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Bike braking vibration modelling and measurement

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Abstract

Modern bicycle disk-brake systems often induce vibration and noise in bike components such as brake rotors, wheels, and even bike frames. When the vibration or noise are excessive, brake performance can be perceived as unsatisfactory. Previous research incorporating bike frame structural dynamics and brake friction modeling has shown that stick-slip friction is likely the cause of much of this vibration and noise. Bicycle design parameters such as brake friction behavior and bike component structural properties are central in producing and/or sustaining these vibrations. The predicted dynamics of these models has correlated reasonably well with the testing of braking systems. This research extends the modelling of previous efforts to improve correspondence with brake noise/vibration testing and gain further understanding into the contributors and possible cures of this unwanted vibration. Specifically, the extended model incorporates torsional wheel dynamics (including rotor/hub, rim, and tire inertias, and spoke, rotor, and tire stiffnesses) into previous models. This new model allows the dynamics of the bike frame and wheel to couple through braking application. To support and validate the modelling, motion/vibration measurements are recorded during noisy braking with a non-contact laser vibrometer in the laboratory and with an accelerometer in field tests. Vibration measurements are studied along with model predictions toward the goal of connecting unwanted noise/vibration with specific design parameters of the bicycle brake-frame-wheel system.

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Keywords: Mountain biking; braking; stick-slip friction; vibrations

1. Background

As mountain bikes have progressed in performance and complexity, more is demanded of each bike system. In braking, power (braking force), weight, and reliability are key requirements. High power, low-weight brakes on

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ever lighter bike frames have resulted in increased brake induced vibration. High forces on rotors and calipers cause compliant bike frames and wheels to deflect in bending and torsion. Calipers are accelerated and rotors are decelerated causing the pad and rotor to momentarily stick together. As the stuck system continues to "wind-up," static friction forces are finally overcome, the pad/rotor interface slips and the caliper and rotor (and other attached components) vibrate independently until they often stick again. Many inputs and design parameters contribute to this stick-slip and locally unstable behavior as bike vibration and acoustic noise can develop. This paper develops a 2-DOF model including frame and wheel compliance to extend previous work on this topic to further understand the mechanisms of noise and vibration. Some experimental measurements are used to evaluate the efficacy of the model. Previous work on bicycle brake vibration is in Redfield (2009), and Redfield and Sutela (2008); work on brake vibration in general is in Abdelhamid (2001), Van der Auweraer et al. (2002), and Jacobsson (2003); and models of sliding friction are found in McMillan (1997), and Popp and Stelter (1990).

2. Dynamic model development

A 2-DOF model is developed of the rear braking dynamics of a mountain bike that includes structural (frame) compliance/inertia, wheel/tire torsional stiffness/inertia, and brake pad/rotor forces. Figure 1 shows the rear sub-systems of a bike including the rear triangle, wheel and tire, and brake components. Significant forces that induce vibrations are the equal and opposite brake forces on the rotor and caliper (through the pads), the resulting traction force between the ground and tire, and the equal and opposite axle forces between the frame and axle. The caliper and axle forces apply an effective couple to the rear frame (two leftmost forces in the figure) causing the frame to deflect in bending; the traction force and the rotor brake force create opposing torsional moments on the wheel about its center resulting in a "wind-up" of the wheel due to the compliance of the spokes and tire.

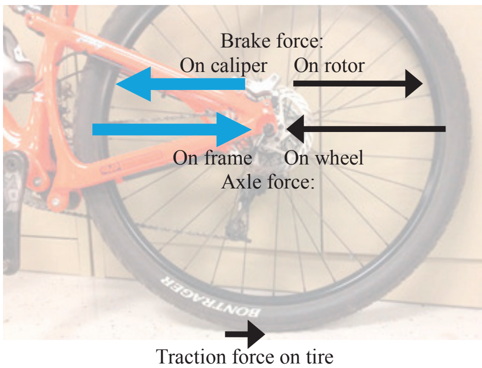

Fig. 1. Braking forces on frame and wheel

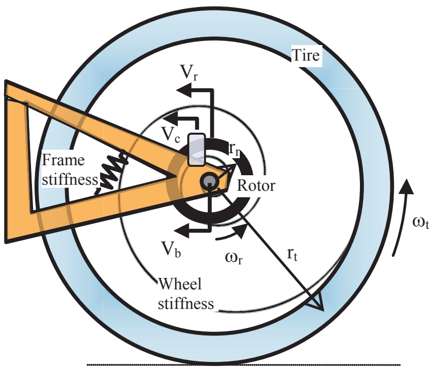

Fig. 2. Schematic of frame and wheel

A schematic of this system with key velocities is in Fig. 2. The spring icon denotes that the rear frame has a bending stiffness and the spiral line inside the wheel represents the wheel/tire's torsional stiffness between the ground and rotor. Important velocities are that of the bike, V_b, the caliper, V_c, and the rotor V_r, all in the horizontal direction, and rotational velocities of the rotor and tire, ω_r and ω_t. Vertical motions are neglected in this analysis.

2.1. Bond graph model and equations of motion

The dynamics of the rear braking is modeled using bond graphs; bond graphs are pictorial representations of key energetic effects and significant power flows in a dynamic system. Figure 3 is the bond graph of our bike vibration system that incorporates wheel/tire and frame dynamics, brake friction force, and input bike (or wheel) speed ($V_b = r_t\ \omega_t$). Power enters the system from the ground contact, is stored in inertias of the tire and rim (J_r), passes through the compliance/damping of the tire and spokes (k_t and b_t) and interacts with the hub and rotor mass

(J_r). Next the power enters the frame through the sliding contact of the brake friction which dissipates much energy. Frame energy is stored in its mass (m_f) and stiffness (k_f) and dissipated in structural damping (b_f).

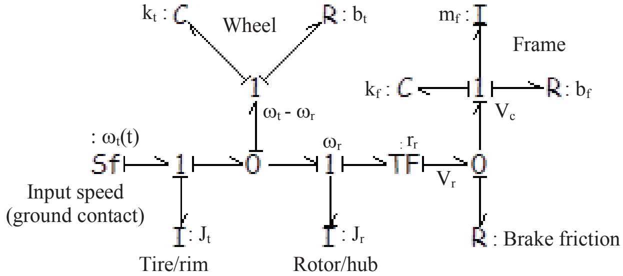

Fig. 3. Bond graph model of braking dynamics

Equations come from the bond graph (Eq. 1) with the state variables being the rotational velocity of the rotor, ω_t, the translational velocity of the frame at the caliper, V_c (relative to the bikes forward constant velocity), the torsional deflection of the wheel, θ_w, and the deflection of the frame at the caliper due to its bending, x_c.

$$\dot{\omega}_r = \frac{1}{J_r}\left[k_t\theta_w + b_t(\omega_t - \omega_r) - r_r F_f\right], \quad \dot{V}_c = \frac{1}{m_f}\left[F_f - k_f x_f - b_f V_c\right]$$
$$\dot{\theta}_w = \omega_t(t) - \omega_r, \quad \dot{x}_c = V_c \tag{1}$$

2.2. Brake force representation

The braking force is a function of the caliper piston normal force, the friction coefficient, and the slip velocity; further, the piston force is a function of the braking application rate (Figs. 4 and 5). Brake force opposes slip velocity and static friction is almost always greater than kinetic. Jacobsson (2003) explains some of the physics that drive the development of brake friction models used in the literature. Fig. 4 shows that friction force transitions quickly from positive to negative (with a change in slip direction). Maximum static friction, $\mu_s N$, shifts to kinetic, $\mu_k N$, more slowly as *slip sensitivity* increases. This negative slope is friction force behavior represents an effective negative damping to the system, a destabilizing effect.

Further, brakes can be applied only so quickly, both because hand force on the brake lever develops over time and because the physics of brake components introduce a delay between lever-pull and the pad-rotor contact. These effects are modeled as a first order lag to approximate the *application rate* of the pad force on the rotor (Fig. 5). Average brake forces of 900 N are measured from a load cell at the caliper during field testing during significantly steep downhill riding and the values of kinetic pad-rotor friction are found in the literature ($\mu_s \cong 0.4$). Engineering experience estimated slip sensitivity and application rate for the brake model; piston forces can be approximated by monitoring pressure in the hydraulic line.

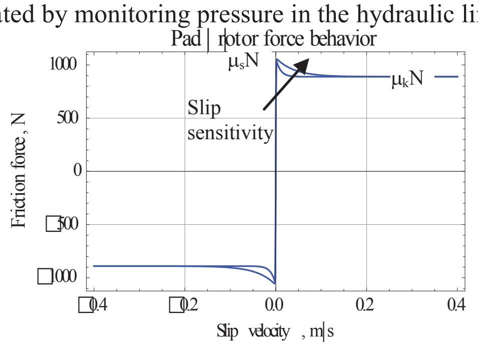

Fig. 4. Pad-rotor forces with slip velocity

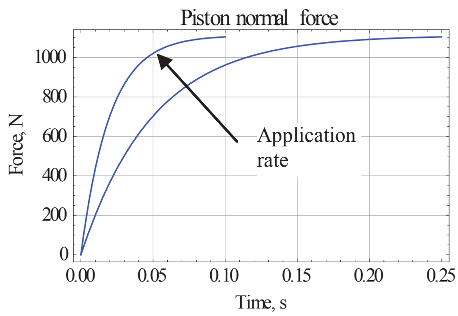

Fig. 5. Caliper piston force application rate

2.3. Model parameterization and execution

Physical parameters for the model were found by measurement, the literature, and engineering experience/judgment. For instance, frame parameters were determined from the work of Redfield and Sutela (2009) where they measured the stiffness of the rear frame structure at 350 N/mm due to a loading mimicking brake force application. Effective translational wheel stiffness (at the rotor/caliper interface) was measured at 90 N/mm. Fundamental frequencies of the frame and wheel (300 and 200 Hz) were also measured that allowed the determination of an effective frame mass assuming a single DOF vibration. Bike geometry was measured such as a 66 cm tire and a 160 mm rotor. Engineering experience assumed damping ratios for the frame and wheel sub-systems (0.02-0.1). Most of these parameters were varied while exercising the model to determine how they impacted braking performance. The model was executed with *Mathematica*, a powerful, computer-based design tool that is a sophisticated mathematical engine and graphics generator. Manipulations were created that allowed continuous changes in model parameters and nearly instantaneous simulation results.

3. Simulation results

Simulations were run of braking dynamics resulting in phase plane and time response plots. Phase plane plots plotted velocity versus displacement normalized by natural frequency, ω_n. Key parameters were varied to investigate how they contributed to stable, unstable, and limit-cycle behavior. It is the unstable and limit-cycle that increases vibration duration and can result in unwanted sound and noticeable bike vibration. Figure 6 shows a damped and stable, rotor and caliper response. The rotor tangential speed starts near 0.9 m/s with the caliper at zero (both relative to the bike); the braking force slows the rotor and speeds the caliper initially and they both deflect (while oscillating) due to system compliances. The caliper eventually deflects a normalized distance of 6 m/s and the caliper to just past 7 m/s. This is the finite deflection due to the steady-state braking force.

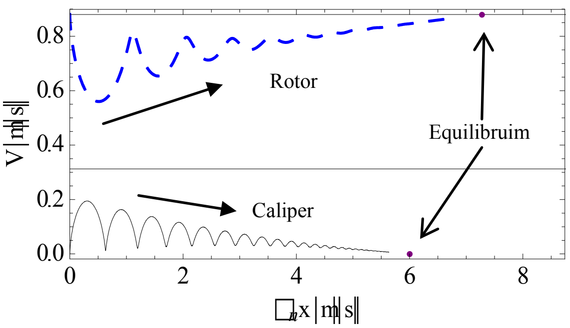

Fig. 6. Well-behaved vibration: velocity vs. normalized deflection

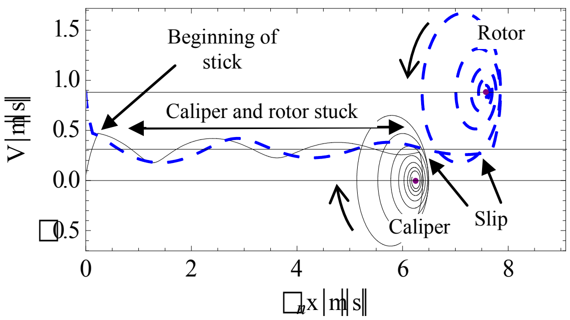

Fig. 7. Single stiction event that damps to zero

If we decrease the frame and/or wheel stiffness, increase the brake force and/or its application rate, or reduce bike velocity, the caliper and rotor can reach the same velocity and pad-rotor stick will occur as in Fig. 7. The stuck pad and rotor will move together until they slip (static friction is overcome) and they will then oscillate independently until motion is damped (as in Fig. 7) or they stick again as in Fig. 8. This re-sticking can result in limit-cycle vibration (Fig. 8a) and, depending on frequency and amplitude, will be heard or felt by the bike rider. Re-sticking is influenced mostly by the ratio of static and kinetic friction and by the damping in the system. The greater the ratio of μ_s to μ_k, the greater the additional energy added to the system while stuck. Also, increased system damping leads to a faster decrease of limit cycle orbit and a less chance of re-stick.

Figure 8b is the time response of this limit-cycle. Characteristic of its motion is a different frequency, that of the coupled system while stuck. Note the discontinuities at stick, slip, and re-stick that add higher accelerations and frequencies to the system; these discontinuities are like small, but consistent impacts on the rear of the bicycle.

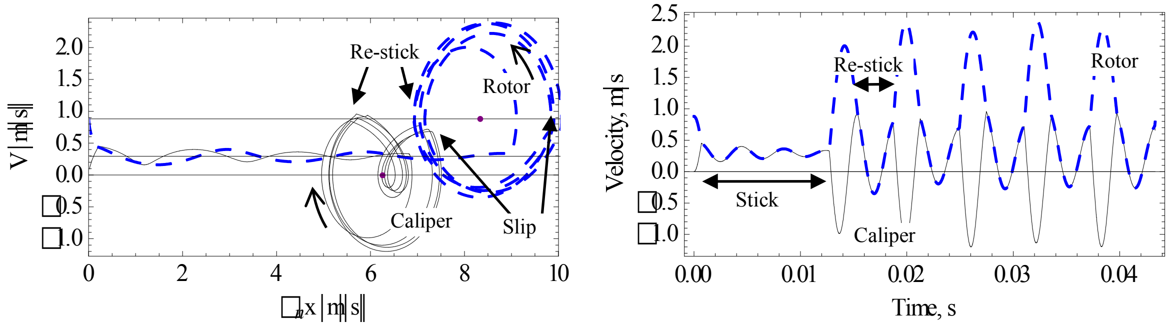

Fig. 8.Stick-slip limit-cycle; a) phase-plane with normalized deflection, b) time response

Another key contributor to system instability is the slip sensitivity of the friction behavior (Fig. 4). The more slowly the static value of friction approaches the steady kinetic value, the more negative damping is added to the system. Figure 9 shows the phase-plane and time response where the effective negative damping increases the vibration amplitude of the caliper until stick occurs which ends in limit-cycle behavior similar to Fig. 8.

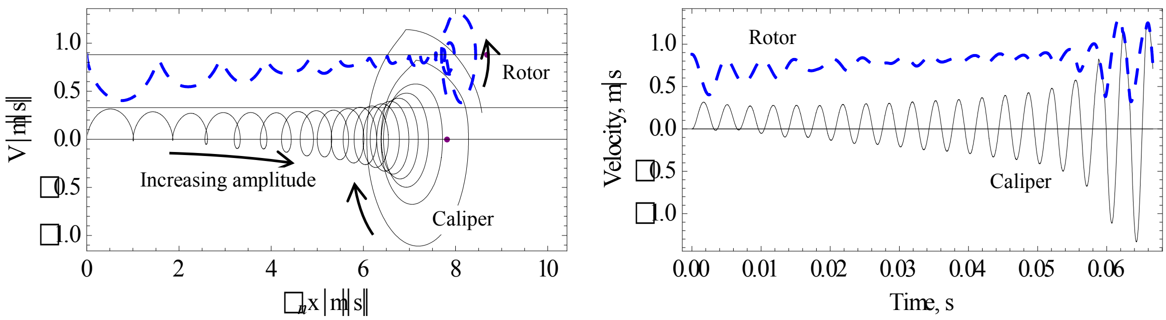

Fig. 9. Effects of increased slip sensitivity; a) phase-plane using normalized deflection, b) time response

4. Experimental results

Braking scenarios that resulted in adverse sound and vibration were measured at the brake caliper for lateral velocity in the lab (using a laser vibrometer) and longitudinal acceleration. The lab set-up had a mountain bike on a trainer with the rear tire disengaged from the roller. The bike was pedaled and braked simultaneously.

Figure 10a shows the lateral velocity measurement before and during a braking event with the bike on the trainer (Two time windows of 0.1 s are shown over a 0.64 s data capture). The low amplitude, low frequency response *before* brake application is caused by the rider pedaling on the bike and shifting weight; the higher amplitude/frequency motion *during* braking is apparent where the amplitude increases quickly (short-term unstable) and then limit-cycle vibration occurs until the brakes are released and the vibration amplitudes drop. This qualitatively compares with the model response of Figs. 8 and 9. Periodograms of the time windows are in Fig. 10b where the *before* spectrum shows dominant frequency content at about 370 Hz and 1800 Hz. 370 Hz is structural noise excited by pedaling, perhaps in the spokes or rotor and the 1800Hz was determined to be ambient noise. The *during* spectrum shows increased power across all frequency but with specific power near 250 Hz., 500 Hz, and between 1200 and 2100 Hz. These are in the range of rotor and spoke harmonics as determined by separate acoustic testing.

Longitudinal acceleration at the caliper was measured in braking events during field testing on level ground. Figures 11c and d show a time response and periodogram with two capture windows as previously. Vibration levels while rolling are amplified during braking and the higher frequencies are excited significantly. Notice that trainer frequency content is much quieter than field data due to lack of knobby tire interaction with the uneven ground; also, certain frequencies match (500, 750 and 1350 Hz.) and many do not. Obviously a bicycle on a trainer has dynamics that somewhat differs from a bicycle in the field.

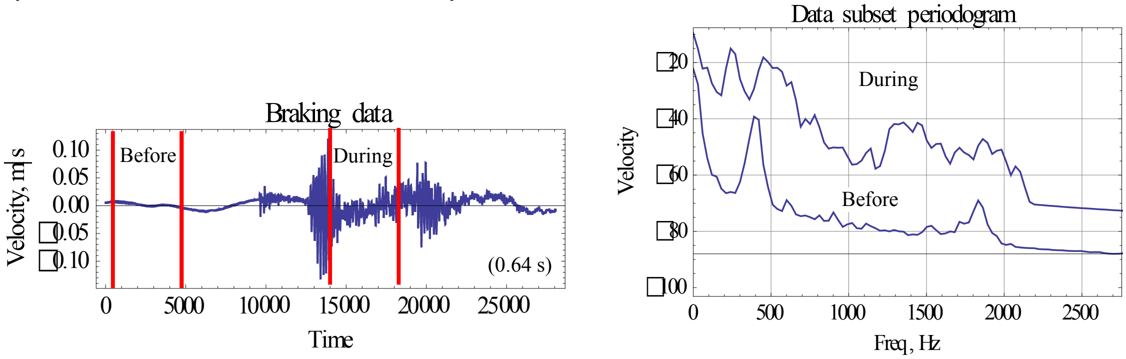

Fig. 10. Trainer velocity measurement; a) time response, b) spectral density approximation

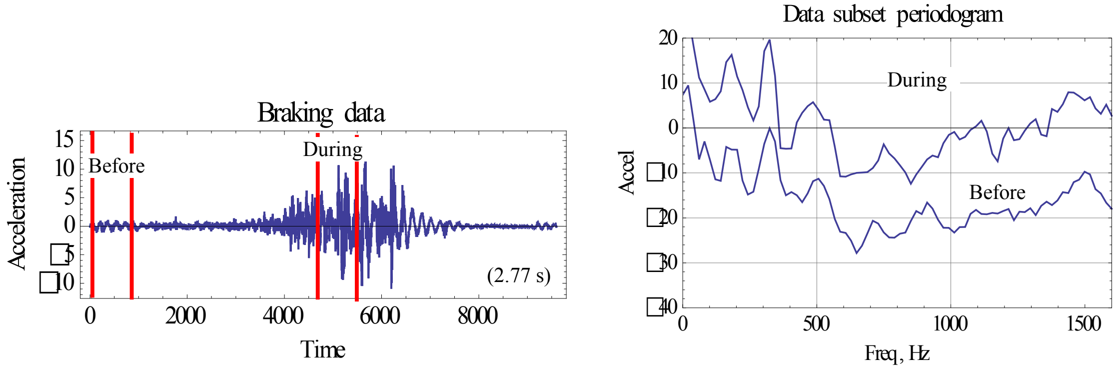

Fig. 11. Field acceleration measurement; a) time response, b) spectral density approximation

5. Conclusions

A 2-DOF model of a bicycle rear frame, wheel, and braking system was developed to investigate design parameter influence on vibration and noise. Vibration and acceleration measurements during noisy braking events were obtained to study the signatures of such noise and to compare to the model dynamics. Key contributors to noise and vibration initiation are the frictional behaviour of pads and rotors, specifically the disparity of static and kinetic forces and the sensitivity of friction forces to slip speed. Inputs and parameters that exacerbate vibrate include high and quickly acting brake forces, low bike speeds, overly flexible frames in bending and wheels in torsion, and low system damping.

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