate as a function of time, from the intake family soils (Eq. 6) (mm/h); $AR(t)$ = the application rate as a function of time (mm/h); and t = time, from the beginning of the application event (hr). Eq. 7 is valid only during the time period before potential runoff begins, i.e., for $0 < t \le t_s$. After potential runoff begins, the intake rate with a modified time will decrease according to Eq. 6, since the soil surface is flooded.

The terms on the right hand side of Eq. 7 can be expanded as follows. The integral of the intake-function can be written as

$$\int_0^t I(t)dt = \frac{kt^{n+1}}{n+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

The integral of the application rate or the applied depth is given by

$$\int_0^t AR(t)dt = \frac{1}{2}\frac{hp}{t_p}\left[(t - t_p)(2tt_p - t^2)^{1/2} + t_p^2 \sin^{-1}\left(\frac{t - t_p}{t_p}\right) + \frac{\pi}{2}t_p^2\right] \quad \ldots\ldots \quad (9)$$

The first variable needed in the calculation of the modified intake rate is the time to surface saturation, t_s. The procedure used to calculate t_s is similar to that proposed by Kincaid, et al. (8). The parameter t_s is the time at which the modified intake function is equal to the application rate (Fig. 3). Thus at $t = t_s$

$$I_m(t_s) = AR(t_s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Combining Eqs. 6–10 with time = t_s results in an expression with t_s as the only unknown. However, its solution is a trial and error process which can be readily completed on a computer or handheld programmable calculator.

While the soil surface is flooded (at times greater than t_s shown in Fig. 3), the modified intake function is given by

$$I_m = k(t - \Delta t)^n \quad \text{at} \quad t > t_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

in which Δt = the amount of time by which the intake function must be delayed so that it will pass through the application rate curve at t_s. The parameter Δt is computed from

$$\Delta t = t_s - \left(\frac{AR(t_s)}{k}\right)^{1/n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

Surface Runoff.—The potential runoff from center-pivot irrigation systems is defined as that portion of the irrigation water that is applied at rates exceeding the soil intake rate. The theoretical potential runoff is calculated by computing the area between the application rate and soil intake rate during the time when the application rate exceeds the soil intake rate. An example of the potential runoff is shown as the cross hatched area in Fig. 3.

While the potential runoff can be quite large in the area near the pivot point, the irrigated area with the largest application rate and thus the area with greatest runoff potential is located toward the distal end of the system. The analysis presented here thus considers only the outer portion of the pivot.

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TABLE 4.—Allowable Surface Storage Values for Various Slopes[a]

Slope, as a percentage (1)	Allowable Surface Storage,	
	in millimeters (2)	In Inches (3)
0–1	12.7	0.5
1–3	7.6	0.3
3–5	2.5	0.1
>5	0.0	0.0

[a]From Dillon, et al. (2).

The actual runoff which takes place beneath the center-pivot system is not only a function of the potential runoff (the cross hatched area in Fig. 3) but also the storage of the applied water on the soil surface. The allowable surface storage is primarily a function of the roughness of the soil surface and the topography of the given site, primarily slope. Values of soil surface storage taken from Dillon, et al. (2) are given in Table 4.

The potential runoff shown in Fig. 3 is given by

$$PR = \int_{t_s}^{t_f} AR(t) - \int_{t_s}^{t_f} I_m(t) \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(13)}$$

in which PR is the potential runoff; and t_f is the second time where the modified intake function equals the application rate (Fig. 3). The parameter t_f is computed from the equation:

$$I_m(t_f) = AR(t_f) \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(14)}$$

in which $I_m(t_f)$ = the value of the modified intake function at time t_f. The calculation of t_f involves a trial and error process using Eqs. 1 and 11 substituted in Eq. 14. Once t_f is determined, the potential runoff can be calculated using Eq. 13. The total irrigation application is the integral of the application rate function between $2t_p$ and 0 and is equal to $\pi/2$ $hp t_p$.

DESIGN GUIDELINES

The potential runoff of the water applied by a center-pivot system is a function of the peak application rate of the system, the time required to complete an irrigation (or the desired irrigation depth), and the soil infiltration rate or soil intake family.

Through a simulation analysis, the relationship between the peak application rate and the maximum irrigation depth for different values of surface storage was determined for several intake family soils. This relationship between the maximum amount of water which can be applied per irrigation and the peak application rate for the 0.1, 0.3, 0.5 and 1.0 family soils is given in Figs. 4–7, respectively. These figures can be used to determine the maximum irrigation depth that can be applied per irrigation without any runoff for the different values of soil surface storage.

Design Example.—The use of Figs. 4–7 can best be illustrated by an example. Suppose the peak application rate of a center-pivot system is
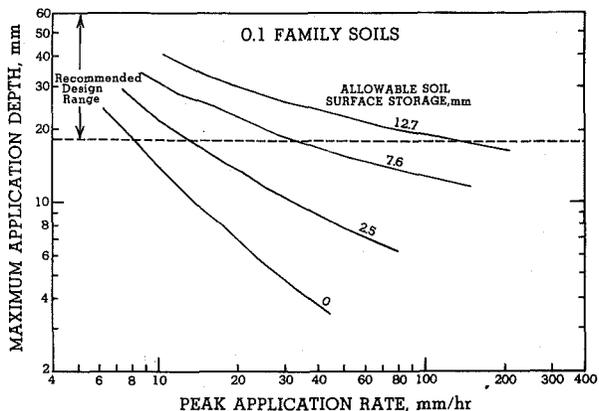
FIG. 4.—Maximum Depth of Water Application per Irrigation for 0.1 Family Soil

30 mm/h (from Eq. 5), and is operating on a 0.3 family soil. If the allowable soil surface storage is 0.0 mm then the maximum depth of application is about 16 mm (Fig. 5). If the allowable surface storage were 2.5 mm, the maximum irrigation depth for zero runoff can be increased to approximately 24 mm. For surface storage values of 7.6 mm and 12.7 mm the corresponding maximum irrigation depths are 38 mm and 49 mm, respectively. These changes in maximum irrigation depths indicate the importance of soil surface storage in reducing the likelihood of runoff of the applied irrigation water.

The allowable soil surface storage can result from natural soil roughness or artificially made storage cavities ("mini basins"). Lyle and Bordovsky (10) described a procedure and the equipment used to develop "mini basins" for use under lateral moving irrigation systems. While the system they described was used under a continuously moving lateral system, the technique may be applicable to center-pivot systems as well.
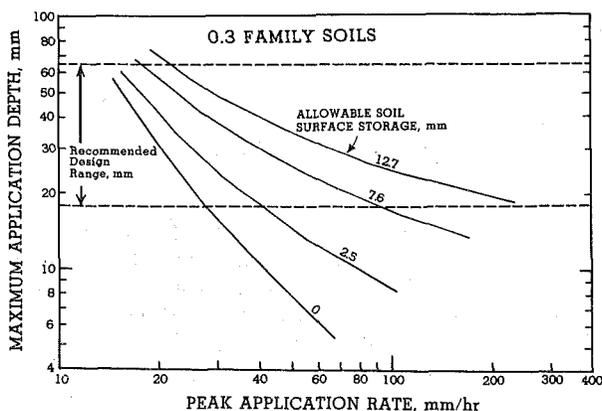

FIG. 5.—Maximum Depth of Water Application per Irrigation for 0.3 Family Soil
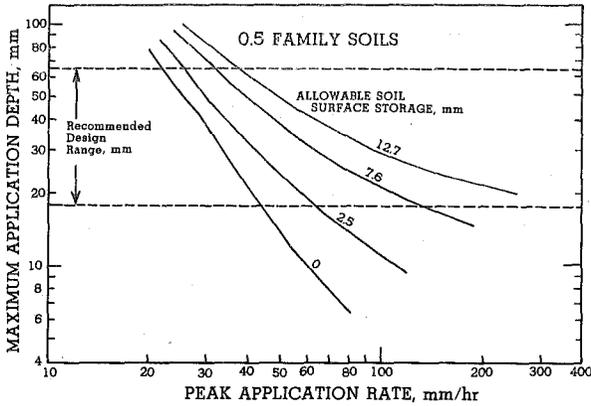
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FIG. 6.—Maximum Depth of Water Application per Irrigation for 0.5 Family Soil

Design Recommendations.—The system discharge used on most standard systems (400 m in length) in the midwest range between 0.1–0.15 L/s/m. Thus the results given in Figs. 4–7 can be further summarized to give the maximum irrigation depth that can be applied without any runoff for the different intake family soils, as a function of soil surface storage and system type. This summary is given in Table 5.

The peak application rate of the system depends upon a number of factors including system length, system flow rate, nozzle spacing and size, and system pressure; thus the data given in Table 5 should be used only as a general guide. Furthermore, the analysis used to develop Table 5 did not include the reduction in soil infiltration rate resulting from a surface seal caused by larger droplets which can be created by larger nozzle sizes operating at reduced pressure. The surface seal development results from a number of factors, primarily droplet energy, application rate, soil texture and soil structure. The surface seal development
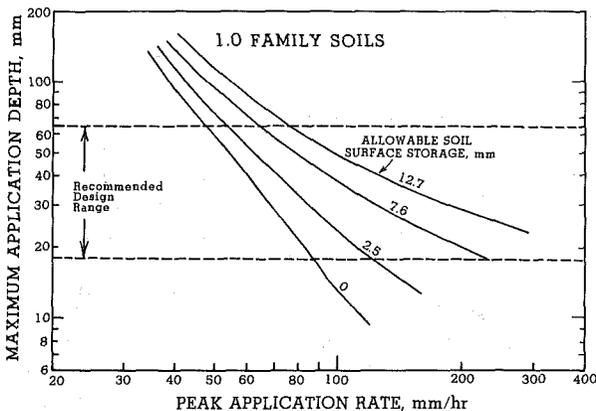

FIG. 7.—Maximum Depth of Water Application per Irrigation for 1.0 Family Soil

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TABLE 5.—Maximum Allowable Irrigation Depths Without Potential Runoff for Center-Pivot Irrigation Systems

Family soil (1)	Unit discharge,[a] in liters per second per meter (2)	System type[b] (3)	Peak rate,[c] in milli-meters per hour (4)	Allowable Surface Storage, in Millimeters			
				0.0	2.5	7.6	12.7
				Maximum Irrigation depth, in millimeters			
				(5)	(6)	(7)	(8)
0.1	0.102	HPI	23	6	11	20	27
		MPI	33	4	9	17	25
		LPI	47	3	8	15	22
		LS1	94	<3	7	13	19
		LS2	156	<3	6	12	17
0.3	0.109	HPI	25	20	29	44	55
		MPI	36	12	18	33	44
		LPI	50	8	14	25	34
		LS1	100	4	8	16	24
		LS2	166	<3	6	13	20
0.5	0.113	HPI	26	52	65	86	102
		MPI	37	23	33	49	61
		LPI	52	14	23	37	48
		LS1	104	<5	11	20	29
		LS2	173	<5	<10	16	22
1.0	0.123	HPI	28	>80	>80	>80	>80
		MPI	40	88	>80	>80	>80
		LPI	56	44	62	84	>80
		LS1	113	10	32	32	43
		LS2	188	5	10	21	30

[a]The results of Heermann, et al. (6) was used to determine net system capacity. An irrigation efficiency of 80% was assumed to obtain the gross system capacity and the systems were 409 m in length.

[b]The system types are: HPI (high pressure impact); MPI (medium pressure impact); LPI (low pressure impact); LS1 (low pressure spray, 360°); and LS2 (low pressure spray, 180°).

[c]Calculated using Eq. 5 with the unit discharge given in Col. 2 and the sprinkler radii in Table 1.

is a complex process presently under study by a number of investigators, and its incorporation into this analysis is underway. As a first approximation, the maximum irrigation amounts for medium pressure and low pressure impact systems given in Table 5 should probably be reduced between 15% and 25% for soil families 0.1 through 0.5.

The results in Table 5 indicate situations where certain types of systems may have serious runoff problems and perhaps should not be used. If the maximum allowable irrigation depths given in Table 5 are less than between 14 and 18 mm, that particular system should probably not be used. Irrigation amounts smaller than this range will result in increased evaporation losses because of the increased number of irrigations during the peak use periods, thereby reducing the irrigation efficiency. For example, the low pressure spray systems used on the 0.1 and 0.3 family

soils will probably have irrigation management problems because the maximum irrigation depths are too small to provide an acceptable irrigation schedule without having excessive runoff. However, if the soil surface storage could be increased by artificial means similar to Lyle and Bordovsky (10) or other techniques, these systems might be used.

CONCLUSIONS

The results of the analysis presented in this paper indicate that many of the available systems ranging from high-pressure to spray can be used on several of the intake family soils. However, the maximum irrigation depth which may be applied without any potential runoff may be too small for some systems to provide an acceptable irrigation amount, and therefore should not be used in that particular location.

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

$AR(t)$ = application rate of the system (in mm/h);
hp = peak application rate (mm/h);
hp_i = peak application rate at location R_i (mm/h);
$I(t)$ = soil infiltration rate at time t (mm/h);
$I_m(t)$ = modified soil infiltration rate at time t (mm/h);
k = empirical constant in the soil infiltration equation (mm/h);
n = empirical constant in the soil infiltration equation;
PR = potential runoff (mm);
Q = system discharge (L/s);
q_i = unit discharge of the system (L/s/m);
r = sprinkler radius of throw (m);
r_i = sprinkler radius of throw at location R_i (m);
R = system length (m);
R_i = distance from the pivot point to a particular sprinkler location (m);
R_n = distance from the pivot point to the last sprinkler on the system (m);
t = time (hr);
t_f = final time when the system application rate is equal to the modified infiltration rate (hr);
t_p = time when the application rate is at its peak rate (hr);
t_s = first time when the system application rate is equal to the modified infiltration rate, and also the time to surface saturation (hr); and
Δt = time shift between the soil infiltration equation and the modified infiltration equation (hr).