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## Commissioners Annuity Reserve Valuation Method (CARVM)

Keith P. Sharp\*

### Abstract

This paper describes the commissioners annuity reserve valuation method (CARVM) and highlights the fundamental contrast with insurance valuation. Numerical examples illustrate methods of applying CARVM to particular annuity designs. The application of NAIC Actuarial Guideline 13 on bailouts is given particular attention.

Key words and phrases: *cash surrender values, single premium deferred annuity, antiselection, election*

### 1 Introduction

Annuity business has shown substantial growth over the past two decades. According to the *Life Insurance Fact Book 1998* the total U.S. industry reserves for annuities were more than twice as high as reserves for life insurance policies (ACLI 1999, p. 114).

Annuity products generally are designed so that one optional benefit is a series of payments till death, a life annuity. Despite their name, however, most annuities bought by individuals are purchased as tax-favored cash accumulation vehicles. Most annuity contracts terminate by surrender rather than annuitization and eventual death.

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The commissioners annuity reserve valuation method (CARVM) was first defined by the National Association of Insurance Commissioners (NAIC) in the 1976 amendments to the Standard Valuation Law (American Academy of Actuaries, 1997). The CARVM reserve was defined to be:

... the greatest of the respective excesses of the present values, at the date of valuation, of the future guaranteed benefits, including guaranteed nonforfeiture benefits, provided for by such contracts over the present value at the date of valuation, of any future valuation considerations derived from gross premiums . . . .

The valuation considerations are defined to be " . . . the portions of the respective gross considerations applied under the terms of such contracts to determine nonforfeiture values . . ." These valuation considerations are here given the symbol  $P^{NFV}$ . This definition of the CARVM reserve will be represented here by the following formula, valid in some simple cases:

$${}_tV_x^{CARVM} = \max_{n \geq t} \{B_n v^{n-t} - P^{NFV} \ddot{a}_{\overline{n-t}|}\} \quad (1)$$

where  $x$  is the issue age,  $t$  is the number of policy years that have elapsed at valuation,  $B_r$  is the surrender benefit at the end of policy year  $r$ ,  $r = 1, 2, \dots$ , and  $n$  is the policy year-end being tested to determine whether it gives the maximum. The maximum is taken over all possible future policy year-ends,  $n$ , including the possibility  $n = t$  of immediate surrender.

All the policy year-ends at which the annuity holder may elect to receive a benefit are included. For most annuities the most important mode of termination is surrender at a date  $n$  chosen by the policyholder, hence  $B_n$ . Detailed consideration of non-elective benefits, for example those available on death, are considered in Sharp (1999).

CARVM differs greatly from the usual prospective definition of a life insurance reserve.

$${}_tV_x^{INS} = \sum_{m=t+1}^{\infty} DB_m \times {}_{m-t-1}p_x \times q_{x+m-t-1} v^{m-t} - \sum_{m=t+1}^{\infty} P^{NET} \times {}_{m-t-1}p_x \times v^{m-t-1} \quad (2)$$

For insurance the dominant benefit may be the death benefit  $DB_m$ .

A related matter is that the insurance reserve in equation (2) uses probabilities (e.g.,  ${}_{m-t-1}p_x$  and  $q_{x+m-t-1}$ ) while the CARVM reserve equation (1) has no probabilities. CARVM replaces the probabilities with a maximum. The CARVM philosophy is to assume that with 100 percent certainty the policyholder will antiselect at the worst time for the insurance company. This is reasonable, though conservative, in the case of an annuity and its surrender benefit. In the life insurance case it would be unreasonable to assume that the policyholder is so eager to antiselect at the optimum time that he or she will die to do so. Thus the CARVM calculations in this paper do not use probabilities.

Jaffe (1983) gave an early analysis of CARVM. Much of that paper and its discussion are still relevant today, though the impact of the relatively recent Actuarial Guidelines 33 and 34 (see e.g., National Association of Insurance Commissioners, 1998) should be kept in mind.

## 2 CARVM Reserve in a Simple Case

The application of CARVM is most readily illustrated through numerical examples. First, consider a single premium deferred annuity (SPDA), which is an annuity more straightforward than would usually be met in practice. In particular there are no explicit loads. The valuation assumptions are listed in Table 1.

The guaranteed credited rates are those specified in the contract at issue, possibly with stronger guarantees made subsequent to issue. The guaranteed annuitization basis is less generous than the valuation basis. Thus, the lump sum equivalent to the guaranteed annuity will be, on the valuation basis, of lesser value than the actual lump sum benefit option. Therefore, we can ignore the annuitization guarantee in calculating the reserve. The converse case will be considered later.

We calculate the reserve required under CARVM if the single premium were \$10,000 and accumulation during 1996 and 1997 were at the guaranteed 8 percent p.a. Thus we calculate the cash surrender values (CSV) that would apply on surrender at the end of each policy year. The present value at the valuation date, two years after issue, is found by discounting the CSVs at the 6 percent p.a. valuation rate. For years when the 8 percent p.a. accumulation is operative, an additional year of accumulation increases the present value because 6 percent is less than 8 percent. Once the accumulation credited rate falls to 5 percent p.a., a year's delay in surrendering reduces the present value because 5 percent is less than 6 percent.

**Table 1**  
**Valuation Assumptions for**  
**A Single Premium Deferred Annuity (SPDA) Without Loads**

Single premium:	\$10,000
Front-end load:	0 percent of single premium
Surrender charge (back-end load):	0 percent
Guaranteed credited rates:	8 percent p.a. for 5 years; 5 percent thereafter
Actual credited rates:	8 percent p.a. in 1996; 8 percent p.a. in 1997
Guaranteed annuitization basis:	4 percent p.a., IA 83 (Table a)*
Issue date:	December 31, 1995
Maturity date:	December 31, 2019
Valuation date:	December 31, 1997
Valuation rate:	6 percent p.a.
Valuation mortality	
Before maturity:	Zero mortality
After maturity:	IA83 (Table a)

*Notes:* p.a. = per annum; \*Published by the Society of Actuaries, Schaumburg, Ill., USA.

The minimum reserve is the largest value in the right column of Table 2. This column corresponds to a surrender on December 31, 2000 and is

$$12,337 = 10,000 \times 1.08^2 \times 1.08^3 / (1.06^3).$$

This surrender contains no probabilities; the maximum occurs at the point at which the guaranteed credited rate falls below the valuation rate.

The above calculation method has sometimes been used if nonzero pre-maturity mortality is assumed, but the death benefit equals the cash surrender value. Then, effectively, the above calculation ignores death as a separate decrement and includes it as a surrender. This is conservative because the annuitant won't choose the most expensive time to die, so the 100 percent certainty philosophy shouldn't apply to the death benefit. Under Actuarial Guidelines 33 and 34 (NAIC, 1998) this conservative approximation is not the approved method.

**Table 2**  
**CARVM Cash Surrender Value (CSV) and Present Value (PV) Calculations**  
**For the Single Premium Deferred Annuity (SPDA) Without Loads**

Dec. 31	Cash Surrender Value (CSV)	PV at Dec. 31, 1997
1995	$10,000 = 10,000$	-
1996	$10,800 = 10,000 \times 1.08$	-
1997	$11,664 = 10,000 \times 1.08^2$	$11,664 = 10,000 \times 1.08^2$
1998	$12,597 = 10,000 \times 1.08^2 \times 1.08$	$11,884 = 10,000 \times 1.08^2 \times 1.08 / (1.06)$
1999	$13,605 = 10,000 \times 1.08^2 \times 1.08^2$	$12,108 = 10,000 \times 1.08^2 \times 1.08^2 / (1.06^2)$
2000	$14,693 = 10,000 \times 1.08^2 \times 1.08^3$	$12,337 = 10,000 \times 1.08^2 \times 1.08^3 / (1.06^3)$
2001	$15,428 = 10,000 \times 1.08^2 \times 1.08^3 \times 1.05$	$12,220 = 10,000 \times 1.08^2 \times 1.08^3 \times 1.05 / (1.06^4)$
2002	$16,199 = 10,000 \times 1.08^2 \times 1.08^3 \times 1.05^2$	$12,105 = 10,000 \times 1.08^2 \times 1.08^3 \times 1.05^2 / (1.06^5)$

### 3 CARVM Reserve in the Presence of Loads

Let us now continue the example with a few complications added:

1. There is a front-end load charged to the policyholder;
2. There is a surrender charge (back-end load) in the early policy years;
3. Accumulation before the valuation date was at a higher credited rate than that originally guaranteed.

The complete set of valuation assumptions is given in Table 3.

**Table 3**  
**Valuation Assumptions for**  
**A Single Premium Deferred Annuity (SPDA) With Loads**

Single premium:	\$10,000
Front-end load:	4 percent of single premium
Surrender charge (back end load):	8 percent for the first six years from issue and zero on and after January 1, 2002
Guaranteed credited rates:	8 percent p.a. for five years; 5 percent p.a. thereafter
Actual credited rates:	9 percent p.a. in 1996 and in 1997
Guaranteed annuitization basis:	4 percent p.a., IA83 (Table a)
Issue date:	December 31, 1995
Maturity date:	December 31, 2019
Valuation date:	December 31, 1997
Valuation rate:	6 percent p.a.
Valuation mortality:	
Before maturity:	Zero mortality
After maturity:	IA83 (Table a)

The reserve required under CARVM is calculated using a single premium of \$10,000 and accumulation during 1996 and 1997 of 9 percent p.a., which is higher than the 8 percent p.a. that was guaranteed at issue. The minimum reserve is the largest value in the right column of Table 4.

**Table 4**  
**CARVM Cash Surrender Value (CSV) and Present Value (PV) Calculations**  
**For the Single Premium Deferred Annuity (SPDA) With Loads**

Dec. 31	Cash Surrender Value (CSV)	PV at Dec. 31, 1997
1995	$8,832 = 0.92 \times 0.96 \times 10,000$	-
1996	$9,627 = 0.92 \times 0.96 \times 10,000 \times 1.09$	-
1997	$10,493 = 0.92 \times 0.96 \times 10,000 \times 1.09^2$	$10,493 = 0.92 \times 0.96 \times 10,000 \times 1.09^2$
1998	$11,333 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08$	$10,691 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08 / (1.06)$
1999	$12,239 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^2$	$10,893 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^2 / (1.06^2)$
2000	$13,219 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3$	$11,099 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 / (1.06^3)$
2001	$13,879 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05$	$10,994 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05 / (1.06^4)$
2002	$15,841 = 1.00 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05^2$	$11,837 = 1.00 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05^2 / (1.06^5)$



This is a competition between December 31, 2000 at

$$11,099 = 0.92 \times 0.96 \times 10,000 \times 1.09^2 \times 1.08^3 / (1.06^3)$$

and December 31, 2002 at

$$11,837 = 1.00 \times 0.96 \times 10,000 \times 1.09^2 \times 1.08^3 \times 1.05^2 / (1.06^5).$$

The latter wins because the effect of removing the 0.92 back-end load factor outweighs the additional two years of discounting at 6 percent p.a. a sum accumulating at only 5 percent p.a. ( $1.05^2/1.06^2$ ). As usual, the valuation is of only the contract guarantees applying at the valuation date. Thus, the 5 percent accumulation is used even though after December 31, 1997 the insurance company may choose to credit more than 5 percent in order to discourage surrenders.

#### 4 New York Continuous CARVM Reserve

Under standard CARVM the maximum in equation (1) is taken over all possible future policy year-ends. Equation (1) is often referred to as *curtate CARVM*. Under New York's version of CARVM (New York Insurance Law, Section 4217(6)(D)) the maximum in equation (1) is taken over each possible future day of surrender; continuous CARVM. This can make a significant difference if there is a back-end load that reduces at the end of a policy year. The valuation assumptions are given in Table 5.

The minimum reserve is the largest value in the right column of Table 6 visualized as being produced for each possible surrender day. This is the continuous CARVM method prescribed by New York. This maximum value is just after the surrender charge is removed, January 1, 2002, in view of the 6 percent discounting beating the 5 percent accumulation:

$$\$11,950 = 1.00 \times 0.96 \times 10,000 \times 1.09^2 \times 1.08^3 \times 1.05 / (1.06^4).$$

Some insurance commissioners in other states including Illinois and Virginia require the use of New York continuous CARVM in at least some situations. NAIC Actuarial Guideline 33 is neutral on this topic.

**Table 5**  
**New York CARVM Reserve Valuation Assumptions for**  
**A Single Premium Deferred Annuity (SPDA) With Loads**

Single premium:	\$10,000
Front-end load:	4 percent of single premium
Surrender charge (back end load):	8 percent for the first six years from issue; zero from Jan. 1, 2002
Guaranteed credited rates:	8 percent p.a. for five years; 5 percent p.a. thereafter
Actual credited rates:	9 percent p.a. in 1996 and in 1997
Guaranteed annuitization basis:	4 percent p.a., IA83 (Table a)
Issue date:	December 31, 1995
Maturity date:	January 1, 2020
Valuation date:	December 31, 1997
Valuation rate:	6 percent p.a
Valuation mortality:	
Before maturity:	Zero mortality
After maturity:	IA83 (Table a)

## 5 CARVM Reserve Under Annuitization Option

Under CARVM it is necessary to consider all options that can be exercised (elected) by the annuity owner. Frequently an annuity contract includes a current settlement provision that at annuitization the company's then in-force annuitization rates will be used if more favorable than those guaranteed in the contract.

We calculate the reserve required under (non-New York) CARVM assuming the single premium was \$10,000 and the accumulation during 1996 and 1997 (i.e., January 1, 1996 to December 31, 1997) was at 9 percent p.a. We are given that at  $m$ , the age at maturity,

$$a_m(6\%) / a_m(7\%) = 1.085$$

and that annuitization is allowed only at the maturity date.

**Table 6**  
**New York CARVM Cash Surrender Value (CSV) and Present Value (PV) Calculations**  
**For the Single Premium Deferred Annuity (SPDA) With Loads**

Dec. 31	Cash Surrender Value (CSV)	PV at Dec. 31, 1997
1995	$8,832 = 0.92 \times 0.96 \times 10,000$	-
1996	$9,627 = 0.92 \times 0.96 \times 10,000 \times 1.09$	-
1997	$10,493 = 0.92 \times 0.96 \times 10,000 \times 1.09^2$	$10,493 = 0.92 \times 0.96 \times 10,000 \times 1.09^2$
1998	$11,333 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08$	$10,691 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08 / (1.06)$
1999	$12,239 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^2$	$10,893 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^2 / (1.06^2)$
2000	$13,219 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3$	$11,099 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 / (1.06^3)$
2001	$13,879 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05$	$10,994 = 0.92 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05 / (1.06^4)$
1/1/2002	$15,086 = 1.00 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05$	$11,950 = 1.00 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05 / (1.06^4)$
12/31/2002	$15,841 = 1.00 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05^2$	$11,837 = 1.00 \times 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05^2 / (1.06^5)$

**Table 7**  
**Valuation Assumptions for an Annuitization Option**  
**Single Premium Deferred Annuity (SPDA) With Loads**

Single premium:	\$10,000
Front-end load:	4 percent of single premium
Surrender charge (back end load):	0 percent
Guaranteed credited rates:	8 percent p.a. for five years; 5 percent p.a. thereafter
Annuitization age:	Allowed only maturity date
Guaranteed annuitization basis:	7 percent p.a., IA83 (Table a)
Issue date:	December 31, 1995
Maturity date:	December 31, 2002
Valuation date:	December 31, 1997
Valuation rate:	6 percent p.a.
Valuation mortality:	
Before maturity:	Zero mortality
After maturity:	IA83 (Table a)

In reality, individual deferred annuity contracts often allow considerable flexibility in the timing of annuitization. The valuation assumptions are listed in Table 7.

The minimum reserve is the largest value in the right column of Table 8. This corresponds to the December 31, 2002 annuitization on a basis more generous (7 percent) than the valuation basis (6 percent p.a.). The amount of annual annuity purchased with an amount  $M$  is  $M/a_n(7\%)$ . When valued at 6 percent p.a. this gives a reserve  $a_n(6\%)/a_n(7\%)$ , discounted back to the valuation date:

$$12,843 = 0.96 \times 10,000 \times 1.09^2 \times 1.08^3 \times 1.05^2 \times 1.085 / (1.06^5).$$

If such a current settlement provision is present, then consideration must be given to Actuarial Guideline 33. Under AG 33 the provision would trigger the application of the reserve floor of 93 percent of the contract fund value at valuation.

**Table 8**  
**Annuity Option Cash Surrender Value (CSV) and Present Value (PV) Calculations**  
**For the Single Premium Deferred Annuity (SPDA) With Loads**

Dec. 31	Cash Surrender Value (CSV)	PV at Dec. 31, 1997
1995	$9,600 = 0.96 \times 10,000$	-
1996	$10,464 = 0.96 \times 10,000 \times 1.09$	-
1997	$11,405 = 0.96 \times 10,000 \times 1.09^2$	$11,405 = 0.96 \times 10,000 \times 1.09^2$
1998	$12,318 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08$	$11,620 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08 / (1.06)$
1999	$13,304 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^2$	$11,840 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^2 / (1.06^2)$
2000	$14,368 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3$	$12,063 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 / (1.06^3)$
2001	$15,086 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05$	$11,950 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05 / (1.06^4)$
2002	$15,841 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05^2$	$11,837 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05^2 / (1.06^5)$
2002	Annuitize	$12,843 = 0.96 \times 10,000$ $\times 1.09^2 \times 1.08^3 \times 1.05^2$ $\times a_n(6%) / (a_n(7%) \times 1.06^5)$

**Table 9**  
**Bailout Significant: Valuation Assumptions for**  
**A Single Premium Deferred Annuity (SPDA) With Loads**

Single premium:	\$100,000
Guaranteed interest rate	
Years 1 to 5:	8 percent
Years 6 to 10:	6 percent
Years 11 and above:	3 percent
Front-end load:	4 percent
Surrender charge	
Years 1 to 4:	5 percent
Years 5 to 10:	2 percent
Years 11 and above:	0 percent
Bail-out rate:	7 percent
Valuation rates:	
This SPDA:	6½ percent
Whole life policy:	5½ percent

## 6 Actuarial Guideline 13 on Bailouts

### 6.1 Bailout Significant

It is common for an annuity contract to include a bailout feature. This is a response to potential policyholder fear that he or she will be trapped by the surrender charge into holding an annuity with a below-market-credited rate. The bailout generally allows for a surrender gross of surrender charge (back-end load) if the credited rate falls below a bailout rate specified in the contract.

There was some confusion about whether CARVM required the reserve in such circumstances to be gross of the back-end load. NAIC Actuarial Guideline 13 (NAIC, 1998) clarifies this by requiring that the reserve be gross of the load only if the bailout is significant. The term 'significant' is not used in the guideline, but it refers to a situation where it is thought that there is chance that the bailout clause will come into play. Under the guideline the bailout is, generally, significant if the bailout rate is higher than the long-life rate. The long-life rate is the

standard valuation law valuation interest rate used for policies with guarantee durations of more than 20 years.

Thus, in reserving for an annuity with bailout, the first step is to determine whether the bailout is significant. In the example below, the bailout is significant and the present value is taken of the surrender value gross of surrender charge. The valuation assumptions are listed in Table 9 and the reserve at issue under CARVM is shown in Table 10.

The bailout rate is 7 percent, greater than the  $5\frac{1}{2}$  percent long valuation rate. The bailout is significant, so the surrender charge is contingent and not available to reduce the reserve. In years 1 through 5 the guaranteed credited rate of 8 percent is higher than the bailout rate of 7 percent, so bailout cannot occur. From year 6 we have 6 percent less than 7 percent and the bailout is significant. The maximum of PV (CSV) for years 1 to 5 and PV (Fund) for years above 5 is \$102,470. (See Table 10.)

## 6.2 Bailout Not Significant

Let us now consider an annuity with a lower bailout rate, 4 percent p.a. Table 11 shows the valuation assumptions. We calculate the reserve at issue under CARVM in light of Actuarial Guideline 13 on Bailouts. The results are shown in Table 12.

The bailout is not significant because the bailout rate, 4 percent p.a., is less than the whole life valuation rate (long life rate) 5.5 percent p.a. Thus we can ignore the bailout and calculate the reserve assuming the surrender charge to be available to reduce the reserve. The reserve thus calculated is \$98,205. (See Table 12.)

**Table 10**  
**Bailout Significant: Fund, Cash Surrender Value (CSV), and Present Value (PV) Calculations**  
**For the Single Premium Deferred Annuity (SPDA) With Loads**

Duration	Fund	CSV	PV (Fund)	PV (CSV)
0	$100,000 \times 0.96 = 96,000$	91,200	96,000	91,200
1	$100,000 \times 0.96 \times 1.08 = 103,680$	98,496	97,352	92,485
2	$100,000 \times 0.96 \times 1.08^2 = 111,974$	106,376	98,723	93,787
3	$100,000 \times 0.96 \times 1.08^3 = 120,932$	114,886	100,113	95,108
4	$100,000 \times 0.96 \times 1.08^4 = 130,607$	124,077	101,524	96,448
5	$100,000 \times 0.96 \times 1.08^5 = 141,055$	138,234	102,953	100,894
6	$100,000 \times 0.96 \times 1.08^5 \times 1.06 = 149,519$	146,529	102,470	100,421
7	$100,000 \times 0.96 \times 1.08^5 \times 1.06^2 = 158,490$	155,320	101,989	99,949



**Table 11**  
**Bailout Not Significant: Valuation Assumptions for**  
**A Single Premium Deferred Annuity (SPDA) With Loads**

Single premium:	\$100,000
Guaranteed interest rate	
Years 1 to 3:	8 percent
Years 4 to 10:	6 percent
Years 11 and above:	3 percent
Front-end load:	2 percent
Surrender charge	
Years 1 to 4:	5 percent
Years 5 to 10:	3 percent
Years 11 and above:	0 percent
Bail-out rate:	4 percent
Valuation rates:	
This SPDA:	6½ percent
Whole life policy:	5½ percent

## 7 Conclusion

CARVM methodology differs from that used for traditional insurance policies and for annuities in payment. The differences are needed because the time of receipt of benefits can be elected by the policyholder without the necessity of dying.

Annuity designs in practice often include the option to take a partial withdrawal. There is usually also a death benefit. Actuarial Guidelines 33 and 34 clarify techniques to be used with such a mix of benefits. These guidelines are the focus of Sharp (1999), which immediately follows this paper in this issue.

**Table 12**  
**Bailout Not Significant**  
**Fund, CSV, and PV Calculations**  
**For the Single Premium Deferred Annuity With Loads**

PY	GR	SC	FV	CSV	PV(Fund)	PV(SCV)
0		0.05	98,000	93,100	98,000	93,100
1	0.08	0.05	105,840	100,548	99,380	94,411
2	0.08	0.05	114,307	108,592	100,780	95,741
3	0.08	0.05	123,452	117,279	102,199	97,089
4	0.06	0.05	130,859	124,316	101,720	96,634
5	0.06	0.03	138,710	134,549	101,242	98,205
6	0.06	0.03	147,033	142,622	100,767	97,744
7	0.06	0.03	155,855	151,179	100,294	97,285
8	0.06	0.03	165,206	160,250	99,823	96,828
9	0.06	0.03	175,119	169,865	99,354	96,374
10	0.06	0.03	185,626	180,057	98,888	95,921
11	0.03	0.00	191,195	191,195	95,638	95,638
12	0.03	0.00	196,930	196,930	92,495	92,495
13	0.03	0.00	202,838	202,838	89,455	89,455
14	0.03	0.00	208,924	208,924	86,515	86,515
15	0.03	0.00	215,191	215,191	83,672	83,672
16	0.03	0.00	221,647	221,647	80,922	80,922
17	0.03	0.00	228,296	228,296	78,263	78,263
18	0.03	0.00	235,145	235,145	75,691	75,691
19	0.03	0.00	242,200	242,200	73,203	73,203
20	0.03	0.00	249,466	249,466	70,798	70,798
21	0.03	0.00	256,950	256,950	68,471	68,471
<b>Maximum</b>						<b>98,205</b>

*Notes:* PY = Policy Year; GR = Guaranteed Credited Rate; SC = Surrender Charge; FV = Fund Value at End of Policy Year; CSV = CSV at End of Policy Year; PV(Fund) = PV(Fund) at Issue Gross of Surrender Charge; and PV(SCV) = PV(SCV) at Issue Ignoring Bailout.

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