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Minimizing the Number of Optical Amplifiers Needed to Support a Multi-Wavelength Optical LAN/MAN *

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Abstract

Optical networks based on passive star couplers and employing wavelength-division multiplexing (WDM) have been proposed for deployment in local and metropolitan areas. Amplifiers are required in such networks to compensate for the power losses due to splitting and attenuation. However, an optical amplifier has constraints on the maximum gain and the maximum output power it can supply; thus optical amplifier placement becomes a challenging problem. The general problem of minimizing the total amplifier count, subject to the device constraints, is a mixed-integer non-linear problem. Previous studies have attacked the amplifier-placement problem by adding the "artificial" constraint that all wavelengths, which are present at a particular point in a fiber, be at the same power level. In this paper, we present a method to solve the minimum-amplifier-placement problem while avoiding the equally-powered-wavelength constraint. We demonstrate that, by allowing signals to operate at different power levels, our method can reduce the number of amplifiers required in several small to medium-sized networks.

1 Introduction

1.1 Network Environment

The focus of this study is on a class of the next-generation optical local/metropolitan area networks (LAN/MAN) which span distances from fewer than a kilometer to a few tens of kilometers and which provide loop-free communication paths between all source-destination pairs. A large-distance version of such a network is depicted in Fig. 1, and it consists of $N = 63$ stations and $M = 4$ passive optical star couplers ("stars"), such that each star is connected to other stars and/or stations via two unidirectional fiber links. The passive star coupler provides a broadcast facility, but it must also be of the "non-reflective" type (to be elaborated below) in order to prevent loops in the network.

Our study will consider the case where each station in the network has a fixed-wavelength transmitter and is set to operate on its own unique wavelength channel. Each station either has a tunable receiver or a receiver

array in order to receive signals from all of the other stations. The objective is to ensure that a station's transmission can be received by every other station after being subject to losses and gains as the signal traverses through different parts of the network. The network consists of optical stars that are non-reflective. A non-reflective star consists of pairs of inputs and outputs, and each output carries all of the wavelengths that were incident on all of the inputs except for the wavelengths that were carried on its own paired input. Such stars have been employed in the Level-0 All-Optical Network (AON) [1]. Non-reflective stars are needed in order to avoid interference due to loops ("echoes") in the network. A star in the network with k input fibers and k output fibers operates such that the power on each wavelength on an input fiber is divided evenly among the $k - 1$ output fibers. This is referred to as the *splitting loss* at a star. (Note that the splitting loss can be different for different-sized stars in the network.)

As the sample network in Fig. 1 shows, these networks can be deployed as part of a metropolitan area network (MAN). We require that each transmitted signal/wavelength be received at all of the other receivers at a power level greater than a station's receiver sensitivity level, denoted by p_{sen}^1 . However, apart from the splitting loss due to the stars mentioned above, there is signal attenuation on the fibers given by the parameter α dB/km. Even though attenuation losses for fiber are relatively low (approximately 0.2 dB/km loss) compared to other transmission media, larger networks (MANs) and networks with numerous splitting/coupling losses will require amplification to allow a transmitted signal to reach the receivers at a detectable level. The constraints on the system are shown in Table 1, along with typical values for each parameter. $P_{NONLIN,max}$ defines the power level, in a fiber, above which a signal encounters significant non-linear effects. However, the total power at any point in the network is usually bounded by a lower value P_{max} , which is the maximum output power of an amplifier and a transmitter. These parameter values (last column of Table 1) will be used in our illustrative numerical examples in Section 3. The gain model for our amplifiers is given by

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¹The signal-to-noise ratio (SNR) of the wavelengths is another important parameter and needs to be investigated in the future.

Figure 1: Example of a passive-star-based optical metropolitan area network (slightly modified version of the one used in [4]).

(from [9])

$$\frac{P_{in}}{P_{sat}} = \frac{1}{G-1} \ln \left(\frac{G_0}{G} \right) \quad (1)$$

where P_{in} is the total input power (across all wavelengths) to the amplifier in mW, P_{sat} is the internal saturation power in mW, G is the actual gain achieved (in absolute scale, *not* dB), and G_0 is the small-signal gain (which is the gain achievable for small values of input power when the amplifier does not saturate, again in absolute scale). We do not consider other system factors that might be relevant in determining the actual system performance, such as amplifier noise and crosstalk at the receivers.

1.2 Problem Definition

In the network setting described above, it is important to quantify the minimum number of amplifiers required to operate the network and to determine their exact placements in the network. In such a network, when signals on different wavelengths originating from different locations in the network arrive at an amplifier, their power levels could be very different. This phenomenon is known as the Near-Far Effect and it results in inefficient utilization of the individual amplifier. The difference in power levels of the input wavelengths can significantly limit the amount of amplification available since the higher-powered wavelengths could saturate the amplifier and limit the gain seen by the lower-powered wavelengths. Also, allowing wavelengths in the same fiber to be at different power levels changes the minimal-amplifier-placement problem from a mixed-integer linear program (MILP) [8] into a mixed-integer non-linear program, as we shall show later in this paper.

Previous optical amplifier-placement schemes [4, 8] bypassed these problems by restricting all of the wavelengths at any given point in a fiber to be at the same power level. Unfortunately, requiring wavelengths to be at the same power level often forces the designer to

Figure 2: Simple two-star network that needs no amplifiers to operate.

add more amplifiers than the minimum necessary in order for the receivers to receive signals at or above the receiver sensitivity level. Since each optical amplifier costs around \$25,000, every attempt should be made to minimize their number in the network. It is also desirable to reduce the number of amplifiers used in the network based on noise, maintenance, and fault-tolerance considerations.

Our study was motivated by the network in Fig. 2. For reasonable network parameters, this network can operate without using any amplifiers. However, if the power levels for all wavelengths must be equal on any given link, as required by the MILP approach in [8], then an amplifier (on one of the links between stars A and B) will have to be added to the network. This is because, if we fix the output power of star A to be some value x , then the signals from stations 3 and 4 must reach star B with an output power higher than x . Without an amplifier, signals from stations 1 and 2 reach star B at a power less than x , which means that wavelengths on the link from star B to station 3 (and similarly on the link from star B to station 4) will have

Parameter	Description	Range	Value used
p_{sen}	Minimum signal power at receiver and the amplifier sensitivity level	-30dBm at 1 Gbps	-30 dBm
G_{max}	Maximum small-signal gain	<25 dB (MQW) [6]	20 dB
$P_{NONLIN,max}$	Maximum total power in fiber	10-50mW	10 mW
P_{max}	Maximum total output power of amp and transmitter		0 dBm
P_{sat}	Internal saturation power of the amp		1.298 mW
α	Fiber attenuation		0.2 dB/km

Table 1: Important parameters and their values used in the amplifier-placement algorithms.

unequal powers. Therefore, requiring equal power on all wavelengths adds an unnecessary amplifier to this network. As we will soon see, allowing wavelengths to be at unequal powers eliminates the need for any amplifiers in this network.

In this paper, we propose a scheme that minimizes the number of amplifiers for the network setting described in [4] without the restriction that wavelengths in the same fiber be at the same power level. The method works as follows: 1) determine whether or not it is possible to design the network taking into consideration the limitations of the devices (e.g., the power budget of the amplifiers), 2) generate a set of constraints to closely describe the problem setting, which turns out to be a non-linear program, 3) pass the set of constraints to a non-linear solver, such as *CFSQP* (C code for Feasible Sequential Quadratic Programming) [7], in order to solve for the minimum number of amplifiers needed for the entire network, and 4) determine the exact placements of the optical amplifiers. Numerical examples will show that this network-wide optimization method without the equal-power constraint often results in solutions that require fewer amplifiers than the solutions in [4, 8].

2 Solution Approach

Given a network as in Fig. 1, we would like to minimize the number of amplifiers used in the network without violating the device capabilities and constraints. Throughout this paper, we assume that the stars are connected together in the form of a tree and that all neighbors have two unidirectional links connecting each other. A mathematical formulation of the problem is provided in Section 2.1. Unfortunately, the resulting mixed-integer non-linear optimization problem is extremely difficult to solve. Hence, we carefully avoid the integral constraints by modifying the formulation, specifically the objective function, and solve the resulting non-linear optimization problem. The description of the solution strategy is provided in Section 2.2. The output from the solver is fed to an Amplifier-Placement Module which outputs the exact positions and gains of the amplifiers. The functionality of the Amplifier-Placement Module is described in Section 2.3.

2.1 Formulation

In this subsection, the amplifier placement problem is formulated as an Integer Non-Linear Constrained Optimization Problem. First, the notation used in the for-

mulation is introduced, and then the constraints and objective functions are described.

2.1.1 Device Parameters

- p_{sen} = Minimum power required on a wavelength for detection in dBm. This represents both the receiver sensitivity level and the amplifier sensitivity level, which have been assumed to be equal.
- P_{max} = Max. power of an amplifier in mW
= Max. power of a transmitter in mW

It is not necessary that the maximum amplifier output and transmitter powers be identical. For simplicity, we have assumed them to be equal.

- G_{max} = Max. (small-signal) amplifier gain in dB
- α = Signal attenuation in dB/km

2.1.2 Problem Variables

This section introduces the variables used in the problem formulation. Note that among the variables representing the power levels, those beginning with lowercase ($p_l^{min,beg}$, $p_{x,i}$, p_i^{xmit}) are measured in dBm and those with uppercase (P_l^{beg} , P_l^{min}) in mW. Also, the variables in lowercase represent the per-wavelength power levels, whereas the ones in uppercase represent the aggregate power over all the wavelengths on the respective link.

- N = number of access stations in the network
= number of wavelengths in the network
 - M = number of stars in the network
 - L = number of links in the network $= 2 \times (N + M - 1)$
- Note that stars are identified by the indices $1, 2, \dots, M$ and stations by the indices $M+1, M+2, \dots, M+N$. As we shall soon see, this provides notational convenience when we refer to the source/destination of a link, irrespective of whether it is a station or a star. Also, the wavelengths in the network are identified by the indices $M+1, M+2, \dots, M+N$ of the source stations. We associate the following parameters with each link l , $1 \leq l \leq L$.
- s_l = Source of link l , $1 \leq s_l \leq (M+N)$.
 - d_l = Destination of link l , $1 \leq d_l \leq (M+N)$.
 - Λ_l = Set of powered wavelengths carried by link l .
 - n_l = Number of amplifiers on link l .
 - L_l = Length of link l in km.
 - SG_l = Actual total Supplied Gain on link l in dB.
 - $p_l^{min,beg}$ = Power level of the least-powered wavelength arriving at link l , in dBm.

- P_l^{beg} = Total power at the beginning of link l , in mW.
- P_l^{min} = Total power on link l when all signals are $\geq p_{sen}$ and at least one wavelength is equal to p_{sen} , in mW.
- $gmax_l$ = Maximum gain available from an amplifier on link l , in dB.
Consider the star i , $1 \leq i \leq M$.
- D_i = in-degree of star i = out-degree of star i .
- $p_{x,i}$ = power of wavelength x at the output of star i , in dBm.
Consider the station i , $(M+1) \leq i \leq (M+N)$.
- p_i^{xmit} = Transmitted power of wavelength i at station i , in dBm.

2.1.3 Useful Functions

The following functions allow conversion between the milliwatt (regular) and dBm (log) scales.

$$\begin{aligned} ToDB(\xi) &= 10 \cdot \log_{10}(\xi) \\ ToMW(\xi) &= 10^{\xi/10} \end{aligned}$$

They are used to express the constraints conveniently in the appropriate scale.

2.1.4 Basic and Non-Basic Variables

Given a network, the values of the topology-specific variables N , M , L , s_l , d_l , Λ_l , L_l , and D_i are fixed, irrespective of the amplifier-placement algorithm chosen. The only basic variables used in the formulation are p_i^{xmit} , SG_l , and n_l . Note that the variables P_l^{beg} , $p_l^{min,beg}$, P_l^{min} , $p_{x,i}$, and $gmax_l$ are non-basic variables and can be expressed in terms of the basic variables as follows.

For link l , whose source is a star, i.e., $1 \leq s_l \leq M$, we have

$$p_l^{min,beg} = \min_{x \in \Lambda_l} p_{x,s_l} \quad (2)$$

and we also have

$$P_l^{beg} = \sum_{x \in \Lambda_l} ToMW(p_{x,s_l}) \quad (3)$$

For link l , whose source is a station, i.e., $(M+1) \leq s_l \leq (M+N)$, we have

$$p_l^{min,beg} = p_{s_l}^{xmit} \quad (4)$$

and we also have

$$P_l^{beg} = ToMW(p_{s_l}^{xmit}) \quad (5)$$

For any link l , the total power drops to its minimum level when at least one of the wavelengths is equal to the sensitivity level (p_{sen}). Hence, on link l , starting with an aggregate power level P_l^{beg} , when the weakest signal is at a power level $p_l^{min,beg}$, after appropriate scale changes, we have

$$P_l^{min} = ToMW(ToDB(P_l^{beg}) - (p_l^{min,beg} - p_{sen})) \quad (6)$$

For links from stations to stars, i.e., $(M+1) \leq s_l \leq (M+N)$ and $1 \leq d_l \leq M$, we have

$$\begin{aligned} p_{s_l,d_l} &= p_{s_l}^{xmit} + SG_l \\ &\quad - \alpha \cdot L_l - ToDB(D_{d_l} - 1) \end{aligned} \quad (7)$$

For links between stars, i.e., $1 \leq s_l, d_l \leq M$, we have

$$\begin{aligned} \forall x \in \Lambda_l \quad p_{x,d_l} &= p_{x,s_l} + SG_l \\ &\quad - \alpha \cdot L_l - ToDB(D_{d_l} - 1) \end{aligned} \quad (8)$$

For any link l ,

$$gmax_l = G(P_l^{min}, G_{max}, P_{sat}) \quad (9)$$

We note that various amplifier gain models can be used to obtain this function G .

2.1.5 Constraints Inequalities.

Consider the link l , $1 \leq l \leq L$. The powers on each of the wavelengths at the beginning of the link l should be at least the sensitivity level, p_{sen} . This can be ensured by requiring that the *weakest* signal has a power level of at least p_{sen} as follows.

$$p_l^{min,beg} \geq p_{sen} \quad (10)$$

The powers on each of the wavelengths at the end of each link l should be at least p_{sen} . This is to enable the receivers to detect the signals correctly. Thus,

$$p_l^{min,beg} + SG_l - \alpha \cdot L_l \geq p_{sen} \quad (11)$$

The above inequalities (Equations (10) and (11)) ensure that the signal powers remain at or above p_{sen} everywhere along the fiber links and throughout the network.

There are upper limits on the maximum power carried by all the signals in a link. This value P_{max} is the same for transmitters and amplifiers, and hence at the beginning of link l , we have

$$P_l^{beg} \leq P_{max} \quad (12)$$

Similarly, at the end of the link l , we have

$$ToDB(P_l^{beg}) + SG_l - \alpha \cdot L_l \leq ToDB(P_{max}) \quad (13)$$

Since we need to divide the total supplied gain SG_l among the n_l amplifiers on link l , we have

$$SG_l \leq gmax_l \cdot n_l \quad (14)$$

However, the gain SG_l should require no fewer than n_l amplifiers; thus,

$$SG_l > gmax_l \cdot (n_l - 1) \quad (15)$$

Integrity Constraints.

Consider the link l , $1 \leq l \leq L$. The number of amplifiers, n_l , on any link l , is an integral value. Hence, we require that

$$n_l \quad \text{is an integer.} \quad (16)$$

2.1.6 Objective function

Minimize

$$\sum_{l=1}^L n_l \quad (17)$$

2.1.7 Complexity

The only basic variables used in the formulation are p_i^{xmit} , SG_l , and n_l . The others can be computed either beforehand from the topology or at run-time as a function of the basic variables. Hence, we have

- number of variables = $2 \cdot L + N$,
- number of integer constraints = L , and
- number of non-linear inequalities = $6 \cdot L$.

2.1.8 Reasons for Non-linearities

The approach presented in this paper differs from the one in [8] in that it allows the different wavelengths on a link to be at different power levels. Whereas the method in [8] needed to place amplifiers whenever all the wavelengths on the link were at their lowest power level, now the placement of the amplifier is constrained by the *weakest* signal on the link. Hence, on each link, we need to identify the wavelength coming in with the lowest power level ($p_l^{min,beg}$). This introduces a non-linear term in the formulation (Equation (2)). Moreover, the maximum gain ($gmax_l$) available at an amplifier on a link is dependent on the precise mix of the power levels on its incoming wavelengths. This computation cannot be performed off-line and results in non-linear constraints (see Equations (14) and (15)).

2.2 Solver Strategies

The mixed-integer non-linear optimization problem resulting from Section 2.1 is an extremely difficult one to solve and is highly computation-intensive. In order to reduce the computation complexity, it is possible to eliminate the integral constraints altogether. This can be done by removing the variables n_l from the formulation, and hence the constraints in Equations (14) and (15) disappear. We define a *new objective function*:

Minimize

$$\sum_{l=1}^L SG_l / gmax_l \quad (18)$$

which is close to the original one, since $n_l = \lceil SG_l / gmax_l \rceil$. The *starting point* of the problem space is especially important for this non-linear search. We initialize the basic variables of the problem, namely, SG_l and p_i^{xmit} such that

$$\begin{aligned} SG_l &= 0 \\ p_i^{xmit} &= ToDB(P_{max}) \end{aligned}$$

i.e., the network is initialized to a state when all the transmitters are operating at their highest powers and all of the links have zero gain. However, we could also use the solution from [8] as a feasible starting point. Since the new objective function is not identical to the

one in the integral case, the solver might end up minimizing the exact function $SG_l / gmax_l$ and not the number of amplifiers in the network. To handle this situation, we adopt a *non-intrusive measurement* approach, where, at every feasible point along the search path to the optimum solution taken by the non-linear program solver, we evaluate the *exact* objective function and remember the point in the search space which resulted in the minimum value for the *exact* objective function thus far.

The ensuing heuristic search has the following interesting properties.

1. It contains significantly fewer variables and constraints. In fact, it has only
 - $L + N$ variables,
 - $4L$ inequalities, and
 - zero integer constraints.
2. All the constraints and the objective function are easily differentiable. Hence, the gradients can be fed to the non-linear program solver to aid it in its search for the optimum solution.

The non-linear program solver, *CFSQP*, which we used for this study achieves the minimization of the smooth objective function subject to general smooth constraints through the generation of feasible iterates. If the starting point is infeasible, it generates a point satisfying the constraints by solving a strictly convex quadratic program (QP). It then uses a nonmonotone line search [3] forcing a decrease of the objective function within at most three iterations. There are, however, limitations to this approach and they are discussed below.

1. Local minima: The non-linear program solver might terminate at a point corresponding to a local minimum for the objective function. This happens, for example, when the starting point corresponds to the Linear Program solution (see Table 2 and the example in Fig. 1).
2. Feasible point generation: When the starting point is infeasible, subject to the constraints, the solver may not be able to locate a feasible point in the problem space. With *CFSQP*, this problem can be fixed by using a different quadratic programming solver to generate the feasible point. However, finding a feasible point becomes increasingly difficult as the number of network elements grow (i.e., more network elements means more variables).
3. Integer variables: The non-linear program solver (*CFSQP*), which we used in this study, is not well-suited to handle integer variables. Hence, its results for this problem could be improved upon by using specialized mixed-integer non-linear program solvers.

The output of the non-linear program solver is fed to the Amplifier-Placement Module which is described next.

2.3 Amplifier-Placement Module

This module uses the values of SG_l and p_i^{xmit} output by the non-linear program solver to determine the exact location and gain of the amplifiers in the network. It operates on a link-by-link basis as follows. It computes the maximum value of the gain available from

each amplifier on a link l ($gmax_l$) using Equation (9) and, hence, the number of amplifiers (n_l) required on that link. It also computes the power levels of the different wavelengths at the output of the stars ($p_{x,i}$). Then, it follows the As Soon As Possible (ASAP) method for the amplifier placement, which operates as follows. For all but the last amplifier on a link, this method places an amplifier on a link as soon as the input power is low enough to allow the *maximum* gain, and for the last amplifier on a link, it places the amplifier as soon as the input power is low enough to allow the *remaining* gain. Several other methods of splitting the gain (SG_l) along the link l , including uniform distribution among the n_l amplifiers, are possible. The ASAP method was chosen to maintain the power levels of the signals as high as possible. Further discussions on various approaches to gain splitting can be found in [5].

3 Numerical Examples

The link-by-link method in [4] was designed to equalize the powers of the wavelengths in the network, as opposed to trying to minimize the number of amplifiers in the network. By forcing the powers of all wavelengths to be equal to p_{sen} at the beginning of most links, the algorithm placed amplifiers simply by knowing how many wavelengths were on a link. If the number of wavelengths on a link is precomputed, this allowed the algorithm to operate on each link individually (locally) without knowing what was happening on other links. This led to a very simple amplifier-placement algorithm. Unfortunately, as was shown in [8] and can also be seen in Table 2, this approach does not minimize the number of amplifiers needed in the network. The transmitter powers can be adjusted to avoid placing amplifiers on the links which originate at a station. However, since signals on all other links start off with the minimum power (p_{sen} on each wavelength), we know that the algorithm will place an amplifier on every single link not originating at a station in the network. We note that there are $L - N$ such links in the network which originate at a star (recall that L = number of links, N = number of stations, and M = number of stars); thus we obtain the lower bound of $L - N = 2 \times (N + M - 1) - N = N + 2 \times (M - 1)$ on the number of amplifiers used by the method in [4]. This algorithm performs the poorest, in comparison to other placement schemes, on networks that have short links because the other algorithms can usually avoid placing an amplifier on a short link simply by exiting the originating star with enough power to traverse the short link. We show the results of this algorithm for various networks in column 2 of Table 2.

The global method in [8], allowed wavelengths at the beginning of the links to be above the absolute minimum allowed, p_{sen} . However, the powers on all of the wavelengths at any given point in the network was required to be equal; this equally-powered-wavelengths constraint enabled the computation of the maximum gain ($gmax_l$) available on a link, by knowing just the number of wavelengths on the link. The amplifier-placement problem can be formulated as a mixed-integer linear program and solved exactly. Consider a pair of adjacent stars in the network. Taking

Figure 3: Mid-sized tree-based network needing no amplifiers to function.

into account the attenuation loss along the links connecting the stars and the splitting losses at the stars, we require that there be at least one amplifier on either of these links. The lower bound on the number of amplifiers required using the Linear-Program (LP) method in [8] is thus $M - 1$, where M is the number of stars in the network. (See [8] for details.)

The method described in this paper (see Section 2) is a global one too; however, unlike the LP method in [8], it allows the wavelengths at any point in the network to operate at unequal powers. The solution obtained to the amplifier-placement problem is not guaranteed to be the optimum because of the presence of local minima. Moreover, the only available lower bound on the number of amplifiers required by this Non-Linear Program (NLP) method is the trivial one (i.e., not needing any amplifier). Next, we compare the results of these three approaches to amplifier placement on certain sample networks (see Table 2).

As mentioned earlier, the network in Fig. 2 motivated this study. While both the earlier approaches (the link-by-link method and the LP method) required a few amplifiers to operate the network, the NLP method described in this paper does not require any.

The network in Fig. 3 is the motivating network, described above, taken to the extreme. This network has many stars and yet it needs no amplifiers to function. Table 2 reveals that the new method was indeed able to come up with the solution of not needing any amplifiers. This is the type of network where the unequally-powered-wavelengths solution is clearly superior to the previous two amplifier placement methods. Although it is arguable whether this network is realistic or not, we have presented it here in order to give the reader some insight as to the conditions in which the new method performs best.

The network in Fig. 4 is meant to be a realistic design of a MAN. This network was designed in a semi-random fashion with some heuristics to guide the design. Table 2 shows that the new method was able to find a solution which required fewer amplifiers than the methods in [4] and [8]. Fig. 4 also provides an insight into how the actual placements of amplifiers differ between the equally-powered-wavelengths method (LP) and the unequally-powered-wavelengths method (NLP). The amplifiers that are filled black are the locations at which the equally-powered-wavelengths method placed the six amplifiers it deemed necessary to operate. The empty, or filled white, amplifiers are the loca-

Network	Link-by-link method [4]	Equally-powered wavelengths (LP) [8]	Unequally-powered wavelengths (NLP) (this paper)
Lower bound	$N + 2 \times (M - 1)$	$M - 1$	0
Simple 2 star (Fig. 2)	6	1	0
Tree (Fig. 3)	44	14	0
MAN (Fig. 4)	38	6	4
Scaled-up MAN	48	16	16*
Scaled-down MAN	38	4	0
Previously-studied MAN (Fig. 1)	79	77	77*

we predicted that our new method might not perform better than the equally-powered-wavelengths solution. We predicted this because the more nodes a network has, the more variables the solver is manipulating and the more local minima the solver can get stuck at. As Table 2 shows, the solver was unable to come up with a better solution than the LP solution, even when given multiple feasible starting points including the solutions found in [4] and [8].

4 Future Work

4.1 Switched Networks

The algorithms described in this paper were designed to operate on "loopless" networks where there is only one path from a source to a destination. In a switched network, there can potentially be multiple paths between a source and a destination. Since the above algorithms operate knowing how many wavelengths are on a given link, they assume that all wavelengths that can possibly reach a link could all be present on that link simultaneously. This approach has the potential to place more amplifiers in the network than is absolutely necessary. A switched network could contain multiple paths between any source-destination pair. Designing links in such networks to carry a few instead of all possible connections can result in a significant savings in the number of amplifiers. We believe it will be possible to modify our current algorithms to allow them to exploit this phenomenon that occurs in switched networks. This is a topic of our future work.

4.2 Gain Model

In the near future, we plan to try and further improve on the optical amplifier gain model. We expect to be able to create a reasonably-accurate gain model of the popular Erbium-doped fiber amplifier (EDFA). Analytical methods for modeling the amplifier gain, gain saturation, and noise described in [2] will be incorporated in the model. We also plan to expand our amplifier gain model to handle per-wavelength gain. This would allow us to model an amplifier that has a non-flat gain spectrum. It would also allow us to model the small gain for wavelengths that are normally considered to lie outside of the "amplifier bandwidth". The formulation of the problem would have to be changed to handle per-wavelength gain too.

5 Conclusion

We considered the problem of minimizing the number of optical amplifiers in an optical LAN/MAN. This study departed from previous studies by allowing the signal powers of different wavelengths on the same fiber to be at different levels. Although this increases the

complexity of the amplifier placement algorithm, numerical results show that certain networks do benefit from this method by requiring fewer amplifiers. Our results demonstrated that smaller networks (in terms of distance) benefited the most from this new method. Larger networks tended not to benefit as much because 1) using unequally-powered wavelengths hurts the efficiency of the amplifiers too much if long links have to be traversed, and 2) larger networks have more local minima causing our solver to sometimes miss the global "optimal" solution.

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