2005

Using multi-stack and variable-speed-drive systems to reduce laboratory exhaust fan energy

Gang Wang
University of Nebraska

Mingsheng Liu
University of Nebraska–Lincoln, mliu2@unl.edu

INTERNATIONAL JOURNAL OF ENERGY RESEARCH
Int. J. Energy Res. 2005; **29**:1–12
Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/er.1032

Using multi-stack and variable-speed-drive systems to reduce laboratory exhaust fan energy‡

Gang Wang and Mingsheng Liu*,†

Energy Systems Laboratory, University of Nebraska, Lincoln, U.S.A.

SUMMARY

In buildings that contain laboratories, fume hoods are normally used to control contaminant concentrations. Exhaust stacks with a constant exit velocity are required to make sure that dangerous concentrations do not occur in occupied areas near the building or on the roof top. To achieve constant velocity when exhaust flow rates are less than design, makeup air is introduced to the system at the inlet of the exhaust fan. Since laboratory exhaust airflow is often significantly less than the design airflow, exhaust fans consume significantly more energy than is necessary. To reduce exhaust fan energy, techniques involving multiple exhaust stacks and a variable speed drive (VSD) can be applied to laboratory exhaust systems. The potential fan energy savings depend on optimal selection of the number of stacks, the sizes of the stacks, and the exhaust system ductwork design. This paper introduces application principles, describes the optimal methods of stack sizing, and presents an example to demonstrate these methods. Published in 2005 by John Wiley & Sons, Ltd.

KEY WORDS: laboratory building; exhaust system; stacks; VSD; energy conservation

1. INTRODUCTION

Laboratory fume hoods are used in nearly all industrial, university, and other institutions that conduct chemical or biological testing and experiments. These hoods draw polluted toxic air into the exhaust system, which creates sufficient negative pressure at the outlet of the fume hood to ensure proper toxic air collection in the interior space. Typically the air is exhausted from the building on the roof, through an exhaust system that prevents unsafe toxic contaminant concentrations in inhabited areas outside of the building. The exhaust system design depends on the type of fume hoods being used, the building's architectural requirements, and other parameters. In this study, exhaust systems are categorized into two groups: makeup air type and non-makeup air type.

Makeup air exhaust systems use a constant speed fan and a makeup air duct at the inlet of the exhaust fan. The fan is sized to maintain the required negative static pressure at the fume hood and the required stack exit velocity under design airflow conditions. The makeup air damper allows outside air to flow directly into the fan inlet to maintain the negative static

*Correspondence to: M. Liu, Professor, Energy Systems Laboratory, University of Nebraska, Lincoln, U.S.A.
† E-mail: mliu2@unl.edu
‡ This article is a U.S. Government work and is in the public domain in the U.S.A.

Received 26 November 2003
Accepted 19 January 2004

pressure set-point when the laboratory exhaust is less than the design value. Thus a makeup air exhaust system maintains the stack exit velocity at the design value or higher, regardless of the laboratory exhaust airflow rate. Makeup air exhaust systems are often used with constant air volume (CAV) fume hoods, or when variable air volume (VAV) fume hoods are used but the stack heights are not high enough to prevent toxic air downwash to inhabited areas under low stack exit velocity.

Non-makeup air exhaust systems use a VSD to adjust the fan speed as the laboratory exhaust airflow changes. When the laboratory exhaust airflow decreases, the VSD decreases the fan speed to maintain the negative static pressure set-point at the fume hoods. The stack exit velocity decreases proportionally as the laboratory exhaust airflow decreases. Non-makeup air exhaust systems are often designed with high stacks such that the downwash under low stack exit velocity would not impose harmful concentrations on inhabited areas outside of the building. Non-makeup air exhaust systems consume significantly less fan energy than makeup air exhaust systems. However, because of architectural requirements, stacks are often not allowed to be designed high enough to prevent downwash under low stack exit velocities. Therefore makeup air exhaust systems have been the more popular exhaust system design.

A number of studies have shown that significant fan energy is wasted in makeup air exhaust systems. Varley (1993) compared the actual laboratory exhaust airflow and the design exhaust airflow in an industrial research laboratory building which had 56 constant-exhaust fume hoods. The investigation found that the actual laboratory exhaust airflow varied from 21 to 43% of the design value. Obviously, significant makeup air was used continuously. Recently, Wang and Liu (2001) investigated the fan airflow versus the laboratory exhaust airflow in makeup air exhaust systems in which constant speed fans are used. When the laboratory exhaust airflow was less than the design value, the fan airflow was *higher* than the design value. The excessive airflow (difference between the fan airflow and the design airflow) increased as the laboratory exhaust airflow decreased. The fan power was often higher than the design fan power under lower laboratory exhaust airflow conditions. Hitchings and Shull (1993) measured laboratory exhaust airflow continuously for over a month in three laboratory buildings where VAV fume hoods were used. They found that the laboratory exhaust airflow varied from 31 to 57% of the design airflow in building 1, 29 to 53% in building 2, and 45 to 70% in building 3. Similar results have been reported by other researchers (Moyer and Dungan, 1987; Rabiah and Wellenbach, 1993). However, although the maximum laboratory exhaust airflow is significantly lower than the design value, it may change from year to year. Engineers have to size the exhaust system using the conventional value to ensure system reliability during its lifetime. Thus makeup air exhaust systems always use a significant amount makeup air regardless of the type of fume hoods used in laboratories.

Wang and Liu (2001) investigated potential fan energy savings using VSD in makeup air exhaust systems. The VSD is modulated to maintain the required stack exit velocity. Fan head is significantly reduced compared with a constant-speed fan system. The makeup air damper is modulated to maintain fume hood negative static pressure. Theoretical analysis shows that the use of a VSD can significantly reduce annual fan energy consumption.

Wang *et al.* (2002) examined the retrofit of existing makeup air exhaust systems by adding one or more smaller stacks combined with the VSD technique. When the laboratory exhaust airflow is lower than the small stack's capacity, the full size stack is closed and the small stack is activated. Both the makeup air and fan airflow are significantly reduced compared with a single stack system. The theoretical study found that adding small stacks, combined with VSD techniques, saves more fan energy than the VSD technique alone. The potential savings depends on the airflow pattern as well as the pressure loss distribution of the exhaust system under design airflow conditions.

For a new facility design, the aggregated stack capacities can be set equal to the conventional design capacity. This design reduces the size and the initial cost of the system. This paper introduces the application of a VSD and multiple stacks to the makeup air exhaust system, describes the optimal selection of stack sizes, and demonstrates potential fan energy savings analysis using an example.

2. VSD AND MULTI-STACK

Figure 1 is a schematic diagram of a VSD and multi-stack application in a makeup air exhaust system. The exhaust system consists of a bundle of stacks, an exhaust, fan, a makeup air damper, an airflow station (FS), and two static pressure sensors (P). One static pressure sensor is located at the fume hood and the other at the stack inlet. The bundle of stacks may consist of two or three stacks. More than three is not necessary. Each stack can be turned on/off by a stack damper.

The laboratory exhaust airflow is measured by the airflow station. The controller activates (opens) the stack/stacks which have the lowest airflow capacity that is higher than the laboratory exhaust airflow. The stack inlet static pressure is measured by a static pressure sensor. If the measured value is lower than the set point, the controller speeds up the fan. If the measured value is higher than the set point, the controller slows down the fan. The constant stack exit velocity is indirectly maintained by the constant stack inlet static pressure. The fume hood negative static pressure is measured by the other static pressure sensor. If the absolute value of the static pressure is lower than the absolute value of the set point, the controller closes the

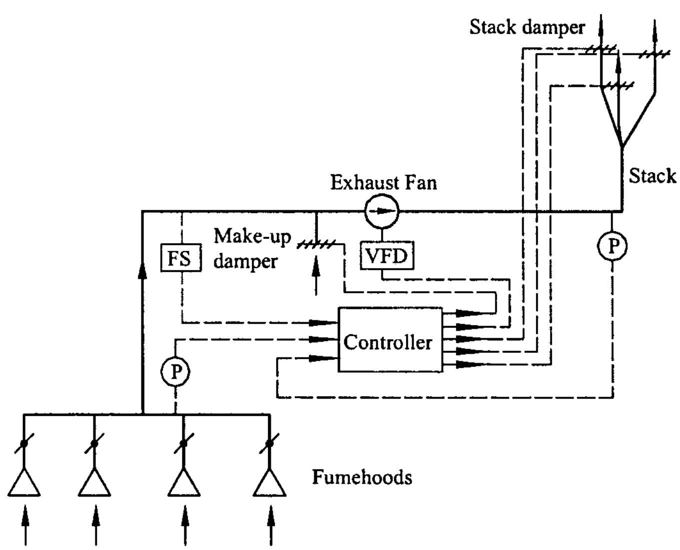

Figure 1. Schematic diagram of the multi-stack exhaust system.

makeup air damper more. If the absolute value of the static pressure is higher than the absolute value of the set point, the controller opens the makeup air damper more.

The annual fan energy performance of the exhaust system with a VSD and multi-stacks strongly depends on the selection of stack capacities. The number and sizes of the stacks must be optimized based on the exhaust airflow pattern for a specific laboratory exhaust system in order to achieve the maximum fan energy savings.

To optimize stack sizes (y_i), the following mathematical models have been developed. Assuming a total number of stacks, m, the controller can select n stack combinations (z_i). The number of combinations can be determined by the number of stacks, using Equation (1).

$$n = \sum_{i=1}^{m} C_i^m = 2^m - 1 \tag{1}$$

The capacity of a particular stack combination depends on the stacks' design capacities. Both the stack combination capacity series $\{z_j\}$ ($j = 1,2,\ldots,n$) and the stack capacity series $\{y_i\}$ ($i = 1,2,\ldots,m$) are arranged in order of increasing capacity.

$$y_i \leqslant y_{i+1} (i = 1, 2, \ldots, m-1)$$

$$z_j \leqslant z_{j+1} (j = 1, 2, \ldots, n-1)$$

In order to minimize the system size and initial cost, the maximum stack combination capacity (the sum of the stack capacities) is set equal to the laboratory exhaust system design airflow.

$$z_n = \sum_{i=1}^{m} y_i = 100\% \tag{2}$$

For a two-stack system, the total number of stack combinations is 3 and the stack combination capacity series is expressed by the stack capacity series.

$$\{z_j\}(j = 1, 2, 3) = \{y_i, y_2, y_1 + y_2 = 1\}$$

For a three-stack system, the total number of stack combinations is 7. The stack combination series depends on the size of the stacks. When the total capacity of the two small stacks ($y_1 + y_2$) is less than the capacity of the third stack (y_3), the stack combinations series are:

$$\{z_j\}(j = 1, 2, \ldots, 7) = \{y_1, y_2, y_1 + y_2, y_3, y_1 + y_3, y_2 + y_3, y_1 + y_2 + y_3 = 1\}$$

When the total capacity of the two small stacks ($y_1 + y_2$) is higher than the capacity of the third stack (y_3), the combinations series are:

$$\{z_j\}(j = 1, 2, \ldots, 7) = \{y_1, y_2, y_3, y_1 + y_2, y_1 + y_3, y_2 + y_3, y_1 + y_2 + y_3 = 1\}$$

If the laboratory exhaust airflow is between the capacities of stack combinations $j-1$ and j, all stacks in combination j are turned on and the rest of the stacks are turned off. The airflow upstream of the fan inlet is equal to the laboratory exhaust airflow. The fan airflow is equal to the capacity of the activated stack combination j.

$$\bar{Q} = z_j (j = 1, 2, \ldots, n) \quad \text{when } z_{j-1} < \bar{Q}_h \leqslant z_j \tag{3}$$

where

$$\bar{Q} = Q/Q_\mathrm{d}$$

$$\bar{Q}_h = Q_h/Q_\mathrm{d}$$

The fan head is expressed in Equation (4) with the assumption that the air steam dynamic head is the same at the fume hood's static pressure sensor location and at the stack exit. This assumption has a minimal impact on the overall modelling accuracy.

$$H = \Delta P_s + \Delta P_2 + \Delta P_3 \tag{4}$$

Assuming turbulent flow, the relative fan head ($\bar{H}$) is expressed using the laboratory exhaust airflow ratio ($\bar{Q}_h$), the exhaust fan airflow ratio ($\bar{Q}$), and the design duct pressure losses ratios (x_1, x_2 and x_3).

$$\bar{H} = x_1 + x_2 \cdot \bar{Q}_h^2 + x_3 \cdot \bar{Q}^2 \tag{5}$$

where

$$\bar{H} = H/H_\mathrm{d}$$
$$x_1 = \Delta P_{\mathrm{s,d}}/H_\mathrm{d}$$
$$x_2 = \Delta P_{2,\mathrm{d}}/H_\mathrm{d}$$
$$x_3 = \Delta P_{3,\mathrm{d}}/H_\mathrm{d}$$

The fan power can be determined directly using fan curves when the fan airflow and fan head are known. The relative fan power (the ratio of fan power to design fan power) is generally expressed as

$$\bar{W} = \bar{W}(\bar{H}, \bar{Q}) \tag{6}$$

where

$$\bar{W} = W/W_\mathrm{d}$$

The annual fan energy consumption is found by integrating fan power over the hours of a year. However, it can also be calculated by summing the product of fan power and coincident operating hours for each airflow rate category. Equation (7) expresses the annual average fan power in relative terms:

$$\bar{E} = \frac{1}{T}\int_0^1 \bar{W} \cdot f(\bar{Q}_h)\,\mathrm{d}\bar{Q}_h = \int_0^1 \bar{W} \cdot \bar{f}(\bar{Q}_h)\,\mathrm{d}\bar{Q}_h \tag{7a}$$

$$\bar{E} = \frac{1}{T}\sum_{k=1}^{l} \frac{\bar{W}_{\bar{Q}_{h,k}} + W_{\bar{Q}_{h,k-1}}}{2} \cdot T_k = \sum_{k=1}^{l} \frac{\bar{W}_{\bar{Q}_{h,k}} + W_{\bar{Q}_{h,k-1}}}{2} \cdot \bar{T}_k \tag{7b}$$

where

$$f\left(\frac{\bar{Q}_{h,k} + \bar{Q}_{h,k-1}}{2}\right) \approx \frac{T_k}{\bar{Q}_{h,k} - \bar{Q}_{h,k-1}} \tag{8}$$

$$T_k = \int_{\bar{Q}_{h,k-1}}^{\bar{Q}_{h,k}} f(\bar{Q}_h)\,\mathrm{d}\bar{Q}_h \approx f\left(\frac{\bar{Q}_{h,k} + \bar{Q}_{h,k-1}}{2}\right)(\bar{Q}_{h,k-1} - \bar{Q}_{h,k}) \tag{9}$$

$$\bar{f}(\bar{Q}_h) = \frac{f(\bar{Q}_h)}{T} \tag{10}$$

$$\bar{T}_k = \frac{T_k}{T} \tag{11}$$

The time density function $f(\bar{Q}_h)$ is a ratio of the number of operating hours in a flow range to the scale of that range. For example, if there were 800 h of fan operation between 30 and 40%, the time density would be 800/(0.4–0.3), or 8000. The time distribution T_k is defined as the number of operating hours in a particular interval, $\bar{Q}_{h,k-1}$ to $\bar{Q}_{h,k}$. In the example, T_k would be 800 h. Both the time density and the time distribution can be expressed in relative terms $\bar{f}(\bar{Q}_h)$ or $\bar{T}_k$ by dividing by the total number of operating hours in a year (T).

3. STACK SIZE OPTIMIZATION

The optimal selection of stack sizes is critical in minimizing fan energy consumption. In this section, both analytical and numerical methods will be developed to optimize stack sizes when the number of stacks, laboratory exhaust usage patterns, and pressure loss distributions are known.

3.1. Analytical method

When a VSD is used, the fan's working point is always near the original design's system curve. The fan efficiency remains at, or close to, the design value at all times. If the fan efficiency is treated as a constant and the time density is expressed as a polynomial of the exhaust airflow, then the annual fan energy consumption can be deduced from Equation (7a).

$$\bar{E} = \sum_{k=0}^{l} \sum_{j=1}^{n} a_k \int_{z_{j-1}}^{z_j} (x_1 \cdot z_j + x_2 \cdot \bar{Q}_h^2 \cdot z_j + x_3 \cdot z_j^3) \cdot \bar{Q}_h^k \, \mathrm{d}\bar{Q}_h \tag{12}$$

$$\bar{f}(\bar{Q}_h) = \sum_{k=0}^{l} a_k \bar{Q}_h^k \tag{13}$$

The partial derivative of annual fan energy consumption with respect to stack capacity can be expressed as

$$\begin{aligned}\frac{\partial \bar{E}}{\partial y_i} = \sum_{j=1}^{n} \sum_{k=0}^{l} a_k \Bigg\{ \frac{\partial z_j}{\partial y_i} \Bigg[\left(\frac{k+4}{k+3} x_2 + \frac{k+4}{k+1} x_3 \right) z_j^{k+3} + \frac{k+2}{k+1} x_1 z_j^{k+1} \\ - \left(\frac{x_1 + 3x_3 z_j^2}{k+1} + \frac{x_2 \cdot z_{j-1}^2}{k+3} \right) z_{j-1}^{k+1} \Bigg) \Bigg] \\ - \frac{\partial z_{j-1}}{\partial y_i} [(x_1 + x_2 z_{j-1}^2) z_j z_{j-1}^k] \Bigg\} = 0 (i = 1, 2, \ldots, n-1)\end{aligned} \tag{14}$$

Using Equation (14) to solve for optimal capacities of a two-stack system gives the following equation set:

$$\begin{aligned}\sum_{j=1}^{3} \sum_{k=0}^{l} a_k \Bigg\{ d_{j,1} \Bigg[\left(\frac{k+4}{k+3} x_2 + \frac{k+4}{k+1} x_3 \right) z_j^{k+3} \\ + \frac{k+2}{k+1} x_1 z_j^{k+1} - \left(\frac{x_1 + 3x_3 z_j^2}{k+1} + \frac{x_2 z_{j-1}^2}{k+3} \right) z_{j-1}^{k+1} \Bigg) \Bigg] \\ - d_{j-1,1} [(x_1 + x_2 z_{j-1}^2) z_j z_{j-1}^k] \Bigg\} = 0\end{aligned} \tag{15}$$

$$z_0 = 0 \tag{16}$$

$$z_1 = y_1 \tag{17}$$

$$z_2 = y_2 \tag{18}$$

$$z_3 = y_1 + y_2 \tag{19}$$

$$y_1 + y_2 = 1 \tag{20}$$

where

$$\{d_{j,1}\}(j = 0, 1, 2, 3) = \{0, 1, -1, 0\}$$

Equation (15) is obtained by setting the derivative of the annual fan energy consumption to zero, and the five equations that follow it express the relationships between stack capacities and stack combination capacities.

For a three-stack system, the optimal stack sizes can be determined using the following equation set.

$$\begin{aligned}\sum_{j=1}^{7} \sum_{k=0}^{l} a_k \Bigg\{ d_{j,1} \Bigg[\left(\frac{k+4}{k+3} x_2 + \frac{k+4}{k+1} x_3 \right) z_j^{k+3} + \frac{k+2}{k+1} x_1 z_j^{k+1} \\ - \left(\frac{x_1 + 3x_3 z_j^2}{k+1} + \frac{x_2 z_{j-1}^2}{k+3} \right) z_{j-1}^{k+1} \Bigg) \Bigg] \\ - d_{j-1,1}[(x_1 + x_2 z_{j-1}^2) z_j z_{j-1}^k] \Bigg\} = 0\end{aligned} \tag{21a}$$

$$\begin{aligned}\sum_{j=1}^{7} \sum_{k=0}^{l} a_k \Bigg\{ d_{j,2} \Bigg[\left(\frac{k+4}{k+3} x_2 + \frac{k+4}{k+1} x_3 \right) z_j^{k+3} + \frac{k+2}{k+1} x_1 z_j^{k+1} \\ - \left(\frac{x_1 + 3x_3 z_j^2}{k+1} + \frac{x_2 z_{j-1}^2}{k+3} \right) z_{j-1}^{k+1} \Bigg) \Bigg] \\ - d_{j-1,2}[(x_1 + x_2 z_{j-1}^2) z_j z_{j-1}^k] \Bigg\} = 0\end{aligned} \tag{21b}$$

$$z_0 = 0 \tag{22}$$

$$z_1 = y_1 \tag{23}$$

$$z_2 = y_2 \tag{24}$$

$$z_3 = \begin{cases} y_1 + y_2 & (y_1 + y_2 \leqslant 0.5), \\ y_3 & (y_1 + y_2 \geqslant 0.5) \end{cases} \tag{25}$$

$$z_4 = \begin{cases} y_3 & (y_1 + y_2 \leqslant 0.5), \\ y_1 + y_2 & (y_1 + y_2 \geqslant 0.5) \end{cases} \tag{26}$$

$$z_5 = y_1 + y_3 \tag{27}$$

$$z_6 = y_2 + y_3 \tag{28}$$

$$z_7 = y_1 + y_2 + y_3 \tag{29}$$

$$y_1 + y_2 + y_3 = 1 \tag{30}$$

where if $y_1 + y_2 \leqslant 0.5$:

$$\{d_{j,1}\}(j = 0, 1, 2, 3, 4, 5, 6, 7) = \{0, 1, 0, 1, -1, 0, -1, 0\}$$

$$\{d_{j,2}\}(j = 0, 1, 2, 3, 4, 5, 6, 7) = \{0, 0, 1, 1, -1, -1, 0, 0\}$$

and if $y_1 + y_2 \geqslant 0.5$:

$$\{d_{j,1}\}(j = 0, 1, 2, 3, 4, 5, 6, 7) = \{0, 1, 0, -1, 1, 0, -1, 0\}$$

$$\{d_{j,2}\}(j = 0, 1, 2, 3, 4, 5, 6, 7) = \{0, 0, 1, -1, 1, -1, 0, 0\}$$

Equations (21a) and (21b) are obtained by setting the partial derivative of the annual fan energy consumption to zero, and the nine equations that follow it express the relationships between stack capacities and stack combination capacities.

3.2. Numerical method

Optimal stack sizes can also be determined using a numerical method that is based directly on Equation (7b). The numerical method can use actual fan efficiency values from fan curves. Also, there is no need to express the time density with a polynomial of the exhaust airflow.

Several options for the numerical method are available. After comparing different methods for this special application, the Hooke–Jeeves method (Bronshtein and Semendyayev, 1998) was used in this study.

4. APPLICATIONS

The potential fan energy savings and applications of the VSD and multi-stack techniques are demonstrated in this section with an example. The facility in the example is a typical university laboratory building, which has five floors including one underground floor. The laboratories are located on the first, second and third floors and have approximately 100 VAV fume hoods. The laboratory design exhaust airflow is $28.3\,\mathrm{m}^3/\mathrm{s}$ (60 000 CFM). Individual fume hood airflow increases from 40 to 100% of the design airflow when the sash position moves from fully closed to fully open.

Table I summarizes the number of hours, relative time distribution and average relative time density for each airflow range. The number of hours was determined based on the operation

Table I. Laboratory exhaust airflow time distribution and time density.

Airflow range $\bar{Q}_{h,k-1}, \bar{Q}_{h,k}$	0.0–0.3	0.3–0.4	0.4–0.5	0.5–0.6	0.6–0.7	0.7–0.8	0.8–0.9	0.9–1.0
Number of hours T_k	0	270	1110	1900	2200	1900	1110	270
Relative time distribution $\bar{T}_k$	0	0.0308	0.1267	0.2169	0.2511	0.2169	0.1267	0.0308
Relative time density $\bar{f}(\bar{Q}_h)$	0	0.308	1.267	2.169	2.511	2.169	1.267	0.308

patterns of similar facilities on the campus. There are a total of 8760 operating hours. The relative time distribution is calculated with Equation (11) using the number of operating at each range, hours and the total operating hours (8760). The relative time density is calculated using Equation (8) according to the relative time distribution and the finite relative airflow interval (0.1).

Equation (31) was obtained using a regression analysis of the relative time density and the relative exhaust airflow information in Table I. The sixth order regression has excellent accuracy for this airflow density except in the low flow range.

$$\bar{f}(\bar{Q}_h) = -0.635 + 20.2\bar{Q}_h - 182\bar{Q}_h^2 + 643\bar{Q}_h^3 - 948\bar{Q}_h^4 + 599\bar{Q}_h^5 - 132\bar{Q}_h^6 \tag{31}$$

The design fan head is 1620 Pa (6.5″H_2O). The design static pressure at the fume hood is −375 Pa (−1.5″H_2O) relative to the occupied space. The stack design pressure loss is 75 Pa (0.3″H_2O). The main duct pressure loss (from the location of the fume hood static pressure sensor to the fan inlet) at design is 1170 Pa (4.7″H_2O). Since the fan is close to the stack, the duct pressure loss after the fan is approximated as zero. The fractions of pressure loss pressure difference in each section are calculated to be 0.28, 0.72, and 0 for x_1, x_2 and x_3, respectively.

The relative fan power and fan head curves are generated from the actual fan performance curve under design conditions, as shown in Figure 2.

The optimal stack sizes were determined using both the analytical and numerical methods. Table II summarizes the optimal stack capacities, stack sizes, and annual average fan power. The analytical and numerical methods produced similar stack capacities. The maximum

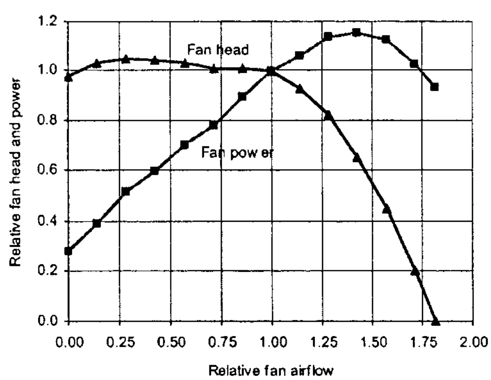

Figure 2. Fan curve.

Table II. Optimal stack capacities and annual fan energy consumption.

		Analytical method		Numerical method	
System	Stack	Capacity (%)	Relative fan energy (%)	Capacity (%)	Relative fan energy (%)
Two-stack	1	25.5	50	25.1	50
	2	74.5		74.9	
Three-stack	1	15.0	46	15.0	46
	2	27.6		27.9	
	3	57.4		57.1	

difference was 0.4% of the design capacity. This indicates that the constant efficiency assumption has little impact on the optimization.

For the two-stack system, the optimal stack capacities came out to be 25.5 and 74.5%. When the laboratory exhaust airflow is less than 74.5% of design, the large stack is activated. The fan airflow is 74.5% of the design airflow. The makeup air varies from 0 to 44.5% of the design airflow. When the laboratory exhaust airflow is higher than 74.5% of the design airflow, both stacks are activated. The fan airflow is 100% of the design airflow. The makeup airflow varies from 0 to 25.5% of the design airflow. The maximum makeup airflow is 44.5% of the design airflow, and occurs when the laboratory exhaust airflow is at 30%.

For the three-stack system, the optimal stack capacities are 15, 27.6, and 57.4%. When the laboratory exhaust is lower than 42.6% of the design airflow, two smaller stacks are activated, and the fan airflow is 42.6% of design. The makeup airflow varies from 0 to 12.6% of the design airflow. When the exhaust is between 42.6 and 57.4%, the largest stack is activated, and the fan airflow is 57.4% of design. The makeup airflow varies from 0 to 14.8% of design. When the exhaust is between 57.4 and 72.4% of design, the largest and the smallest stacks are activated and the fan airflow is 72.4% of design. The makeup airflow varies from 0 to 15% of the design airflow. When the exhaust is between 72.4 and 85% of design, the two largest stacks are activated, and the fan airflow is 85% of design. The makeup airflow varies from 0 to 12.4% of the design airflow. Finally, when the exhaust is higher than 85% of design, all stacks are activated and the fan airflow is 100% of design. The makeup airflow varies from 0 to 15% of the design airflow. The makeup airflow is never higher than 15% for this three-stack system.

The VSD and multi-stack techniques reduce the annual fan energy consumption significantly. The annual average fan power is reduced to 50% for the two-stack system, and 46% for the three-stack system—a savings of 50% or better. This comparison is for a conventional system, in which the exhaust fan power is assumed to be at the design value for all exhaust airflow conditions. The savings is due to reduced makeup airflow and to reduced fan head. Under partial airflow conditions, the pressure loss between the fume hood's static pressure sensor and the fan inlet decreases proportionally to the square of the airflow ratio. This reduction in pressure loss translates to a fan head reduction under partial airflow conditions because of the use of a VSD.

The makeup airflow of the three-stack system is significantly less than that of the two-stack system because the three-stack system has more capacity combinations. Consequently, the three-stack system uses less fan energy. The potential annual fan energy savings of the three-stack system is 4% greater than for the two-stack system.

It appears that more stacks means higher fan energy savings. However, the control of a three-stack system is much more complex than for a two-stack system, and a three-stack system switches more frequently than a two-stack system. This issue must be considered when the final design decision is made.

5. CONCLUSIONS

The application of VSD and multi-stack techniques can significantly reduce the annual fan energy of laboratory makeup air exhaust systems, where constant stack exit velocity is required. The potential fan energy savings depends on an optimal selection of the number of stacks and the size of each stack, and on the exhaust system ductwork design (pressure loss distribution under design flow conditions).

Optimal stack sizes can be determined using either analytical or numerical methods provided in this paper. The necessary inputs for this optimization are the laboratory exhaust flow pattern, and the exhaust system pressure loss distribution.

The greater the number of stacks, the higher the potential fan energy savings. However, the complexity of controlling the system increases sharply as the number of stacks increases. The final decision for an optimal system design must take this issue into consideration. Typically, a two-stack system can achieve the majority of the available energy savings with minimal control system implementation.

The VSD energy loss was not considered in the modelling or in the example. The power loss of the VSD may cause the fan energy savings to be lower than what was presented in this paper.

NOMENCLATURE

a_k	= coefficient for the time density regression
$\bar{E}$	= relative annual fan energy consumption
$f(\bar{Q}_h)$	= time density versus relative laboratory exhaust airflow (h)
$\bar{f}(\bar{Q}_h)$	= relative time density versus relative laboratory exhaust airflow
H	= fan head (Pa or in wg)
H_{d}	= design fan head (Pa or in wg)
$\bar{H}$	= relative fan head
n	= stack combination number
m	= stack number; segment of airflow interval
Q	= fan airflow (m^3/s or CFM)
Q_{d}	= design fan airflow or exhaust system design airflow (m^3/s or CFM)
Q_h	= laboratory exhaust airflow (m^3/s or CFM)
$\bar{Q}$	= relative fan airflow
$\bar{Q}_h$	= relative laboratory exhaust airflow
T	= total operating hours (h)
T_k	= operating time distribution in a range from $\bar{Q}_{h,k-1}$ to $\bar{Q}_{h,k}$ (h)
$\bar{T}_k$	= relative time distribution in a range from $\bar{Q}_{h,k-1}$ to $\bar{Q}_{h,k}$
W	= fan power (kW or hp)
W_{d}	= design fan power (kW or hp)
$\bar{W}$	= relative fan power
x_1	= relative static pressure difference between the sensors at design
x_2	= relative design pressure loss in the duct upstream of the fan
x_3	= relative design pressure loss in the duct downstream of the fan
y_i	= stack design capacity series
z_j	= stack combination design capacity series
ΔP_2	= duct pressure loss from the fume hood to the fan (Pa or in wg)
$\Delta P_{2,\mathrm{d}}$	= design pressure loss in the duct upstream of the fan (Pa or in wg)
ΔP_3	= duct pressure loss from the fan to the stack (Pa or in wg)
$\Delta P_{3,\mathrm{d}}$	= design pressure loss in duct downstream of the fan (Pa or in wg)
ΔP_{s}	= static pressure difference between the two static pressure sensors (Pa or in wg)
$\Delta P_{\mathrm{s,d}}$	= static pressure difference between the sensors at design (Pa or in wg)

REFERENCES

Bronshtein IN, Semendyayev KA. 1998. *Handbook of Mathematics*. Springer-Verlag: Berlin-Heidelberg, New York.

Hitchings DT, Shull RS. 1993. Measuring and calculating laboratory exhaust diversity—three case studies. *ASHRAE Transactions* **99**(2):1059–1071.

Moyer R, Dungan J. 1987. Turning fume hood diversity into energy savings. *ASHRAE Transactions* **93**(2):1822–1832.

Rabiah TM, Wellenbach JW. 1993. Determining fume hood diversity factors. *ASHRAE Transactions* **99**(2):1090–1096.

Varley JO. 1993. The measurement of fume hood use diversity in an industrial laboratory. *ASHRAE Transactions* **99**(2):1072–1080.

Wang G, Cui Y, Yuill D, Liu M. 2002. Development of multi-stack exhaust systems for laboratory buildings. *Proceedings of ASME Solar Energy Conference*, Reno, NV.

Wang G, Liu M. 2001. Energy efficient single stack exhaust fan systems (E^3S^3F). *Proceedings in International Conference for Enhanced Building Operations*, Austin, Texas, 179–184.