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Recognizing Actuarial Assumptions
Victoria Stachowski* and Alice Underwood†

Abstract‡
As assumptions underlie every aspect of actuarial calculations, actuaries must be aware of the assumptions they are using and understand their importance and the possible effects of changing assumptions on the results of their calculations.

This paper explores the nature of assumptions in: (i) mathematical models, (ii) data selection, (iii) actuarial methods, and (iv) the business environment. We examine reasons for making assumptions such as convenience, tradition, indications in the data, or lack of data. In addition, we discuss (i) how actuaries can judge whether these reasons are sufficient; (ii) methods that can help actuaries quantify the impact of their assumptions, such as what-if scenarios, simulation, and stress testing; and (iii) the circumstances for which testing is most important.

Key words and phrases: hypotheses, postulates, methods, standards, models, and analysis

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1 The Importance of Assumptions

Understanding actuarial assumptions is a requirement of the professional standards of practice. According to one actuarial compliance guideline, "If there is a change in the actuarial assumptions or methods from those previously employed, the change should be mentioned in the actuarial statement of opinion" (Actuarial Standards Board 4).

Actuaries are often told to check their assumptions but they may not know what checking assumptions entails. This article suggests ways to check assumptions and also explains where to look for assumptions, as some assumptions are less obvious than others. We explore common assumptions in several areas: mathematical models, data selection, actuarial methods, and the business environment. Methods to quantify the impact of assumptions, such as what-if scenarios, simulation, and stress testing are discussed, as are the circumstances in which such testing is most important. We examine reasons for making certain assumptions—such as convenience, historical practice, indications in the data, or simply lack of data—and discuss how actuaries can judge whether these reasons are sufficient.

Understanding assumptions can assist actuaries in choosing the most appropriate methods for pricing, reserving, and other tasks; deciding the next steps to take in an analysis; determining the level of confidence for estimates; and in creating financial products that protect against the chance that certain assumptions are incorrect.

2 Reasons for Making Assumptions

When actuaries recognize their assumptions, they must also recognize their reasons for making them. Though there are many reasons for making actuarial assumptions, we will group our examples into two general categories: (i) assumptions are dictated by external or internal factors affecting the analysis and (ii) assumptions are dictated by convenience or circumstance.

2.1 Assumptions Dictated by the Analysis

The data may suggest or necessitate certain assumptions. For example, perhaps you are fitting a claim size severity distribution to experience data. Having trended the historical claims, you use one or several statistical methods to fit different distributions: perhaps a lognormal, a two-parameter Pareto, and a loggamma. If
you determine the lognormal is the best choice, then assuming a lognormal distribution is logical, at least for claim sizes within or not far outside the range of available data.

**Gaps in the data may force certain assumptions.** Suppose you need to make a calculation using the standard deviation of annual loss ratios by line of business, but you have only the plan loss ratios. Some assumption will therefore be necessary. One possibility would be the use of related data, such as standard deviations from industry data—but then you are assuming that the variability of your own book is similar to that of the industry book.

**Anecdotal evidence.** In some lines of business, common knowledge plays a large role. You may encounter assertions such as “everyone knows motor liability business has Pareto severity with alpha parameter 2.5,” or “claims in that line take five years to pay out.” In the absence of strong evidence, either supporting or contradictory, it may be reasonable to heed the general wisdom. Although you should not let folklore override empirical evidence, informed actuarial judgment is one of the cornerstones of the profession.

Anecdotal evidence, however, often contains implicit assumptions that have not been tested recently, if ever. Investigating these assumptions may lead to new insights.

**Significance.** If the value of a particular parameter will have only a minor effect on the final result, estimating the parameter's value precisely may not be worth the effort. In such cases, you may consider the tradeoff between time and accuracy with respect to making a reasonable or standard assumption for the value. The accurate assessment of the significance of the value in question distinguishes this type of assumption from one made solely for convenience's sake.

### 2.2 Assumptions of Convenience

**The problem can't be solved otherwise.** Some calculations are intractable without simplifying assumptions. For example, assuming independence of property losses is often not correct. Unfortunately, there is rarely good information about the correlation between the frequency and/or severity of individual losses. Even if the correlation were known, including it in your calculations could be complicated. For these reasons we often assume independence. If the correlation is weak, this assumption may be harmless. But
if losses are materially correlated, the extent of the dependence should be considered somehow—possibly through a Monte Carlo simulation or judgmental loading.

**Simplicity.** Ockham's Razor states that, all things being equal, a simple hypothesis is preferable to a complicated one. This is also called the principle of parsimony. A simpler assumption is easier to work with and easier to explain to others. For example, we may select a distribution with few parameters rather than a distribution with many, if the goodness-of-fit is similar. In this case we gain the advantage of simplicity and also avoid possible over-fitting.

**It's what you can do.** Suppose you only learned one method for doing a particular analysis. To take a far-fetched example, maybe the only kind of average you can calculate is a simple average. Perhaps you have heard that there are exotic types of averages such as weighted average or average ex high/low, but you don't know how they work. Still, if you recognize the assumptions underlying the method you do know—for a simple average, all the data points are assumed to be equally valid—and feel confident that these assumptions are satisfied, there may be no problem. If you suspect that your assumptions are violated, however, seek assistance and advice from more experienced colleagues, research papers, or outside experts.

**It's what the relevant authority will accept.** This is the flip side of the situation above. Here are two examples: (i) your boss, or another executive in your company, may be comfortable only with one particular method—such as a simple average; or (ii) regulatory bodies may require a certain calculation method, thereby dragging along its fundamental assumptions. When regulation explicitly requires certain assumptions, such as a specific interest rate for discounting, we have “prescribed assumptions” (Actuarial Standards Board 1996). In such cases you must make the calculation using the assumptions required by those directing the work product. If you believe a prescribed assumption is unrealistic or unwarranted, however, you may want to recalculate using the assumption you consider best. Then you can compare the results and, if appropriate and material, explain any differences to the relevant person or authority.

**It's what the client will accept.** Insurance companies are in the business of making money, and one part of this process is selling
products to clients. Sometimes the client is in a position to dictate which assumptions the actuary will use—and sometimes the actuary has insufficient information to refute the client’s demands. The historical treatment of credibility illustrates this situation. Credibility methods are used in property-casualty business to balance a client’s observed experience against the a priori expectation. Simply put, credibility is the percentage weight given to the client’s observed experience. Hewitt (1989) explains that while the theoretical Bayesian credibility formula never assigns 100 percent weight or full credibility to the data, historically “buyers with better-than-average experience wanted full recognition in their rates.” These buyers were by definition the better risks and usually were the larger customers as well. So, “an arbitrary assignment was made—the point at which exposures were sufficient to admit of ‘full’ credibility—and, of course, on the basis of convenience.” Such methods allowing full credibility remain part of the standard actuarial repertoire.

Resources. The investigation and refinement of assumptions take time and money. These resources are finite, however, particularly for those trying to analyze an issue that appears at the last minute. Sometimes an answer that is too late is as bad as no answer at all. This is why it is important for actuaries to determine which of their assumptions will have the most impact on their model and to prioritize their efforts accordingly.

3 Assumptions in the Mathematical Methods

Because assumptions are fundamental to mathematics and statistics, whenever actuaries use mathematics, they rely on fundamental mathematical assumptions. In addition, actuaries must make assumptions about which mathematical methods are appropriate in given situations.

The latter issue is of more practical concern. While it would be impossible to discuss all such instances, we will give some examples to illustrate what we mean.

3.1 Choice of Techniques

Suppose you want to make a best estimate of the coming year’s loss ratio. Premium and loss data for several years have been on-leveled
and trended to produce loss ratios as if at today’s premium rates and trended loss levels.

Table 1

<table>
<thead>
<tr>
<th>Underwriting Year</th>
<th>As-If Loss Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>On-Level Premium</td>
</tr>
<tr>
<td>1</td>
<td>1,000,000</td>
</tr>
<tr>
<td>2</td>
<td>1,250,000</td>
</tr>
<tr>
<td>3</td>
<td>1,300,000</td>
</tr>
<tr>
<td>4</td>
<td>1,400,000</td>
</tr>
<tr>
<td>5</td>
<td>1,500,000</td>
</tr>
<tr>
<td>6</td>
<td>1,600,000</td>
</tr>
<tr>
<td>7</td>
<td>1,800,000</td>
</tr>
<tr>
<td>8</td>
<td>1,750,000</td>
</tr>
<tr>
<td>9</td>
<td>2,000,000</td>
</tr>
<tr>
<td>10</td>
<td>2,000,000</td>
</tr>
</tbody>
</table>

You want to take some sort of average as your best estimate for next year. There are several types of averages used by actuaries; what follows is a look at the implicit assumptions made by a few of these averaging methods. The data used are displayed in Table 1.

**Straight average.** This method assumes that: (i) all the historical years are equally predictive or equally credible, (ii) the fundamental loss situation has not changed over time, and (iii) none of the historical data points is an outlier (a fluke) that should contribute less to the final prediction. This average of Column (4) of Table 1 is

\[
\frac{62 + 68 + 50 + 57 + 105 + 76 + 65 + 41 + 71 + 80}{10 \times 100} = 67.5\%.
\]

**Average excluding high and low values.** This method accepts the first two assumptions of the straight average. It assumes, however, that the high and low values are outliers and thus not predictive. This average of Column (4) of Table 1 is

\[
\frac{62 + 68 + 50 + 57 + 76 + 65 + 71 + 80}{8 \times 100} = 66.1\%.
\]
**Premium-weighted average.** This method assumes years of larger premium volume are more predictive than years with smaller premium volume. If there are large year-to-year differences in on-level premium, this method can be more appropriate than others. This average is

\[
\frac{\sum (\text{premium}_t \times \text{loss ratio}_t)}{\sum \text{premium}_t} = \frac{\sum \text{losses}_t}{\sum \text{premium}_t} = \frac{\sum \text{Column (3)}}{\sum \text{Column (2)}}.
\]

Summing down Columns (2) and (3) of Table 1 yields

\[
\frac{620 + 850 + 650 + \cdots + 717.5 + 1420 + 1600}{1000 + 1250 + 1300 + \cdots + 1750 + 2000 + 2000} = 68\%.
\]

**General weighted average with greater weight on more recent years.** This method assumes that recent years are more predictive. This assumption may be warranted if there is a trend in the data. A common method is using only the last three years, with more weight on the most recent years. For example, take weights for the last three years of 20 percent, 30 percent, and 50 percent. Our result is then

\[(0.2)41\% + (0.3)71\% + (0.5)80\% = 69.5\%.
\]

One problem with this approach, especially for longer tailed lines of business, is that the most recent years are not as reliable because the development of losses to ultimate is based on immature data.

These examples demonstrate that a procedure as common as taking an average may be full of assumptions.

### 3.2 Choice of Distributions

A common and important actuarial task is the selection of probability distributions for the frequency and severity of individual losses or for the aggregate amount of annual losses.
The most commonly used distribution to describe frequency of claims is the Poisson process distribution. If \( N(t) \) is the number of claims in \((0, t]\), then

\[
\Pr[N(t) = n] = \frac{(\lambda t)^n e^{-\lambda t}}{n!}, \quad n = 0, 1, 2, \ldots
\]

where \( \lambda > 0 \) is a parameter. The mean and variance of this distribution are both equal to \( \lambda \).

The Poisson process is characterized by three assumptions:

1. In an infinitesimally small time interval \((t, t + dt)\), the probability of having one claim is approximately \( \lambda dt \);

2. In an infinitesimally small time interval \((t, t + dt)\), the probability of two or more claims is essentially zero; and

3. The numbers of claims in non-overlapping time intervals are independent.

The Poisson process is popular in part because of its simplicity and mathematical tractability. Unfortunately, in many real world situations the variance is not likely to be equal to the mean. And, as indicated by Hogg and Klugman (1984, Chapter 2, p. 25),

... while Poisson postulates (1) and (2) are reasonably accurate in [many] kinds of situations, assumption (3) concerning the independence of the number of [claims] in non-overlapping intervals is often questionable. For example, a car may be so badly damaged at a given time that it has no chance of being damaged again in the near future because it is being repaired.

The Poisson process may nonetheless be appropriate for an insurer with a large portfolio of auto policies and auto claims. If a situation arises where assumption (3) is clearly false, the Poisson process may be modified as a mixed Poisson process or a blocked Poisson process—the above quote from Hogg and Klugman refers to a blocked Poisson process; see Ramsay (1991)—or other frequency distributions must be investigated.

Loss frequency is only one aspect of the actuarial analysis. It may also be necessary to select a form for the severity distribution. There may be several possibilities including the gamma, Pareto, or lognormal. The assumptions behind each of these models are important and
should be considered. For example, the Pareto and lognormal are heav­ily skewed and have thick right tails; this results in relatively high prob­abilities of catastrophic claims. See, for example, Panjer and Willmot (1992, Chapter 4) for other distributions that may be appropriate for loss frequency and loss severity and the various Poisson processes.

Clustering of claim sizes is another potential problem. The mathem­atical models are generally smooth, but actual claims tend to cluster at round numbers. You may need to smooth the data or group the data in order to avoid distortion. Beware of assuming that your real-world data are driven only by the purity of a mathematical process.

3.3 Model Risk and Parameter Risk

It is common knowledge that models, regardless of their complex­ities, are only idealizations of the real world phenomena they purport to describe. Following Daykin, Pentikäinen, and Pesonen (1994, p. 18) we define the following:

Model risk arises from the fact that a model is only an approximation to reality. This results in unavoidable errors because some meaningful variable has been omitted from the model. As some degree of model risk is always present, this risk must be recognized.

Parameter risk arises because we must rely on statistical estimates from observed data to determine the parameters of the model.

Process risk arises because the actual data (losses, investment returns, etc.) are inherently random, even if the model and parameters were exactly correct.

Parameter risk and model risk are important components of the risk taken by an insurance company. Unlike process risk, they are to some extent under the actuary’s control through the assumptions about model and parameters and deserve special consideration.

4 Assumptions in the Data

Once your mathematical model is created, it must be tested with real data in order to determine whether it is useful in a practical setting. Use of these data may involve as many assumptions as those in your mathematical model.

The concern with respect to data quality and appropriateness is im­portant. An Actuarial Standards Board bulletin on this issue states that although it is not the actuary’s task to audit the data, he or she
may be aware that the data are incomplete, inaccurate, or not appropriate as desired. In such cases, the actuary should consider whether the use of such imperfect data may produce material biases in the results of the study, or whether the data are so inadequate that the data cannot be used to satisfy the purpose of the study (Actuarial Standards Board 1993b, p. 4).

The following are some of the assumptions most relevant to data selection and use.

4.1 Assuming the Data Are Clean

It is common to assume that the data are clean, i.e., that the person(s) who entered the data corrected any typographical errors. If the data set is not too large, you can inspect it visually for obvious errors.

Sometimes the data that are initially free of errors are sullied at a later stage, as might happen when printed information is converted to machine-readable form via scanning. Scanned data may contain errors resulting from misinterpreting numbers and letters. Data sent by fax can present similar difficulties. Sometimes data stored on magnetic media such as hard drives and floppy disks may become corrupted if they are used often, improperly stored, or are left unattended for a long time.

A comparison to the original data, including checking totals, is advisable to make sure that such errors are found and corrected.

4.2 Assuming the Data Are Appropriate

Even if you are confident that the data are clean, they still may not be appropriate for your particular situation. Here are some examples to consider.

Using data from a statistical collection agency. The use of data from a statistical collection agency such as Insurance Services Office (ISO) assumes that the companies submitting data to the statistical collection agency reflect the same sort of business that your own company writes. Is this reasonable? For example, if your company engages in target marketing, the ISO classification relativities might not be appropriate. If you have excellent loss control measures, the ISO relativities might be correct, while the overall loss levels are too high.
Using data from another state or country. Regional differences may affect the insurance data. Consider the amount of litigation in California versus the amount in North Carolina. On an international scale, the definition of workers compensation in the United States means something quite different from the “accidents du travail” found in Belgium.

Data behind standard tables. There may be data behind standard relationships, formulas, or tables you plan to use. If these data are not applicable to your situation, consider whether and how you should use the results derived from them.

For years, actuaries in the United States used the Salzmann Tables in pricing property excess-of-loss reinsurance. These tables were compiled by Ruth Salzmann and based on 1960 accident year data for homeowner fire claims. As Ludwig (1991) has pointed out, these tables have been used to price many exposures and perils not similar to those studied by Salzmann.

4.3 Different Assumed Meanings Behind the Data

It is important that you understand the meaning of the data. Differences in meanings can arise in several ways.

Definitions. Did the person who created the data use the definition you have in mind? For example, there are several different meanings for IBNR. Many Europeans use the term IBNR to refer to pure IBNR or incurred but not yet reported (IBNYR). In the United States IBNR is used more commonly to refer to broad IBNR or all future development to come: this would include the IBNYR and also pipeline claims, incurred but not enough reported (IBNER), and so on. There may also be definitional differences with respect to claim counts, exposure units, earned premium, incurred claims, and so on.

Interpretation. Even when definitions are the same, interpretations may differ. For example, reserving actuaries may be interested in expected loss ratios (ELR) by line of business. Suppose in a company’s computer system these ELRs have been entered by the pricing actuaries. A problem will ensue if pricing actuaries think that they are supposed to enter a conservative ELR estimate whereas the reserving actuaries interpret the entries as the most likely value. Ideally the two different actuarial groups would work with the
same assumptions (e.g., ELR estimates are always best point estimate, neither conservative nor aggressive). If this is impossible or impractical, the two groups should at least be aware of their differences.

**Multiple Meanings.** Particularly when working with data from different sources, you may have to combine items that you would prefer not to combine. Pinto and Gogol (1987), in their paper on excess development factors, discuss the data from the Reinsurance Association of America (RAA). The RAA information goes back more than 20 years for some lines of business and is calculated by pulling together statistics supplied by member companies. But the member companies have written different types of policies over the years. In order to combine and compare the information, assumptions are needed with regard to the treatment of ALAE, different policy limits, different attachment points, different reporting patterns, and so on.

One way to avoid data-meaning difficulties is by asking questions, to make sure that there are no ambiguities. Another good idea is to check through the numbers and formulas. A calculation can be worth a thousand words.

## 5 Assumptions in the Actuarial Methods

Most actuarial methods involve assumptions, whether explicit or implicit. In this section we explore a few examples of assumptions in common reserving and ratemaking techniques.

### 5.1 Reserving Methods

Calculating loss reserves, i.e., the amount of money that must be set aside in the present to make claim payments in the future, is an important actuarial task. The actuary must consider the reported value of claims, any development likely to occur on these claims, and the projected additional amount for claims which have not yet been reported.

Here we will consider three different commonly used methods for creating the reserve for broad IBNR (including development on known claims, pipeline claims, etc., in addition to IBNYR). These are the loss ratio method, the link ratio method, and the Bornhuetter-Ferguson method. Assorted variations on these methods exist, but we will focus on basic versions of each.
The **loss ratio method**: This method is described by the following equation:

\[
IBNR_{\text{LossRatio}} = \text{Selected ELR}\% \times \text{Premium} - \text{Reported Incurred}
\]

where ELR is the expected loss ratio. Notice that this method makes a simple but powerful assumption: the selected ELR is the correct ultimate loss ratio, and thus ultimate losses can be calculated as the premium multiplied by ELR. The IBNR is just the difference between the losses originally expected and those losses that have already been reported.

This assumption has consequences. For example, the IBNR is inversely related to the reported incurred. The larger the reported incurred loss amount, the smaller the calculated IBNR—the IBNR may even be negative.

The **link ratio method**: This method (based on reported incurred) can be expressed as follows:

\[
IBNR_{\text{LinkRatio}} = \text{Reported Incurred}_t \times (LDF_t - 1)
\]

where \(LDF_t\) is the factor needed to develop losses at time \(t\) to their ultimate value.

The key assumption behind this method is that ultimate losses are directly related to the reported incurred at time \(t\) through a multiplicative loss development factor. For a larger reported incurred loss amount, the calculated IBNR will also be larger. The fundamental assumption leads in the opposite direction from the assumption for the loss ratio method.

The **Bornhuetter-Ferguson method**: This method is a combination of the two previous methods:

\[
IBNR_{\text{BF}} = \text{Selected ELR}\% \times \text{Premium} \times \left(1 - \frac{1}{LDF_t}\right).
\]

The assumption behind this method is a combination of the previous two. The ultimate loss is assumed equal to the reported incurred at time \(t\) plus an IBNR that is independent of the reported incurred. To produce the IBNR, the ELR is applied to that percentage of the premium that is not yet reported according to the development pattern.
If development patterns are steady from year to year, and if you pick the same (and the correct) percentage of premium as your loss ratios for all three methods, then they will give the same results. Tables 2 and 3 provide a simplified example, with only a few accident years.

**Table 2**  
A Perfect World (in $000s)

<table>
<thead>
<tr>
<th>AY</th>
<th>Earned Premium</th>
<th>Reported Losses</th>
<th>Age-Ult LDF</th>
<th>Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50,000</td>
<td>39,375</td>
<td>1.0000</td>
<td>Loss</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>50,000</td>
<td>31,500</td>
<td>1.2500</td>
<td>Link</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7,875</td>
</tr>
<tr>
<td>3</td>
<td>50,000</td>
<td>18,000</td>
<td>2.1875</td>
<td>B-F</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21,375</td>
</tr>
<tr>
<td>Total</td>
<td>150,000</td>
<td>88,875</td>
<td></td>
<td>29,250</td>
</tr>
</tbody>
</table>

Selected ELR 78.75%

Notes: AY = Accident Year; Age-Ult = Age to Ultimate; and B-F = Bornhuetter-Ferguson

Why did we call this a perfect world? All three methods produce the same estimated IBNR, but that is no coincidence. The oldest year, Year 1, is fully developed: its age-to-ultimate factor is 1.000 so its losses are already at ultimate. Its loss ratio is 78.75 percent. Year 2 is not completely developed; if we apply the age-to-ultimate factor of 1.250 to the reported incurred (stated in thousands) of 31,500 as we would in the link ratio method, we obtain an estimated ultimate loss of 39,375. The ultimate loss ratio is again 78.75 percent. Similarly, developing Year 3 losses to ultimate yields $2.1875 \times 18,000 = 39,375$ for an ultimate loss ratio of 78.75 percent once more.

There is simply no place for the three methods to differ. In a world as perfect as this, where losses develop consistently over time and we have a known and unchanging ultimate loss ratio for all years, the assumptions of all three methods are equivalent and the calculations must produce identical estimates for IBNR.

But most real world situations will not produce such clean results. Most likely you will obtain the selected ELR and the loss development factors from industry data, or from a larger body of historical data at your own company, or from a credibility weighting of the two. It is improbable that they will fit together with your three years of data.
as beautifully as in the last example. Something like this is far more realistic:

**Table 3**

A More Realistic World (in $000s)

<table>
<thead>
<tr>
<th>AY</th>
<th>Earned Premium</th>
<th>Reported Losses</th>
<th>Age-Ult LDF</th>
<th>Loss Ratio</th>
<th>Link Ratio</th>
<th>B-F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50,000</td>
<td>39,375</td>
<td>1.0000</td>
<td>-1,875</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>50,000</td>
<td>31,500</td>
<td>1.3000</td>
<td>6,000</td>
<td>9,450</td>
<td>8,654</td>
</tr>
<tr>
<td>3</td>
<td>50,000</td>
<td>18,000</td>
<td>2.3400</td>
<td>19,500</td>
<td>24,120</td>
<td>21,474</td>
</tr>
<tr>
<td>Total</td>
<td>150,000</td>
<td>88,875</td>
<td>23,625</td>
<td>33,570</td>
<td>30,128</td>
<td></td>
</tr>
</tbody>
</table>

Selected ELR 75.00%

*Notes: AY = Accident Year; Age-Ult = Age to Ultimate; and B-F = Bornhuetter-Ferguson*

As you can see, small changes in the loss development factors or loss ratios can make a big difference. Using the data and selections in this table, there is a difference of about $10 million between the IBNRs calculated by the link ratio method and the loss ratio method—a difference of about 42 percent in the reserves.

What can you do in this situation?

**Consider the fundamental assumptions of each method.** The loss ratio method assumes the selected ELR is correct. Do you have strong confidence in this? If so, this method is a reasonable choice. But even if you don’t, you may still select the loss ratio method. You might choose it because you have little confidence in the values of the loss development factors or have reason to believe that losses could be reported significantly faster or slower than the selected development pattern indicates.

The link ratio method has no ELR assumption, but instead assumes a direct relationship between reported losses and the ultimate loss value. This method is more appropriate if you think something has changed in the environment to cause losses to be higher or lower than the original expectation—as opposed to losses simply being reported faster or slower than expected. If you have higher confidence in your development pattern and link ratios than in your ELR, this method is preferred.
The Bornhuetter-Ferguson method uses a link ratio assumption to determine what portion of the premium represents unreported losses at a certain point in time; it then applies an ELR assumption to this portion of the premium to produce the IBNR. This method assumes that IBNR is independent of the losses reported to date. The assumption is reasonable if you think that nothing has changed in the environment or the reporting of losses—but instead you were just lucky or unlucky in the low or high level of losses reported to date.

**Use more than one method.** You may opt to take a weighted average of results produced by the different methods. Or you may use the link ratio method for the older periods, where the losses are most completely developed; the loss ratio method on the younger periods, where the link ratios are most uncertain and unstable; and the Bornhuetter-Ferguson method on the intermediate time periods.

**Test methods under different assumptions.** If you are unsure about that 75 percent expected loss ratio, try the methods with 70 percent and 80 percent and get a feel for the sensitivity. The same goes for the loss development factors.

If you can, ask for more information, such as possible changes in premium rates and reporting patterns over time, to refine your estimates or at least develop a range of estimates.

**Consider changes in the environment.** Perhaps the assumed 75 percent loss ratio was appropriate in past years, but is now deteriorating. On the other hand, the claims department may have changed its policies about setting reserves for known claims—calling into question the assumed development pattern. Fisher and Lester (1975) explain how these three methods perform under two different situations: loss reserve strengthening and a deteriorating loss ratio.

### 5.2 Tail Factors

The selection of the tail factor—the development factor that takes losses from the oldest reported age to their ultimate value—is critical in reserving. According to one American Academy of Actuaries survey, the selection of the loss development tail factor was second out of four major causes of reserve deficiency (American Academy of Actuaries 1995). Unfortunately, tail factor selection is hampered by the fact that
there are scarce data on older accident years and that the older accident years may bear little resemblance to more recent accident years. Thus, assumptions are necessary, and informed actuarial judgment is critical.

In both of the examples in Subsection 5.1, the tail factor (age-3-to-ultimate) is 1.000. But we did not discuss how these tail factors, or the age-to-age development factors in general, were chosen. If you're not simply copying development factors out of an ISO circular, you probably will analyze a triangle similar to that in Table 4.

### Table 4
Data for Short-Tail Line

<table>
<thead>
<tr>
<th>UY</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23,000</td>
<td>25,000</td>
<td>27,000</td>
<td>27,000</td>
<td>27,000</td>
</tr>
<tr>
<td>2</td>
<td>29,000</td>
<td>35,000</td>
<td>36,000</td>
<td>36,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>40,000</td>
<td>47,000</td>
<td>58,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>34,000</td>
<td>38,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>29,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UY</th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
<th>48-60</th>
<th>60-Ult.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.087</td>
<td>1.080</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.207</td>
<td>1.029</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.175</td>
<td>1.234</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.118</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Age-to-Age Ratios**

**Notes:** UY = Underwriting Year
In Table 4, the paid losses are our given data. We have constructed the triangle of age-age ratios: for example, the 12 to 24 ratio for year 1 is \( \frac{25,000}{23,000} = 1.087 \) and so on. At the bottom of Table 4 we show some averages of the age-to-age ratios thus obtained. Considering the assumptions behind different averaging methods, we have made the final selections shown in the last row (Selected) of Table 4.

We have selected a factor of 1.000 for 60-Ultimate, even though we have no information about how the losses develop beyond age 60. In this case the assumption may be reasonable, because the previous two age-to-age factors were already 1.000, suggesting that loss development has stopped.

A factor of 1.000 was reasonable above. What might we do with a (more realistic) triangle such as the one in Table 5?

What link ratio for 120-Ultimate—in other words what tail factor—do these data suggest?

The point here is that we should not take a tail factor of 1.000 for development 120-Ultimate simply because the data end at age 120. Though your software may suggest 1.000 as a default tail factor, you must not make the software’s default assumption your own assumption without giving the matter some thought. With Table 5, as the development has not ended by age 120 and the last few link ratios do not show a strongly decreasing pattern, an assumption of 1.000 for the tail factor may be too optimistic.

On what could you reasonably base your tail factor assumptions? Here are a few possibilities: industry data; data from related business in your company; informed actuarial judgment; or fitting a curve to the selected age-age factors and extrapolating.

Our point here is not to tell you how a tail factor should be selected. Rather, we want to call your attention to the fact that assumptions will be necessary and deserve careful consideration.
Table 5  
Data for Long-Tail Line

<table>
<thead>
<tr>
<th>UY</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>60</th>
<th>72</th>
<th>84</th>
<th>96</th>
<th>108</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12,354</td>
<td>18,386</td>
<td>22,553</td>
<td>26,588</td>
<td>32,204</td>
<td>33,329</td>
<td>34,891</td>
<td>35,084</td>
<td>35,682</td>
<td>36,788</td>
</tr>
<tr>
<td>2</td>
<td>16,381</td>
<td>21,863</td>
<td>30,020</td>
<td>31,119</td>
<td>31,576</td>
<td>35,088</td>
<td>35,455</td>
<td>37,693</td>
<td>38,934</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15,478</td>
<td>26,142</td>
<td>36,586</td>
<td>38,387</td>
<td>43,819</td>
<td>46,166</td>
<td>49,140</td>
<td>50,535</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>17,457</td>
<td>17,854</td>
<td>23,476</td>
<td>24,580</td>
<td>26,561</td>
<td>26,683</td>
<td>27,841</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>16,453</td>
<td>25,084</td>
<td>29,315</td>
<td>35,299</td>
<td>35,695</td>
<td>38,072</td>
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<td></td>
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<tr>
<td>6</td>
<td>17,864</td>
<td>33,809</td>
<td>39,996</td>
<td>46,772</td>
<td>57,971</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>18,934</td>
<td>24,829</td>
<td>33,962</td>
<td>38,400</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>18,462</td>
<td>20,076</td>
<td>28,765</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>19,035</td>
<td>35,080</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>19,512</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Age-to-Age Ratio

<table>
<thead>
<tr>
<th></th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>60</th>
<th>72</th>
<th>84</th>
<th>96</th>
<th>108</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Avg</td>
<td>1.466</td>
<td>1.308</td>
<td>1.117</td>
<td>1.116</td>
<td>1.054</td>
<td>1.041</td>
<td>1.032</td>
<td>1.025</td>
<td>1.031</td>
<td></td>
</tr>
<tr>
<td>Avg Ex Hi Lo</td>
<td>1.468</td>
<td>1.311</td>
<td>1.115</td>
<td>1.112</td>
<td>1.052</td>
<td>1.045</td>
<td>1.028</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Weighted Avg</td>
<td>1.464</td>
<td>1.301</td>
<td>1.117</td>
<td>1.124</td>
<td>1.056</td>
<td>1.043</td>
<td>1.032</td>
<td>1.025</td>
<td>1.031</td>
<td></td>
</tr>
<tr>
<td>Selected</td>
<td>1.466</td>
<td>1.308</td>
<td>1.117</td>
<td>1.117</td>
<td>1.054</td>
<td>1.043</td>
<td>1.031</td>
<td>1.025</td>
<td>1.031</td>
<td></td>
</tr>
</tbody>
</table>

Note: We have omitted the triangle of age to age factors.
5.3 Catastrophe Pricing and the Excess Wind Procedure

Before the advent of computer modeling, actuaries used other methods to price catastrophe-exposed business. These methods typically involved isolating the catastrophe-related portions of historical losses and then spreading these losses over a long time period to produce an average catastrophe load for the rates. One of the best known such methods, the excess wind procedure, is still in use today. But, as Musulin (1997) has pointed out, the excess wind procedure rests on at least four assumptions that may not be appropriate and which, therefore, call its accuracy into question.

Before we explore these assumptions, here is a brief explanation of the excess wind procedure. In this method, the actuary collects 20 to 30 years of statewide loss data by accident year and separates these data into wind and nonwind components. A yearly ratio of wind to nonwind losses is computed. For those years having an excessively high wind-to-nonwind ratio, the actuary removes the excess wind losses from the yearly totals and spreads the excess losses over the time period to produce an average yearly wind loading. This procedure smooths the rates and prevents large swings in the rate indication. [See Musulin (1997) or Homan (1990).]

As Musulin (1997) points out,

... this method makes several assumptions about the 20-30 year period used in the 'excess' calculation including:

- Catastrophic activity was 'normal';
- Population demographics were stable;
- Insured losses by peril were stable;
- Changes in coverage or construction practices did not affect the ratio of wind to nonwind losses.

Musulin shows each of these assumptions to be questionable. An examination of weather history over a 100-year time frame shows that the hurricane activity in the period 1960-1987 was unusually low. Population demographics have certainly not remained stable in the past few decades; a far higher percentage of people today live in coastal areas, especially in Florida, than 30 years ago. And there have been changes in standard insurance coverages and construction practices that render the last two assumptions dubious as well.

Consider these points carefully and make appropriate adjustments before using the excess wind procedure in your own ratemaking cal-
culations. But also be aware that similar assumptions underlie other
traditional catastrophe rating procedures.

In recent years the trend has been toward using computer models
to assess likely catastrophe losses. Actuaries who use such models still
must be cognizant of crucial assumptions. Catastrophe models, due to
their computational complexity and their sophisticated meteorological
and/or seismological underpinnings, are notoriously prone to inducing
a black box mentality.

A black box mentality occurs, in large part, due to a user's failure to
recognize and understand a method's assumptions (coupled with igno­
rance or incomprehension of the calculations based on those assump­
tions). Actuaries need not be experts in seismology and meteorology
to use computerized catastrophe models, and they need not be able to
follow all the details of the programming to obtain reasonable results.
Users of such a model, however, should have a good grasp of the fun­
damental assumptions and methods stuffed into the box in order to
provide the proper input and then correctly interpret the output.

5.4 Parallelogram Method for On-Leveling of Premium

The parallelogram method is often used to convert premium from
policies written and earned in the past to what the premium would be
if those policies were written today (or some other selected date). This
step is necessary for ratemaking. Losses and premium both need to be
made current in order to begin the ratemaking process.

The parallelogram approach graphically demonstrates how policies
with a term of one year that are written after January 1st will be earning
during the next calendar year. Using the relative areas indicated by the
parallelogram method simplifies the calculation of on-level factors. For
example, suppose your company takes rate changes on January 1, 1996
and again on July 1, 1996. We can use the parallelogram method to see
what percentage of the calendar year 1996 earned premium is at the
different rate levels. In Figure 1, time runs along the horizontal axis
while the vertical axis represents the percentage of a policy that has
been earned. Each policy can be thought of as a diagonal line running
from lower left to upper right.

The parallelogram method is elegant, but it makes an important as­
sumption: that policies are written uniformly throughout the year. De­
pending on the line of business, this may not be the case. Commercial
policies tend to clump around January 1st and the start of the other
quarters. In the following example, we compare calculations to create
on-level factors for the 1996 accident year earned premium.
Using Figure 1 and assuming uniform inception dates produces Table 6. You can see that 50 percent of the premium earned in calendar year 1996 comes from policies that started in calendar year 1995. This means that the on-level factor of 1.155 needs to be applied to 50 percent of the earned premium from calendar year 1996. Another 37.5 percent of the earned premium comes from those policies that started between 01/01/96 and 07/01/96. We use the on-level factor 1.050 to adjust this amount of the calendar year 1996 premium. Finally, the remaining portion, 12.5 percent of the premium earned in calendar year 1996, needs no adjustment because it is already on-level.

The accuracy of this method depends on the assumption of uniform premium writings throughout the year. If this assumption fails, the parallelogram's sub-areas do not represent the right proportion of earned premium. For example, if 40 percent of the premium was written on January 1 (a fairly reasonable assumption), and the rest was written evenly throughout the year, we would use the method similar to that of Table 7 to calculate a calendar year on-level factor.1

1Here's clarification on the math: 40 percent of the premium starts on January 1. This means that 60 percent of the premium remains and we assume it is written in a uniform manner. The amounts of premium for each category use the same proportions as shown in Table 6, but are simply adjusted for the reduced amount. According to
Table 6
On-Leveling Premium to 12/31/96: Uniform Assumption

<table>
<thead>
<tr>
<th>Date of Rate Change</th>
<th>Amount of Rate Change</th>
<th>Amount of Premium</th>
<th>Interval On-Level Factor*</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/01/95</td>
<td>0.0%</td>
<td>50.0%</td>
<td>1.155</td>
</tr>
<tr>
<td>01/01/96</td>
<td>10.0%</td>
<td>37.5%</td>
<td>1.050</td>
</tr>
<tr>
<td>07/01/96</td>
<td>5.0%</td>
<td>12.5%</td>
<td>1.000</td>
</tr>
<tr>
<td>Weighted Average for 1996</td>
<td></td>
<td></td>
<td>1.096</td>
</tr>
</tbody>
</table>

*The on-level factor is calculated by adding 1.00 to each of the future rate changes and taking the product; e.g., 1.100 × 1.050 = 1.155.

Table 7
On-Leveling Premium to December 31, 1996
40 Percent on January 1, Uniform Thereafter

<table>
<thead>
<tr>
<th>Date of Rate Change</th>
<th>Amount of Rate Change</th>
<th>Amount of Premium</th>
<th>Interval On-Level Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/01/95</td>
<td>0.0%</td>
<td>30.0%</td>
<td>1.155</td>
</tr>
<tr>
<td>01/01/96</td>
<td>10.0%</td>
<td>62.5%</td>
<td>1.050</td>
</tr>
<tr>
<td>07/01/96</td>
<td>5.0%</td>
<td>7.5%</td>
<td>1.000</td>
</tr>
<tr>
<td>Weighted Average for 1996</td>
<td></td>
<td></td>
<td>1.078</td>
</tr>
</tbody>
</table>

This may not seem like a large difference. But let's go a little further with this calculation and track the difference in the indicated rate change caused by the two assumptions.

The difference between a rate indication of 10.5 percent and 8.6 percent may not seem large, but it is a large difference to the marketing department, underwriters, agents, state regulators, and your customers. This amount of difference in the future premium can have a significant impact on the bottom line. Finally, the uniform writings assumption is not appropriate in this situation, so why use it?

With the power and availability of today's computers, it may not be necessary to make these approximations when bringing premium on-level. The extension of exposures method, which re-rates each past policy at today's rates, is more accurate. Systems limitations or signifi-

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*each date of rate change, we have the following: 1/1/95: 60% × 50% = 30%; 1/1/96: 60% × 37.5% + 40% = 62.5%; 7/1/96: 60% × 12.5% = 7.5%.*
Table 8
Comparison of Indicated Rate Changes
Under Different Writings Assumptions

<table>
<thead>
<tr>
<th></th>
<th>Uniform Writings</th>
<th>40% on January 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Losses trended to future accident period:*</td>
<td>500,000</td>
<td>500,000</td>
</tr>
<tr>
<td>(2) Earned premium for 1996 accident year:*</td>
<td>700,000</td>
<td>700,000</td>
</tr>
<tr>
<td>(3) Calculated on-level rate factor (Table 7):</td>
<td>1.096</td>
<td>1.078</td>
</tr>
<tr>
<td>(4) Converted premium ((2) × (3)):</td>
<td>767,200</td>
<td>754,600</td>
</tr>
<tr>
<td>(5) Calculated loss ratio ((1) ÷ (4)):</td>
<td>65.2%</td>
<td>66.3%</td>
</tr>
<tr>
<td>(6) Target loss ratio:*</td>
<td>60.0%</td>
<td>60.0%</td>
</tr>
<tr>
<td>(7) Calculated rate indication: ((5) ÷ (6))</td>
<td>1.086</td>
<td>1.105</td>
</tr>
<tr>
<td>(8) Selected rate increase:</td>
<td>8.6%</td>
<td>10.5%</td>
</tr>
</tbody>
</table>

Notes: *These items were selected arbitrarily; we did not need them in the parallelogram calculations in Table 7.

cant changes in the class plan, however, may make it difficult to update premium on this detailed basis. In these cases, understanding the impact of the uniform writings assumption may save you from avoidable errors.

6 Assumptions in Software Tools

Increasing numbers of actuaries are studying complex problems using computer programs that are either off-the-shelf or written specifically for the task. Examples include the modeling of catastrophes, asset liability management, and dynamic financial analysis. Using such programs can save time and effort, enabling a more in-depth analysis with greater speed. As pointed out earlier in Section 5.3, blindly using software tools as black boxes can lead to misuse and error.

The needed level of understanding depends on the use of the tool. For example, generally it is not necessary to question how spreadsheet functions have been implemented. But if you are performing extensive Monte Carlo simulations, however, you should understand the workings of the random number generator. In particular you must be certain that the generator has a sufficiently large period and is capable of producing a sufficiently random stream. It may be prudent to perform statistical
tests on the generator to judge how well its output mimics the behavior of a truly random sequence.²

It's important to be aware of general software issues such as the default values, the functional approximations used, and the number of significant digits used in calculations. Also, to the extent that the tool incorporates actuarial or statistical techniques, the user should be aware of assumptions inherent in these—for example the probability distributions selected, the type of IBNR calculation used, assumptions about parameter uncertainty, and the theory behind and practical effect of any statistical tests.

7 Assumptions about the Business Environment

Actuaries must take into account the business and economic environment when performing calculations; this entails making assumptions. These assumptions can be broken down into three categories: (i) departures from the past, (ii) the future economic environment, and (iii) company-specific assumptions.

7.1 Departures from the Past

Occasionally a new situation arises and the old actuarial assumptions are no longer valid and you require new assumptions. In such cases the new assumptions become very significant, as there are few, if any, data to support them. For example:

**Brand new coverages.** When a new coverage is introduced, actuaries generally have few, if any, relevant historical data. In order to create a price for this new coverage, underwriters and actuaries must estimate frequency and severity of future claims. Selecting appropriate frequencies, severities, and exposure bases, however, means relying heavily on assumptions.

**Law changes and unique settlements.** Pollution liabilities have proven difficult to estimate due to Superfund, Superfund reform, and the prospect of further Superfund reform. Additionally, there are data and data interpretation problems.

One suggestion when dealing with new situations is to use the same methods being used by others in the industry. For example, you might

²For more on random number generators see, for example, Kalos and Whitlock (1986, Appendix) and Bratley, Fox, and Schrage (1983, Chapter 6).
use the multiple-of-current-payments approach described by Bouska and McIntyre (1994) in evaluating pollution liabilities. A standard approach may prevent unpleasant situations with regulatory bodies and rating agencies. But if the question is critical for the insurance company, the actuary may want to consult with outside experts about the validity of applying standard assumptions and methods to the particular situation.

7.2 Assumptions about Future Economic Environments

The economic environment, particularly its future outlook, affects a number of actuarial areas including valuations, the discount of loss payment patterns, and estimation of future inflation rates for pricing. Given the complexity of our economy, some assumptions are necessary.

**Interest rate curve or fixed interest rate.** Although life and pension actuaries have always been aware of the importance of interest rates, property and casualty actuaries have historically paid less heed to the interest rate assumption. U.S. statutory accounting principles do not allow discounting of many property-casualty loss reserves. With the advent of asset-liability management and dynamic financial analysis, property-casualty actuaries are increasingly involved in creating interest rate assumptions. Issues to consider include flat versus upward sloping yield curves, the use of a risk-free versus a risk-adjusted rate, inclusion or exclusion of inflation, and methods for measuring duration. For a discussion of these issues see, for example, Panjer et al., (1998) and, from a European perspective, Daykin, Pentikäinen, and Pesonen (1994, Part 2).

With life, pensions, annuities, or any other long-tail business, actuaries also make assumptions about reinvestment rates of return. Reinvestment risk may be the greatest risk to profitability. Assets that are intended to support future liabilities often provide

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3This may seem surprising to life actuaries. But the neglect of interest rate assumptions in the older property-casualty literature should be put in context. Developing as it did from fire and marine insurance, property-casualty practice was originally much more concerned with the shorter-tail lines of business. Even for the longer-tail property-casualty lines, the uncertainty in the amount to be paid has generally been much greater than the potential amount of interest discount. Because the interest discount was perceived to have a relatively small effect, it did not receive much attention in the early literature. The emphasis on appropriate interest rate selection has changed as time to payment has lengthened, as inflation has become more important, and as techniques have become more refined.
interim cash flows, e.g., bond coupons. Assuming that future income flows can be reinvested at today’s prevailing interest rates could lead to poor decisions. Yield curves may shift or change shape. Actuaries need to take into account relevant economic forecasts and the expected pattern of future cash flows. Margins for model risk, parameter risk, and process risk should be considered; see, for example, McClenahan (1990) and Geske (1999) for more on this.

Trends for premiums and losses. In the 1980s and the early 1990s, U.S. insurer losses in their workers compensation line of business appeared to be out of control. Annual medical costs were climbing, and actuaries could clearly see an upward trend.

Then things changed. Whether you credit the U.S. federal government, the health care system, or the various state legislatures around the country, workers compensation costs suddenly decreased. This is an example in which the assumption that tomorrow would be like yesterday was incorrect.

As this example illustrates, inflation rates for losses (and premiums) do change (sometimes suddenly) over time. Nevertheless, a constant trend is assumed in many actuarial models to project future cost levels. This can produce distortions in loss development, loss ratio estimates, and payment patterns.

Other economic situations. There are many more situations in which actuaries have to make economic assumptions: future currency exchange rates for international business; change in property values for residual value insurance; and the effect of the broader economy on sensitive lines of business such as credit and surety.

7.3 Assumptions about Your Business

It is important for actuaries to keep abreast of the changes in their business and to translate these changes into appropriate actuarial assumptions.

Here are some areas that may be worth inspection.

The underwriting mix. This refers to the types of products sold, where the products are sold, the demographics of those purchasing, and/or the limits of the policies. Most actuarial methods assume that the underwriting mix is constant. If the mix changes it may affect how the losses and premiums develop.
For example, an actuary may warn that rates are too low for a certain territory. The underwriters, in response to the warning, reduce their writings in this territory. Unless this action is communicated to the actuary, inappropriate assumptions may be used in the next analysis.

Other possibilities abound. For example, if offered policy limits are increased, the actuary must allow for increased development of higher losses. A decision to stop offering higher limits would have the opposite effect. Turnover in the underwriting staff may shift the amounts of business written in certain territories or lines of business. Demographic shifts over time, such as increasing percentages of the population moving to urban or coastal areas, also can have a major impact.

The cost of reinsurance. Many actuarial methods are based on an analysis of direct data, so they assume implicitly that the cost of reinsurance is zero. The transaction costs associated with reinsurance, however, can be significant. If no allowance for such costs is made in the analysis, the overall return realized on the net book of business may not achieve the goals outlined in the rate filing.

The company's business plan. One possible source of information is your company's business plan. You need to take care in using this plan; some business plans are compiled as a matter of form and are not adequate reflections of the intentions of management. You also need to be cognizant of the many assumptions used in making the plan and whether these assumptions are reasonable.

Assumptions about your business may find their way into your work, possibly through communication—or lack of it—with other departments in your company. To be aware of these and other business effects, you need regular contact with underwriting and other departments. You may need to improve your communication with other actuaries.

8 You are Aware of Assumptions—What Next?

Once you are aware of your assumptions you must (i) document them, (ii) check to see that they are correct and appropriate, (iii) check for consistency among them, and (iv) quantify their impact on your work.

Documentation: The objective here is to make sure that others who rely on your work are aware of your assumptions. You should
state and explain all assumptions, emphasizing the most significance ones. A good rule of thumb to follow is to make sure that another actuary practicing in your field can follow and understand your work. This is the level of documentation expected by the Actuarial Standards Board of the American Academy of Actuaries (1991). Even if no one else examines your work, going through this process may help you carefully choose your assumptions.

**Correctness:** Each assumption should be checked to see if it is correct and appropriate. Some assumptions, though technically incorrect, may be made in order to simplify calculations.

**Consistency:** There are two types of consistency to look for: consistency within an analysis and consistency across analyses. Consistent assumptions within an analysis simply means that, for the particular analysis, the assumptions make sense. Consistent assumptions across different analyses, however, mean that similar assumptions must be used in different analyses. For example, suppose the loss ratios and development patterns used to set your reserves are markedly different from the ones used to calculate rate indications. Then you might forecast a bad profit result for the year even if your whole rate change is approved. Similarly, if you are using one set of interest rate assumptions for estimating your liabilities, you may want to use the same set for valuing your assets. Consistent assumptions across analyses may not be required but should be considered.

Another important area for consistency is in financial planning. Anderson (1998) discusses this aspect of actuarial practice. He argues that if planning assumptions “such as catastrophe loads, loss trends and the effects of variability are not explicitly linked to the assumptions used for ratemaking on the product and state level, a built-in bias may be created for either rate adequacy or rate redundancy.” Thus the planned results are not achieved.

Choosing planning assumptions consistently with pricing and reserving assumptions may help management to better understand the company’s results and to see why operating differences from plan arise.

**Quantification:** In order to understand the importance of your assumptions you must be able to quantify their impact. The greater the impact of your assumption, the more important it is that you test the assumption. But what does it mean to test an assumption?
How can you go about quantifying its importance? Here are some possibilities.

- **What-if scenarios (stress testing).** One way to judge the importance of assumptions is through stress testing. You can test and quantify the impact of assuming different loss ratios, different lapse rates, and different tail factors. How much does the tail factor have to change in order to create a 5 percent change in the estimate of the reserves? Is such a change in the tail factor likely or not?

- **Different methods.** Using various approaches to solving a problem, such as link ratio versus Bornhuetter-Ferguson versus loss ratio reserving methods, allows you to determine whether different methods are giving approximately the same answer. If they do, you can work with greater confidence. If they do not, check to determine which assumptions underlying the methods might be leading to the discrepancy.

- **Different data sets.** This is similar to the idea behind using different methods. Using various parts of your data—such as incurred versus paid losses, incremental versus cumulative, primary versus excess—is another way to test your results and your assumptions. You also may want to investigate data resampling techniques such as the jackknife and the bootstrap which are based on repeated sampling of parts of the data; see Efron (1982).

- **Simulations.** Stochastic simulations are often the only way to proceed when working with relationships that are complicated and involve many variables. Simulation can help you get a handle on the results. Additionally, simulation allows you to perform stress tests using thousands of different what-if scenarios, whether the underlying relationships are complex or simple.

- **Effect of changing assumptions over time.** Sometimes assumptions change from one actuarial report to the next. Khury (1980) describes the idea of actuarial gain or loss, i.e., the change in a final amount (such as a reserve) that is due to a change in an assumption rather than due to changes in the data. It is important to communicate the size and nature of the actuarial gain or loss to management, so that they have a clear understanding of the current business situation.
• **Range of estimates.** It is standard actuarial practice to establish a range of values for estimates, not just a single number. The range may reflect (i) statistical confidence interval, (ii) the impact of different assumptions, or (iii) uncertainty about your assumptions.

### 9 Benefits from Understanding Assumptions

There are many benefits to better understanding your assumptions. We have divided them into three categories: professional, practical, and business. The boundaries between these categories, however, are fluid.

**Professional Benefits:** Understanding and testing your assumptions is a requirement of actuarial standards of practice.

**Practical Benefits:** Knowing the assumptions behind the various methods can help determine which one is most appropriate or which method will lead to greater certainty and/or reliability regarding the final outcome. Understanding your assumptions helps you determine the level of confidence you can have in your estimates. If you have too little confidence, an understanding of the assumptions can help you determine the next steps you ought to take (if any) in an investigation.

**Business Benefits:** These are many; only a few are cited here.

• **Early warning system.** Thoroughly understanding your assumptions can show the areas in your company that need the greatest attention. Key assumptions can be monitored as experience emerges, possibly allowing for corrections to your analysis before the experience is mature enough for a complete review. This helps to prevent surprises when the situation does not develop as originally planned.

Awareness of these assumptions may also allow for strategic action with respect to the business. Of course, there may be some things over which the company has little control, such as inflation or mortality rates. But other items may be open to company influence. For example, a scenario test may demonstrate that a lower lapse rate among term life policyholders is required in order to reach a profit goal. Efforts can then be directed toward achieving this lower lapse rate.
• **Creating a safety margin.** In other situations, you may be able to safeguard future results. For example, if you are creating a specialty insurance product, you may be able to make sure that the product design protects you in case your more significant assumptions are incorrect.

Suppose that you are designing a reinsurance cover, but your client has only limited data. You are forced to rely on industry data and assume that your client will have average results. If you are uncomfortable with this assumption, maybe you can build in a special sub-limit for the lines of business with the scantiest data or the lines where you fear your client may have worse-than-average experience. Or perhaps you can charge a higher up-front premium and offer a generous profit share in the event that ceded losses are small.

• **Planning.** As discussed earlier, understanding your assumptions can help you ensure that your financial plan is consistent with your pricing and reserving operations (Anderson 1998). This makes your company's operations more transparent and can help management to more easily identify reasons for deviations from plan.

Actuarial science is inexact at best. By understanding your assumptions you can avoid unnecessary errors, increase your level of certainty in your results, and improve the decisions made by your company.

**References**


