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Chromatic Aberration of Eyepieces in Early Telescopes

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Abstract

The twofold objective of this study is (1) to identify and give a brief review of the historical development of the various designs of early (pre-1850) telescope eyepieces, and (2) to determine by measurements and calculations the axial and lateral chromatic aberrations of a number of extant eyepieces from that period in order to provide basic data on which to judge the relative quality of different eyepiece forms. Eight distinct types of eyepieces containing one to five lens elements are discussed and illustrated. The second objective was addressed by measuring the focal lengths of the individual lens elements and the lens spacings in each eyepiece. The data were processed by a ray-tracing program that yields the chromatic aberration (CA) and focal length of each of the eyepieces. Twenty-one telescopes from that period were studied in this way. Similar calculations were also made using data available in the literature on several additional historic telescopes. The conclusions drawn from the data are: (1) Telescopes with Galilean eyepieces have a smaller CA than those with Keplerian or Huygens eyepieces. (2) The two- lens Keplerian terrestrial eyepieces are demonstrated to have larger axial and lateral CA than any of the other eyepiece types. (3) Calculations from examples of telescopes by Divini and Campani for which data were available indicate that while the axial CAs of the two were the same, Campani's instrument had a much smaller lateral CA. (4) Five element eyepieces were found to be better corrected for CA than the early four-lens systems that replaced them. (5) This study tends to confirm the speculation that the reason makers of early achromatic telescopes overcorrected the objective lenses was to compensate for the CA of the eyepieces.

Contents

1. Introduction

Among the many inventions to come out of the seventeenth century, the telescope must surely rank among the most important in terms of its subsequent uses. Almost immediately, it found service in the military and commercial worlds. Soon after, its obvious aid to astronomy became known through the spectacular observations by Galileo and others, observations that ultimately led to a whole new conception of the universe.

Unfortunately, early telescopes were not considered to be trustworthy, since they did not produce clear, sharp images due to surface irregularities of the lenses, lack of internal uniformity of the glass, and other defects. As these problems were solved, the images improved, but one stubborn condition remained unsolved for a long time. It is now called "chromatic aberration" (CA), but its origin remained a mystery until Isaac Newton showed that white light is composed of light of many colors and that lenses, however carefully made, produce fuzzy, colored images because different colors are focused differently by lenses. Newton despaired of ever being able to correct the problem, and it was another half century before an effective method was devised, and the achromatic telescope was born. While this resulted in a marked improvement of images and allowed greater magnifications, there was still room for additional improvements that continued on into the nineteenth century.

While much attention has rightly been paid to the invention of achromatic objectives for telescopes and to their eighteenth-and nineteenth-century development, there have been few studies of the other end of the telescope.[1] The telescope eyepiece plays an important role in determining the clarity and quality of the image as well as the field of view and the ease of use, so it too deserves study, both of its historical development and of the comparative qualities of its different forms.

Albert Van Helden has written an authoritative article about the development of the compound eyepiece, but he restricted his historical study to the thirty-year period ending at 1670.[2] Rolf Willach has published some very important data on the three forms of the telescope in use before 1650.[3] This is illustrated by his detailed optical tests on four extant

1 M. Eugene Rudd, Duane H. Jaecks, Rolf Willach, Richard Sorenson and Peter Abrahams, "New light on an old question: Who invented the achromatic telescope?" *Journal of the Antique Telescope Society,* 19 (2000) 3–12.

2 Albert Van Helden, "The development of compound eyepieces, 1640 1670," *Journal of the History of Astronomy,* 8 (1977), 26–37.

3 Rolf Willach, "The development of telescope optics in the middle of the seventeenth century," *Annals of Science*, 58 (2001), 381–98.

telescopes from that era. In addition, there is a critical discussion of the early methods of polishing lenses and their effect on the quality of the telescopes. Henry King's well-known history of the telescope has descriptions of the various forms of eyepiece, but they are scattered throughout the book.[4] Deborah Jean Warner included a one-page historical discussion of eyepieces in an article on terrestrial telescopes.[5] This included references to specific examples of three-, four-, and five-element eyepieces made by Dollond and a brief discussion of the Ramsden eyepiece. Reginald Cheetham's little book on old telescopes surveys the various types of eyepieces and is especially useful, since it includes ray diagrams and brief histories of each.[6] Thomas Court and Moritz von Rohr's paper presents a great deal of valuable historical information on eyepieces.[7] But of all these sources, Willach's is the only one that provides numerical data on the actual performance of early telescope eyepieces, but these only for a very restricted early period.

The first section of this paper briefly discusses the development and usage of different kinds of telescope eyepieces in use up to the middle of the nineteenth century. The second section describes measurements and calculations made on actual and idealized (or generic) telescopes to determine the CA of each type of eyepiece in order to see how it depends on the design and how it relates to that of other designs. Attention is also paid to the relationship between the CA of the eyepiece and that of the objective.

Chromatic aberration is the result of the variation of the index of refraction with wavelength that is characteristic of all transparent media such as glass. The result is that different component colors of light from a point source are focused differently by a simple lens resulting in a blurred image and color fringing along the edges of objects. A combination of two or more lenses is needed to correct this aberration. In the case of objective lenses, lens elements of different kinds of glass placed in contact or close together are usually used. However, compound eyepiece designs achieve a degree of achromatism with lens elements of the same type of glass by choosing the optimum combinations of focal lengths and lens separations.

The axial (or longitudinal) CA is the spread dx of the image distances along the axis of the lens for different colors, while the spread of image sizes perpendicular to the axis is the lateral (or transverse) CA. The latter is described by the quantity df, the variation of the focal length, since the magnification of a lens is directly related to its focal length f. For a single thin lens, the quantities dx and df are equal, but in a multi-lens system, they can be quite different. In eyepieces, the lateral CA is the more important quantity, since it describes the color fringing resulting from unequal magnifications for different colors of the spectrum.

Obviously, other considerations such as field of view, eye relief, and flatness of field are also important in understanding eyepiece design, but the focus of this study is on CA which historically was one of the greatest hurdles in improving telescopes.

4 Henry C. King, *The History of the Telescope* (New York, 1955).

5 Deborah Jean Warner, "Telescopes for Land and Sea," *Rittenhouse: Journal of the American Scientific Instrument Enterprise,* 12 (1998), 33–54.

6 Reginald J. Cheetham, *Old Telescopes* (Southport, UK, 1997).

7 Thomas H. Court and Moritz von Rohr, "A history of the development of the telescope from about 1675 to 1830 based on documents in the Court Collection," *Transactions of the Optical Society,* 30 (1929) 207–60.

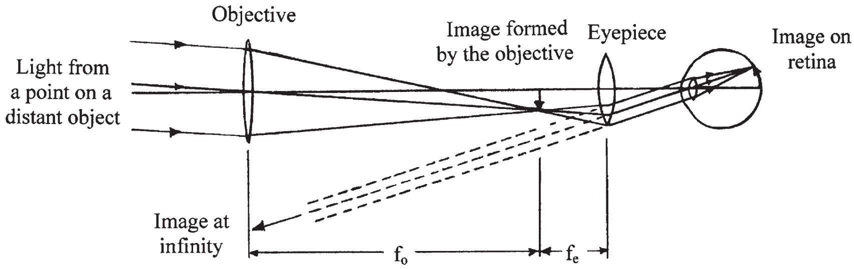

Figure 1. Simplified schematic diagram showing the overall operation of a telescope.

2. The role of the eyepiece in a telescope

Basically, the objective lens forms a real image of a distant object on its focal plane at a distance from the lens equal to its focal length f_o. Instead of catching that image on a screen as in a camera, it is viewed by the eyepiece used as a magnifier is used to view small objects. Since the relaxed eye needs to have parallel rays entering its lens for clearest vision, the eyepiece is positioned at a distance f_e from the image formed by the objective where f_e is the focal length of the eyepiece. The lens of the eye then focuses the final image on the retina.

Note from Figure 1 that if $f_o > f_e$, as in the usual case, the angle between the optical axis and the rays entering the eye is larger than the angle between the optical axis and the rays from the object. It is this angular magnification, given by the relation $M = f_o/f_e$, that makes the telescope the useful instrument that it is.

In Figure 1, the eyepiece is represented by a single lens, but, as we shall show, the single lens was replaced by a compound lens system early in the history of the telescope. It is also clear from the diagram that in this simple telescope, the eye views the image (along the dashed lines) in a downward direction, whereas the incident light from the object is from above the axis. Thus, the image appears inverted, a condition that can be corrected by the use of an erecting eyepiece.

Figure 2. A Galilean eyepiece on an English telescope from c. 1660.

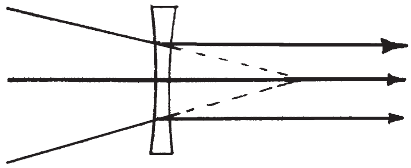

Figure 3. Schematic ray diagram of a Galilean eyepiece. The rays converging from the left are from the objective lens which is not shown.

3. Early telescope eyepiece types

Most of the early telescope eyepieces fall into one of the eight categories listed and described below. The figures show the layouts and ray diagrams for each system. The converging light rays from the objective lenses are shown approaching the eyepieces from the left. The emerging parallel rays on the right go to the eye.

The designs shown are the generic ones. For example, the generic three-lens Schyrlean eyepiece design has three equal focal length lenses with spacings of twice the focal length. But for various reasons, individual makers used focal lengths and spacings that varied around the generic values.

3.1. Galilean eyepiece

The eyepiece of a Galilean (or "Dutch") telescope consists of a single diverging or concave lens and was the type used from 1608, when the telescope was invented in Holland. Figure 2 shows an example of such an eyepiece, and Figure 3 is the ray diagram for it. Note that in these ray diagrams, each time the outer rays cross the axis, the image is inverted. Therefore, an even total number of crossings (usually two, or as in the Galilean eyepiece, zero) are necessary to form an erecting eyepiece. While an inverted image is acceptable in astronomical work, an erect image is usually demanded from telescopes used in terrestrial applications.

Figure 4. Single-lens Keplerian eyepiece with a prism for use with an unsigned English astronomical transit from the last half of the 19th century.

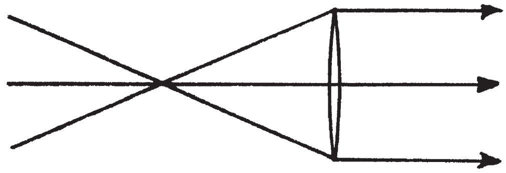

Figure 5. Ray diagram of a Keplerian eyepiece.

Although the two forms of the Keplerian eyepiece (see below) became available early in the seventeenth century, the Galilean type still remained popular. Some preferred it for astronomical work, even after the Huygens eyepiece (see below) was invented. For terrestrial applications, it was used almost exclusively until mid- century, at which time it was superseded by the Schyrlean erecting eyepiece. But in the early eighteenth century, it found a new use in the "prospect" or "perspective" glasses, which were small, low-power telescopes that were popular for casual viewing. The Galilean eyepiece is still used today in opera glasses and toy telescopes. It has the advantages of simplicity and an erect and relatively aberration-free image. Its disadvantages are its small field of view, especially at higher magnifications, and the fact that it cannot be used with cross hairs or reticles.

3.2. Keplerian eyepiece

Johannes Kepler, in his book *Dioptrice* (1611), described a telescope with a single positive or convex lens as an eyepiece, as shown in Figures 4 and 5. In this case, the rays make a single crossing of the axis indicating the formation of an inverted image.

While Kepler never built a telescope, one of this form that dated from 1613 was in the Dresden Mathematical-Physical Salon until it was destroyed during the Second World War. The first recorded use of the Keplerian eyepiece was by the astronomer Christoph Scheiner in 1617. Francesco Fontana began making such telescopes in Italy around 1637.[8] Starting in the 1640s, it was used in the very long astronomical telescopes of Christiaan Huygens, Johannes Hevelius, Giuseppe Campani, and others. For example, in 1655, Constantine Huygens, in collaboration with his brother Christian, made a Keplerian telescope 12 feet long that was used to examine the planet Saturn and finally determine its true ring structure.[9]

The Keplerian eyepiece has the advantage of a larger field of view than the Galilean form and may be used with cross hairs or a reticle. However, its lateral CA is large, and its inverted image makes it unsuitable for terrestrial purposes. Thus, it was used primarily in astronomical applications.

3.3. Two-lens Keplerian terrestrial eyepiece

In *Dioptrice*, Kepler also suggested the use of an additional positive lens in the eyepiece to reinvert the image. In compound eyepieces, the lens closest to the objective is called the

8 Maria Luisa Righini Bonelli and Albert Van Helden, "Divini and Campani: A forgotten chapter in the history of the Accademia del Cimento," Supplement to *Annali dell' Istituto e Museo di Storia della Scienza* (Florence, 1981), 7.

9 R. T. Gunther, *Early Science in Oxford, Vol. 2, Astronomy* (Oxford, 1923), 296.

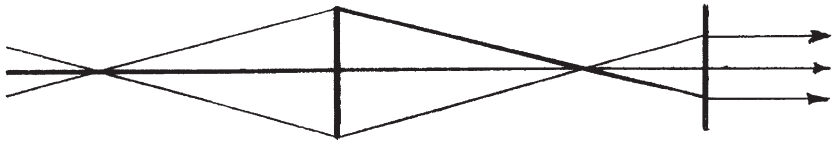

Figure 6. Ray diagram of a two-lens Keplerian terrestrial eyepiece. The focal length of each lens is f and the spacing is $2f$.

field lens, since the field of view of the telescope is dependent on the diameter of this lens. The lens closest to the eye is, of course, the eye lens. The ray diagram of Figure 6 shows the even number of crossings necessary for an erect image.

It is also known as the Scheiner eyepiece, since Christoph Scheiner had such an eyepiece in 1630. However, this form has rarely been used because of its excessive lateral and axial chromatic aberration.

3.4. Three-lens Schyrlean eyepiece

In 1645, the Capuchin monk Anton Maria Schyrl de Rheita published his *Oculus Enoch et Eliae*, in which he described a three-lens design that became the first successful erecting eyepiece.[10] An example is shown in Figure 7, and a ray diagram is given in Figure 8.

Rheita himself made some telescopes to that pattern, and they were also made by the Augsburg telescope maker, Johannes Wiesel. The Schyrlean eyepiece was such an important advance that it quickly spread through Germany to France, Italy, and England, and was produced by many instrument makers. For the next hundred years, it was by far the most widely used terrestrial eyepiece.

In its basic form, it has three equal focal length lenses with spacings equal to twice the focal length. John Herschel, who called it "Rheita's eyepiece," described it by saying that the usual single convex lens was followed by a short two-lens telescope.[11]

Some examples examined have two lenses with unequal focal lengths mounted at opposite ends of a short inner assembly that could be reversed to provide two different magnifications. This inner assembly typically included instructions to insert it one way for clearer, smaller images and the other way for greater magnification but less clarity.

The Schyrlean eyepiece appeared in vellum-covered pasteboard telescopes with wooden lens cells well into the first-half of the eighteenth century. In the later part of that period, it was also used in wooden sea telescopes. The well-known seventeenth- century telescope maker, Giuseppe Campani, used the Schyrlean three-lens eyepiece exclusively. This type of eyepiece was also used in 1683 on the only complete existing telescope made by Christiaan Huygens.[12] He named it Campanine because he learned about the three-lens eyepiece from Campani.

10 Schyrl de Rheita, A.M., *Oculus Enoch et Eliae save Radius Sidereomysticus* (Antwerp, 1645).

11 Sir John F. W. Herschel, *The Telescope (from the Encyclopedia Britannica)* (Edinburgh, 1861), 52.

12 Peter Louwman, "Christiaan Huygens and his telescopes," in *Titan: From Discovery to Encounter,* ed. Karen Fletcher (Noordwijk, the Netherlands, 2004), 103–14 (113).

Figure 7. A three-lens Schyrlean eyepiece from an unsigned Italian telescope from the early 18th century.

3.5. Huygens eyepiece

In 1662, while in Paris, Christiaan Huygens invented an important two-lens eyepiece that has been used with both refracting and reflecting telescopes. His design consisted of two plano-convex lenses, as shown in Figures 9 and 10.

The focal length of the field lens is typically 1.5–3 times that of the eye lens. If the two lenses are spaced by half the sum of their focal lengths, the lateral CA is completely eliminated. Furthermore, the portion of the field of view that is sharp is greater than that of the Keplerian eyepiece. Its shortcomings are its curvature of field, short eye relief, and the fact that it is non-erecting. Also, since the image formed by the objective lens must come to a focus at a point between the two lenses, any reticle or cross hairs must be placed between the two lenses, an undesirable arrangement, since the reticle is then viewed only by the uncorrected eye lens. However, the field of view is moderately good, and although many other astronomical eyepieces have been invented since that time, the Huygens eyepiece still finds uses today.

3.6. Ramsden eyepiece

In 1782, Jesse Ramsden proposed a non-erecting eyepiece that was an improvement over the Huygens design in that the image to be viewed was in front of the first lens, as shown in Figure 11.[13]

In its basic form, the focal lengths of the two lenses are equal, and their spacing is again equal to half the sum of their focal lengths, the arrangement that preserves the correction for lateral CA. However, in this case, the field lens is at the focal plane of the eye lens, and any scratches, dust or specks on the former lens are seen magnified in the field of view. Therefore, the spacing is usually changed to about two-thirds of the focal length of either lens. However, this change entails the loss of the advantage of complete achromaticity. The Ramsden eyepiece is still in use today in astronomical telescopes.

13 J. Ramsden, "A description of a new construction of eye-glasses for such telescopes as may be applied to mathematical instruments," *Philosophical Transactions of the Royal Society* 73 (1782) 94–99.

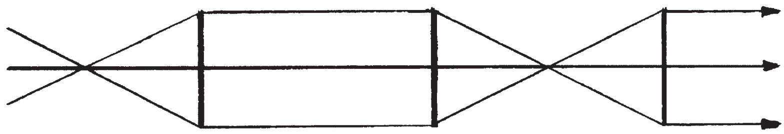

Figure 8. Ray diagram of the three-lens Schyrlean eyepiece. The focal length of each lens is *f* and the spacings are 2*f*.

3.7. Four-lens terrestrial eyepiece

In 1651, Schyrl de Rheita suggested the use of four lenses in an erecting eyepiece.[14] Such telescopes were known to have been made by Wiesel in the first-half of the seventeenth century, and in 1665 Balthasar de Monconys described the optical arrangement of such an instrument in *Journal des voyages de Monsieur de Monconys.*[15] Eustachio Divini of Rome, who flourished in the period 1646–1685, made telescopes with four-lens eyepieces. However, the four-lens system was little used until the middle of the eighteenth century.

John Herschel claimed that Dollond was responsible for replacing the eye lens of the three-lens Schyrlean eyepiece with a Huygens combination, thus making a four- glass eyepiece.[16] However, he gave no reference, and we do not even know which member of the Dollond dynasty he was referring to or when this innovation took place. What is known is that in 1753, John Dollond noted that telescopes with four- lens eyepieces were being "sold in the shops."[17] A ray diagram of it is shown in Figure 12. In the early version, this type of eyepiece was housed in a segmented brass draw tube, sometimes with one segment for each lens element. Later, four-lens eyepieces (with the first two lenses forming an erector and the last two forming a Huygens eyepiece) were made by Dollond, Jesse Ramsden, and many others. By about 1820, these were made with an un-segmented eyepiece tube that housed two smaller two-lens sub-assemblies, one screwed into each end. This design subsequently became a standard for most makers of hand telescopes. The example shown in Figure 13 is a transitional model with just one sub-assembly and two segments.

By changing the spacing of the sub-assemblies, the magnification could be varied. A system to do this conveniently led to what John Herschel called "Mr. G. Dollond's Pancratic Telescope."[18] It can be thought of as a precursor to the zoom lens. However, in his *Economy of the Eyes,* William Kitchiner claims that he made the invention in 1820.[19]

3.8. Five-lens terrestrial eyepiece

Although telescopes from the late seventeenth century with five-lens eyepieces are known, the Schyrlean form dominated at that time. In his 1753 paper, John Dollond

14 Willach (note 3), 390.

15 Willach (note 3), 392–94.

16 Herschel (note 11), 59.

17 John Dollond, "A letter from Mr. John Dollond to Mr. James Short, F.R.S. concerning an improvement of refracting telescopes," *Philosophical Transactions of the Royal Society,* 48 (1753), 103–107.

18 Herschel (note 11), 60.

19 William Kitchiner, *The Economy of the Eyes, Part II. Of Telescopes* (London, 1825), 200–209.

Figure 9. The Huygens eyepiece from a Berge Late Ramsden telescope, c.1800. L to R: field lens, eye lens, red filter for solar viewing.

touted the advantages of a five-lens eyepiece, claiming that aberrations were reduced.[20] Telescopes with that design were made by Dollond, John Gilbert, James Ayscough, and perhaps others in the 1760s (Figures 14 and 15). However, the four- lens design was simpler than the five-lens design, less expensive to manufacture, and had less light loss from reflections from lens surfaces. So, it is not surprising that by about 1780, the four-lens eyepiece had become the standard in terrestrial telescopes.

4. Determination of eyepiece properties

4.1. Method

Except for the simplest single-lens construction, eyepieces are fairly complex optical systems, some consisting of as many as ten or more optical surfaces. They are best studied by the use of ray tracing, made easier by a number of computer programs that are available. For the present purpose, a relatively simple paraxial ray- tracing spreadsheet program has proven to be easy to use and sufficiently accurate for the calculations needed to understand the differences among the various types of eyepieces. In this program, all of the elements are assumed to be thin lenses.

The input data needed are the spacings of the lenses and the focal length and CA of each lens. The dispersion of transparent media is described by the Abbe number, which is defined as

$$\nu = (n_D - 1)/(n_F - n_C),$$

where n is the refractive index, and F, D, and C denote the blue, yellow, and red Fraunhofer wavelengths (486 nm, 589 nm, and 656 nm, respectively). Tables of refractive indices and Abbe numbers are available for many glass types.

The CA of a lens is taken to be $dP = P_F - P_C$, where P is its power (the reciprocal of its focal length). If the actual value of dP for a lens element has been measured, that value can be used in the program. But if not, the CA can be approximated by using the relation $dP = P/\nu$. The value of ν can be estimated, since the individual elements of early eye-

20 Dollond (note 17).

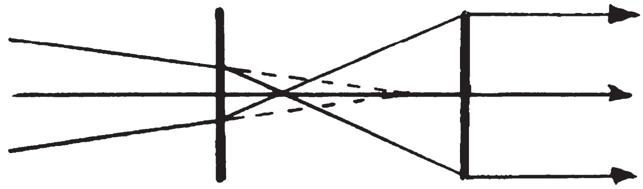

Figure 10. Ray diagram of the Huygens eyepiece. The focal length of the field lens is 2*f*, that of the eye lens is *f*, and the spacing is 1.5*f*.

pieces are always simple crown lenses. The average Abbe number from many measurements of early crown glass is $\nu = 58$ so that value was chosen for the lenses in this study.

The output data from the program are the focal length *f* of the compound eyepiece, the variations of its focal length, d*f*, and of the axial position of the image point, d*x*, over the F–C wavelength interval. The program yields a positive value of *f* for non-erecting eyepieces and a negative value for erecting eyepieces. The quantity d*f* is especially important, since it is a measure of the variation of the magnification and thus of the lateral CA that describes the color fringing of the image.

The program was tested by making direct measurements of the quantities *f* for all of the eyepieces and d*x* for about half a dozen of them. These were compared with the calculations using the spreadsheet program. In spite of the approximations and estimates, the agreement was quite satisfactory in every case.

4.2. Results

The calculated values of axial and lateral CA relative to the focal length are summarized in Table 1, along with the total values of d*x* for the eyepiece and the objective relative to the objective focal length. The eyepieces of twenty-one telescopes made during the period from about 1700 to 1825 were studied. In addition, dimensional data given by Rolf Willach on telescopes by Campani, Wiesel, and Eustachio Divini were entered into the program.[21] The Campani telescope is one in Willach's collection. The measurements on the Wiesel and Divini telescopes are from a book by Balthasar de Monconys.[22] Results for the generic eyepiece designs for the various numbers of lens elements, as well as the generic Keplerian, Galilean, and Huygens eyepieces, are also listed. Focal lengths of 50 and 500 mm were assumed for the eyepieces and objectives, respectively, for all of the generic telescopes and the objectives were assumed to be uncorrected crown lenses. Actual CA measurements were made for the objectives of the measured telescopes.[23]

The results of the calculations can be used to shed light on several historical questions. We now take up five such questions. While some of them have already been answered qualitatively, this study furnishes numerical data to solidify the earlier conclusions.

21 Willach (note 3).

22 Balthasar de Monconys, *The voyages of M. de Monconys* (Paris 1665), appendix.

23 M. Eugene Rudd and Eric P. Rudd, "A new method of measuring chromatic aberration in lenses," *Journal of the Antique Telescope Society,* 25 (2003), 3–8.

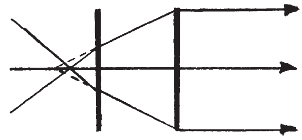

Figure 11. Ray diagram of the Ramsden eyepiece. The focal length of each lens is f and the spacing is about $2/3\,f$.

4.2.1. Keplerian vs. Galilean vs. Huygens eyepieces

In spite of the fact that Kepler's converging lens eyepiece provided the advantages of a larger field of view with greater magnification than the concave, some observers still preferred the Galilean eyepiece because it reportedly gave a clearer view. As late as 1826, Kitchiner devoted an entire chapter in his book to a discussion of the relative merits of the three and quoted both Benjamin Martin (who preferred the concaves) and William Herschel (who discovered the satellites of the planet Uranus with a Galilean telescope).[24] In 1776, Herschel prepared a set of concave eyepieces in order to study the relative merits of the two single-lens eyepieces. He reported that "The glasses, both double and plano-concaves, were alternately tried with convex lenses of an equal focus, and the result, for brightness and distinctness, was decidedly in favor of the concaves."[25] The chief objection to the Galilean form, he wrote, was not being able to use a micrometer with it. Kitchiner himself preferred a compound eyepiece (i.e. the Huygens) over either single one because of its ease of use. He also noted that even though the single-lens eyepieces have clearer images, they were only clearer in the center of the field.[26]

From Table 1, we can see that both the generic single-lens eyepieces have the same axial and lateral CA, as expected. But when combined with a non-achromatic objective lens, the Galilean telescope has a smaller overall axial CA. This is because the value of df for the Galilean eyepiece, being a negative lens, is negative and therefore subtracts from the positive aberration of the (uncorrected or under- corrected) objective, while with the Keplerian eyepiece, the aberrations add. The importance of the effect depends on the telescope's magnification M because the value of df is proportional to the focal length for each lens. Assuming that both lenses are made of the same type of glass, it follows that the ratio of aberrations with the two eyepieces is given by the equation df(Keplerian)/df(Galilean) = $(M + 1)/\ (M - 1)$. Thus, if the magnification is ten, the ratio is 11/9, and little difference would be noted between the two types. But at a magnification of two, the Galilean has only one-third the CA of the Keplerian. So, as far as CA is concerned, the Galilean telescope is better than the Keplerian, but at high magnifications the difference is small.

The singlet objective lenses used before the invention of the achromatic lens in 1758 typically had CAs of about 1.7%. When used with eyepieces other than Galilean, the total CAs of the telescopes are typically 2%. The use of a Galilean eyepiece for a magni-

24 Kitchiner (note 19), 52–66.

25 William Herschel, "A series of observations of the satellites of the Georgian planet ... with an introductory account of the telescopic apparatus that has been used on this occasion ...," Philosophical Transactions of the Royal Society, 105 (1815), 293–362 (297).

26 Kitchiner (note 19), 62.

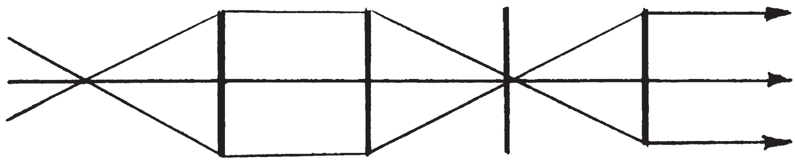

Figure 12. Ray diagram of a generic four-lens eyepiece. The focal length of each lens is *f* and all the spacings are f.

fication of 10 reduces the total axial CA to about 1.5% but for 2 or 3 power, it will be as low as 1%.

Another possible reason that the Galilean is clearer than the Keplerian is that the concave lens of the former has a considerably smaller thickness than the convex lens of the latter and therefore should have less of a problem with striations and other irregularities in the glass.

4.2.2. Keplerian terrestrial eyepiece aberrations

This eyepiece was rarely used, the reason being, according to Willach, that the CA was so large.[27] We can now check that claim against the numbers from a comparison of the calculations made in the current investigation, as shown in Table 1. Indeed, the two-lens Keplerian terrestrial eyepiece has a much larger axial and lateral CA than nearly every other eyepiece in the table, confirming Willach's assertion. Because this type was rarely used, none could be found on which to make an actual measurement except for a modern toy telescope called the "Space Telescope."

4.2.3. Campani vs. Divini

The two Italians, Eustachio Divini and Giuseppe Campani, were great rivals in seventeenth-century telescope making. Contests were even staged to compare their telescopes' abilities to read printing at various distances.[28] Campani was generally considered to be the winner at these events, and it was his telescopes that were used by Giovanni-Domenico Cassini in many of his important astronomical discoveries at the Paris Observatory.

Using Willach's data from telescope eyepieces by these two makers,[29] we can see how they compare to each other and to those by other makers. It can be seen from the table that the axial CA in Campani's three-element eyepiece was slightly smaller than that of Divini's four-element eyepiece but the more important lateral CA was much smaller. Obviously, Campani took greater pains than Divini in designing his eyepieces.

Measurements were made on six telescopes from the first-half of the eighteenth century that also used the three-lens eyepiece arrangement. There were only small variations from the generic design of three lenses of focal length *f* with spaces between the lenses

27 Willach (note 3), 383.

28 Bonelli and Van Helden (note 8).

29 Willach (note 3), 394, 397.

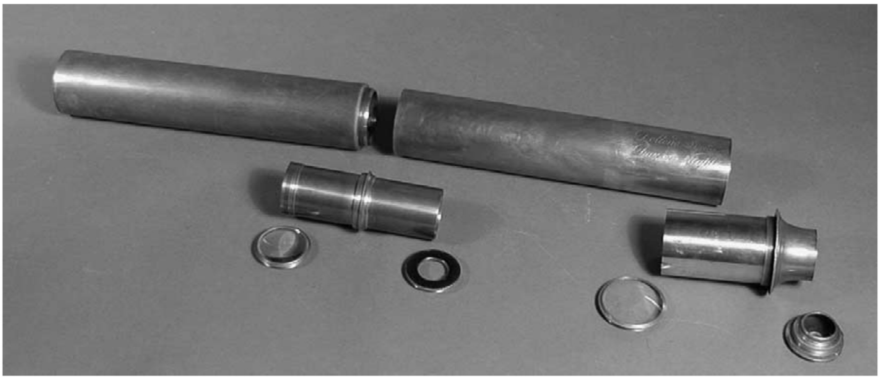

Figure 13. The four-lens eyepiece from a Dollond Day or Night telescope.

equal to 2*f*. Using this prescription, calculations were made with $f = 50$ mm. The lateral CA of the generic system is $df/f = 1.7\%$, which compares well to the 1.5% average of the six telescopes measured.

It is a simple matter to use the spreadsheet program to vary the input parameters to see whether it is possible to improve the CA on our hypothetical eyepiece. As shown in the graphs of Figure 16, by changing the spacing of two elements from the normal value of 100 mm (i.e. 2*f*) to 125 mm, it is possible to eliminate completely the lateral CA. However, this comes at the expense of a small increase in the axial CA. Of course, other aberrations must also be taken into consideration, not to mention the contribution of the objective lens, so the adjustment of all the spacings and focal lengths to obtain an optimum image is a formidable problem, one that Campani evidently came closer to solving than did Divini.

4.2.4. Five-element vs. four-element eyepieces

The four-element form became popular during the twenty years following its introduction by Rheita in 1651 but then fell out of favor until the late eighteenth century, when it was used extensively in the sea telescopes of that time. By about 1800, it had become the standard type and has been used in terrestrial hand telescopes ever since. However, in 1753, John Dollond claimed that a five-lens eyepiece system was superior to the four.[30] A few makers began using it, but it was not well received by most, and it soon fell out of use.[31] What do the numbers say about the comparison of the four-and five-lens eyepieces?

The average lateral CA of the eight four-lens eyepieces measured is $df/f = 0.59\%$, while for four five-lens eyepieces, it averaged 0.25%. Since the five-lens form also has a smaller axial CA it seems to be the better of the two, at least when CA is the only criterion. Nevertheless, it was abandoned in favor of the four-lens system, probably for the reasons mentioned previously. But both were considerable improvements over the three-lens system with its average value of $df/f = 1.5\%$.

30 Dollond (note 17).

31 Court and von Rohr (note 7), 217.

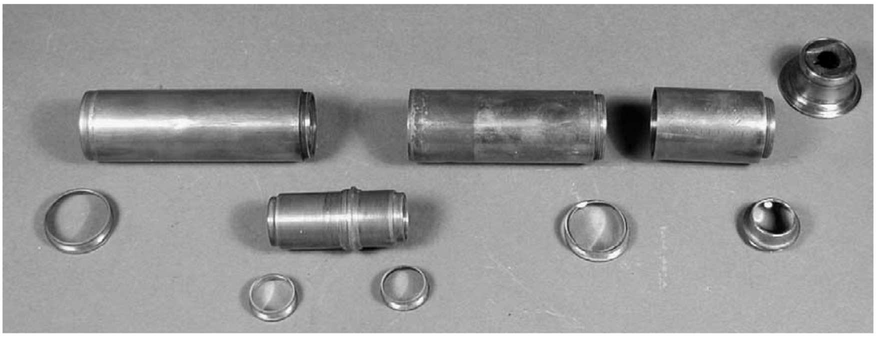

Figure 14. The five-lens eyepiece from a telescope by J. Gilbert from the late 1770s.

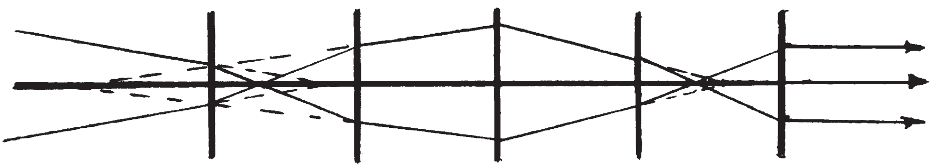

Figure 15. Ray diagram of a generic five-lens eyepiece. The focal length of the field lens is $2f$, that of the eye lens is $2/3\,f$, and that of the other lenses is f. The spacings are all equal to f.

4.2.5. Over-correction of achromatic objectives

Measurements of the chromatic aberrations of many achromatic telescope objectives show that nearly all of those from the period 1758–1850 are markedly over-corrected, meaning that their focal lengths for blue light are greater than for red light, contrary to the behavior of simple uncorrected lenses. One explanation for this is that the telescope makers were attempting to compensate for the axial CA of the eyepieces with over-correction of the objectives.[32] This would require the CA of the objective to be equal in magnitude but opposite in sign to that of the eyepieces. Indeed, in all but one of the achromatic telescopes measured, the two axial CAs did have opposite signs and were of the same order of magnitude.

During the period of use of five-element eyepieces, say 1758–c.1775, the objective lenses had over-corrections of 0.6–0.7%, which, with the CA of the eyepieces, resulted in a total axial CA of 0.3–0.4%. After about 1775, when four-element eyepieces were universally used with terrestrial telescopes, the objectives typically had smaller over-corrections and better compensation. Overall values from 0.3% under-correction to 0.2% over-correction were common in that period, and some had near-perfect compensation. One might conjecture that the telescope makers of the early achromatic period started by choosing an eyepiece configuration to give the smallest lateral CA consistent with other requirements.

32 See, e.g., H. Dennis Taylor, *The Adjustment and Testing of Telescope Objectives*, 5th edition (Bristol, 1983), 44.

Table 1. Chromatic aberration of various eyepieces

Telescope	f_e (mm)	dx_e/f_e (%)	df_e/f_e (%)	dx_e (mm)	dx_o (mm)	dx_{tot}/f_o (%)
One-element eyepieces						
Generic Galilean	−50.0	1.7	1.7	−0.86	8.6	1.5
Generic Kepler	50.0	1.7	1.7	0.86	8.6	1.9
Two-element eyepieces						
Generic Kepler terrestrial	−50.0	−8.6	6.9	4.31	8.6	2.6
DJ1 Space Telescope (Kepler terr.)[a]	−16.3	−14.8	5.0	2.40	3.52	2.9
Generic Ramsden	50.0	1.7	0.0	0.86	8.6	1.9
Generic Huygens	50.0	2.6	0.0	1.29	8.6	2.0
579 Dollond Night Glass (Huygens)[b]	99.9	2.8	−0.39	2.8	11.1	2.2
594 Dollond Library (Huygens)[b]	11.5	3.5	−0.31	0.40	−1.12 –	0.096
Three-element eyepieces						
Generic 3-lens	−50.0	−5.2	1.7	2.59	8.6	2.2
Campani[c]	−70.7	−6.1	0.045	4.30	31.69	2.0
557 Black Italian[b]	−46.9	−5.2	2.0	2.41	9.06	2.1
551 Brown Italian[b]	−50.9	−5.1	1.3	2.61	9.74	2.2
574 Le pére[b]	−63.9	−5.2	1.9	3.33	22.1	2.0
J-2 Ayscough[a]	−55.1	−5.4	0.96	2.99	16.9	1.8
J-3 Mann[a]	−54.2	−5.4	1.5	2.91	14.0	1.8
J-4 Mann[a]	−54.8	−5.4	1.2	2.94	16.1	2.0
Four-element eyepieces						
Generic 4-lens	−50.0	−5.2	3.4	2.59	8.6	2.2
10643 Wiesel[d]	−69.18	−4.2	0.04	2.88	−1.50	0.072
Wiesel[e]	−71.0	−4.5	0.63	3.19	12.6	2.2
Divini[f]	−122.2	−8.4	3.5	10.2	58.3	2.0
533 Nairne & Blunt[b]	−34.3	−5.4	0.94	1.84	−2.04	−0.040
541 Dollond Day-Night[b]	−53.8	−5.5	1.0	2.96	−1.32	0.33
563 Dollond Military[b]	−23.1	−5.2	0.44	1.21	−1.18	0.0008
568 Wm. Harris (terrestrial)[b]	−35.9	−5.0	0.59	1.80	−0.10	0.34
591 Utzschneider & Fraunhofer[b]	−19.8	−6.7	0.34	1.33	0.15	0.27
592 Long Dollond[b]	−29.8	−5.1	0.92	1.52	−0.33	0.10
594 Dollond Library (terrestrial)[b]	−20.1	−5.6	0.70	1.14	−1.12	0.0020
J-7 Nairne[a]	−32.4	−5.8	−0.23	1.86	−1.37	0.10
Five-element eyepieces						
Generic 5-lens	−50.0	−5.2	0.00	2.59	8.6	2.2
521 Gilbert[b]	−42.2	−7.1	−0.08	3.12	−5.75	−0.32
586 Dollond Reverse Taper[b]	−45.9	−7.6	−0.28	3.49	−3.24	0.05
J-5 Ayscough[a]	−49.4	−7.8	0.35	3.83	−7.33	−0.33
J-6 Unsigned[a]	−45.4	−7.3	0.57	3.31	−6.68	−0.32

The columns are as follows: Identification of the telescope; focal length of the eyepiece; axial CA of the eyepiece as a percentage of its focal length; lateral CA of the eyepiece as a percentage of its focal length; axial CA of the eyepiece; axial CA of the objective; total axial CA of the telescope as a percentage of the focal length of the objective. The symbols in the first column indicate the origin of the telescopes measured; [a] from the collection of Duane H. Jaecks, Lincoln, NE; [b] from the collection of M. Eugene Rudd, Lincoln, NE; [c] from the collection of Rolf Willach, Tägerwilen, Switzerland (see Figure 12 of Willach, note 2); [d] from the Skokloster Castle collection in Sweden, described by Monconys (see Figure 9b of Willach, note 2); [e] described by C. Huygens (see Figure 10a of Willach, note 2); [f] described by Monconys (see Figure 10b of Willach, note 2).

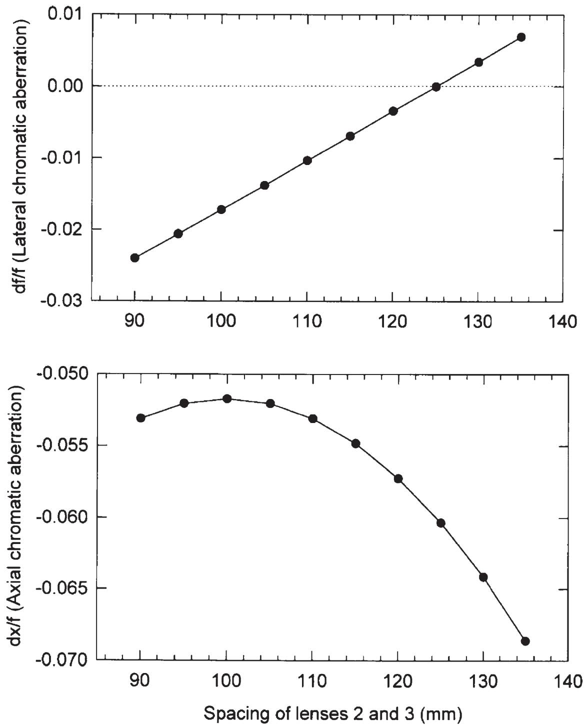

Figure 16. The effect on the chromatic aberration of a 3-lens Schyrlean eyepiece from varying one of the lens spacings.

Then, the resulting axial CA of the eyepiece was corrected by a corresponding overcorrection of the objective. We must remember, however, that at this time, analytical methods of lens design were in a primitive stage of development, and makers had to rely on their skills and experience to achieve their goals.

After about 1850 interchangeable eyepieces came into general use, so makers began to separately correct the objectives and eyepieces for CA.

5. Conclusions

Our historical survey indicates that concave Galilean eyepieces were used from the time of the invention of the telescope, 1608, up to the present time, although after about 1780, they were no longer employed in astronomical work. After 1611, they also had to compete with the convex Keplerian form. Kepler also suggested the two- lens terrestrial form of the eyepiece in 1611, but it was seldom used because of its very large CA. The Huygens eyepiece from 1662 and the Ramsden form of 1782 have both survived up to the present. The Schyrlean three-lens eyepiece, introduced in 1645, was the popular choice until the 1750s. Dollond's five-lens eyepiece of 1753 had less than a quarter of a century of usage before telescope makers abandoned it in favor of the four-lens form that became the standard for hand-held telescopes.

The measurements and calculations made of the CA of the eyepieces in many telescopes lead us to several conclusions. The Galilean eyepiece has a smaller CA than the Keplerian, which helps account for the preferences of such careful observers as William Herschel. The two-lens terrestrial Keplerian form was found to have an unacceptably large CA, which explains its rarity. Cassini's choice of Campani's telescopes over those of Divini were explainable on the basis of Campani's much better correction of lateral CA, although other factors may have been involved. In the late eighteenth century, the four-element eyepiece displaced the five-element type, even though our CA measurements favored the former, just as John Dollond had claimed. Other considerations such as economics probably provided the motivation.

As in other aspects of science, progress in the development of early telescope eyepieces was international in scope. Important contributions were made by workers in England, France, the Netherlands, Bohemia, Germany, and Italy, to mention only those noted in this paper.

Isaac Newton's remark that he was able to see farther because he stood on the shoulders of giants might be paraphrased in the present context by noting that mankind was able to see farther through the use of telescopes as they were developed and improved over the centuries.

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