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Mathematical Communication through Written and Oral Expression

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Mathematical Communication through Written and Oral Expression

Abstract

In this action research study of my classroom of sixth grade mathematics, I investigated the use of communication of mathematics through both written and oral expression. Giving my students the opportunity to communicate mathematics both in writing and orally helped to deepen the students' understanding of mathematics. The students' levels of comprehension were increased when they were presented with a variety of instructional methods. Through discussion and reflection the students were able to find methods that worked best for them and their learning ability. Students' understanding increased from probing questions that made the students reflect and re-evaluate their solutions. This learning took place when students were made aware of different solutions or ways of doing things from the class discussions that were held. I discovered that when students are challenged to express their thinking both in writing and orally, the students found that they could communicate their thinking in a new way. Some of my students were only comfortable expressing their thoughts in one of the two ways but by the time the project was completed, they all expressed that they enjoyed both ways, and maybe changed the original way they preferred doing mathematics. As a result of this research, I will continue to require students to communicate their thinking and reasoning both in writing and orally.

I have been teaching elementary mathematics for the past 10 years and during this time I assigned mathematics homework daily. I expected students to complete the assignment by the next day so that I could move on to the new concept. I always corrected the homework looking for the correct answer, paying little attention to how the students got that answer. I required students to show their work but did not paid attention to that work on a regular basis. I handed those problems back to the students to correct and still I only checked to see if they had the correct answer. I had only assigned the problems in the book because they were the "easier" problems to correct. They were "easier" because the answers were in the book. I could quickly look at the students' solutions and know if they got the solution correct. I assigned these problems because they were easier for the students, usually. They were simple algorithms and did not require students to do a lot of in-depth thinking. These problems were rote in style and usually one step.

Once in a while I would assign a problem out of the book, such as a Habits of Mind problem. Habits of Mind are problems taken from the Math in the Middle program. These open-ended problems are problems that lend themselves to different solution strategies. They are problems that are above and beyond the typical story problems. These problems require students to analyze data, formulize a hypothesis, and solve. Unfortunately, these types of problems took more time to go over, and so I found myself straying away from them. These problems were harder to grade because they took extra time that I did not feel I had to correct. If students missed one of these problems and asked them how they got that answer, very few of them could communicate their thinking to me. The students would show their work, but when asked to reflect back on it, they could not follow their own thinking. This caused added frustration for the students and me.

It was through the harder problems that I came up with the topic for my research project. The fact that students could not communicate their thinking to me or to their classmates got me thinking. Math in the Middle has made me realize that just getting the solution is not as important as the journey of how I arrived at that solution. It was from this that I decided to do my research project on written and oral communication in mathematics.

Ideally, my classroom would be full of mathematical conversation. Students would be knowledgeable in mathematical concepts, confident in their abilities to solve problems, and willing to share their ideas. In order to achieve this I gave students problems two times a week. I selected students to present their solutions in writing for one of the problems and then selected students to present their solutions orally for the other problem.

Problem Statement

Too many times in mathematics, the problems are strictly computational and require nothing more than memorization of facts and process to complete. These were the types of problems I typically assigned to students. This style does not allow for much discussion because usually, students do not know why they do these types of problems. They memorize the process and because of this, they cannot explain what they did, since they do not know why they did the steps. After experiencing Math in the Middle, I have become aware of what real problem solving is and how to explain the process to achieve understanding. I realized that almost all of problems that I have students work on are far from these types of problems.

This year, the students that are in my class are more like me, “black and white” thinkers. When I first had to explain my steps in Math in the Middle, I would typically answer, “I don’t know how I did the problems,” or “I don’t know how I got the answer, but I did.” I was content with not knowing how to achieve the understanding. My students were the same way. When I

asked them to tell me how they got their solution or how to find the solution, they responded in many of the same ways that I would have. It was not until recently that I realized that I have been using the wrong types of problems.

Looking at the National Council of Teacher of Mathematics (NCTM) process standards, the issue of communicating through written and oral expression in problem solving is addressed in several areas. Everything I am hoping to achieve in problem solving is addressed in the description of problem solving in the process standards. I want students to be able to use a variety of strategies to solve a problem and be able to look back and discuss their success.

Obviously, the biggest NCTM process standard I want to achieve is that of communication. Students need to be able to communicate their mathematical thinking to their peers for two reasons: one is to help them build confidence in their mathematical thinking and the other is to help their peers understand a concept that might be hard for their peers. Students are able to explain things in a way that their peers can understand when a teacher is having trouble reaching them. I also notice that students need to make connections, which is another NCTM process standard. Students need to connect their thinking with the process they are doing in order to communicate it to others. All three of these process standards are key ingredients to success in communicating mathematical concepts.

The NCTM content standards that are also addressed in communicating responses through written and oral expression in problem solving are number and operations, algebra, geometry, measurement, and data analysis and probability. In true problem-solving style all of these types of problems can be addressed.

The overall goal that I want for my students is for them to be able to communicate their thoughts through oral and written expression. I have felt, first-hand, the feeling of frustration

when I could not explain how or why I did something. I also felt first-hand success when I could finally explain how and why I did something. I want my students to feel that success on a daily basis and then be able to transfer that communication skill into other aspects of their lives.

Literature Review

As a mathematics instructor, I have been a teacher with an inappropriate approach to the nature of mathematics and learning. I have been the teacher who does not require students to think for themselves. I have just taught the lesson and expected the students to complete the homework without any questions. I have been the teacher who has not allowed the students to make connections or given them the opportunity to ask "Why?"

There are not many teachers who expect and require mathematics to be meaningful for students and still fewer teachers see mathematics as a creative process; rather, they seem to believe that mathematics is simply memorizing facts and the processes. Numerous teachers are satisfied with teaching mathematics as doing routine problems, without ever making sure that their students truly understand the process. Students normally learn and trust what they are taught. I believe student confidence toward mathematics is created by their teachers. Therefore, a change needs to take place in mathematics instruction from just memorizing and performing problems to critical thinking and understanding in order to improve student confidence in mathematics. I believe this can happen by implementing written and oral communication into the mathematics classroom.

Referring to the NCTM standards, I read that communication is an essential part of mathematics and mathematics education. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others in writing or orally, they learn to be clear and convincing (NCTM, 2000). When challenging students to think and

reason about mathematics, it allows them to come up with their own meaning for mathematics and then develops a deeper understanding. Allowing the opportunity for students to look at a mathematics problem, apply what they know, solve the problem, and communicate their results will not only apply meaning to the process, but also develop a deeper understanding of mathematics.

Through my review of the research on written and oral communication in mathematics, I found five common themes. The first theme in the reading is that teachers need to have sound mathematical knowledge and have mastered their content in order to create a meaningful learning environment for mathematics. A second theme I found is that teachers need to value the constructivist perspective if they want to create their own mathematical understanding. A third theme is that teachers need to base their instructional practice on what they know about their students' understanding from the students' written and oral communication in order to create a curriculum that best fits the students needs. A fourth theme I found is that students can get a deeper conceptual understanding by reflecting on their own learning, as well as listening to their peers' thinking, rather than only listening to their teacher. The final theme found is that students can learn as much or more by communicating conflicting ideas and arguments as they can by only being presented with correct solutions.

The proceeding sections will take a closer look at each of these themes in more depth and also explain the ways in which my project differs from the published reading in these areas.

Teacher Knowledge

The argument has been made that the use of written and oral communication in mathematical learning should increase deeper conceptual understanding. However, this kind of change will require major changes in the teaching practices that most teachers use. Teachers

need to develop their mathematical thinking and knowledge before they can expect any change to take place. In Borasi, Siegel, Fonzi, and Smith's study (1989) the teachers participated in a semester-long seminar to receive training in the three strategies that were to be implemented in the project. They then "joined a research team to help them plan and implement some innovative units involving reading experiences in one of their mathematics classes" (p. 280-281).

These teachers knew that they needed to have further knowledge on the strategies in order to effectively teach the material. Without it, the changes would have been minimal. Borasi, Siegel, Fonzi, and Smith (1989) performed a study on the "integration of transactional reading strategies, on one hand, and mathematics-related texts other than textbooks and word problems, on the other, into mathematics instruction" (p. 280). The project started with the semester-long course for the teachers and then continued into the next school year for 32 weeks. It involved students in eighth through eleventh grades.

> Wood (1999) states that,
> …for teachers to bring this theory successfully to practice in their classrooms requires more of them than an awareness of originality in children's thinking. In addition, teachers need to understand, and to some extent accept the fundamental tenets that distinguish a constructivist theory of learning from other theories. (p. 171)

In other words, teachers need to be aware of how the children think and work, in order to successfully reach them. If teachers have a small understanding of mathematical principles and concepts, then their teaching is only following the "rules" for the process. If teachers want changes to occur then they must change their instruction to put an emphasis on the concepts of mathematical and encourage students to be active learners instead of "little robots" just doing what they are told.

Through Math in the Middle I have learned that mathematics is more than just doing the problem and following a set of rules. I have learned that my mathematics instruction needs to

focus on the meaning and understanding that must be applied to each problem. In Borasi, Siegel, Fonzi, and Smith's (1998) study, the teachers made a change in their teaching methods by attending the semester-long seminar. These teachers knew, in order for the project to work, they needed to make those changes if they wanted to see any change in the students. I believe my mathematics focus and instruction has changed because of my involvement in Math in the Middle. My research project was based on those changes. My project promoted written and oral communication in mathematics in order to gain a deeper understanding of the concepts. My study did not investigate the teacher, but it did investigate the students and how they learned from expressing their own thinking in written expression and out loud.

Constructivist Perspective

Looking at the constructivist perspective, meaning is not inherent in written or oral language. Meaning is constructed by each individual on the basis of his or her own experience and involves individual interpretation (Yackel, Cobb, & Wood, 1993). Words do not mean anything by themselves. It is when a thinker makes use of the words that they have meaning. Teachers need to create a classroom climate that encourages communication and promotes thinking of mathematics. It is then that students can construct their own mathematical meanings.

Wood (1999) conducted a research project on reform-oriented classrooms in which teaching and teachers' thinking and reasoning were of central interest. This investigation of teaching was collected over 18 months in a second grade mathematics classroom. Eighteen teachers participated in this project and they designed their lessons around the constructivist perspective. According to Wood, "Children understand mathematics best if they are actively involved in their own learning" (p. 173). Communication in the classroom is important in students' development of understanding concepts. Encouraging pupils to talk about their

conceptions and to justify their own strategies of exploration or proof represents a shift in the social relations in the classroom from a "teacher-centered" to a more "pupil-centered" approach (Hoyles, 1985). By changing to a constructivist and child-centered approach to learning, the emphasis is placed on the children's ways of knowing and the creativity of their own mathematical thinking. When students make the connection to math through their own experiences, they become investigators and are more likely to take charge of their own learning.

In my research project, the students were expected to construct their own mathematical meaning by orally presenting their mathematical thinking and solutions to homework problems, as well as explaining their mathematical understanding in writing in a mathematics journal. In Adler's (1999) study, students were to use verbalization as a tool for thinking and to display their mathematical thinking to their peers. This study observed a classroom of 30 students in tenth and eleventh grade, while I observed a classroom of five students in sixth grade. In Adler's study, as well as in mine, the meaning is constructed by each individual on the basis of his or her own experience and has individual meanings to those students. Adler (1999) goes on to say that, "For talk to be a resource for mathematics learning it needs to be transparent; learners must be able to see it and use it" (p. 63). When students can connect to their thinking, they will then be able to apply that knowledge in other instances.

Instructional Decisions

In a classroom that is full of communication, the instructor has one main role, which is to watch over that discussion and to keep it going in the right direction. The teacher needs to encourage the students, push them to think "out of the box," and help them with any wrong information, when necessary. By letting the students share their ideas in written or oral explanations, teachers can make instructional decisions based on their students' understanding of

the content. If teachers see that they are not reaching some students it is then they may change their instruction to best meet those students' needs. Wheatley (1992) researched elementary age students in the area of problem-centered learning. Wheatley believed that teachers needed to just listen to their students. Teachers need to change and adapt to the students and not the other way around. My research project was intended to study my students' written and oral communication and through this project I found myself changing the way I was instructing, based on each student. I only have five students in sixth grade, so this was easily done. I would not have expected each student to learn in the same way, although a lot of times, teachers teach that way. One way I learned about the students' learning abilities was by having them journal. By requiring my students to present their mathematical thinking and solutions in these journals, I could look back at their thoughts, frustrations, and successes and know where to go from there.

In Borasi and Rose's (1989) investigation of journal writing and mathematics instruction, the journals provided the teachers with a lot of information about the students and the course. Without these journals this information could have gone unknown. The teacher could read the student's journal and get to know that student on an individual basis. The teacher could also learn of any problem areas and figure out how to address them. Written and oral communication plays an important part in a teacher's instructional plan. When students are allowed to communicate their mathematical ideas and thought process, a teacher can better judge how things are going for each student. This also helps to ensure the teacher is meeting the students' needs.

Reflection

A teacher needs to encourage students to express their thinking in writing and orally. This forces students to reflect on their own thinking and decision-making. Hoyles (1985) researched

group discussion in mathematics. Hoyles states, “We can all recall how a chance comment of another person can trigger off a whole new avenue for exploration” (p. 207). Giving students the opportunities to work in groups can help students in many ways. Group discussions can help them by letting them rethink, explain, and justify their solutions. It is through the written and oral expression that students are able to deepen their mathematical understanding.

Borasi and Rose (1989) stated,

> The teacher is expected to read these entries and occasionally comment and respond to them in a supportive and non-evaluative way – for example, by sharing personal thoughts on the matter, or by suggesting new questions for reflection and exploration. (p. 348)

It is very important for the teacher to reflect on the students work so that they know how to change or improve their thoughts. The teacher can pose a question to deepen a student’s thinking. Hoyles (1985) stated, “The role of the teacher has therefore been found to be important in the learning process in order to encourage reflection on what has been done and on what further outcomes could arise” (p. 210).

The teacher should also reflect on a regular basis. I found this journaling to be a great way to reflect on my lesson, throughout my research project. I could look back and see what was working and how I could change the things that did not work. Writing in journals help to reflect on materials learned in mathematics class. While reflecting in writing, students are actively engaged in the problem.

Pugalee (2004) conducted a study of 20 ninth grade algebra students in the area of oral or written description of their problem-solving practice. The students were divided into two groups for a 10-class session. Pugalee (2004) found that of these two strategies, he did not see a lot of variance in the outcomes. Both written and oral expression had about the same amount of success. The author stated, “Writing is posited as providing a level of reflection that promotes

students' attending to their thinking about mathematical process" (p. 28). Even though both strategies have about the same success rate, Pugalee believed reflection was important.

The three articles that I mentioned in this section related to my research project in several ways. I required my students to report orally their solutions to assigned problems and they had to reflect on that thinking. I required my students to write in their mathematics journals, where they also had to reflect about their thinking of that problem, just like in Borasi and Rose's (1989) study. One difference was that I did not have my students reflect on each day's problem, but rather had them justify their mathematical solutions to open-ended mathematics problems related to my school's curriculum.

Conflicting Ideas

When students listen to one another's thought processes and reflect on their mathematical ideas, conflicting ideas and opinions are surely going to come up. In Hoyles's (1985) study on group discussion in mathematics, the researcher found that through these group discussions conflicts arose. In this study Hoyles studied "10- to 11-year-old children and the need to explain their solution in order that their partners could follow the arguments and participate in the 'game'…" (p. 208). Hoyles stated, "Conflict too was found to play an ambiguous role; sometimes conflict ended with a 'better' final solution, sometimes it was avoided by the production of a joint answer incorporating both suggestions side by side and sometimes, it was 'resolved' by authority argument" (p. 209).

Since conflicting ideas are sure to show up during these discussions the teacher has a vital role to play. The teacher needs to create a safe environment for the students to help avoid hurt feelings. The teacher needs to "teach" the students on how to discuss their opinions in a manner that is not degrading of another student. Hoyles (1985) says, "Children must be helped to

work together, to tolerate disagreements, to challenge each other and to avoid premature closure of argument" (p. 209). If these steps are not followed by the teacher, it is inevitable that someone's feelings will be hurt.

Yackel, Cobb, and Wood's (1993) study of four students over the course of a year was studying the understanding of small-group problem solving in elementary school mathematics by focusing on the relationship between children's individual mathematical conceptions and the nature of their social interactions as they work together in small-group problem-solving settings. Through this study the authors also saw some conflict in these group situations. Yackel, Cobb, and Wood state, "…The latter might include challenging their partner's explanations, asking for clarification, explaining why they disagree with their partner's solution, and taking the perspective of their partner" (p. 46). If these students were not taught to handle these situations correctly, the group would not have been as productive, due to students arguing or shutting down. The teacher needs to teach the students to be patient and let the others in the group finish their explanations before they interrupt.

Yackel, Cobb, & Wood (1993) used an example that stated,

> If one partner wants to explain his or her solution method while the other child wants to think through his or her own method, then one of the children must temporarily suspend the attempt to fulfill his or her obligation. (p. 47)

If the students try to speak while the other is thinking, a struggle will likely begin. Although disagreements can be good, they need to be handled in a positive way. Students need to be taught to wait their turn and to listen to their peers.

I chose these research papers because they related to my research project. I had my students working with partners or in small groups just as in Yackel, Cobb, and Wood's (1993) study. Disagreements in the students' thinking did occur between partners. I was able to teach

my students to reason through their differing solutions. My students even shared conflicting views during their oral explanations of a mathematical problem. Unlike Pugalee's (2004) study of 20 ninth grade students, my focus group was five sixth grade students. However, as in both studies, my students were actively involved in their own and others' learning.

Conclusion

The eight articles that I referenced were all written with a different purpose. Their results were different from one another, yet each study provided valuable information to me to support my research project. It is obvious from these studies that allowing students the opportunities to verbalize their thinking or journal about it is challenging yet empowering for the students. By ensuring students to communicate their thinking either in writing or orally, to justify and reflect on their mathematical ideas, and to talk out disagreements on solutions allows students the opportunity to create their own mathematical meaning. In my study, students had to express themselves in writing and orally, and they became better throughout the project at communicating their mathematical thinking.

Purpose Statement

The purpose of my study was to investigate what happens to students' written and oral communication in the area of mathematical problem solving. I examined the following research themes:

- The quality of students' written communication in mathematics,
- The quality of students' oral communication in mathematics,
- The quality of teacher questions to probe student thinking and reasoning.

I investigated the following research questions:

- What happens to the quality of student written communication related to mathematical problem solving when I give students non-standard word problems to solve?
- What happens to the quality of students' oral explanations of problem-solving solutions when I focus on asking more question to probe student thinking and reasoning?
- What happens to my teaching when I ask more questions to probe student thinking and reasoning related to mathematical problem solving?

Method

I collected data from my sixth grade class in the second semester of the 2008-2009 school year. I used a variety of data collection methods to examine my research questions. Each research question was looked at using three pieces of data collection for triangulation of evidence. For my first research question, I conducted individual interviews, copied student quizzes, and used a rubric to assess the students' oral presentations of given problems. Two students were interviewed and expected to communicate their reasoning orally on open-ended problems that focused on similar topics that were being covered in class (see Appendix A for interview questions). For the quizzes, I chose problems that were similar to those that were required for class work. By giving the quizzes I was able to understand the students' levels of understanding and check to see if we were making any growth. In order to score students' quizzes I used a rubric (see Appendix B). I required each student to present orally twice a week.

I had to alter the total points on the rubric for the oral section since the students were not doing any writing, which changed the total points of the rubric from 16 to 12. When giving the

oral portion of the quiz, I had to give the students the questions in writing and let them study the question before expecting them to present.

For my second research question I used student journals, quizzes, and interviews. In the student journals students were required to write journals about one Habits of Mind type problem a week. I used the rubric to assess their journals (See Appendix B). Through the quizzes I was able to assess student understanding of math topics that were written in journals through open-ended test items similar to the problems that the students had written previously. I conducted the interviews with two students where I expected them to communicate their reasoning through writing on open-ended problems that focus on similar topics. At the end of each week of the duration of the project, I gave the students a quiz. These quizzes contained two questions that were similar to the problems that were assigned during the week.

My third research question focused on me and my changes throughout the project. In order to note the changes and questions I reflected using my personal journal. (See Appendix E for personal questions). In the beginning the personal journals seemed pointless and took a lot of time to complete. As I completed the Analytic Memos these journals became very important. I could reflect back on what students had done or said, in order to remind me of what was exactly going on. Without my personal journals, it would have been impossible to think back to events that took place weeks ago. After class ended, I would take five minutes to quickly jot down any notes and thoughts and then at the end of the day, I would go back and write out better notes in my smaller notebook. Then, on the weekends, I would take these notes and use them to complete the personal journal questions (See Appendix E). To conclude the project, I gave each student a copy of the ending interview questions and asked them to write their responses in their journals.

Through this research project I did run into a few challenges in collecting data. One of my biggest struggles was trying to get everything done. It seems like second semester goes really fast and seemed to go even faster with this research project; I also had state standards that I needed to get to. With all of our field trips and programs, the students' days were constantly being interrupted. Once I started collecting the data, I found that I had trouble keeping it all straight. My classroom is quite small, so I just stacked everything on a shelf behind my desk. I had each student's journal, their quizzes, and my journal organized in alphabetical order. Any loose papers for each student were placed inside that student's journal. I found this project and the data I collected to be interesting, useful, and thought provoking.

Findings

At my elementary school the classes are a mixture of boys and girls of varying learning abilities. My classroom consisted of both fifth and sixth grade students. Since I had to keep both grades busy my typical math class generally lasted around 30 minutes. To start math class I had a rectangular table at which students sat. I usually stood up at one end of the table with my dry erase board and gave the lesson. I gave students sample problems for them to practice. They each had a small dry erase board, marker, and eraser that they are able to use to do any figuring. Once I believed that students had an understanding of the lesson I assigned them their homework. Their homework usually consisted of 15 problems from that day's lesson and then I assigned an open-ended math problem that they were to do either in writing or orally, depending on which I assigned to them. The students then had time to get into groups to discuss how they should "tackle" that problem. I usually liked the students to work on these problems for 10-15 minutes before I allowed students to ask me any questions. I believe that students are quick to ask for help from an adult if they are not forced to first think for themselves. Students were then sent

back to their desks to work on homework. They could either work on the 15 problems or work with a partner on the open-ended problem. Students presented their solutions at the beginning of the next day's math class.

After being involved in Math in the Middle I have changed my thinking about how students work on math problems. Before my involvement I believed that students needed to work independently and that I was the only one they could ask for help from. If they were to ask a classmate for help, they might "cheat." I had never experienced a class that encouraged me to work with a group in order to solve a problem. I know that it was from these interactions that I was able to learn about many difficult concepts. I then realized that if I was learning from my peers, it only made sense that the students could learn something valuable from their peers. Now I encourage the students to talk with a classmate before coming to talk to me. I have tried to create a safe environment for the students so that they are comfortable talking to one another.

My research project was condensed down to four weeks due to a personal situation, so I had a lot of work to do in a short period. Although this was not a lot of time to collect data I do believe that I discovered many findings that will change my way of teaching. I believe the overall discovery was that the increased communication in the classroom impacted the students' learning. They were able to gain a deeper understanding of mathematics through the written and oral communication. Through this process I was able to see a change in each student but was not able to collect data to truly measure the growth in student understanding. I confirmed my findings with evidence for each research question.

To conclude the project, I gave each student a copy of the ending interview questions and asked them to write their responses in their journals. By having the students write their answers down, I was able to carefully go back and read each student's response. One of the questions that

I asked the students was, "*Do you think communicating mathematics verbally and in writing has increased your understanding of math topics*?" All five students believed that in some way, they had gained a deeper understanding of certain math topics because they had to communicate in writing and orally. I also asked them if they thought I should require future fifth and sixth graders to continue communicating their math solutions in writing and orally. Once again, they all agreed. Three students commented that they had fun through this project and the other two felt that they were better math students because of the project. All five students made reference to having more confidence in mathematics than they did before the project started.

Written Communication

During my research project, I realized that written and oral communication went together. I found that requiring the students to justify their answers in writing before the presentations allowed them to be better prepared for those presentations and helped them to develop a clearer picture of the mathematics topics. I was able to support these findings by looking back to my student interviews. I asked the students the following question: "*Do you think it would be easier to present a solution if you first explained your solution in a math journal?"* All five of my sixth graders believed that it would be easier. I have listed some of the students' responses:

- *"Yeah, I need time to think by myself before doing a problem. I don't want to have talk in front of the others and sound like I don't know what I am doing. By writing it out I can see if it makes sense to me" (Trent[1]).*
- *"Yes, I think it would be easier to give the answer if I first had time to go over the problem alone. I would be able to write down stuff and no one else would have to see if I was doing it wrong" (Madison).*
- *I like writing so I think it would be easier for me to write down my thinking before I had to present it to others" (Brenna).*

[1] All names are pseudonyms.

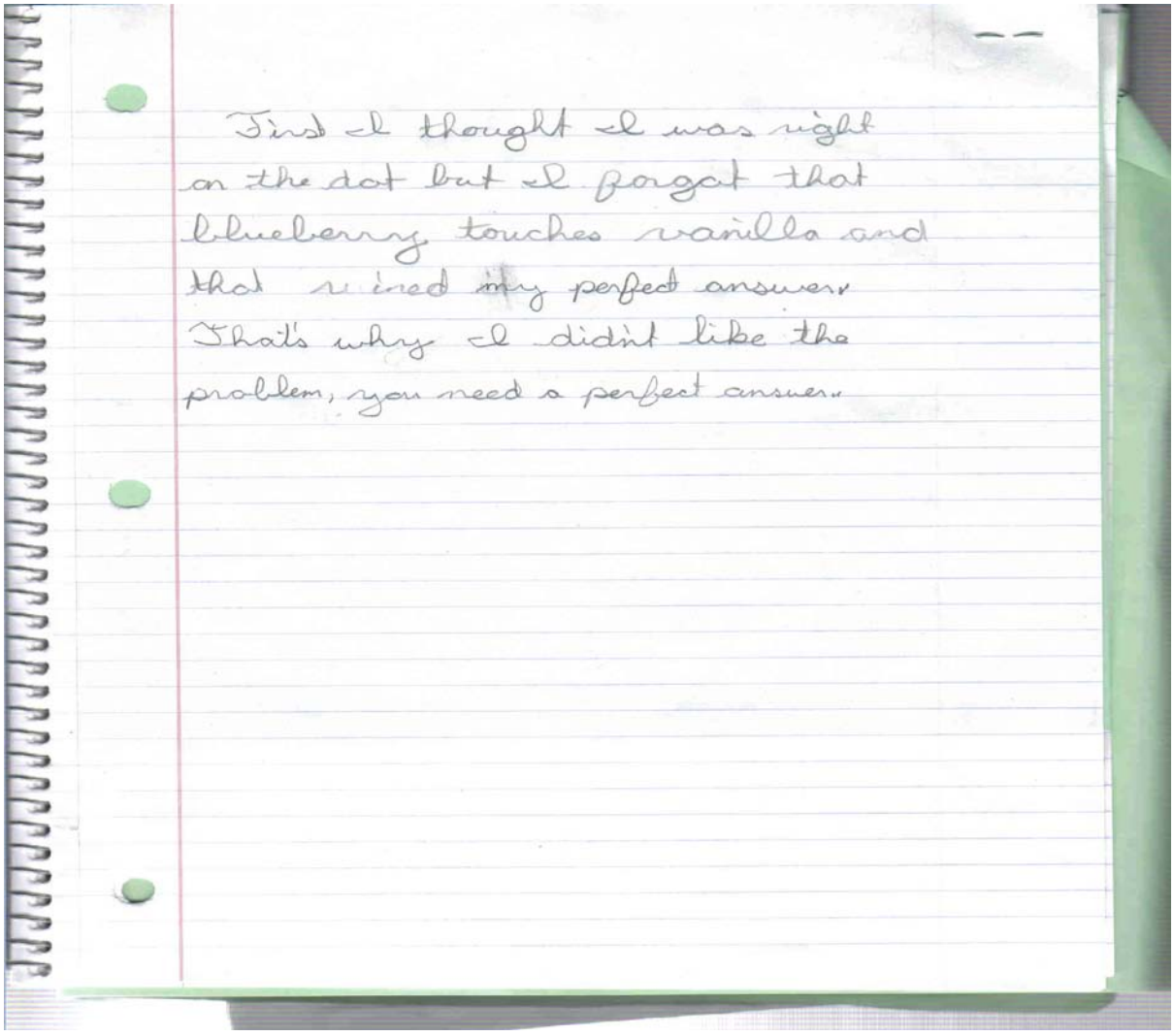
These quotes, from the students, proved to me that they saw the benefit of justifying their solutions in writing before having to present them to their peers. Students all believed that writing down their thoughts helped them get organized and gave them an opportunity to clarify any wrong thoughts.

Twice a week I had students present their solutions to a math problem that they had solved in their written journals. I found a quote from my personal journal that commented on the success of the student's written communication.

> *This week I noticed that students are really doing a nice job in their journals. Elizabeth has turned in one of her best pieces of writing. Her first two pages were just random thoughts that she was having but then on the third page, she wrote out a detailed description of what she was thinking and why. It was so well written that I had a fifth grader read over what she had done, to see if she could follow what Elizabeth had written. The fifth grader had no trouble trying to solve the problem based on the description. Even Madison, who does okay in writing out her thoughts, really stepped up her game this week. She is getting more confident in her math abilities and is not seeming to be as afraid to participate. (*Personal Journal, April 16, 2009).

Toward the end of the project I was able to see changes in each student. This made me realize how important it was to continue the written and oral communication in my math classes next year.

During the second week I did run into frustration with the students' written responses. Alex was a student who typically did what he was told. Therefore, I was surprised at his effort during the second week. I had explained to the students what was required for a written solution. Alex believed that there was nothing wrong with his written solution to a problem. His solution lacked description, complete strategy for solving the problem, and a clear solution. The scanned copy of his work shows this finding.



When students write their explanations, I want to be able to first know what problem they are trying to solve. The students should start each written piece with a brief description of the assigned problem, so that I can refresh myself with the problem, without having to try and guess what they are solving. Alex's submission is a perfect example of some of the work students were turning in on occasion. These lacks of effort led to frustration for me. In my journal I commented on this frustration.

> *The students are getting "lazier" in their responses. They are just doing the bare minimum in their written responses and not truly painting a picture in my mind that they understand the process. I wonder if the students started out doing more writing and oral explanations in the beginning because it was new and exciting, but now the newness has*

> *worn off and they are not as into this style. As I look through the work the students have turned in, I am not as impressed with their effort. What do I need to do as their teacher to get them motivated again?* (Personal Journal, April 9, 2009)

One of the changes I made was in how I taught my problem-solving lessons. In the past I would just give the problem to the students and expect them to give me an answer. The changes I made were we would discuss the problem as a group and I would tell them my expectations to achieving their solutions. I also made sure that I had some type of manipulatives to help them reach their conclusion. By doing this, I was able to get students back to writing clearer explanations.

Another component of the writing portion of my project was the quizzes. I would conclude each week with a written quiz that each student had to take. This quiz consisted of two questions that were similar to the problems that were assigned during that week. Below is an example of a quiz and a copy of how Alex scored.

The two quiz problems were:

1. Lorenzo asked every tenth student who walked into school to name their favorite pizza topping. Their responses were: 18 pepperoni, 9 cheese, 3 sausage, and 2 mushroom. What is the probability that a student will prefer pepperoni pizza?

2. If there are 384 students at the school Lorenzo attends, use a proportion to predict how many students prefer pepperoni pizza.

I did not know what to expect as I looked over the students' quizzes. I was hopeful that students would score well on this quiz but was surprised at the results. Alex received a score of 3 on his first quiz. He showed a limited understanding of the problem, with major computational errors; his communication omitted significant parts of the problem. He did not attempt to write 3 - 5 complete sentences and had excessive mechanics and conventional errors. Alex's solution was not clearly stated and was incorrect. Below is how I scored his quiz.

Problem Solving Journal Rubric

Alex	0	1	2	3	4
Mathematical Knowledge	Shows no understanding of the problem.	Shows very limited understanding of the problem, major computational errors.	Shows understanding of some of the problems mathematical concepts may contain computational errors.	Shows nearly complete understanding of the problems mathematical concepts and principles, no computation errors.	Shows complete understanding of the problems mathematical concepts and principles, no computation errors.
Communication	Words do not reflect the problem, may include drawings which completely misrepresent the problem.	Has some satisfactory elements, but fails to complete or may omit significant parts of the problem.	Makes significant progress towards completion of problem, but ambiguous and unclear.	Gives a fairly complete response with reasonably clear explanations or descriptions.	Gives a complete response with clear and appropriate diagrams.
Writing	No/little attempt to write a paragraph with 3-5complete sentences; excessive mechanics and convention errors.	Some complete sentences, but not a 3-5 sentence paragraph; many mechanics and convention errors.	Complete sentences, but not a 3-5 sentence paragraph; some mechanics and convention errors.	3-5 complete sentence paragraph; minor mechanics and convention errors.	More than 5 complete sentences with accurate mechanics and conventions.
Solution	No solution stated. .	Solution is not stated, and is incorrect.	Solution is stated, and is incorrect.	Incorrect solution is stated, but is reasonable.	Correct solution is stated.

The highlighted parts are where Alex scored. I realized after scoring other students' quizzes that I needed to show students the rubric so that they knew how they were being scored, since most students made some of the same mistakes as Alex. At the conclusion of the fourth week, I was able to see some growth in Alex's quiz scores. He was not the most improved student but it was more obvious to see his growth throughout the project. Since Alex did not do well on the first

quiz and did not get the solution correct, I decided to give him the same quiz to see if he improved.

The first time Alex took the quiz he scored a 3. He struggled with how to set the problem up and had a lot of computational errors. After the fourth week he realized that he knew how to find probability, he knew that he needed to write 18/32, which he reduced to 9/16. The second question was about setting up a proportion. He wrote:

> To find the number of students that liked pepperoni pizza when there are 384 students, I will start by using the answer from #1 which was 9/16. I need to set up a proportion and then cross multiply to find the total number of students that liked pepperoni pizza in that school.
>
> $\frac{9}{16} \times \frac{n}{384} = 16n * 3456$
>
> Next, I need to divide 3456 by 16 to get n alone.

Alex was doing everything correctly but he did come up with the wrong answer. He got 215; after looking at his work, I was able to see he just miscalculated. As I scored his rubric this time, I saw improvement.

Problem Solving Journal Rubric

Alex	0	1	2	3	4
Mathematical Knowledge	Shows no understanding of the problem.	Shows very limited understanding of the problem, major computational errors.	Shows understanding of some of the problems mathematical concepts may contain computational errors.	Shows nearly complete understanding of the problems mathematical concepts and principles, no computation errors.	Shows complete understanding of the problems mathematical concepts and principles, no computation errors.

Communication	Words do not reflect the problem, may include drawings which completely misrepresent the problem.	Has some satisfactory elements, but fails to complete or may omit significant parts of the problem.	Makes significant progress towards completion of problem, but ambiguous and unclear.	Gives a fairly complete response with reasonably clear explanations or descriptions.	Gives a complete response with clear and appropriate diagrams.
Writing	No/little attempt to write a paragraph with 3-5complete sentences; excessive mechanics and convention errors.	Some complete sentences, but not a 3-5 sentence paragraph; many mechanics and convention errors.	Complete sentences, but not a 3-5 sentence paragraph; some mechanics and convention errors.	3-5 complete sentence paragraph; minor mechanics and convention errors.	More than 5 complete sentences with accurate mechanics and conventions.
Solution	No solution stated. .	Solution is not stated, and is incorrect.	Solution is stated, and is incorrect.	Incorrect solution is stated, but is reasonable.	Correct solution is stated.

When Alex first started this project, he scored 3 out of 16 points. Upon completion of the project he scored 11 out of 16 points. I believed that there was satisfactory improvement by Alex.

When looking at the rubrics from Alex's quiz scores I believe they show a growth in the students' written communication. By having students write down exactly what they were thinking, students started to understand mathematical concepts better and hopefully became more confident in their abilities.

Oral Communication

During the oral explanations it was easy to check for understanding based on the class discussion. If the students had completed the problems that were to be presented before the class presentations, then the class would be able to have a more involved discussion. Getting the

students involved in the discussion helped in deepening student understanding as students learned of their own mistakes as well as their classmates.

Some evidence from my group interviews supported this finding. One of my group interview questions was, “Do you think you could learn more by listening to your peers solving problems?” Students gave these responses:

- *“Yeah, I guess. Sometimes they have different ideas than me so I might get some different ideas.” (Madison)*
- *“I suppose so. I think Elizabeth is pretty good in Math so I think I could learn more from listening to her.” (Alex)*
- *“If I was able to work the problem first, then I think I would have a better understanding of what was being discussed. After listening to the others I might decide that I did my problem wrong or differently.” (Brenna)*

Another question that I asked the group was, “Will it be easy or difficult to understand other students’ explanations during the verbal presentations?” Students had mixed feelings about this question. Some of their responses are listed.

- *“I think it will be hard because not everyone thinks like me and I think I am going to have a hard time understanding them.”(Trent)*
- *“I think that it is going to be a little of both. I struggle in math and think that some of the students will only confuse me more.” (Alex)*
- *“I need to be able to see what someone is doing if I am going to follow along. I think it is hard for me to understand something that is just said aloud.”(Brenna)*
- *“If I have worked on the problem before the discussion, I think I will have an easier time following their explanations.” (Elizabeth)*

I found it interesting how the students responded to this question. Overall I believed, from these responses, that these students do not have a lot of confidence in their own abilities when it comes to mathematics.

I found two instances where I had written in my personal journal about students working on the problems before we met as a whole group to present.

> *While listening to the class's discussion I am wondering what would have happened if the students had not worked on the problems before getting together as a whole group. I wonder what direction our discussion would have gone if the only person who had seen the problem before would have been Elizabeth. If Elizabeth would have been the only one to see the problem and was presenting to them for the first time, would the students still have caught her error? I believe because they had all seen the problem and had an opportunity to work on it, our class discussion was better.* (Personal Journal, March 26, 2009).

It is apparent from this entry that I was seeing a change in our mathematics discussion when the students were exposed to the problem before the class discussion. I found more proof of this finding in another journal entry.

> *What an exciting math class! The students were assigned a problem to work and then present in class. Since students are getting more comfortable with the verbal presentations, I asked Trent and Madison to give their solutions. When they were done giving their solution, the class started their discussion about the solution and amazingly, they all got the same answer. The exciting part was when Brenna asked if she could share how she solved the problem. Although the students all got the same answer, after hearing Brenna's method, several of the students agreed that this method made more sense to them. What a ego booster for Brenna! She really struggles with math.* (Personal Journal, April 2, 2009).

Looking back at my journal entries I was able to confirm that the classroom discussions were deeper when students had previous access to the problems before the class presentation/discussion time. I noticed that several of the students looked forward to presenting the problems orally. One student who especially liked to share her thoughts was Elizabeth. She really shined during the oral explanations. One day I gave her the problem:

> Joanie purchased her school uniform for $135. This was the price after a 15% discount. What was the original price of her uniform?

Elizabeth thought for a while and after doing some figuring, she said,

> To solve this problem, I need to figure out what the original price of the problem was. I didn't know how to start so I just started picking different numbers, higher

than 135 and multiplied them by 0.15. I tried starting at 190 and multiplied it by 0.15. I got 28.5. Next, I subtracted it from 190 and got 161.5. That was too high so I continued this process until I got an answer of 158.85. When I multiplied it by 0.15, I got 23.8275. I subtracted it from 158.85 and got 135.0225. I decided that was pretty close so I believe $158.85 was the original cost of the uniform.

Below is Elizabeth's rubric:

Problem Solving Journal Rubric

Elizabeth	**0**	**1**	**2**	**3**	**4**
Mathematical Knowledge	Shows no understanding of the problem.	Shows very limited understanding of the problem, major computational errors.	Shows understanding of some of the problems mathematical concepts may contain computational errors.	Shows nearly complete understanding of the problems mathematical concepts and principles, no computation errors.	Shows complete understanding of the problems mathematical concepts and principles, no computation errors.
Communication	Words do not reflect the problem, may include drawings which completely misrepresent the problem.	Has some satisfactory elements, but fails to complete or may omit significant parts of the problem.	Makes significant progress towards completion of problem, but ambiguous and unclear.	Gives a fairly complete response with reasonably clear explanations or descriptions.	Gives a complete response with clear and appropriate diagrams.
Writing	No/little attempt to write a paragraph with 3-5complete sentences; excessive mechanics and convention errors.	Some complete sentences, but not a 3-5 sentence paragraph; many mechanics and convention errors.	Complete sentences, but not a 3-5 sentence paragraph; some mechanics and convention errors.	3-5 complete sentence paragraph; minor mechanics and convention errors.	More than 5 complete sentences with accurate mechanics and conventions.
Solution	No solution stated. .	Solution is not stated, and is incorrect.	Solution is stated, and is incorrect.	Incorrect solution is stated, but is reasonable.	Correct solution is stated.

Elizabeth scored 10 out of 12 points on the oral quiz. I believed that although her method for solving the problem was not the fastest or maybe the best, she did do a nice job. By

the end of my project, I could see a real improvement in the scores of the students' oral communication.

Of course, I did find a few problems with students orally communicating their explanations through the class presentations. Through these discussions, students were exposed to more than one strategy for solving a problem. Although the exposure was good, I saw that some students also became frustrated with the different strategies, if they could not comprehend them. On March 26, 2009, I wrote in my journal about an instance of this frustration.

> *I do enjoy listening to the students as they discuss the solutions to these problems. We have just started this project but I am slowing seeing a change in how the students communicate with each other. A problem did arise this week when Alex had a minor melt down. He was trying hard to understand the way the Trent and Elizabeth solved their problem, but just could not understand. He felt like he was the only one who couldn't do it in the same way as his classmates and tried to shut down. This is not the first time Alex has been frustrated with this project.*

Although it took extra time to explain to Alex that it was okay to solve a problem in a different way, I believed it was a great opportunity to explain to the others that this will probably happen again, in the future. I also tried to point out to Alex that he should feel good about the fact that he had a way that none of the other students had thought of.

Overall, my findings on oral communication were positive. On April 8 I wrote, "Madison is getting so much better at explaining her thought process as compared to week one. She stated to me that she is better at written explanation and doesn't like to speak in front of her peers. I have noticed a terrific change in her. She has developed confidence in her abilities and isn't shutting down when I ask her for more clarification on her explanation." I believe that students really are gaining a new confidence from these oral explanations, based on my observations.

A few minor setbacks have occurred but the understanding that the students gained far outweighed the setbacks. Getting the students involved in oral communication helped students

gain a deeper understanding during these presentations when the students had been given the problem ahead of time to work on it before the class discussion. Students learned that there was more than one way to solve a problem through these discussions.

Changes in Teaching

Throughout this study I was also trying to make changes in myself. I do not know if I necessarily improved in the area of asking questions, but I will continue the process. I wonder if I received the responses I was hoping for even when I changed the style of questions I was asking. I am always trying to improve myself as a teacher, but still need a lot of improvement. Although the amount of explanations in the students work diminished, as I stated before, I felt students kept digging into the questions more because of the questions I asked. On April 17, I interviewed students before finishing my project. Several of the students said things like:

- "*You used to assign problems that were from the book. They were okay, but then you started giving us these new problems and you made us do extra thinking. When we came up with an answer, you would ask more questions. At first this made me mad because I did not like having to explain my thinking so much. But then I realized I was starting to "get" how to do the problem better from those questions. Thanks!" (Trent)*

- *"I like how you made us think "out of the box." (Madison)*

- *"I liked this project because you didn't just accept our answer the first time. You would ask another question that would make us question our first answer.*" (Elizabeth)

Their responses made me feel good. Not only were the students changing but they were seeing the change in me. In my personal journals I wrote about some of the changes I was making in attempt to better myself as a teacher. After the first week, I wrote in my journal about a change I was making in myself.

> *After completing this week I see that I need to show students the scoring rubric before giving them the problem. I have noticed that students are unclear about my expectations of their work. I need to take time to slow down and be very specific. We will start on Monday by carefully going through a problem and I will demonstrate exactly what I want. (*Personal Journal, March 26th, 2009).

By stopping and being specific with my expectations will help in achieving the results that I am requiring of the students written explanations. I want students to be successful in mathematics and I realize I need to be clear in what I expect in mathematics.

Another instance when I wrote about the changes in my teaching was on April 9. During this week I found myself frustrated with the students' lack of effort in the work they were turning in. I know through Math in the Middle, the one thing that I appreciated from the professors and assistants was their patience when I was struggling. It was from these experiences that I realized another important change.

> *I am very frustrated with the laziness of the students and them submitting poor work. I am struggling with how to get the students to regain their focus and keep my composure. I remember my Math in the Middle experience and a time when I was not putting forth the effort that I should have been. I wanted to give up and just wait until class presentations where I could copy the work. It was the professor that came over and calmly asked me what was going on. She patiently sat there with me and got me refocused. It was from this experience that I realized that I need to be more patient. I know I will get more out of this project as will the students if we can all remain calm. (*Personal Journal, April 9, 2009).

From developing more patience with my students I was able to get them to be more patient with each other. I believe the students will benefit in many ways if everyone can calmly discuss issues and disagreements.

During this week I also wrote:

> *I have noticed that through this process I have changed my thinking and understanding of mathematics. I have worked with these five students for three weeks on written and verbal communication and although the results of the work turned in have "slipped" I do feel that students have changed their thinking process with math. I have worked with them on specifically explaining their thinking and I have sat with the students to challenge their solutions. Sometimes they don't appreciate me making them think about something else, but when the students make a new connection with a method, they thank me.* (Personal Journal, April 9, 2009).

In four weeks, I can see a change in myself as a teacher and I believe that continuing written and oral communication with my future classes will only benefit me as a teacher in all areas, not just mathematics.

Conclusions

I have come to realize that challenging my students to communicate mathematics, both in writing and orally, allowed my students the opportunities to get more comfortable with mathematical concepts. Through these presentations I believe that students were able to learn new methods for doing open-ended problems. They were able to gain a deeper understanding of the process. This deeper understanding happened when students were probed with questions that made them reflect on their learning and re-evaluate their reasoning. The written and oral communication helped students realize that their understanding does not come from merely completing homework problems, but from "talking" through the problem with their peers. The students had to re-evaluate their thinking as well as their classmates, and these things helped reach that deeper understanding.

My conclusions are consistent with the research done by Adler's (1999) case study. Adler found that, "For talk to be a resource for mathematics learning it needs to be transparent; learners must be able to see it and use it" (p. 63). Through my research project I wanted to achieve this same statement. I wanted students to be able to see the process and be able to use it to connect with the problems that I had them work on.

Borasi and Rose's study (1989) of a college mathematics course had similar conclusions to what I found. Borasi and Rose found,

> Journal writing in fact introduces new important dimensions in the mathematics classroom: by writing in the journals, students make use of writing as a learning tool in the context of mathematics; by reading students' journals, teachers access a wealth of information usually unavailable to them; and by commenting on

> students' entries, responding to specific questions and posing new ones, teachers engage in a unique and continuous dialogue with each individual student throughout the course. (p. 362)

I believe the journals were very powerful for me, as the teacher, to "see" more about my students that I don't normally get to know about them.

In Wood's (1999) study, the students' disagreements were resolved by arguing. Students were about to experience mathematics as a subject that relied on reasoning for the justification of ideas. These second grade students were expected to follow the thinking and reasoning of others, and then voice their disagreement. When they challenged another student's reasoning this was an indication, to the researcher, of deeper understanding. I believe this did occur with my students as well. However, I am not convinced that the disagreements always resulted in a deeper understanding for all of my students. I believed that, for at least two students, they left many presentations more confused than before we started the discussions. I found myself getting more involved to help clarify a situation when I saw the frustration starting to arise. On April 2 I wrote:

> *This week I had to get involved in the class's discussion about the strategies they could use in solving a problem involving several different operations. The students were talking about whether they should all do it on their own and then get back together to discuss everyone's solution. Now this problem had 5 different parts and I noticed that Madison was not really listening to what the other students were saying. She was counting the problems and then had an idea. She suggested that each student take one of the five parts and solve just that one problem and then once everyone had time to work on their problem they would come back and meet as a group. She said that way not everyone had to do all of the work and each person could focus on their problem instead of all of the problems. From the expressions on Brenna and Elizabeth's faces, I could tell that they did not like that idea. Those two girls believed that their ideas were better and I don't believe that they even listened to Madison, because they just kept talking about how to go about solving the long problem. (Alex and Trent typically just sit back and let the girls make all of the decisions.) Madison was patiently waiting for Elizabeth and Brenna to quit talking so she could again give her idea, but the two girls would not listen. Madison was starting to get mad. I decided to sit down beside the class and ask them what they were thinking. Of course,*

> *Elizabeth led the discussion. I asked if anyone else had any thoughts and Madison was reluctant to share her strategy. I had to work with her to open back up. Again, I noticed that Elizabeth and Brenna did not seem interested in her idea. I finally asked the girls why they did not like this idea. After a lengthy discussion, I found out that Brenna did not really understand the method that Madison came up with but did not want to let anyone know that she did not understand what Madison was suggesting. I believe Elizabeth and Madison must be fighting and that is why Elizabeth would not listen to Madison. The boys thought that Madison's idea would work best for them.*

I noticed throughout the project that I had to intervene more when Madison and Elizabeth did not agree. Even with these few instances that resulted in my getting involved, our end result was not affected.

Overall, I believe my research resulted in a deeper understanding for the students when the students were able to discuss their ideas with one another.

Implications

When thinking about how I am going to use the conclusions from my research, as I look to the future teaching of mathematics in my classroom, I believe I am going to listen to my students. I am going to be more aware of the students' frustration levels, and make sure I work on intervening in situations before someone is brought to tears. I am going to encourage the students not to be afraid of being different in their thinking. I know from being involved in Math in the Middle the confidence that comes from talking about mathematics. From my findings I have witnessed, just after four weeks, that students' confidence has increased, their conversations have been rich in mathematical discussion, and they appear to understand more components of math.

Based on student responses, I do plan to continue using written and oral communication in mathematics in the years to come. I will continue having students use the journal, not only to help me see where the student's level of understanding is, but also to help the students to be able

to reflect on their own growth. They will be able to go back throughout the year and hopefully see a change. At the conclusion of my research I believe that I know my students better than I did before the project started. I believe that by the time the research study was over my students were more confident talking math with me. I believe that my research has given me new inspiration for teaching mathematics, and I am very excited to get started with the incoming fifth and sixth grades students in the future.

As I think about my future classes and the continuation of written and oral communication in mathematics, I have decided that I want to let the other elementary teachers in my building know about my project and the success that I believe I have achieved from having students communicate mathematics in writing and orally. I believe that if I can convince the teachers who teach below me to implement this process in their classes, students will gain more understanding about mathematical concepts. I want the other elementary teachers to help their students get more involved in mathematics by allowing them the opportunities to talk about math in more depth. I would like to also talk with our high school math teacher about my project and see if he would be interested in trying it with the older students. I believe students can only benefit if all of the teachers at my school would teach mathematics using the techniques of my study.

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Appendix A

Math Performance Interviews

Oral Communication

Say to the student:

"I would like you to work on this problem, saying aloud whatever it is you are thinking as you work through the problem. I especially want to hear you talk about how you decide what to do to solve the problem."

(Choose an appropriate open-ended problem that focuses on similar topics that are being covered in class at the time of the interview. Score students with the presentation rubric. I will record the observation in my journal.)

Written Communication

Say to the student:

"I would like you to work on another problem, only this time, explain your thinking and justify your solution in writing. I especially want you to write about how you decide what to do to solve the problem."

(Choose an appropriate open-ended problem that focuses on similar top topics that are being covered in class at the time of the interview. Score students with the presentation rubric. I will record the observation in my journal.)

Example problems that could be used for math performance interviews:

3. Kaylee spent 3/5 of her money on music cds and 1/6 of the money she had left on breakfast. She left the house with $35 and the only money she spent was on the music cds and breakfast. How much money did Kaylee return with?
4. The school store ran out of Wolves basketball shirts on Monday. The store received an order of shirts on Wednesday. During that day, the store sold 22 shirts. On Thursday, the store sold 1/3 of the shirts that were left. When the school store opened on Friday, there were 112 available. How many shirts did the order contain?
5. The New Year has just begun and "Wil-Mart" wants to sell last year's movies to make room for the new releases. They are reducing the price of the movies from $25 to $16. What is the rate of the discount?
6. When Braden opens and lays a cracker box out flat, he sees that the top and bottom of the box both measure 4" by 4" and the sides of the box measure 4" by 9.5". What is the surface area of the cracker box?

7. A square public statue measures 45 feet on each side. 1’ square tiles will surround the base of the statue to make a walkway for the spectators. It will measure 15’ wide. What is the area to be covered by the tiles?
8. A baby calf weighs 65 pounds at birth. It gains 3.0 pounds per day. At this rate, how old will the calf be when it reaches two times it birth weight?
9. A survey was conducted on a school’s drama program. During this survey they found that 145 students wanted an after-school drama program, 600 students wanted drama to be during the school day, and 320 students thought that there should not be a drama program. Create a circle graph to represent these student responses.
10. Lorenzo asked every tenth student who walked into school to name their favorite pizza topping. Their responses were: 18 pepperoni, 9 cheese, 3 sausage, and 2 mushroom. What is the probability that a student will prefer pepperoni pizza?
11. If there are 384 students at the school Lorenzo attends, use a proportion to predict how many students prefer pepperoni pizza.
12. Joanie purchased her school uniform for $135. This was the price after a 15% discount. What was the original price of her uniform?

Appendix B

Problem Solving Journal Rubric

	0	1	2	3	4
Mathematical Knowledge	Shows no understanding of the problem.	Shows very limited understanding of the problem, major computational errors.	Shows understanding of some of the problems mathematical concepts may contain computational errors.	Shows nearly complete understanding of the problems mathematical concepts and principles, no computation errors.	Shows complete understanding of the problems mathematical concepts and principles, no computation errors.
Communication	Words do not reflect the problem, may include drawings which completely misrepresent the problem.	Has some satisfactory elements, but fails to complete or may omit significant parts of the problem.	Makes significant progress towards completion of problem, but ambiguous and unclear.	Gives a fairly complete response with reasonably clear explanations or descriptions.	Gives a complete response with clear and appropriate diagrams.
Writing	No/little attempt to write a paragraph with 3-5complete sentences; excessive mechanics and convention errors.	Some complete sentences, but not a 3-5 sentence paragraph; many mechanics and convention errors.	Complete sentences, but not a 3-5 sentence paragraph; some mechanics and convention errors.	3-5 complete sentence paragraph; minor mechanics and convention errors.	More than 5 complete sentences with accurate mechanics and conventions.
Solution	No solution stated. .	Solution is not stated, and is incorrect.	Solution is stated, and is incorrect.	Incorrect solution is stated, but is reasonable.	Correct solution is stated.

Appendix C

Individual Student Interview Questions

- How would you describe yourself as a mathematics student?
- How would you describe yourself as a student in other school subjects?
- Tell me about your feelings and opinions toward giving presentations on daily mathematics problems.
- Tell me about your feelings and opinions toward justifying math solutions in a math journal on daily mathematics problems.
- Do you think most other people in the class feel the same way? Please explain.
- What do you like about presentations and verbal communication?
- What don't you like about presentations and verbal communication?
- What do you like about journaling and written communication?
- What don't you like about journaling and written communication?
- How easy or hard is it to understand students' explanations during their presentations?
- Have presentations and journals helped you understand more mathematics this semester? Please explain why or why not?
- What advice would you give me about whether I should have my class next year do presentations and journals or not? Please explain.

Appendix D
Group Interview Questions

I will interview groups of two students at the beginning and at the end of the research project to determine how their attitudes towards mathematics will change or has changed by being challenged to reason both verbally and in writing, and which method they prefer.

Beginning Interview:

- Why do you think I have started having students do problem solving presentations?
- Why do you think I have started having students justify their solutions in writing?
- Do you think communicating mathematics verbally and in writing well increase your understanding of math topics?
- Will it be easy or difficult to understand other students' explanations during problem solving presentations?
- Do you think you could learn more by listening to your peers solving problems?
- Do you think it would be easier to present a solution if you first explained your solution in a math journal?
- Do you think you'd prefer to communicate your mathematical thinking and understanding verbally or in writing? Why?
- Do you have any questions about increasing your mathematical understanding through verbal and written communication?

Ending Interview:

- Why do you think I required students to do problem solving presentations?
- Why do you think I required students to justify their solutions in writing?
- Do you think communicating mathematics verbally and in writing has increased your understanding of math topics?
- Was it easy or difficult to understand other students' explanations during problem solving presentations? Why?
- Do you think you learned more by listening to your peers solve problems?
- Do you think it was easier to present a solution when you first explained your solution in a math journal?
- Do you prefer to communicate your mathematical thinking and understanding verbally or in writing? Why?
- As I consider if I will require problem solving presentations and math journals next year, what advice would you give me?
- Think back to when we first began doing problem solving presentations and math journals:
    - How have the presentations and journal entries changed throughout the semester?
    - Describe the pros and cons of doing the problem solving presentations, hearing the presentations, and writing about your thinking and solutions.
- Is there anything you would like to ask me about increasing your mathematical understanding through verbal and written communication?

Appendix E

Personal Journal Prompts

I will complete a form weekly for my personal journal notebook given to us.

Brandee Wilson
Personal Journal
Date:

1. What are two observations you made this week regarding students' attitude and responses toward daily presentations and journal writing?

2. What are two observations you made this week regarding preparedness in presentations and journaling?

3. What are two observations you made this week regarding students' attitudes while working together in class?

4. How has this week's presentations and journaling influenced your lesson plans (if at all)?

5. What is one question you have after this week?

6. Other comments or observations: