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CARVM and NAIC Actuarial Guidelines 33 & 34

Keith P. Sharp*

Abstract

Annuity valuation under the NAIC Standard Valuation Law is determined according to methods different from those methods used for life insurance. The CARVM assumption of efficient policyholder selection is clarified under NAIC Actuarial Guidelines 33 and 34 to allow for non-elective (e.g., death) benefits. In particular, Actuarial Guideline 34 is oriented toward variable annuities and prescribes methods to be used in the presence of a minimum guaranteed death benefit. In this paper these methods are examined and illustrated with examples.

Key words and phrases: *annuity, elective benefit, valuation, reserves*

1 Introduction

In the previous article in this volume, Sharp (1999) explained the calculations involved in determining annuity reserves under the commissioners annuity reserve valuation method (CARVM). These reserves are calculated by a method different from that used for insurance reserves (American Academy of Actuaries, 1997). CARVM assumes that for elective benefits such as surrender, the policyholder will select with 100 percent efficiency the best time to make the election, if the comparisons are made using the company's valuation rate of interest. More

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concisely, this is the worst time for the insurance company. In some simple cases the CARVM reserve is calculated using formulas containing no probabilities.

In this paper we consider the treatment of annuities with a (non-elective) benefit on death under National Association of Insurance Commissioners (NAIC) Actuarial Guideline (AG) 33 (NAIC, 1998). In two examples we consider the case of a fixed (nonvariable) annuity. The treatment is extended in later examples to the valuation under NAIC Actuarial Guideline 34 of variable annuities with a minimum guaranteed death benefit (MGDB).

2 Actuarial Guideline 33

After its 1976 introduction there was some disagreement about how CARVM should be applied to the situation where there were potentially elective and non-elective (e.g., death) benefits. After the issue of Actuarial Guideline 33 (formerly GGG) (see, e.g., Lalonde, 1995) there continued to be some confusion on this issue. The method of using CARVM in complicated situations, however, now has been largely resolved.

At its September 1995 meeting, the NAIC Life and Health Actuarial Task Force interpreted Actuarial Guideline 33 to require consideration of integrated benefits in the CARVM stream(s). Here *integrated* refers to the consideration of the present value of benefit streams under which certain proportions of policyholders are dying and the remaining policyholders are selecting the optimum time of surrender. Under the revised version of Actuarial Guideline 33 effective December 31, 1998, benefits are classified as either elective or non-elective. Each possible set of elections then is considered. This may result in a large tree of possible sets of elections.

For example, there may be a policy provision for annual surrender of up to 10 percent of the annuity value without imposition of a back-end load (surrender charge). One possible branch of the tree would correspond to a 10 percent surrender at the end of policy year one, a 5 percent surrender at the end of policy year two, a 10 percent surrender at the end of policy year three, etc. Typically one can use linearity to cut the number of branches to be tested. In other words, the reserve candidate is likely to be a linear function of the surrender proportion. Hence, the reserve candidate is a monotonic (increasing or decreasing) function of the surrender proportion. In this case the maximum corresponds to either the lowest or the highest possible surrender proportion. In this example, the CARVM maximum would likely correspond to either a 0

percent or 10 percent surrender at the end of year two, not a 5 percent surrender. Superimposed on this structure would be the probabilities of death, a non-elective benefit for which the use of expected values is appropriate. A valuable intuitive analysis of complicated situations like this is given by Backus (1998).

3 Example 1: Simple CARVM with Zero Deaths

The following notation is used throughout all examples:

$$\begin{aligned}
 SC_t &= \text{Surrender charge;} \\
 AV_t &= \text{Account value at the end of year } t; \\
 &= AV_{t-1} \times 1.06 \\
 AVAVG_t &= \text{Average account value at the end of year } t \\
 &= (AV_{t-1} + AV_t)/2; \\
 CSV_t &= \text{Cash surrender value at the end of year } t; \\
 &= AV_t \times (1 - SC_t); \\
 DB_t &= \text{Death benefit at the end of year } t; \\
 NAR_t &= \text{Net amount at risk at the end of year } t; \\
 NARAVG_t &= \text{Average net amount at risk at the end of year } t \\
 &= (NARAV_{t-1} + NARAV_t)/2; \\
 P_t &= \text{Probability of survival from} \\
 &\quad \text{Dec. 31, 1999 to Jan. 1, } t; \\
 q_t &= \text{Assumed annual mortality in year } t; \\
 PV_2(NARAVG_t) &= \text{Present value at Dec. 31, 1999 of } NARAV_t \\
 &\quad \text{paid on midyear death in year } t; \\
 &= NARAV_t \times P_t \times q_t \times 1.07^{-(t-1999.5)} \\
 PV_2(CSV_t) &= \text{Present value at Dec. 31, 1999} \\
 &\quad \text{of future Dec. 31 CSVs in year } t; \\
 &= CSV_t \times P_t \times (1.07)^{-(t-1999)} \\
 PV_2(AVAVG_t) &= AVAVG_t \times P_t \times q_t \times 1.07^{-(t-1999.5)}
 \end{aligned}$$

We first present an example using simple CARVM assuming zero deaths. The assumptions are given in Table 1.

Table 1
Valuation Assumptions for
A Single Premium Deferred Annuity (Fixed)

Issue date:	January 1, 1998
Single premium:	\$60,000
Accumulation	
Guaranteed:	6 percent per annum
Actual for 1998 and 1999:	6 percent per annum
Death benefit:	\$100,000
Front end load:	0 percent
Back end load	
Policy year 1:	8 percent
Policy year 2:	4 percent
Policy year 3:	0 percent
Policy year 4:	0 percent
Valuation date:	December 31, 1999
Valuation mortality rate	
Policy year 1:	0.000
Policy year 2:	0.000
Policy year 3:	0.000
Policy year 4:	0.000
Valuation interest rate:	7 percent per annum

Specifically, assume a January 1, 1998 issue of a \$60,000 single premium deferred annuity credited with a guaranteed 6 percent per annum. In other words, the account value (fund value) visible to the policyholder is credited at a rate of at least 6 percent per annum.

Assume that the contract specifies that the policy matures after four years. To motivate a later discussion of a variable minimum guaranteed death benefit we discuss a policy with (perhaps unrealistically) a \$100,000 minimum death benefit.

A valuation is to be performed on December 31, 1999, and we need to consider possible surrender on December 31, 1999, December 31, 2000, or December 31, 2001. The immediate December 31, 1999 cash surrender value (CSV) forms a floor for the CARVM reserve. The two dates December 31, 2000 and December 31, 2001 represent two candidates for the status of maximum present value at December 31, 1999.

The CARVM reserve is the greater of these two but with a floor of the immediate CSV. The reserve calculations are shown in Table 2.

Surrender on December 31, 1999

We have accumulation at 6 percent per annum so $AV_t = AV_{t-1} \times 1.06$, where AV_t is the account value at the end of year t . By December 31, 1999 the account value (Rows (3) and (4) of Table 2) has grown to $60,000 \times 1.06^2 = \$67,416$. An immediate surrender would be for $67,416 \times (1-0.04) = \$64,719$, which is a floor to the CARVM reserve.

Surrender on December 31, 2000

A surrender at December 31, 2000 is projected to give a CSV of $60,000 \times 1.06^3 = \$71,461$ and hence a reserve candidate at December 31, 1999 of $71,461/1.07 = \$66,786$.

Surrender on December 31, 2001

A surrender at December 31, 2001 is projected to give a CSV of $60,000 \times 1.06^4 = \$75,749$ and hence a reserve candidate at December 31, 1999 of $75,749/1.07^2 = \$66,162$.

Table 2
Reserve Using Assumption of Zero Mortality

	Policy year from Jan. 1, (t)			
	1998	1999	2000	2001
SC_t :	8%	4%	0%	0%
AV_{t-1} at Jan. 1:	60,000	63,600	67,416	71,461
AV_t at Dec. 31:	63,600	67,416	71,461	75,749
$AVAVG_t$:	61,800	65,508	69,438	73,605
CSV_t :	58,512	64,719	71,461	75,749
$PV_2(CSV_t)$:		64,719	66,786	66,162

CARVM Maximum of the Candidates:

The largest of these candidates is \$66,786, which is the CARVM reserve at December 31, 1999 if we are assuming zero mortality. Here we

are using strict noncontinuous CARVM, examining only surrenders on the last day of each contract year.

We assume that no decision has been made to use continuous CARVM, that is, to use the maximum over all possible days of surrender. New York requires the use of continuous CARVM. Many actuaries (including the author) believe that continuous CARVM gives more appropriate reserves. In this case a surrender on January 1, 2000, the day after valuation, would give a CSV of \$67,416 because the surrender charge is then zero. In reality, it would be preferable to use the \$67,416 as floor to the standard CARVM reserve which otherwise is \$66,786.

4 Example 2: Assuming Non-Zero Deaths

Let us extend our example to highlight the elective/non-elective distinction. Now the previous contract is revalued assuming nonzero deaths, as indicated below. The fixed \$100,000 death benefit is now integrated into the reserve calculation. The assumptions are given in Table 3.

The reserve will consist mainly of the present value of the elected cash surrender value (CSV), but we add also the value of deaths by those who otherwise would make the optimal selection. Surrender on December 31, 2000 or 2001 gives two candidates for the status of maximum present value at December 31, 1999. With consideration of the floor of the immediate December 31, 1999 CSV we have three candidates for the CARVM reserve: surrender on December 31, 1999, surrender on December 31, 2000, or surrender on December 31, 2001.

Surrender on December 31, 1999

Here we consider an immediate surrender on the valuation day. As in Table 2, we have a CSV of \$64,719. Under New York (continuous) CARVM (New York Insurance Law, Section 4217(6)(D)) we consider also a surrender a day later, on January 1, 2000. Under noncontinuous CARVM, however, we do not consider a January 1, 2000 surrender even though that would give a higher value because the 4 percent load has then become zero. The December 31, 1999 candidate to be the CARVM maximum is \$64,719.

Surrender on December 31, 2000

A proportion $(1 - 0.019)$ of the policyholders survives to the end of calendar year 2000 and under this candidate they then surrender for a

Table 3
Valuation Assumptions for
A Single Premium Deferred Annuity (Fixed)

Issue date:	January 1, 1998
Single premium:	\$60,000
Death benefit:	\$100,000
Valuation interest rate:	7 percent per annum
Accumulation	
Guaranteed:	6 percent per annum
Actual for 1998 and 1999:	6 percent per annum
Front end load:	0 percent
Back end load	
Policy year 1:	8 percent
Policy year 2:	4 percent
Policy year 3:	0 percent
Policy year 4:	0 percent
Valuation date:	December 31, 1999
Valuation mortality rate	
Policy year 1:	0.015
Policy year 2:	0.017
Policy year 3:	0.019
Policy year 4:	0.022
Deaths occur in the middle of the year	

CSV of $\$60,000 \times 1.06^3 = \$71,461$. The present value at December 31, 1999 is

$$71,461 \times (1 - 0.019) \times 1.07^{-1} = \$65,517.$$

Those who die (deaths are assumed to occur on June 30, 2000) receive a death benefit of \$100,000; thus the present value of the death benefit is $100,000 \times 0.019 \times 1.07^{-0.5} = \$1,837$. This can also be calculated (Table 4) as the rounded sum of the present value ($1,275 + 561 = \$1,837$) of two components:

- The average account value at death in 2000,

$$(67,416 + 67,416 \times 1.06)/2 = \$69,438,$$

with present value

$$69,438 \times 0.019 \times (1.07)^{-0.5} = \$1,275.$$

- The average excess of the death benefit over the average account value,

$$(100,000 - 69,438) \times 0.019 \times 1.07^{-0.5} = \$561.$$

The Table 4 approach is comparable to that used later in valuing a minimum guaranteed death benefit.

Hence the total value of this candidate is $65,517 + 1,837 = \$67,354$.

Surrender on December 31, 2001

A proportion $(1 - 0.019) \times (1 - 0.022)$ of the policyholders survives calendar years 2000 and 2001 and under this candidate they then surrender for a CSV of $\$60,000 \times 1.06^4 = \$75,749$. The present value at December 31, 1999 is

$$75,749 \times (1 - 0.019) \times (1 - 0.022) \times (1.07)^{-2} = \$63,477.$$

Those who die during 2001 (on June 30, 2001) receive \$100,000; thus the present value of the year 2001 death benefit is

$$100,000 \times (1 - 0.019) \times 0.022 \times 1.07^{-1.5} = \$1,949.$$

Some fellow cohorts of those surrendering on December 31, 2001 die also in 2000, so we add that present value also (above), $1,837 + 1,949 = \$3,787$, rounded to agree with the $1,076 + 2,711 = \$3,787$ of Table 4. Hence the total value of this candidate is $63,477 + 3,787 = \$67,264$.

Table 4
Integrated Reserve Including Fixed \$100,000 Death Benefit

Policy year commencing Jan. 1 (t):	1998	1999	2000	2001	2002
CSV_t at Dec. 31:	58,512	64,719	71,461	75,749	
DB_{t-1} at Jan. 1:	100,000	100,000	100,000	100,000	
DB_t (death benefit) at Dec. 31:	100,000	100,000	100,000	100,000	
NAR_{t-1} (benefit top-up) at Jan. 1:	44,800	38,944	32,584	28,539	
NAR_t (benefit top-up) at Dec. 31:	41,488	35,281	28,539	24,251	
$NARAVG_t$ (benefit top-up):	43,144	37,112	30,562	26,395	
q_t :	0.015	0.017	0.019	0.022	
P_t :			1.000	0.981	0.959
$PV_2(NARAVG_t)^*$:			561	515	
$PV_2(AVAVG_t)$:			1,275	1,435	
$CUMPV_2(NARAVG_t) = \sum_{s=1999}^t (NARAVG_s)$:			561	1,076	
$CUMPV_2(AVAVG_t) = \sum_{s=1999}^t (AVAVG_s)$:			1,275	2,711	
$PV_2(CSV_t)^{**}$:		64,719	65,517	63,477	
Total-Integrated reserve (V_2^{INT}):		64,719	67,354	67,264	

Notes: V_2^{INT} is the maximum of this row of candidates $V_{2,t}^{CAND}$, and is the sum of the previous three rows of this table.

* $PV_2(NARAVG_t) = NARAVG_t \times P_t \times q_t \times 1.07^{-(t-1999.5)}$.

** $PV_2(CSV_t)$ at Dec. 31, 1999 of Dec. 31 $CSV = CSV_t \times P_t \times 1.07^{-(t-1999.5)}$.

CARVM Maximum of the Candidates

Hence we have for our valuation at December 31, 1999 three possible elections for surrender:

- December 31, 1999: \$64,719
- December 31, 2000: \$67,354
- December 31, 2001: \$67,264.

For this fixed annuity under CARVM, in marked contrast with life insurance valuation, we use the greatest of these three candidates, \$67,354, as our CARVM reserve at December 31, 1999.

5 Example 3: Minimum Guaranteed Death Benefits

Under certain annuity designs, often called *variable annuities*, the account value, and hence the CSV, varies with the investment performance of the underlying assets. Commonly the contract specifies that on death the benefit will be the greater of the account value and a minimum guaranteed death benefit.

Consider a single premium variable annuity with valuation assumptions given in Table 5. We will look at various possible contract provisions defining the death benefit. The benefit on surrender on August 14, 2000 will be the account value net of a 4 percent back-end load:

$$10,000 \times (1 + 0.12) \times (1 - 0.13) \times (1 - 0.04) = \$9,354.$$

If the contract provisions provide for the surrender charge to be waived on death, then the benefit on death on August 14, 2000 is:

$$10,000 \times (1 + 0.12) \times (1 - 0.13) = \$9,744.$$

If on death the surrender charge is waived and there is a minimum benefit of the return of premium (one possible design of minimum guaranteed death benefit), the benefit on death on August 14, 2000 is \$9,744 with a floor of \$10,000; hence the death benefit is \$10,000.

If on death the surrender charge is waived and there is an annual reset of the minimum guaranteed death benefit on the policy anniversary, the benefit on death on August 14, 2000 is \$11,200. It was reset

Table 5
Valuation Assumptions for
A Single Premium Deferred Annuity (Variable)

Issue date:	August 15, 1998
Single premium:	\$10,000
Surrender charge	
During policy year 1:	6 percent
During policy year 2:	4 percent
During policy year 3:	2 percent
Thereafter:	0 percent
Actual credited rate	
August 15, 1998 to Aug 14, 1999:	12 percent
August 15, 1999 to Aug 14, 2000:	-13 percent
August 15, 2000 to Aug 14, 2001:	-8 percent
August 15, 2001 to Aug 15, 2002:	2 percent

August 15, 1999 to the then fund value $10,000 \times (1 + 0.12) = \$11,200$ and at August 15, 2000 will be set to $11,200 \times (1 - 0.13) = \$9,744$.

If on death the surrender charge is waived and there is an annual ratchet of the minimum guaranteed death benefit on the policy anniversary, then the benefit on death on August 14, 2001 is \$11,200. On August 15, 1999 the minimum guaranteed death benefit was ratcheted up to the then fund value $10,000 \times (1 + 0.12) = \$11,200$ and was left unchanged at August 15, 2000—the ratchet means that the minimum guaranteed death benefit cannot be reduced.

The design of the minimum guaranteed death benefit can vary widely; the above set of illustrations is only a small subset of the possible designs. Actuarial Guideline 34 is intended to apply to all such designs.

6 NAIC Actuarial Guideline 34

AG 34 (NAIC, 1998) requires that minimum guaranteed death benefits be projected by assuming an immediate drop in the values of the assets supporting the variable annuity contract, followed by a subsequent recovery in asset values at a net assumed return until the maturity of the contract. The amounts of the drops and subsequent increase are specified and depend on the types of assets. This immediate drop methodology was adopted for AG 34 after discussion of the risk

of a long-term bear market in stocks (American Academy of Actuaries, 1996).

Not all observers would agree, however, that it is appropriate to assume a recovery at a rate higher than the rate of return that would apply if there had been no drop.

The basic reserve for the annuity is to be calculated by methods consistent with CARVM provisions in the standard valuation law and AG 33. This reserve is held in a separate account. For the projection of account values, most companies use the valuation rate of interest less asset charges or, more commonly, mortality and expense charges. The base policy reserve generally equals the CSV obtainable at the date of valuation.

Under AG 34 any additional reserve held for the minimum guaranteed death benefit is held in a general account. We consider an example to illustrate the workings of AG 34.

7 Example 4: No Minimum Guaranteed Death Benefit

Consider the example of a variable SPDA with no minimum guaranteed death benefit. The valuation assumptions are described in Table 6. This example is based partly on that given in American Academy of Actuaries (1996). We are performing a valuation at December 31, 1999, two years after issue. The actual credited rate is known: 9 percent in 1998 and -3 percent in 1999, after reduction by the 1.75 percent asset charge. The results of the reserve calculations are given in Table 7.

The account value at December 31, 1999 is

$$60,000 \times (1 + 0.09) \times (1 - 0.03) = \$63,438.$$

The CSV at December 31, 1999 is $63,438 \times (1 - 0.05) = \$60,266$ where the 0.05 is for the 5 percent surrender charge (back-end load). For projections of years after 1999 we use the assumed investment return of 5.25 percent (= 7 - 1.75 percent). The projected CSV at December 31, 2001 is

$$63,438 \times (1 + 0.0525)^2 \times (1 - 0.02) = \$68,868$$

because the surrender charge has dropped to 2 percent. (See Table 7.)

Table 6
Valuation Assumptions for
A Single Premium Deferred Annuity (Variable)
No Minimum Guaranteed Death Benefit

Minimum guaranteed death benefit rollup rate:	6 percent
Single premium:	\$60,000
Issue date:	Jan. 1, 98
Asset charge:	1.75 percent
Investment return:	
Policy year 1 (net of 1.75 percent):	9.00 percent
Policy year 2 (net of 1.75 percent):	-3.00 percent
Future assumed (net of 1.75 percent):	5.25 percent
Immediate drop:	-23.00 percent
Subsequent:	15.00 percent
Valuation rate:	7.00 percent

Assume that the reserve at December 31, 1999 is the greatest of the present values at December 31, 1999 of all possible future surrender values without reduction for probability of death. In effect, we make a valuation assumption of zero deaths. This is equivalent to an assumption that on death the CSV is paid if we ignore the small correction for the fact that a death may occur in a non-optimal year. Assume we are using noncontinuous CARVM, so we are considering only surrenders on the last day of each policy year.

The valuation rate of 7 percent exceeds the assumed accumulation rate of 5.25 percent. Therefore most likely to be the greatest is the immediate CSV at December 31, 1999 of \$60,266 or the present value

$$\$68,868 \times (1 + 0.07)^{-2} = \$60,152$$

after the surrender charge drops from 5 percent to 2 percent. This is taken at December 31, 2001, although a surrender at January 1, 2001 would also have a charge of only 2 percent and so would give a higher reserve. Thus \$60,266 is the greater of the two values and is confirmed by Table 7 to be the greatest of all values.

In the absence of a minimum guaranteed death benefit, this may have been considered an appropriate CARVM reserve before AG 33 and AG 34. The rough treatment of the death benefit would now not conform with AG 33 and AG 34.

Table 7
Reserve Calculation Ignoring Minimum Guaranteed Death Benefit

	Policy year from Jan. 1 (<i>t</i>)									
	1998	1999	2000	2001	2002	2003	2004	2005	2006	
(SC_t) :	5%	5%	5%	2%	1%	0%	0%	0%	0%	
AV_{t-1} at Jan. 1:	60,000	65,400	63,438	66,768	70,274	73,963	77,846	81,933	86,235	
AV_t at Dec. 31:	65,400	63,438	66,768	70,274	73,963	77,846	81,933	86,235	90,762	
$AVAVG_t$:	62,700	64,419	65,103	68,521	72,119	75,905	79,890	84,084	88,498	
CSV_{t-1} at Jan. 1:	57,000	62,130	60,266	65,433	69,571	73,963	77,846	81,933	86,235	
CSV_t at Dec. 31:	62,130	60,266	63,430	68,868	73,224	77,846	81,933	86,235	90,762	
$*PV_2(CSV_t)$:		60,266	59,280	60,152	59,772	59,389	58,417	57,462	56,522	

Notes: $*PV_2(CSV_t)$ at Dec. 31, 1999 of Dec. 31 = $CSV_t \times 1.07^{-(t-1999)}$.

It is common for the valuation of variable annuities to be performed using a credited rate equal to the valuation rate minus charges. In the absence of a major drop in back-end load in some subsequent year, the CARVM maximum often corresponds to surrender on the valuation date. Hence the CSV often forms the reserve.

8 Example 5: Guaranteed Minimum Death Benefit

We again consider the valuation at December 31, 1999 of the single premium deferred annuity of Example 4 above. Now the contract specifies that on death the benefit equals the greater of:

- Asset value, and
- Minimum guaranteed death benefit of the single premium of \$60,000 accumulated at 6 percent p.a.

The results of the reserve calculations are shown in Table 8. The minimum guaranteed death benefit at, for example, December 31, 2001 is given by:

$$60,000 \times (1 + 0.06)^4 = \$75,749.$$

As part of the calculation of the integrated reserve we follow Actuarial Guideline 34. We calculate base asset values on the assumption that at January 1, 2000 for our particular fund type there is an immediate drop of 23 percent in the asset value, followed by a recovery at 15 percent per annum. These particular values are not among those given in AG 34, but are adequate for illustrating the process. The base asset value at December 31, 2001 is:

$$60,000 \times (1 + 0.09) \times (1 - 0.03) \times (1 - 0.23) \times (1 + 0.15)^2 = \$64,601.$$

The asset value is assumed to be subject to a maximum of (capped by) the asset value calculated assuming no immediate drop. The base uncapped asset value at December 31, 2003 thus calculated is:

$$60,000 \times (1 + 0.09) \times (1 - 0.03) \times (1 - 0.23) \times (1 + 0.15)^4 = \$85,434.$$

Table 8
Calculation of Minimum Guaranteed Death Benefit Amounts

1.	Policy year from Jan. 1 (<i>t</i>)							
	1998	1999	2000	2001	2002	2003	2004	2005
2. (SC_t) :	5%	5%	5%	2%	1%	0%	0%	0%
3. AV_{t-1} at Jan. 1:	60,000	65,400	63,438	66,768	70,274	73,963	77,846	81,933
4. AV_t at Dec. 31:	65,400	63,438	66,768	70,274	73,963	77,846	81,933	86,235
5. $AVAVG_t$:	62,700	64,419	65,103	68,521	72,119	75,905	79,890	84,084
6. CSV_{t-1} at Jan. 1:	57,000	62,130	60,266	65,433	69,571	73,963	77,846	81,933
7. CSV_t at Dec. 31:	62,130	60,266	63,430	68,868	73,224	77,846	81,933	86,235
8. $PV_2(CSV_t)$		60,266	59,280	60,152	59,772	59,389	58,417	57,462
9. Base AV at Jan. 1: ¹	60,000	65,400	48,847	56,174	64,601	74,291	85,434	98,249
10. Base AV at Dec. 31: ²	65,400	63,438	56,174	64,601	74,291	85,434	98,249	112,987

Notes: * $PV_2(CSV_t)$ at Dec. 31, 1999 of Dec. 31 is equal to $CSV_t \times 1.07^{-(t-1999)}$; ¹Base AV at Jan. 1 if Jan. 1, 2000 drop, no cap; ²Base AV at Dec. 31 if Jan. 1, 2000 drop, no cap.

Table 8 (Continued)
Calculation of Minimum Guaranteed Death Benefit Amounts

	Policy year from Jan. 1 (<i>t</i>)							
	1998	1999	2000	2001	2002	2003	2004	2005
11. Base AV at Jan. 1: ³	60,000	65,400	48,847	56,174	64,601	73,963	77,846	81,933
12. Base AV at Dec. 31: ⁴	65,400	63,438	56,174	64,601	73,963	77,846	81,933	86,235
13. Base AV average: ⁵	62,700	64,419	52,511	60,387	69,282	75,905	79,890	84,084
14. $MGDB_{t-1}$ at Jan. 1:	60,000	63,600	67,416	71,461	75,749	80,294	85,111	90,218
15. $MGDB_t$ at Dec. 31:	63,600	67,416	71,461	75,749	80,294	85,111	90,218	95,631
16. DB_{t-1} at Jan. 1:	60,000	65,400	67,416	71,461	75,749	80,294	85,111	90,218
17. DB_t at Dec. 31:	65,400	67,416	71,461	75,749	80,294	85,111	90,218	95,631
18. NAR_{t-1} at Jan. 1:	-	-	18,569	15,287	11,148	6,330	7,265	8,285
19. NAR_t at Dec. 31:	-	3,978	15,287	11,148	6,330	7,265	8,285	9,396
20. $NARAVG_t$:	-	1,989	16,928	13,217	8,739	6,798	7,775	8,840

Notes: ³Base AV at Jan. 1 if Jan. 1, 2000 drop, cap of non-drop AV (line 3); ⁴Base AV at Dec. 31 if Jan. 1, 2000 drop, cap of non-drop AV (line 4); ⁵Base AV average if Jan. 1, 2000 drop, cap of non-drop AV (average of lines 11 and 12); $MGDB_{t-1}$ at Jan. 1 = $60,000 \times 1.06^{(t-1998)}$; $MGDB_t$ at Dec. 31 = $60,000 \times 1.06^{(t-1997)}$; DB_{t-1} at Jan. 1 = max(line 11, line 14); DB_t at Dec. 31 = max(line 12, line 15); NAR_{t-1} at Jan. 1 = (line 16 - line 11); NAR_t Dec. 31 = (line 17 - line 12).

The cap on asset value at December 31, 2003 is given by:

$$60,000 \times (1 + 0.09) \times (1 - 0.03) \times (1 + 0.0525)^4 = \$77,846$$

so the cap is binding; the capped asset value at December 31, 2003 is \$77,846. The minimum guaranteed death benefit at December 31, 2003 is:

$$60,000 \times (1 + 0.06)^6 = \$85,111.$$

The capped asset value is \$77,846. Hence the net amount at risk (NAR benefit top-up) because of the minimum guaranteed death benefit at December 31, 2003 is $85,111 - 77,846 = \$7,265$.

Following the usual CARVM philosophy, we test a set of candidates to determine which is greatest. This candidate will be the legal minimum reserve at December 31, 1999. The candidates correspond to potential surrender at December 31, 1999, 2000, 2001, etc.

For example, we consider the possibility of the reserve at December 31, 1999 being given by a CARVM maximum occurring at December 31, 2001. Therefor we find the present value at December 31, 1999 (or January 1, 2000) of the NAR payouts on deaths in 2000 and 2001. We assume mid-year deaths and rates of mortality as given in Table 9.

For 2000, $q_t = 0.019$ and average $NAR = 16,928$ from Table 8:

$$PV = 0.019 \times 16,928 / 1.07^{0.5} = 311.$$

For 2001, the probability of dying is $(1 - 0.019) \times 0.022$ and average $NAR = 13,217$ from Table 8.

$$PV = (1 - 0.019) \times 0.022 \times 13,217 / 1.07^{1.5} = 258.$$

The total for 2000 and 2001 is $311 + 258 = \$569$. All present values (PV) are at the valuation date, Dec. 31, 1999.

As a further part of examining the December 31, 2001 candidate policy termination date, we find the present value at December 31, 1999 (or January 1, 2000) of the unreduced asset value payouts in 2000 and 2001. In other words, we consider the present value of a death benefit of the unreduced (no-drop) asset value.

The use of unreduced asset payouts on death is consistent with the use of the unreduced asset payouts in calculating the present value of surrenders.

At first sight this is inconsistent with using the reduced asset value in the value of the minimum guaranteed death benefit guarantee. Unreduced amounts are used for benefits, however, where the benefit is proportional to the assets accumulated at the actual credited rate. If the investment return is -50 percent, then these benefits are halved. But this doesn't mean that we need half the reserve. We needed to be holding the full amount of assets in the separate account because these also were halved in value.

The same logic does not apply to the NAR and the value of the minimum guaranteed death benefit. The minimum guaranteed death benefit is specified in dollars independent of asset performance. This is like the death benefit under a traditional whole life policy. Correspondingly, the minimum guaranteed death benefit reserve is held in the general account.

For 2000, $q_t = 0.019$ and the average base account value (AV) is 65,103 from Table 8:

$$PV = 0.019 \times 65,103 / 1.07^{0.5} = 1,196.$$

For 2001, the probability of dying is $(1 - 0.019) \times 0.022$ and the average base account value is \$68,521 from Table 8.

$$PV = (1 - 0.019) \times 0.022 \times 68,521 / 1.07^{1.5} = 1,336.$$

The total for 2000 and 2001 is $1,196 + 1,336 = \$2,532$. All the present values (PV) are at valuation date, Dec. 31, 1999.

The major portion of this candidate to be the reserve is the present value at December 31, 1999 of surrenders by all survivors at December 31, 2001.

We are assuming that everyone who survives to December 31, 2001 surrenders then. Hence the PV at December 31, 1999 is, using the Table 7 no-drop CSV of \$68,868:

$$PV = (1 - 0.019) \times (1 - 0.022) \times 68,868 / 1.07^2 = \$57,711.$$

Table 9
The Integrated Reserve

	Policy year from Jan. 1 (<i>t</i>)						
	1998	1999	2000	2001	2002	2003	2004
<u>Account Value</u>							
1. $AVAVG_t$ (line 5 of Table 8):	62,700	64,419	65,103	68,521	72,119	75,905	79,890
2. CSV_t (line 7 of Table 8):	62,130	60,266	63,430	68,868	73,224	77,846	81,933
<u>Integrated Reserve Calculation Including MGDB</u>							
3. $NARAVG_t$ (line 20 of Table 8):	-	1,989	16,928	13,217	8,739	6,798	7,775
4. (q_t) :	0.015	0.017	0.019	0.022	0.024	0.027	0.030
5. P_t :			1.000	0.981	0.959	0.936	0.911
6. $PV(NARAVG)_t$:			311	258	170	136	157
7. $PV(AVAVG)_t$:			1,196	1,336	1,402	1,514	1,610
8. Cumulative total of line 6:			311	569	739	874	1,031
9. Cumulative total of line 7:			1,196	2,532	3,934	5,449	7,059
10. $PV(S\&SCSV)_t$:		60,266	58,154	57,711	55,970	54,109	51,628
11. Total of lines 8, 9 and 10:		60,266	59,661	60,812	60,643	60,432	59,718
The integrated reserve is maximum of the above row.							

Notes: P_t is the probability of survival from Jan. 1, 2000 to Jan. 1, t ; $PV(NARAVG)_t$ at Jan. 1, 2000 of NARs paid on death and is equal to $NARAVG_t \times q_t \times P_t \times 1.07^{-(t-1999.5)}$; $PV(AVAVG)_t$ of average unreduced account values paid at death (mid year discounting) and is equal to $AVAVG_t \times q_t \times P_t \times 1.07^{-(t-1999.5)}$; $PV(S\&SCSV)_t = CSV_t \times P_{t-1} \times 1.07^{-(t-1999)}$, where $PV(S\&SCSV)_t$ is the present value of the cash value of those who survive and surrender at year end.

We add the PV at December 31, 1999 of the sum to December 31, 2001 of deaths and of 2001 surrenders, from above:

$$PV = 569 + 2,532 + 57,711 = \$60,812.$$

We consider all candidate policy termination dates in deciding the minimum reserve to be held at December 31, 1999. But we notice that this \$60,812 at December 31, 2001 is the highest number in the integrated-reserve-is-the-maximum line (line 11); it so happens that we calculated for the correct year. In reality all years (all candidates) are calculated and the maximum taken.

We then take the maximum also of the separate account reserve, line 10. This is \$60,266, and it applies to a December 31, 1999 surrender. This is less than \$60,812, which applies to the December 31, 2001 candidate; therefore, the reserve held is \$60,812. Despite the difference in dates, \$546 of the \$60,812 is held as a general account minimum guaranteed death benefit reserve.

Note the CARVM philosophy that we assume the worst case about the elective decrement that is controlled by the policyholder, surrender. Deaths are calculated according to a mortality table. Policyholders won't elect to die to get the best return from their annuity. We have to add for each possible surrender year the value of deaths that would occur previous to that surrender date.

9 Conclusion

Actuarial Guideline 34 clarifies CARVM. Its provisions are consistent with the idea that surrender benefits will, with 100 percent efficiency, be timed by the policyholder to maximize his or her return. Death is a non-elective benefit, and the calculations resemble the traditional actuarial discounting of a product of a death probability and a benefit amount.

The logic of this view may be clearer if we consider an insurance company to be valuing a cohort of 100,000 policyholders. Perhaps the optimum strategy for the policyholders is to elect to surrender after three years. Only 98,801 of them, however, will be alive to do so, for example. The other 1,199 will have died and received the death benefit. Thus we value this cohort at issue assuming a 0.98801 probability of receipt of CSV after three years. The valuation must also take into account the benefits paid on death with probability 0.01199.

The traditional view is that an insurance company spreads risks over many individuals. It is possible to spread the mortality risk, but the CARVM view is that antiselective surrender will be performed efficiently and simultaneously by all living policyholders.

References

- American Academy of Actuaries. *June 1996 Report on Reserving for Minimum Guaranteed Death Benefits for Variable Annuities*, reproduced as Society of Actuaries Study Note 443-55-97. Washington, D.C.: American Academy of Actuaries, 1996
- American Academy of Actuaries. *Life and Health Valuation Law Manual*, Third Edition. Washington, D.C.: American Academy of Actuaries, 1997
- Backus, J.E. "Visual CARVM: Multiple Benefit Streams in Pictures." *The Financial Reporter*, Society of Actuaries; accompanying spreadsheet Viscarvm.xls may at the time of writing be available at www.soa.org (March 1998).
- Lalonde, R.J. "An Overview of New Actuarial Guideline 33." *The Financial Reporter* (August 1995). Reproduced as Society of Actuaries Study Note 443-53-96.
- National Association of Insurance Commissioners (NAIC). *Financial Examiners Handbook*. Kansas City, Missouri: National Association of Insurance Commissioners, 1998.
- Ravindran, K., and Edilist, A.W. "Valuing Minimum Death Benefits and Other Innovative Variable Annuities." *Product Development News*, Society of Actuaries, (December 1994). Reproduced as Society of Actuaries Study Note 443-52-96 (1994).
- Sharp, K.P. "Commissioners Annuity Reserve Valuation Method (CARVM)." *Journal of Actuarial Practice* 7 (1999): 107-124.