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EXPLORING THE PRESENCE OF NONLINEAR DETERMINISTIC DYNAMICS IN COMMODITY PRICES

by

Sagar Dahal

A THESIS

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EXPLORING THE PRESENCE OF NONLINEAR DETERMINISTIC DYNAMICS IN COMMODITY PRICES

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University of Nebraska, 2023

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Determining whether commodity prices (and volatility) are driven by linear stochastic processes or low-dimensional nonlinear deterministic dynamics ("chaos") is crucial for policymaking, forecasting, production, storage, investment, risk management, and hedging decisions. Previous studies that used Lyapunov exponents and correlation dimensions to identify chaotic structures in price series may be unreliable in practical applications because these methods rely on asymptotic properties that require large, noiseless data which is often not available. We applied nonlinear time series analysis approaches to empirically detect the underlying market dynamics using the daily futures prices of ten agricultural commodities. We used phase space reconstruction to reconstruct the empirical attractor from the observed price series, as well as nonlinear predictive skill and permutation entropy measures to distinguish between linear stochastic and nonlinear deterministic dynamics. We find evidence of low-dimensional nonlinear deterministic dynamics in commodity price series. Our results suggest that the observed volatility is most likely endogenously generated by an inherently unstable market which cannot be expected to self-correct, suggesting the need for government intervention to stabilize prices and reduce volatility. It also suggests that long-term forecasts are unlikely due to the nature of the dynamics, but short-term forecasts may be improved using nonlinear prediction methods.

Key words: Chaos, nonlinear dynamics, phase space reconstruction, price volatility

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1. INTRODUCTION

1.1 Motivation of the study

The temporal behavior of commodity prices has been investigated with numerous approaches and methods for several decades. Many efforts have been made to model and forecast commodity prices with mixed results (Antwi et al. 2022; Labys 2003; Tomek and Myers 1993). Overall, empirical findings suggest that agricultural commodity prices exhibit a significant level of autocorrelation over time and persistent, aperiodic (non-repeating), and random-appearing fluctuations (Huffaker, Berg, and Canavari 2018). They are characterized by high levels of volatility that are also time-varying, and the distribution of observed prices is skewed and shows substantial kurtosis.

The dominant view in agricultural economics is that commodity prices can be represented by linear stochastic processes. This view associates the observed volatility and non-periodic behavior to exogenous random shocks or stochastic processes leading to random walk theory or linear stochastic models. These models assert that prices are determined by the interaction of demand and supply factors. Changes in these factors will lead to price fluctuations, which eventually settle at an equilibrium point where the quantity demanded equals the quantity supplied. This equilibrium process takes time, during which external factors such as climate, consumer preferences, and technology may impact the market stochastically. This view thus hypothesizes that price volatility is due to the effort of the inherently stable market to reestablish equilibrium after exposure to exogenous shocks (Berg and Huffaker 2015; Galtier 2013).

Economic theories are adamant about the stable-market hypothesis relying on random shocks to explain the observed volatility as a temporary phenomenon. On the empirical

side, exogenous random error terms were incorporated to address the inability of linear models to generate observed aperiodic cycling. Economists are hesitant to change their models, even though the assumptions underlying them are frequently violated. Instead, they try to justify these models by arguing that they are simple, convenient, and have historically worked well under most market conditions. Rather than completely revamping their models, many economists are attempting to address these issues by just trying to patch these old models using "fancy" names like GARCH and FIGARCH (Mandelbrot and Hudson 2005).

However, this method becomes severely inadequate if the observed price volatility is a persistent behavior resulting endogenously due to inherently unstable markets (Berg and Huffaker 2015; Chavas and Holt 1993). According to the precise definition of a random event, which refers to an occurrence without any prior cause and happens without warning, nothing that explains commodity prices can truly be considered random (Bernard and Streeter 1993). Climatic, environmental, economic, geopolitical, and sociological factors affecting commodity prices that are treated as random and exogenous by standard economic literature are not considered intrinsically random in their respective disciplines. This suggests that these factors collectively and deterministically interact to determine food production and consumption that in turn determine commodity prices (and volatility). Thus, the theory of economics and these disciplines must be integrated rather than considered a matter of random chance (Su et al. 2014; Huffaker, Canavari, and Muñoz-Carpena 2018).

In fact, advances in nonlinear dynamics have taught us that the complexity observed in a series can emerge deterministically from simple low-dimensional nonlinear ("chaotic")

systems (Huffaker, Berg, and Canavari 2018). The apparent randomness in the price series would not have been generated by exogenous shocks, but rather by the sensitivity to the initial condition which is endogenous to the low-dimensional nonlinear deterministic system (Huang, Yu, and Ban 2014). A nonlinear deterministic system with a few degrees of freedom has been shown to produce complex nonlinear patterns that mimic stochastic behavior from the point of view of conventional time series analysis (Faggini 2014). Thus, chaos theory helps us with the rejection of randomness and acceptance of deterministic interaction of the factors influencing commodity prices. We must, however, be cautious about the difference between determinism and predictability. Determinism does not imply predictability, and it is the important difference between random and chaotic behavior (Bernard and Streeter 1993).

1.2 Objective

The objective of this research is to investigate whether commodity prices (and volatility) are generated by linear stochastic processes or low-dimensional nonlinear deterministic dynamics ("chaos"). In other words, this study will explore whether the observed price volatility is due to exogenous random shock to the otherwise stable market or endogenously created due to an inherently unstable market.

1.3 Contribution to the literature

The presence of chaos in commodity price series has already been explored and little evidence of chaotic behavior has been found. However, there are several issues in this literature.

Previous studies looking into the chaotic behavior of price series relied on the BDS test to explore the presence of nonlinearity as well as the topological measures like correlation

dimension and Lyapunov exponents to identify chaos as the source of nonlinearity if present. They tested for chaotic behavior using data filtered with either linear or nonlinear models (Frank and Stengos 1989; Blank 1991; Yang and Brorsen 1993; 1992; Chatrath, Adrangi, and Dhanda 2002; Chatrath, Adrangi, and Shank 2001). Their approach involved filtering the price series using ARIMA models, testing for dependence on the residuals, and treating any dependence found on the residuals of such linearly filtered series as nonlinear. Then they proceed to apply ARCH/GARCH models to detect the source of nonlinearity. If unexplained nonlinearity remains even after applying ARCH/GARCH filters, they would test for the presence of chaos in such filtered residuals (Decoster, Labys, and Mitchell 1992; Chavas and Holt 1991; Frank and Stengos 1989). However, simulation studies have shown that linear and nonlinear filters distort potential chaotic structure (Wei and Leuthold 1998a) and may affect the dimensionality of the original data giving unreliable results (Panas 2001; Panas and Ninni 2000) and making it harder to identify chaos. Chen (1993) argues that correlation dimension is not invariant to linear filters since they may introduce noise into original data and may erase any fractal or chaotic structure in the underlying process making the estimation of correlation dimension questionable.

Another problem faced by these studies is the short length of data and high level of noise in the price series. The presence of noise in data is problematic in any attempt to verify chaos as a data-generating process for a time series. Most of the tests for chaos were focused on identifying topological features of chaos like self-similar geometry (fractal correlation dimension) and sensitive dependence on initial conditions (positive Lyapunov exponents). These measures are based on asymptotic properties which are developed and

meant to be used by physicists who can generate very large samples of high-quality experimental data (Faggini 2009; Schreiber 1999). They require large and clean datasets and are highly sensitive to noise. Whenever noise is present and goes over a threshold, which is a rule and not an exception in economic time series (Medio and Gallo 1995), the calculation of these indicators is unreliable (Chatrath, Adrangi, and Dhanda 2002; Schreiber 1999; McSharry 2005). High level of noise can destroy a subtle signal of deterministic chaos in a time series (Chen 1993).

The presence of measurement noise and a small dataset in previous studies hindered any attempt to identify chaotic behavior from the stochastic processes. This might be the reason for the failure of previous studies attempting to detect chaotic structures in commodity price series. Medio and Gallo (1995), Faggini and Parziale (2016) and Guégan (2009) suggested eliminating or reducing noise by using appropriate filtering techniques without distorting the original signal to obtain reliable results. Most of the prevalent statistical methods that involve smoothing of data (eg: using simple moving averages) to deal with noise may not be appropriate to study chaotic dynamics because important information may be lost during this process (Savit 1988). In addition, as cited in Beker (2014), Ruelle (1995) states that " the separation between noise and the deterministic part of the evolution is ambiguous because one can always interpret 'noise' as a deterministic time evolution in infinite dimension". Thus, it is important to filter out noise from records, but this should be done carefully to avoid unintentionally removing aperiodic nonlinear dynamical structure that might be mistaken for noise. Our research employs singular spectral analysis (SSA) which retains aperiodic oscillations in the

isolated signals and thus has the ability to filter noise present in the data from the deterministic component.

Moreover, given the numerical challenges and unreliability in the estimation of correlation dimension and Lyapunov exponent using short and noisy time series, we have used nonlinear predictive skill (Kantz and Schreiber (2004)) and permutation entropy as discriminating statistics. Since chaotic systems obeys rules and are governed by deterministic laws, one can predict the future of chaotic systems by examining the behavior of past values (Sugihara et al. 1997). So, nonlinear prediction can be used as a signature of chaos and for the identification of deterministic chaos in timeseries (Tsonis and Elsner 1992; Sugihara and May 1990). Similarly, permutation entropy is an invariant measure of complexity of the dynamics and can be used to distinguish between regular, chotic, and random behavior (Bandt and Pompe 2002). They are better than dimension and exponents measures because they can reliably be computed from short and noisy time series (Huffaker, Bittelli, and Rosa 2017).

Another problem with the previous studies is that they tried to prove the presence of chaos in economic time series using statistical tests. However, we can never prove the result about underlying dynamics but can only calculate the probabilities that the particular finding is unprobeable using a null hypothesis (Bradley and Kantz 2015). Schreiber (1999) and Schreiber and Schmitz (2000) also recommend focusing less on trying to prove the presence of deterministic chaos and focusing more on evaluating if the observed volatility is likely to be generated by low-dimensional nonlinear deterministic dynamics. So, we employ surrogate data tests to provide evidence if a nonlinear chaotic dynamic is more plausible for the data.

Finally, the current research will use a larger dataset compared to previous studies, both in terms of number of commodities and especially regarding sample size. This will provide more opportunities to explore the dynamics of many commodity prices over a long period of time.

1.4 Relevance of this research

Proper characterization of the behavior of commodity prices is important in various ways. First, distinguishing between exogenous and endogenous volatility has important implications for policymakers. It is very crucial to understand the cause of volatility in the market since price volatility has been identified as a serious threat to food security. Further, governments develop and implement price stabilizing policies because price volatility can also affect agricultural development, trade and consumption of agricultural commodities, income level of farmers, industrial costs and prices, and social stability (Su et al. 2014). Government seeks to reduce the price volatility or at least minimize its negative impact through public policies. In order to do so they employ either *market-based strategies* that aim to improve the allocation of commodities through storage and trade, protect producers by hedging price risk in futures markets, and protect consumers by emergency aid during food crisis; or *public intervention* by using price floors and ceilings, taxes and subsidies, import quotas, and buffer stocks (Galtier 2013). The ability of the government and international organizations to evaluate the threat of price volatility to food security, and to choose appropriate policies to counteract it depends on the ability to identify the agricultural price dynamics driving this volatility (Huffaker, Canavari, and Muñoz-Carpena 2018). Policymakers want to know if they should plan for stabilizing volatility that emerges endogenously from a nonlinear system, or if they need to guard

against exogenous forces that destabilize the otherwise stable market (in which case policies developed to stabilize prices are actually damaging since they interfere with intrinsic stabilizing forces of the market) (Huffaker 2015). The dominant doctrine of a stable market with exogenous shocks opposes public intervention and blames it for interfering with the natural correction process of markets (Gouel 2012). The ability of the market to stabilize prices on its own relies on private storage, trade, and production responses to price movement. However, if the market volatility is inherently generated due to endogenous causes, the ability of private storage to stabilize prices is severely reduced due to their incorrect expectations (Galtier 2013), and the market does not provide a natural correction process to stabilize prices (Huffaker, Canavari, and Muñoz-Carpena 2018). More importantly, if the price is nonlinear deterministic (or chaotic), any control policy might have unpredictable consequences and thus the government should take extra caution while choosing stabilization policies (Su et al. 2014).

Second, learning about price dynamics improves our understanding of the underlying economic phenomenon and helps with realistic data-driven modeling. Linear stochastic models are often judged on their ability to accurately predict historical data or in-sample observations. However, models that perform well on this criterion do not perform well making out-of-sample forecasts. Relying solely on conventional linear methods such as ACF plots for identifying the underlying process that generates time series data is inadequate because they are incapable of detecting non-linear deterministic structures (Huffaker, Canavari, and Muñoz-Carpena 2018; Huffaker 2015; Kaplan and Glass 1995). On the other hand, merely demonstrating that a stable linear stochastic model accurately fits the data is not sufficient to confirm that real-world dynamics are stochastic, as there

could be other models with distinct structures and representation that can also be parametrized to produce a good fit (Rykiel 1996; Oreskes, Shrader-Frechette, and Belitz 1994; Hornberger and Spear 1981). Therefore, fitting data with a linear stochastic model alone as followed by a conventional model-centric approach does not necessarily validate the stochasticity of real-world dynamics. Oreskes et al. (1994) recommended that modelers should be able to demonstrate the degree of correspondence between their model and the real world it aims to simulate when they have significant real-world implications. Inadequate representation of real-world dynamics may leave the real problem unaddressed and waste resources (Saltelli and Funtowicz 2014). Moreover, relying solely on linear models may result in erroneous comprehension of underlying phenomena and may lead to incorrect economic policies (Faggini 2009). Fitting linear stochastic models to the data when in fact the true system is nonlinear tends to overestimate noise (Blank 1991; Scheinkman 1990). When the underlying dynamics are nonlinear, the linear model may not be able to capture the complexity of the system and may miss important nonlinear relationships. As a result, the linear model may attribute the variability in the data to noise, leading to the overestimation of noise. Although some variation in observed data may have been due to measurement error and can be considered white noise, substantial variability that would otherwise be considered noise by linear stochastic models may be due to nonlinear deterministic dynamics. Therefore, identifying whether the observed time series data are generated by linear-stochastic dynamics or nonlinear deterministic dynamics using pre-modeling data diagnostics is important to inform the specification of theoretical models used to simulate this behavior.

It is also a way of showing correspondence between the model and real-world dynamics (Berg and Huffaker 2015).

Third, a better understanding of the underlying price dynamics can help improve price predictions which can in turn help in the formulation of trading mechanisms, optimal trade execution strategies, risk management and hedging decisions, and production decisions. Accurate forecasting of agricultural prices reduces uncertainty and can have significant implications in decision-making for farmers, suppliers, hedgers and speculators, industry, policymakers, and government. It plays an important role in sustainable agricultural production, supply chain management, and consumers' purchasing decisions (Kurumatani 2020). It determines the expected price of the commodities and helps producers with storage, production, and marketing decisions (Jayaramu 2015). The success of any model is contingent upon the underlying data-generating process. Inappropriate models may lead to large forecasting errors (Hamulczuk et al. 2013). The common approach to forecasting commodity prices using stochastic models resulted in poor forecasting since the estimates of the parameters in these models are prone to change with changing samples and models used (Jayaramu 2015; Kohzadi et al. 1996; Tomek and Myers 1993). There are some cases of success of short-term forecasts of time series using linear models, which suggests that prices are not random. However, the failure of long-term forecasts shows that these models are not fully capturing the dynamics of price behavior (Blank 1991). Chaotic systems show the property of sensitive dependence on initial conditions i.e., even an infinitesimal change in initial conditions may lead to very different outcomes. This property makes the chaotic system very difficult to forecast in a long run since even a minute error in measurement

today, which is not uncommon given the difficulty to measure the value of a parameter precisely, may grow exponentially to give erroneous forecasts. This difficulty in forecasting coupled with the difficulty in distinguishing between chaos and random behavior due to their similarity may have led economists to believe that prices are random (Butler 1990). However, unlike random walks, chaotic systems are deterministic, and thus short-run forecasting can greatly be improved by recognizing the presence of chaos (Blank 1991). Identifying whether the price series is chaotic can help us choose the appropriate prediction methods. If we can identify chaos in the price series, we could use chaotic or nonlinear prediction methods like neural networks, whereas failure to do so should result in the use of non-chaotic prediction methods (Su et al. 2014).

It has long been theorized that trading strategies based on technical analysis will be more successful if the time series is nonlinear or even chaotic (Chatrath, Adrangi, and Dhanda 2002; Clyde and Osler 1997). Studies have also demonstrated that the common trading rules that are inherently nonlinear produce superior outcomes compared to trading rules based on linear models. For example, Clyde and Osler (1997) found that technical analysis works better on nonlinear data, and it generates more hypothetical profit when applied to nonlinear systems. Kyrtsou, Labys, and Terraza (2004) claim that traders that are more comfortable in taking higher risks in their investments are more likely to benefit from investing in markets with chaotic behavior since a skilled trader may be able to identify and exploit underlying structures or patterns. In terms of risk management, Gao and Wang (1999) suggest that understanding the dynamic behavior of prices helps in selecting appropriate model assumptions that is crucial to accurately calculate the market risks. Amini et al. (2021) claimed that neglecting non-linearity may underestimate value-

at-risk, i.e., market participants could be taking more risk than they realize. According to Kyrtsou et. al. (2004), risk management requires the formation of expectations, and although rational expectations have been a standard assumption, recent studies show the heterogeneity in the expectations of traders that confirms with the chaotic structure of prices. Therefore, the identification of the data-generating process has been a goal of economists, traders, hedgers, and market analysts for a long time. It is impossible, however, to find a precise price-generating function as it may not even exist in any form, and it is not realistic to hope that long-term forecasts are possible since the future decisions of all traders are not known. However, it may still be possible to identify a relationship or function, probably based on chaos theory, that confirms to the past prices to some extent and can potentially replicate future behavior over a short time period (Bernard and Streeter 1993).

Forecasting volatility has become a new obstacle in the field of nonlinear economics, despite the fact that the increasing availability of economic data offers opportunities to identify the exact behavior of commodity prices (Prokhorov 2008). We use a data diagnostics framework based on nonlinear time series analysis methods to detect whether the observed time series data are generated by linear-stochastic or low-dimensional nonlinear deterministic systems. It helps us to identify if the observed volatility in the price series is generated due to random exogenous shocks or endogenous market dynamics. Policymakers can then make empirically guided decisions about whether laissez-faire policies or interventions are required to manage price volatility in the market. Moreover, a nonlinear dynamical system is often characterized by chaotic attractors and the nonlinear time series analysis approach used in our analysis can help us

empirically determine the properties of such attractors, which in turn can help formulate models that are able to simulate the complexity of real-world dynamics. It can also help us decide whether chaotic or non-chaotic prediction methods should be used to forecast the prices.

This thesis is structured as follows. Chapter 2 will provide a background discussion about nonlinear models. Chapter 3 will discuss the literature on the analysis of commodity prices and highlight the gaps that this research will address. Chapter 4 will present the methods used to analyze the price series in this study. Chapter 5 will then present and discuss the results, while chapter 6 will present the conclusions on this project.

2. THEORETICAL BACKGROUND

2.1 General ideas

Modeling of economic phenomena using concepts and tools from exact sciences such as physics has been in practice among economists for a long time. Exact sciences considered that the causal mechanism of natural processes was linear and characterized by the superposition principle. Then the total effect is the sum of individual effects, and it is possible to analyze a complete system by separately studying its parts and reconstructing the behavior of the system by re-aggregating them (Faggini 2009). The work of economists in the 19th century is characterized by the linearization of economic phenomena and decomposing these phenomena into elementary components. Economists relied on Newtonian mechanics, which is characterized by attributes like stable equilibrium, smooth curves, easy turning points, non-disruptive changes of state, and representative agents (Faggini 2005).

Traditionally, financial and economic studies assumed that the time series observations are generated by underlying linear stochastic processes. Economists have used linear equations to model most economic phenomena because they are easy to manipulate and yield a unique solution. However, with the availability of new mathematical and statistical tools, economists have realized that many important and interesting phenomena cannot be explained by linear treatment. Linearization and simplifying the economic processes created three basic misunderstandings. Firstly, linearization assumed that the economic agents have similar expectations and behave rationally in processing the available information and optimizing accordingly. However, this is not the case in the real world because the world is made of heterogeneous and irrational agents. Secondly,

linearization assumes equilibrium as a natural end of an economic system. But equilibrium might not approach a steady state, instead could end in limit cycles or highly irregular chaotic paths. Thirdly, linear models fail to appropriately include economic phenomenon such as depressions and recessions, stock market price bubbles and crashes, persistent exchange rate movement, or the occurrence of regular and irregular business cycles (Faggini 2005; 2009).

Traditionally, economists view economic processes as a stable equilibrium system. The concept of equilibrium plays an important role in mainstream economics and any departure from equilibrium is considered to be only temporary. Assumptions of conventional economics such as perfect rationality, identical representative agents, and convexity have been chosen to ensure equilibrium and an analytical solution, and not necessarily for their reality. Most effort in traditional economics is devoted to finding out the conditions in which unique and stable equilibrium exists (Beker 2014).

Linear systems either converge to a fixed point or explode. In dynamical systems, trajectories are represented in the phase space (or state space), which "is an abstract mathematical space in which coordinates represent the variables needed to specify the phase (or state) of a dynamical system. The phase space includes all the instantaneous states the system can have." (Williams 1997, p.23). The fixed point that systems converge to is known as fixed point attractor. As defined by Williams (1997, p.165), "an attractor is the phase space point or points that, over the course of time (iterations), attract all trajectories emanating from some range of starting conditions […]." Therefore, the traditional equilibrium approach in economics is interested only in fixed-point attractors, which are essentially the equilibrium points that economics systems converge to.

However, nonlinear dynamical systems do not necessarily need to converge to a single point. They can converge instead to other sets of the phase space. For example, a system can evolve towards an isolated periodic orbit (such that the system eventually stays in that orbit), which is known as limit cycles or periodic attractors. Quasiperiodic attractors and chaotic attractors are other examples of attractors in the phase space towards which a nonlinear dynamical system can evolve. So nonlinearity has the ability to model fluctuations in the economic and financial markets and offer more options beyond the linear model's alternatives between a stable and explosive path (Beker 2014).

The real-world economic phenomenon is characterized by nonlinearity and discontinuity. Focusing on linear dynamics is of limited interest as it does not resemble the complex behavior of economic processes. Although there is little to no evidence of simple linear dynamics and convergence to a steady state or regular cyclical behavior, the linear approach of modeling is still the mainstream of economics.

2.2 Linear models in economics

Linear stochastic models like ARIMA have been considered an important tool for financial analysis and forecasting. Empirical analysis of prices has also been employing the tools of linear stochastic models. The assumptions of these models are: (i) the dependent variable is the linear function of a set of independent variables plus some error/noise; (ii) the expected value of the error term is zero; and (iii) the error terms are homoscedastic and uncorrelated. These types of models consider that the price series always approaches stable equilibrium, but constantly faces external shocks which deviate it from the equilibrium. Fluctuation in prices is due to these external shocks, while their absence will make the price settle into a steady state. If economists find irregular

behavior of some nonlinear form, they do not appreciate these behaviors because they are difficult and intractable to deal with and are explained as stochastic (Faggini 2009). In commodity price analysis, anything that is left unexplained after applying linear stochastic models is deemed noise. Linear models were able to explain symmetrical cyclical fluctuations but were not able to address large shocks, shifting trends, and structural changes.

Linear models assume that the data is normally distributed. However, the normality assumption of commodity price changes has been rejected by various authors (Cornew, Town, and Crowson 1984; Helms and Martell 1985; Jin 2007; S. J. Taylor and Kingsman 1979). In addition, commodity prices have time-varying volatility (conditional heteroscedasticity) which cannot be explained by linear stochastic models.

The failure of linear stochastic models to explain significant features of time series has led to the search for alternative answers using nonlinear approaches. Nonlinear models do not make any assumptions regarding the distribution of the data. Many nonlinear stochastic models were proposed to explain the behavior of data that cannot be generated by linear models. Robinson (1977) proposed a simple nonlinear moving average model of the form,

$$y_t = x_t + \alpha\, x_{t-1} + \beta x_t . x_{t-1}\, , t = 1, 2, \ldots$$

which is just a modification of the linear first order moving average model with β = 0. This model allows for the possibility of a very large value of y_t given two successive large values of x_t. Tong and Lim (1980) explored the idea of piece-wise linearization of a nonlinear model over state space by introducing thresholds and making these models locally linear, and then developed threshold autoregressive models such as,

$$x_t = \begin{cases} \alpha\, x_{t-1} + \epsilon_t, & if\ x_{t-1} \leq 1 \\ \beta\, x_{t-1} + \epsilon_t, & otherwise \end{cases},$$

where ϵ_t is independently and identically (normal) distributed random variable.

Granger and Andersen (1978) introduced the bilinear time series model of the following form,

$$x_t = \epsilon_t + \alpha x_{t-1}.\epsilon_{t-1},$$

where ϵ_t is independently and identically (normal) distributed random variable (Hsieh 1989).

Engle (1982) proposed an ARCH model where the variance is time-dependent and the error is heteroskedastic. The simple ARCH (1) model can be represented as,

$$y_t = h_t.\epsilon_t;\text{ where,}$$

$$h_t^2 = \alpha_o + \alpha_1 y_{t-1}^2 \quad and \quad \epsilon_t \sim ind.(0,1).$$

Bollerslev (1986) extended the ARCH process to the GARCH process to allow for a more flexible lag structure. GARCH (1,1) model can be represented as,

$$y_t = h_t.\epsilon_t;\text{ where,}$$

$$h_t^2 = \alpha_o + \alpha_1 y_{t-1}^2 + \beta_1 h_{t-1}^2 \quad and \quad \epsilon_t \sim ind.(0,1).$$

There is limited literature using nonlinear moving average models and bilinear time series models in price analysis. The threshold autoregressive model has received some attention in the literature. Aiyagari, Eckstein, and Eichenbaum (1989) used this model and they concluded that the price of storable commodities will switch between two linear stochastic processes, based on whether inventory is positive or zero. Ng (1996) looked for evidence of nonlinearity in commodity prices, and tested the theory in the context of threshold autoregressive models and was able to find evidence for regime-specific behavior.

ARCH and GARCH are the most popular nonlinear stochastic models in empirical economic studies because they are very useful in describing heteroskedasticity in economic time series. They assume that data are a nonlinear stochastic function of their past values. The use of these models allows us to explain the time-varying volatility, systematic noise, and nonstationary trends in commodity prices. They can also explain some kurtosis present in commodity prices. For example, Beck (1993) studied four commodities, both storable and non-storable, and was able to find evidence of ARCH-type processes for storable commodities but not for non-storable commodities. Similar results were reported by Beck (2001), who studied 20 commodities consisting of both storable and non-storable (including soybeans, corn, and wheat) and were able to detect the ARCH process only for storable commodities. Though ARCH and GARCH processes were able to address conditional heteroskedasticity, some studies indicated that these nonlinear stochastic models like ARCH and GARCH still left some unexplained nonlinearity in the data (Frank and Stengos 1989; Decoster, Labys, and Mitchell 1992; Yang and Brorsen 1993). This kind of results highlighted the question of whether nonlinear deterministic processes could generate the observed data.

2.3 Nonlinear processes

Most dynamical systems are nonlinear and many of them are chaotic. All chaotic systems are nonlinear, but not all nonlinear systems are chaotic. A nonlinear deterministic system can yield highly complex time paths (trajectories) that may appear random and even pass some statistical test of randomness. We can have a deterministic aperiodic system with seemingly unpredictable behavior in the long run that seems random and displays

sensitive dependence on initial conditions[1]. Such random looking but deterministic series are called chaotic. Chaos theory explains how seemingly random phenomena can actually follow deterministic patterns despite their apparent unpredictability. These patterns can be described by a small set of essential variables, representing a low-dimensional system dynamics. In contrast, a completely random process is assumed to have infinite dimensions (Blank 1991).

As Feldman (2012) points out, the terminology can be misleading because the word 'chaos' is generally used to indicate the absence of order. But, in nonlinear dynamics, "chaotic systems are, in a sense, perfectly ordered, despite their apparent randomness" (Feldman 2012, p.3). The three properties of chaotic systems are: (i) sensitive dependence to initial conditions, (ii) stochastic appearance even though they are generated by a completely deterministic system, and (iii) universality[2]. We can better understand these properties with the help of a logistic map, which is a function commonly used to demonstrate chaotic phenomena (Hsieh 1991; Baumol and Benhabib 1989; Chatrath, Adrangi, and Dhanda 2002). Let us consider a nonlinear equation (logistic map) with a single parameter, r:

$$x_{n+1} = F(x_n) = r\, x_n(1 - x_n)$$

where, $x_t \in [0,1]$ is a state variable[3], function F maps the interval to itself, r $\in [0,4]$ is the parameter, and the map blowsoff after $r = 4$

[1] small errors amplify exponentially fast and a small difference in the initial condition of a system can lead to a completely different final condition.

[2] Universality suggests that regardless of the specific mathematical equations used to describe chaotic systems, there are common statistical and qualitative characteristics and patterns that can be observed. These shared properties include concepts like fractal geometry, self-similarity, and sensitivity to initial conditions.

[3] state variables are the fundamental qualities that are needed to fully describe the dynamical system. Example: angular position (θ) and velocity (ω) for a pendulum (Bradley 2003).

The dynamics of the system depend on the parameter, r. The system is stable and well behaved for a small value of r, but the bifurcation diagram shows how the long-run behavior of the system changes as the parameter changes (Figure 2.3.1). The bifurcation diagram is the final state diagram for different values of the parameter. The sudden qualitative change in the behavior of dynamical systems as the value of the parameter is varied is called bifurcation.

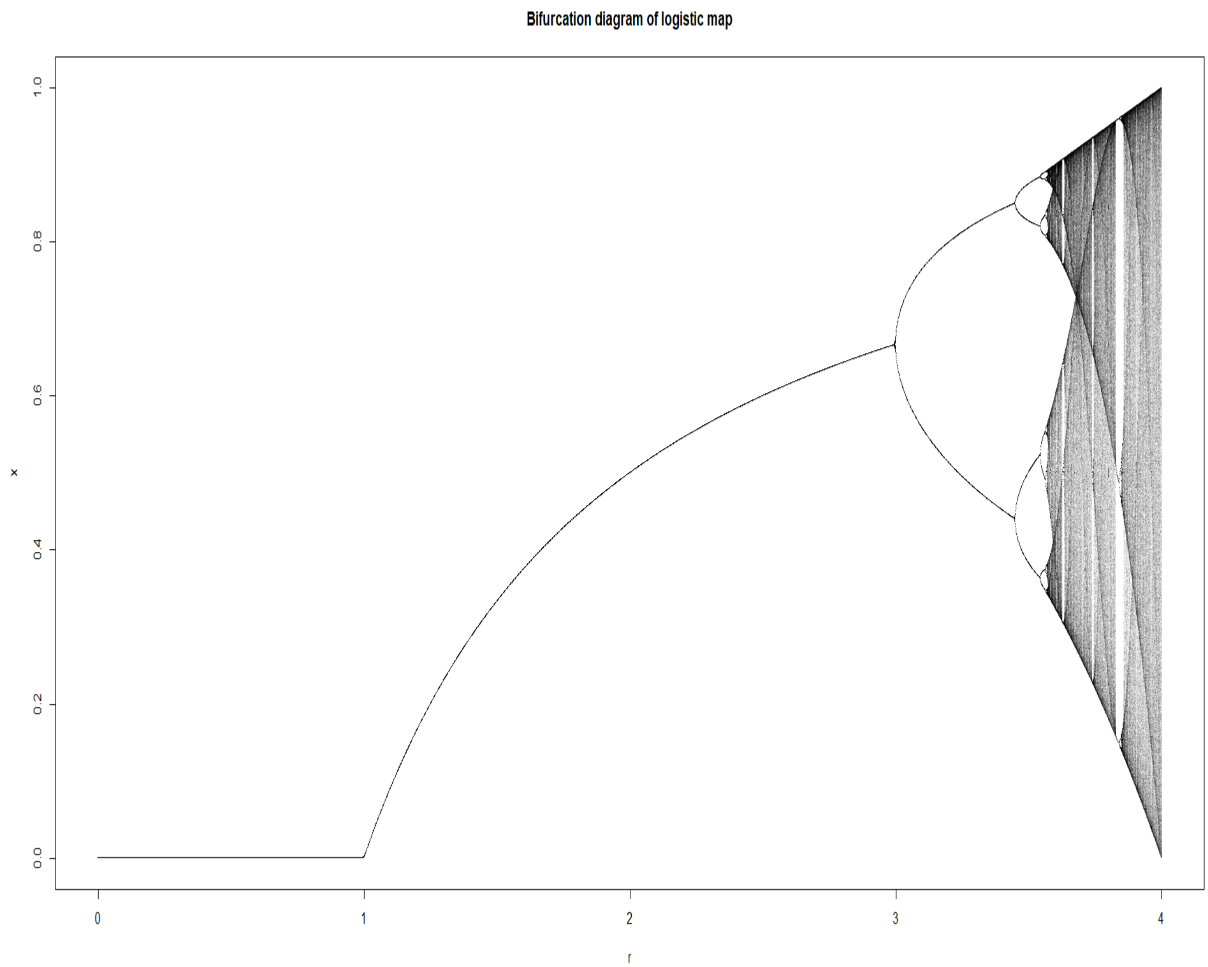
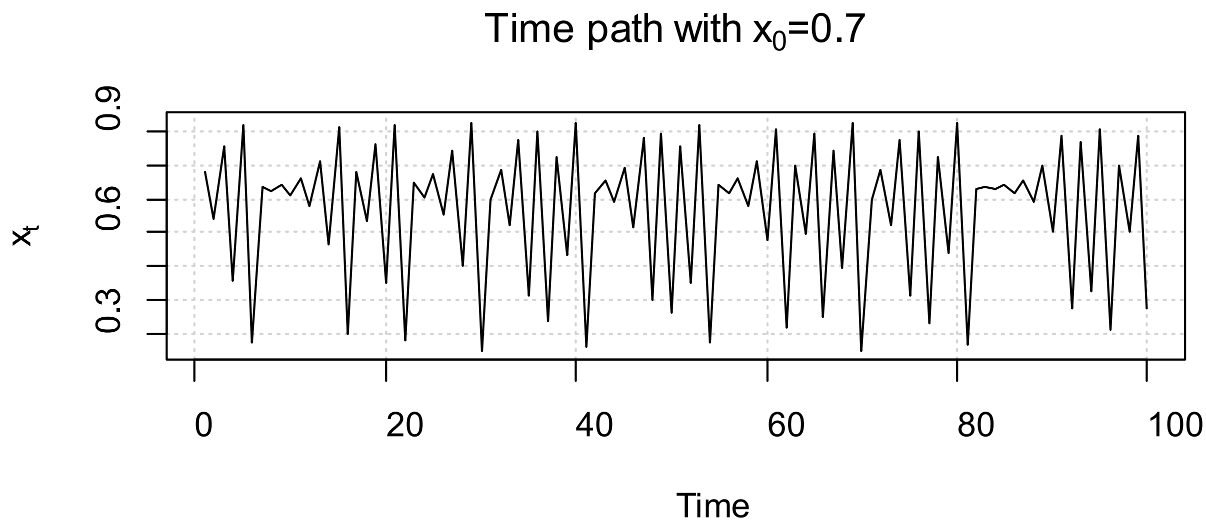


Figure 2.3.1: Bifurcation diagram of a logistic equation $x_{n+1} = r\, x_n(1 - x_n)$, $x_0 = 0.7$.

Figure 2.3.1 represents the bifurcation diagram for the logistic map with the initial condition, $x_0 = 0.7$. We can see that the system would converge to a fixed point when the value of the parameter is between 1 and 3. At r = 3, the system bifurcates and fluctuates between two values, i.e. the system is a period 2 cycle. If r keeps increasing, the system bifurcates and becomes period 4 cycle, period 8 cycle, and so on until $r \approx 3.57$, after which the system becomes chaotic. Chaotic dynamics do not have any periods which can be verified in the picture. However, there are regions with stable periodic orbits (for example, there is a stable period 3 cycle near $r \approx 3.83$) within the chaotic region which are known as the periodic window (Strogatz 2018).

Since it has been determined that the logistic map becomes chaotic when the parameter r > 3.57, we can illustrate the property of sensitive dependence on initial condition by plotting and comparing the time paths for this system with r = 3.71, $x_0 = 0.7$ and r = 3.71, $x_0 = 0.701$. Figure 2.3.2 shows that even an error of 0.001 in the initial condition caused the time path to move differently and led to a completely different final state. The final state after 100 periods in the future is around 0.4 for the initial condition of 0.7 and around 0.9 for the initial condition of 0.701. The paths are similar for the first few observations (up to t = 7), then there is an occasional slight deviation between the two paths up to t = 28, and after that the two paths are vastly different.



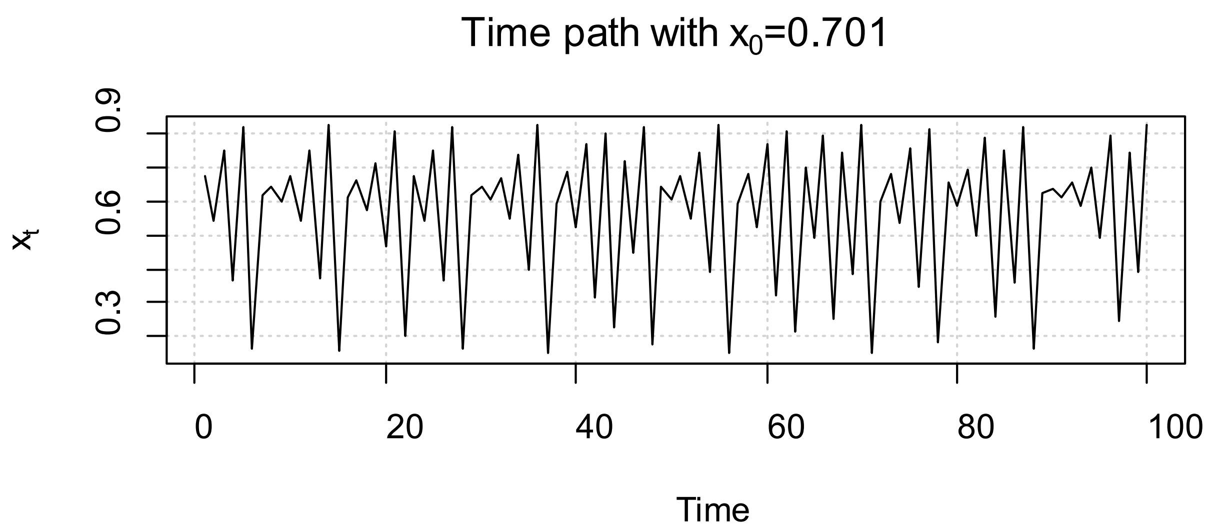


Figure 2.3.2: Time path of the logistic equation $x_{n+1} = r\, x_n(1 - x_n)$, $x_0 = 0.7$ and $x_0 = 0.701$.

A chaotic price series is practically unpredictable or very difficult to predict in the long run. Though the equation generating chaotic structure is completely deterministic, we should have perfect knowledge of the initial price to accurately predict. However, it is almost always impossible to precisely measure the initial price. Due to sensitive

dependence on initial conditions, any small error in the initial price (say roundoff error) will lead to a very large prediction error after some time for a chaotic system (Savit 1988). However, a chaotic system will still be quite predictable over a short time horizon. The interest in chaos theory in finance and economics is due to its ability to generate a path that mimics the fluctuations observed in market prices (Beker 2014; Faggini 2005). These prices show oscillatory behavior that is not cyclic, making it difficult to identify any discernible patterns. Moreover, these oscillations cannot be attributed to external shocks or influences. Chaos theory suggests that these behaviors are due to endogenous fluctuations inherent in nonlinear deterministic systems (Beker 2014). These models could help detect nonlinearity that remained even after applying a nonlinear stochastic model such as ARCH/GARCH.

The investigation of chaos in economics is important from both theoretical and practical points. If a system is chaotic, we can construct mathematical models that provide a deeper understanding of its dynamics. Chaos theory can help in realistic modeling that may explain phenomena such as fluctuations, crashes, crises, and depression in economics. In addition, despite its unpredictability, the deterministic nature of a chaotic system makes it exploitable and controllable. Its sensitivity to initial conditions, the fundamental characteristics of chaos, can be used to control the dynamical system. If chaos can be identified in economic processes, we can achieve a specific goal of economic policy with a smaller amount of resources than using traditional techniques of control by exploiting the sensitive dependence characteristics to choose different trade-offs (Faggini 2005; 2009). Chaos theory implies that the future is perfectly predictable provided one can measure the current state perfectly. However, even the small incorrect

measurements in the state today will amplify exponentially in the future and the forecast becomes worthless. So, chaotic dynamics are considered unpredictable in the long run because we can never measure the present state with absolute accuracy (Brock, Hsieh, and LeBaron 1991).

Whether the seemingly random behavior exhibited by commodity prices is explained by chaotic processes has received considerable attention and studies have explored chaotic explanations for fluctuations in price series (Chatrath, Adrangi, and Dhanda 2002; Yang and Brorsen 1993). However, as will be discussed in the next chapter, the literature has not reached a consensus on the existence of chaotic dynamics in economics because economic data is noisy and limited.

3. LITERATURE REVIEW

Econometric analysis of agricultural commodity prices has a long history. The studies on the behavior of commodity prices saw the changing and increasing complexity of forecasting methods with time. Earlier research was focused on the structural competitive model of price determination which assumed that the prices of commodities are determined by demand and supply. Analysts would use demand and supply equations for forecasting and exploring the dynamic properties of prices. They supported the hypothesis that current production is a function of lagged prices and current production is a determinant of the current price, leading to the development of the cobweb model. A detailed review of these structural models can be found in Tomek and Robinson (1977) and Labys (2005). The use of structural models requires a large amount of data for many variables, which is not always possible. The focus of price analysis was then shifted to a non-structural approach i.e., time series econometrics. Commodity prices started to be viewed as being composed of trends, seasonality, cycles, and random components (Jha and Sinha 2013; Tomek and Robinson 1977; Tomek and Peterson 2000).

The earlier attempts to determine if commodity prices followed random, deterministic linear or nonlinear process mark the origin of the non-structural approach to commodity price analysis. Linear models of price behavior come from the efficient market hypothesis, whereas the dependence and long memory of price series explore the nonlinearity in the price series. Test of efficient market hypothesis in futures prices had revealed short-term price movements to follow random walk or martingale which implied independence of price changes.

Random walk theory is one of the simplest models which states that prices are identically and independently distributed over time, and the history of prices does not contain any information that can be used to predict the future.[4] Working (1958) argued that the continuous flow of information into the market causes the price to change randomly. Samuelson (1965) postulated that the futures price follows a Martingale process and suggested that futures prices are stochastic and the expected price in the next period equals the current price.[5]

The random walk theory and martingale process were tested by various authors. Stevenson and Bear (1970) used the futures prices of soybeans and corn to determine if they follow a random walk by investigating whether significant autocorrelation existed. If the price series followed a random walk, there should not exist any pattern in the sign or size of correlations, and the correlations should not be significantly different from zero. They also applied mechanical trading techniques (filters) to test the validity of the random walk hypothesis.[6] If the random walk hypothesis is valid, the mechanical trading

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[4] Random walk is the sum of independent and identically distributed random variables. The random walk hypothesis assumes that the successive price changes have the same distribution but are independent of each other and the past observations of prices cannot be used to predict its future movements.

[5] A martingale is a stochastic process where the conditional expectation of the future value given all the information accumulated until now is equal to its current value. A sequence of price $P = \{P_t, t = 0, 1, 2, \ldots\}$ having finite means is a martingale if $E\left(P_{\{t+1\}} \middle| I_t\right) = P_t$ for all t, where I_t represents the history of the price available at time t (H. M. Taylor 1990). It does not require successive price changes to be drawn from the same distribution (Mann and Heifner 1976). It assumes that the level of any variable at time t+1 is equal to the level of the same variable in time t on an average using all the past information set. The martingale hypothesis suggests that the best forecast for the price tomorrow is today's price.

[6] Mechanical trading rules or filters are simulation models which test for the possibility of the existence of nonlinear dependence in the price data. There are various forms of mechanical trading rules. The popular one is buying the contract if the price of that contract moves up at least by x-percent and holding it until its price

rule would not yield returns consistently above the buy-and-hold policy. They found the tendency of negative dependence in short periods and positive dependence in long periods, which led them to conclude that there is a tendency for reversal. Similarly, the success of large filters led them to believe the presence of a long-term trend and the profitability of trading rules led them to conclude that the random walk hypothesis was dubious.

Leuthold (1972) performed spectral analysis to test the validity of the random walk hypothesis for live cattle futures price. They found mixed results from this analysis, i.e., the random walk model was consistent for some of the contracts but not for others. They also tested for non-linear patterns and dependence in this data using mechanical trading rules (i.e., profitable mechanical trading rules for investment are only possible if prices do not follow a random walk). They found a high gross return from the small filter rules which indicated trading based on mechanical decisions can be very profitable provided one does not have to pay commission. They also found that profits are possible even after commission for some of the trading rules. These results gave evidence that there exist price trends in the data and led them to cast doubt on the random walk hypothesis. They argued that spectral analysis looks at periods of fixed length whereas filter rules allow the time period to vary and thus capture nonlinear dependency. There may be short-run trends in the data that can be detected by filters but not by spectral analysis. The lack of consistency between conclusions drawn from spectral analysis and filter rules in testing

moves down by x-percent from the subsequent high. Similarly, the short position is maintained until the price rises at least x-percent above the subsequent low.

the random walk hypothesis using identical data led them to postulate that using only one approach may mislead a person to believe that prices followed a random walk.

Cargill and Rausser (1975) tested the random walk hypothesis using futures prices of 7 commodities including corn, soybeans, and wheat from 1960 to 1972 and live beef cattle, and pork bellies from 1966 to 1972. They employed six tests of serial correlation which included both time domain and frequency domain tests for randomness. Independent and uncorrelated futures price change is a necessary condition for the validity of the random walk hypothesis and their rejection is a sufficient condition to reject random walk. The results from three run tests for corn (65 contracts), wheat (65 contracts), and soybean prices (89 contracts) indicated that at most 10 % of the contracts are nonrandom at a 5 % level of significance. The integrated periodogram results showed more than 10 % of the D-Values significantly different from zero at a 5 % level for these commodities. This result is more unfavorable to the random walk compared to the results from run test. The D-values indicate the largest distance between the estimated integrated periodogram and an integrated periodogram for white noise. If the random walk hypothesis is valid, price changes are independent and identically distributed (IID), which means there is no significant differences between the estimated integrated periodogram and the integrated periodogram of white noise[7]. They also examined the spectral density function by plotting 95 % confidence intervals around theoretical values of white noise and found that more than 20 % of the contracts for each of these commodities appeared nonrandom. They also found that many of the contracts for these commodities have more than 10 % of correlation coefficients over 50 lags significant at a 5% level which is inconsistent

[7] Sequence of Independently and Identically distributed random variables with mean zero is called white noise.

with random walk hypothesis. Similar results hold for live beef cattle (47 contracts) and pork bellies (42 contracts). They were able to strongly reject the random walk hypothesis for the commodities under study.

Mann and Heifner (1976) studied the daily closing prices of 9 commodities including corn, soybeans, wheat, pork bellies, and live beef cattle using data from 1959 to 1971. They tested the random walk hypothesis using two non-parametric tests, the turning point test and the phase length test. The turning point test rejected the hypothesis of randomness for over 97 % of the contracts (557 out of 574 contracts), and the phase length test rejected the randomness hypothesis for over 90 % of the contracts (519 out of 574 contracts). They argued that the results support the theory of continuity in price movement (i.e., price changes in successive periods tend to be in the same direction). They concluded that commodity prices exhibit regular patterns rather than behaving randomly.

Peterson, Ma, and Ritchey (1992) also tested the random walk hypothesis, this time with cash prices for 17 commodities (including corn, wheat, and soybeans) and using the variance ratio test. Similar to previous studies, they found long-term price dependence between successive daily commodity prices, which led them to reject the random walk hypothesis. They further suggested that the level of serial correlation observed in the data indicates that past prices can be used to predict future prices.

Along with the work previously discussed, some studies had already hinted at the presence of nonlinear dependence rather than linear dependence in commodity prices, which shifted the domain of price analysis to include nonlinear and chaotic behavior. Houthakker (1961) used daily spot prices and futures prices for cotton from October 1944

to July 1958 to estimate a linear stochastic process to model the data. However, they found that the daily change in price was not log-normal, but rather slightly skewed and highly leptokurtic. They also found that the variance of price changes was not constant over time. This prevented them from applying the available methods of time-series analysis to address their original purpose. They computed the correlogram up to 120 lags, which showed little positive correlation and no obvious pattern. These results made them suggest that a non-linear stochastic process might be more appropriate to model the data. Mandelbrot (1963), following the work of Houthakker (1961), studied the distribution of price changes. He observed that the distribution of price changes is leptokurtic i.e., they have a higher concentration of observations in the tails compared to a normal distribution. He studied cotton prices and postulated that long series of price changes should be represented by a mixture of stable Paretian laws. He proposed stable Paretian distribution as a more general representation of observed price changes.

Hudson, Leuthold, and Sarassoro (1987) studied the distribution of futures price changes using daily closing prices for soybeans, wheat, and live cattle from 1973 to 1982. Normal distribution assumption was tested with kurtosis, R/S test, characteristic exponent, and Bartlett test of homogeneity of variance and the independence of price changes was tested using nonparametric turning point test and phase length tests. The Kurtosis test rejected the normality assumption for 84 of the total 180 contracts studied. The R/S test indicated 64 of the total 180 contracts were not normally distributed. The results of characteristic exponent showed 135 out of the total contracts had estimated alpha values below 2, indicating a nonnormal distribution and raising concerns about the validity of using variance as a measure of variability. The Barnett test rejected the hypothesis that

non-normality is caused by heteroscedasticity. Furthermore, turning point test and phase length test rejected the hypothesis of randomness for 23 and 26 of the total contracts studied respectively. Compared to the results of Mann and Heifner (1976), they found that the number of contracts exhibiting nonnormality has decreased. They concluded that while there is still evidence of leptokurtosis and non-randomness in the data, there is a tendency towards normality and randomness. Fang, Lai, and Lai (1994) suggests that leptokurtosis and a significant deviation from normality can be the signature of nonlinear dynamics.

Frank and Stengos (1989) used the correlation dimension technique and Kolmogorov entropy to study the rate of return on gold and silver based on daily and weekly prices from 1975 to 1986. They failed to reject the Martingale hypothesis using standard techniques, which they argued to be misleading because those techniques cannot capture the true structure of the series. From the estimation of correlation dimension and Kolmogorov entropy, they concluded that the rates of returns of these commodities were generated by a nonlinear process of correlation dimension between 6 and 7. They also performed Brock's Residual Test which suggests that if a series is generated by deterministic chaos, the residuals from a linear or smooth nonlinear transformation of data should yield the same correlation dimension as the original series. They found out that the correlation dimension calculated for residuals of the ARCH (12) process was also between 6 and 7. This provided evidence of chaotic structure in addition to ARCH-type non-linear process.

Decoster, Labys, and Mitchell (1992) followed the analysis of Frank and Stengos (1989) to investigate the presence of nonlinear structure in the daily futures prices of sugar,

coffee, silver, and copper. They used the correlation dimension technique and performed a series of tests (e.g., residual test and shuffle test) to explore the presence of nonlinear structure. They further tested for an ARCH structure of the data and were able to find evidence of the presence of a nonlinear process with a deterministic component in these price series. They concluded that the nonlinear structure is not only due to heteroskedasticity and that there is also evidence of chaos in the data. They suggested future research to identify the actual nature of nonlinearity in data.

Blank (1991) evaluated the futures prices for S&P 500 index and soybeans using nonlinear methods to determine the presence of a non-linear deterministic system (chaos). They used correlation dimensions and Lyapunov exponents for the dynamic analysis. They rejected the null hypothesis of a linear process at a 95% confidence level and concluded that both S&P 500 index and soybean prices were generated by a nonlinear process. They also calculated the maximum Lyapunov exponents, which indicated that the nonlinearity in these futures prices is deterministic and not stochastic. However, they postulated that this result is only a necessary but not sufficient condition to prove the existence of deterministic chaos due to the lack of statistical tests of significance for estimated Lyapunov Exponents.

Yang and Brorsen (1993) studied the first differenced natural logarithms of daily closing futures prices for 15 commodities (including corn, soybeans, wheat, and coffee) and tested them for GARCH and deterministic chaos processes. They applied Brock's residual test and BDS test for deterministic chaos, and Kolmogorov-Smirnov test of fit for GARCH. They found that Brock's residual test showed no evidence of low-dimension chaos, while the BDS test gave mixed results. The BDS test applied to raw data rejected

the null hypothesis of independent and identical distribution for all commodities, which they claimed to be consistent with nonlinear dependence and the presence of deterministic chaos. However, the BDS test for rescaled data (standardized residuals of GARCH (1,1)) rejected the i.i.d. hypothesis in support of deterministic chaos for only half of the commodities. They also found that prices are skewed and leptokurtic, and the GARCH (1,1) model could not remove all the dependence, skewness, and leptokurtosis in the data, suggesting the inadequacy of the GARCH model. The results of Kolmogorov-Smirnov test of fit suggest that a higher order GARCH process may be more appropriate to model the data. They concluded that price changes are nonlinear, and their results provided support for conditional heteroskedasticity but not for deterministic chaos.

Barkoulas, Labys, and Onochie (1997) used monthly prices for 21 internationally traded commodities (including wheat, corn, soybeans, coffee, cotton, and sugar) from 1960 to 1994 and fractional order tests to investigate long-term dependence or long memory. Their results support the presence of long memory in the price series. They argued that the discovery of fractional dynamics for spot commodity prices questions the linear models because fractional dynamics is characterized by irregular cyclic fluctuations with long-term dependence. Because such fractional dynamics can be better represented by nonlinear models, they concluded that most of the commodities are generated by nonlinear processes.

Wei and Leuthold (1998) studied the price series of soybeans, corn, wheat, hogs, coffee, and sugar over 21 years and found time-varying volatility, high skewness, and kurtosis. They also found long-range dependence despite the time series being stationary with respect to covariance, nonnormal distribution, and heteroskedasticity, which suggested

the presence of non-linear dynamics. They tested for ARCH, long memory, and chaotic structure. Although the standard ARCH tests initially indicated the presence of ARCH effects in the series, the slow decay of autocorrelations of variance over time contradicts the behavior expected from ARCH processes. Furthermore, all the price series display asymmetry (Skewness) that cannot be adequately explained by regular ARCH processes. These led them to reject the presence of ARCH processes in this time series. The ARFIMA model was then applied to test for the existence of a long-memory process. However, all price series except sugar appeared to be stochastic processes and not long-memory processes. This failure of ARCH and long memory processes opened the possibility for chaotic structure in the price series. They tested and found evidence of the presence of chaos in these commodity prices.

Chatrath, Adrangi, and Shank (2001) studied the nonlinear dynamics of gold and silver futures prices and were able to find strong evidence in favor of non-linear structure which could be explained by the ARCH model, and thus rejected the presence of chaos in these series. In a subsequent study, Chatrath, Adrangi, and Dhanda (2002) tested for the presence of low-dimensional chaos in the futures prices for corn, wheat, soybean, and cotton using 25 years of data. They employed three tests: correlation dimension, BDS statistics, and Kolmogorov entropy. The BDS statistics strongly rejected the null hypothesis of no non-linearity in residuals of AR series with seasonal correction, which was evidence that commodity futures prices have non-linear dependencies. However, the BDS test applied to standardized residuals of ARCH-type models showed that the nonlinearity in commodity prices is explained by dynamics other than chaos. Correlation dimension estimates showed the absence of saturation (i.e., it kept increasing) up to 20

embedding dimensions, which was inconsistent with chaos. For a series to be chaotic, the value of the correlation dimension should stabilize at some value with an increasing embedding dimension. However, if the correlation dimension keeps increasing with an increase in embedding dimension, the system can be regarded as stochastic. Similarly, Kolmogorov entropy estimates also confirm that there is very little evidence of low-dimension chaos in commodity prices. They concluded that ARCH-type processes, after controlling for seasonality and contract-maturity effects, were able to explain much of the nonlinearity present in the data.

Cromwell (2004) studied the price behavior of commodities such as bananas, beef, coffee, soybean, wool, and wheat using monthly prices from January 1960 to June 1994. They were able to detect nonlinearity in these prices and the results showed the presence of conditional heteroskedasticity. They found that neural network models outperformed the ARCH-GARCH model in terms of forecasting error, suggesting that GARCH-ARCH could only account for some of these nonlinear dependencies. They also used Brock's residual test and the BDS test. However, the test of low dimension chaos is not supported by Brock's residual test, whereas results from BDS test statistics were mixed. They concluded that there may be chaos in the price of these commodities and suggested the need for more robust test methods.

Overall, linear stochastic models such as ARIMA assume that commodity prices are normally distributed. However, as discussed earlier, many studies have shown that the distribution of commodity prices is not normal, but rather exhibit leptokurtosis and skewness. Moreover, linear methods attribute all irregular behavior in a time series as random. Instead, chaos theory has shown that irregularity is not only because of

randomness. It can also be produced due to nonlinear chaotic systems which are completely deterministic (Kantz and Schreiber 2004). Despite these findings, the normality assumption is taken for granted and linear models such as ARIMA are the most widely used methods in forecasting and modeling agricultural commodity prices.

Further, the existing literature largely suggests that at least high-frequency price series of commodities (data sampled at daily, weekly, or monthly) show stochastic trends (unit roots), have time-varying volatility (conditional heteroskedasticity) and are not normally or log-normally distributed. ARCH and GARCH models are a common way of accounting for this conditional heteroskedasticity; however, results from various studies have shown that GARCH and ARCH models cannot account for all the volatility and kurtosis present in commodity prices (Tomek and Myers 1993). This suggests that while there is consensus among scholars that price changes are nonlinear, there is no concrete evidence of the presence of deterministic chaos. In recent years, various studies have been conducted to compare the forecasting performance of ARIMA, nonlinear ARCH and GARCH models, and machine learning models. These studies also found support for nonlinearity in prices and even chaotic structure in the data.

Kohzadi et al. (1996) compared the forecasting performance of ARIMA and the feed-forward neural network (which is theoretically supposed to capture nonlinearity) using monthly prices for live cattle and wheat from 1950 to 1990. They found that the mean squared error (MSE), absolute mean error (AME), and mean absolute percentage error (MAPE) were lower for the neural network model compared to the ARIMA model. They also found that the mean price levels of ARIMA and neural network forecasts are statistically different. They further observed that the neural network model had greater

power to capture a statistically significant number of turning points. These results led them to conclude that neural networks outperformed ARIMA models in forecasting and suggested this may be due to the presence of nonlinearity or even chaotic structure.

Jha and Sinha (2013) used monthly wholesale prices in India for soybeans for a period of 228 months (October 1991 – September 2010) and rapeseed-mustard for a period of 372 months (January 1980 - December 2010) to assess and compare the forecasting accuracy of neural network models and traditional statistical models such as ARIMA. They compared the best ARIMA model (based on the lowest Akaike Information Criterion and Bayesian Information Criterion) with time delay neural networks (TDNN) in terms of modeling and forecasting. They found that the root mean squared error (RMSE) and mean absolute deviation (MAD) values were lower for the neural network model. Similarly, they found that the neural network model performed better than the linear model in predicting turning points. They concluded that TDNN was better than ARIMA in terms of forecasting accuracy.

Jayaramu (2015) compared the forecasted price with the actual price for corn, soybeans, and wheat using 23 years of monthly data from 1991 to 2013. They found that the distribution of prices was highly skewed, and they showed kurtosis. They forecasted the price for 2014 using the AR model with 3 lags based on the lowest Akaike Information Criterion (AIC). They found that the forecasted prices varied significantly compared to the actual price for all the commodities.

Ouyang, Wei, and Wu (2019) compared various traditional models such as ARIMA and VAR (Vector Autoregression) with LSTNet (long and short-term time series network) in terms of their prediction ability using price series from 2006 to 2019 for 12 commodities

(including corn, wheat, and soybean oil). They found that LSTNet outperformed ARIMA and VAR by 91.70 % and 91.69 % respectively in terms of root relative squared error (RSE), and 93.67 % and 93.66 % respectively in terms of relative absolute error (RAE) for a forecast horizon of 24 months. They also found that the price series were nonstationary and prediction for a long forecasting horizon was poor in general. They claimed, *"LSTNet method is a new prediction model that is able to predict multivariate time series simultaneously with full consideration of a mixture of extremely long-term and short-term patterns, linear and non-linear structures."* (Ouyang, Wei, and Wu 2019, p.480).

Silva, Barreira, and Cugnasca (2021) used daily prices for sugar (2004 - 2019) and corn (2003 – 2019) to compare predicting ability of ARIMA, SARIMA, support vector regression (SVR), AdaBoost, and long short-term memory (LSTM) models. The comparison was made using three metrics i.e., mean absolute error (MAE), mean squared error (MSE), and R2 score. They found that the traditional econometric models (ARIMA and SARIMA) performed the worst in predicting the price of both commodities. They concluded that these models were unable to capture the trend in the dataset and argued that volatility of prices and nonstationary plus varying trend were the major reason behind their poor performance.

Although many authors have repeatedly suggested the possibility of chaotic dynamics in commodity prices, relatively little research has been conducted to study the chaotic behavior of commodity prices. The literature exploring the presence of chaos in economic time series also provides limited conclusions due to the presence of high level of noise in the series, relatively small sample sizes, and weak robustness of chaos tests (Faggini

2014). However, chaos theory is still a well-qualified candidate to model fluctuations and other phenomenon in economics and finance, including commodity prices (Beker 2014). We have observed that instead of searching for chaotic behavior of commodity prices, researchers are simply adopting experimental machine learning and deep learning approaches such as neural networks to forecast commodity prices. These methods have been shown to learn the nonlinear dynamics, possibly chaotic behavior in the time series, and thus has better forecasting power (Su et al. 2014). Therefore, there is still a need to explore the chaotic dynamics of these commodities.

4. METHODOLOGY

Proper characterization of the behavior of commodity price series is important for agricultural price forecasting and modeling purposes, as discussed in previous chapters. The current chapter provides the details of the tests and methods used in the empirical analysis.

4.1 Methods

We use nonlinear time series analysis (NLTS) to investigate the presence of low dimensional, deterministic nonlinearity in price series. NLTS uses various statistical tests to identify nonlinear dynamic structures in random-appearing time series data (Kantz and Schreiber 2004). NLTS is powerful because it can detect and characterize real-world dynamics using time series on a single observed variable. This is possible because a single variable in an interdependent dynamic system contains information about the history of its interactions with other system variables, allowing NLTS to diagnose, reconstruct, characterize, and model the underlying data-generating system. Farmer stated that "… the evolution of [a variable] must be influenced by whatever other variable it is interacting with. Their values must somehow be contained in the history of that thing. Somehow their mark must be there." (Huffaker, Bittelli, and Rosa, 2017; Huffaker, 2015). NLTS involves several steps to ensure the accuracy of the analysis. Figure 1 illustrates the steps of NLTS adopted in the empirical analysis, as will be discussed in the next sections. The first step is singular spectrum analysis (SSA), which separates structural components (signal) from noise in the data. Noisy data can make it difficult to detect the oscillatory patterns that may have been generated by a nonlinear system. While linear filters can remove observation or measurement errors (white noise), they may mistakenly

treat significant variability resulting from complex deterministic dynamics caused by nonlinear interactions among system variables as noise and remove them. SSA addresses this problem by distinguishing and extracting such variability, preserving important patterns in the data that should be reproduced by the theoretical models.

The second stage involves testing for nonlinear stationarity, which allows us to accurately study the system's dynamics. This step examines whether the isolated signal components exhibit consistent behavior over time. If the signal is stationary, we can reasonably conclude that it represents a single dynamic system. Essentially, this means that the system's behavior remains relatively constant over time, enabling us to analyze it more accurately.

In the third stage, we reconstruct the underlying dynamics of the data using a technique called phase space reconstruction. This process helps us visualize and understand how the different variables in the system interact and evolve over time. It provides us with a mathematical representation of the relationships and patterns among variables.

Lastly, in the fourth stage, we perform a surrogate data test. This involves generating surrogate data that preserves the statistical properties of the original data. It serves to test whether the observed structure detected in the signal is truly a result of low-dimensional nonlinear dynamics inherent to the system.

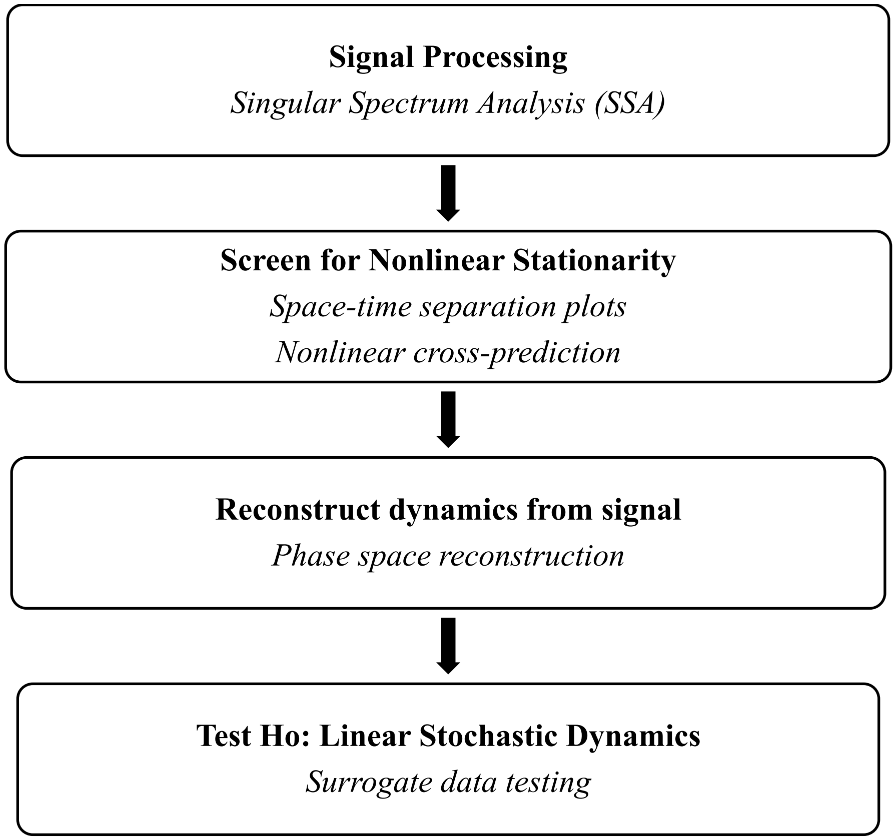

Figure 4.1.1: Nonlinear time series analysis framework.

4.1.1 Signal processing

The initial step is to apply signal processing techniques to (i) separate structured variation (signal) from unstructured variation (noise) and (ii) remove trends in the data. Separating the data into signal and noise and removing trends address problems with noise and nonstationarity, because noise-free data helps with accurate reconstruction of phase space attractors and stationarity ensures that the signal is generated from a single dynamic system, preventing the reconstructed phase space trajectories from jumping from one attractor to the other. Signal processing also measures signal strength by calculating the percentage of total variation it (signal) explains in the observed data. A signal is considered strong if it accounts for a large portion (more than 50 %) of total variation in

the data. It signifies that the structured component explains the majority of variation in the data justifying the use of nonlinear timeseries analysis approaches to characterize the underlying structure. Conversely, a weak signal with signal strength less than 50 % indicates that the time series is primarily composed of noise and suggest that the traditional linear stochastic methods may be more effective to analyze the data (Huffaker, Bittelli, and Rosa 2017).

We apply *Singular Spectrum Analysis (SSA)* which is a data-driven signal processing technique that can handle irregular time series with highly anharmonic or potentially non-sinusoidal oscillations (Elsner and Tsonis 2010; Vautard 1999; Ghil et al. 2002; Golyandina, Nekrutkin, and Zhigljavsky 2001). SSA separates the observed time series x_t into the signal, which is an endogenous structured variation composed of trends and oscillations, and the noise, which is an exogenous unstructured variation without losing dynamic structure.

$$x_t = \underbrace{trend + oscillations}_{signal} + noise$$

SSA proceeds in three steps: decomposition, grouping, and reconstruction. In the decomposition stage, SSA first embeds the price series, $X_t\ (x_1, x_2, \ldots, x_N)$, where N is the number of observations, to construct a 'trajectory matrix'[8], $Y_{L \times K} = [Y_1, Y_2, \ldots Y_K]$ *where* $K = N - L + 1$ *and L is the window length,* whose columns contain the observed series and its forward delayed copies. The vectors $Y_i = (x_i, \ldots, x_{i+L-1})^T$, *where* $i = 1, \ldots, K$, are called L – lagged vectors. The only parameter required to run SSA is the window length L, which sets the number of rows in the trajectory matrix. The selection of window length depends

[8] The term "trajectory matrix" is used because it represents the path of the time series data over time. Each row of the trajectory matrix represents a point in the path of the time series, and each column represents a different point in time.

on preliminary information about the time series and the objective of the research. Theoretically, the window length should be large enough but not greater than N/2 and thus is restricted by $2 \leq L \leq N/2$ (Hassani 2007; Golyandina, Nekrutkin, and Zhigljavsky 2001). Then singular value decomposition (SVD) decomposes the trajectory matrix Y as a sum of rank-one bi-orthogonal elementary matrices $\boldsymbol{Y_1}, \boldsymbol{Y_2}, \dots, \boldsymbol{Y_r}$. The intuition behind this is based on the idea that a time series can be represented as a combination of basic patterns (trends, oscillations, etc.). SVD breaks down matrices into singular vectors and singular values. The singular vectors represent evolution of each component of the time series and the singular values represent the importance or magnitude of those components i.e., the % of variability explained by each component. If $\lambda_1, \lambda_2, \dots., \lambda_L$ represents eigenvalues of YY^T in decreasing order of magnitude ($\lambda_1 \geq \lambda_2 \geq \dots. \lambda_L \geq 0$), EV_i represents the orthogonal system of eigenvectors corresponding to these eigenvalues, r represents the rank of $\boldsymbol{Y}$, and V_i represents the principal component, the empirical orthogonal functions (EOFs) can be written as:

$$Y_i = EOF_i = \sqrt{\lambda_i}.EV_i V_i^T$$

EOFs essentially show the structure of the major components that account for the behavior of the price series over time. Then, the SVD of the trajectory matrix can be written as:

$$\boldsymbol{Y} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{r}} EOF_i = \boldsymbol{Y_1} + \boldsymbol{Y_2} + \cdots + \boldsymbol{Y_r}$$

The sum of all eigenvalues measures the total variability in the time series, and the eigenvalues of each component measure the proportion of variability explained by that component. The cumulative proportion of variability explained by signal components (i.e., the sum of eigenvalues of each component) provides the measure for signal strength.

Next, in the grouping stage, the elementary matrices Y_i are split into several groups, and matrices within each group are added to obtain trend, oscillatory, and noise components. The set $(\sqrt{\lambda_i}, EV_i, V_i)$ is called ith eigentriple of matrix Y. The EOFs are arranged in rank order and grouped according to the magnitude of their respective singular values $(\sqrt{\lambda_i})$. The EOFs with slowly varying eigenvectors form the trend components because a trend is a slowly varying component of a time series.

Subsequent eigentriples whose eigenvector oscillates with identical frequencies are grouped to form oscillatory components. The trend and oscillatory components represent the signal (S). The remaining eigentriples which do not contain trend and oscillatory elements are grouped to form a noise component (N). The detrended signal removes the trend component and hence includes only the oscillatory components.

The final stage of the SSA technique is diagonal averaging for the reconstruction of the time series. It converts the grouped EOF matrices to the vector time series corresponding to trend, oscillatory, and noise components (Golyandina, Nekrutkin, and Zhigljavsky 2001; Hassani 2007). Strong stationary signals are used to reconstruct the real-world dynamics of the system, as will be discussed in the next section. The existence of weak signals indicates that conventional linear stochastic methods may be more effective for modeling purposes (Huffaker, Bittelli, and Rosa 2017).

4.1.2 Phase space reconstruction

Phase-space reconstruction is a fundamental technique in the analysis of nonlinear time series, as it offers a geometric representation of the real-world dynamics that attractors[9] produced by theoretical models should reproduce (Kantz and Schreiber 2004). Phase

[9] Definition of attractors is discussed in Chapter 2 above.

space is the space of all possible states of a system, i.e., it captures the state of system variables at each point in time. The resulting phase-space trajectory constructed by connecting the phase-space points shows how the system evolves over time. Before the 1980s, constructing a phase space for a dynamic system was assumed to require data on all the system's variables, making it challenging to apply this method in practice, as it is difficult to identify or measure all the variables at play in real-world systems. However, Packard et al. (1980) proposed a technique of reconstructing the phase-space by using the observations on a single time series of any dynamical system using the delay coordinate embedding method, later formalized by Takens (1981) and now known as Taken's theorem or delay embedding theorem. Takens mathematically proved the conditions under which the phase space reconstructed from a single observed time series variable was diffeomorphically equivalent to the real-world phase space that would have been obtained if all the state variables were known. The reconstructed phase space is not geometrically identical to internal dynamics[10]; however, it is topologically similar - if done right – and can be extremely useful. Conclusions drawn about the reconstructed dynamics hold for the true dynamics of the system because many properties of a dynamic system are invariant under diffeomorphism[11] (Bradley and Kantz 2015).

A strong signal that explains a substantial portion of the total variation in observed price series is used to reconstruct the 'shadow' version of the attractor using the time delay embedding method of phase space reconstruction (Takens 1981). Basic idea of delay

[10] It refers to the complete set of variables and their interactions that determine the behavior (or evolution) of the dynamic system over time. It means original state space dynamics of the system.

[11] Mathematically, it means one to one, onto, invertible and differentiable mapping. Let's say we have a doughnut, or a coffee mug made from dough. We can shape and mold the dough without tearing or creating any new parts or holes, so they can be transformed from one shape to the other. This kind of transformation, where the dough retains its overall structure while changing its shape, is called a diffeomorphism in mathematics.

embedding method is to plot the timeseries data against the delayed version of itself to obtain the reconstructed phase space. The detrended and filtered observed price series P_t is segmented into a sequence of delay coordinate vectors:

$$P_t, P_{t-d}, P_{t-2d}, P_{t-3d} \dots, P_{t-(m-1)d}, t = 1,2, \dots . N$$

where *d* is the 'time delay' or 'embedding delay', and *m* is the 'embedding dimension' or the number of delayed coordinate vectors or delayed copies of the observed variable. Time delay is the interval between successive time series observations which determines the coordinate of the point in the phase space. It helps us to track how the system changes allowing us to visualize the evolution of the system over time. Embedding dimension represents the number of measurements required at each time step i.e., the number of variables needed to describe system's behavior. It determines the number of axes in the reconstructed space.

This approach of phase-space reconstruction requires the selection of two parameters: (i) embedding dimension, *m,* and (ii) embedding delay, *d*. If *d* is very small, the *m* coordinates in each of these vectors are highly correlated and the embedded dynamics lie very close to the diagonal of the reconstructed phase space. Therefore, the optimum time delay *d* should be carefully selected in order to properly reconstruct the dynamics of the price series.

4.1.2.1 Estimation of embedding parameters

The original embedding theorem of Takens (1981) requires that the time delay *d* must be nonzero and not a multiple of periods of any orbits. This is only valid for an infinite amount of noise-free and real-valued integer data. However, when there is a finite amount of floating-point (decimals) and noisy data, a higher *d* is required to properly unfold the

series dynamics. Improperly unfolded dynamics are not topologically similar to true dynamics (Bradley and Kantz 2015). Too short embedding delay may not provide sufficient time to capture the evolution of reconstructed dynamics, whereas too long delay may make the reconstructed dynamics miss important structures. Generally, the embedding delay is selected to create statistical independence between a time series and its successive delayed copies (Huffaker, Berg, and Canavari 2018).

The other requirement of the embedding theorem to assure topological similarity is $m \geq 2n + 1,$ where n is the true dimension of the real-world attractor. Sauer, Yorke, and Casdagli (1991) relaxed this requirement to $m > 2\ d_A$ where d_A is the capacity dimension of the attractor. However, neither n nor d_A is known in the real world because d_A cannot be calculated without first embedding the data.

Various methods have been developed for the estimation of time delay d and embedding dimension $m.$ The convention is to select the embedding delay d such that the mutual information function reaches its first minimum (Williams 1997). Mutual information tries to measure the extent of the relation between the values P_t and P_{t+l} at a given lag using probabilities rather than a linear basis (such as autocorrelation). The mutual information is high if P_{t+l} is strongly related to P_t for a given lag, and low if P_{t+l} is weakly related to P_t for a given lag. For the reconstruction of the attractor, we don't want P_t to provide more information about P_{t+l} i.e., we want the mutual information to be minimum. A high mutual information suggests that consecutive observations in a time series contain redundant information, leading to oversampling of the system's dynamics. Therefore, selecting an embedding delay that minimizes the mutual information ensures the avoidance of oversampling and enables a more accurate representation of the system's

underlying behavior. Mutual information decreases with the increase in lag up to a point where it eventually stops decreasing. We will take that point - the first minimum – as our embedding delay *d*.

After determining the appropriate embedding delay (*d*), the next step is to estimate the embedding dimension (*m*). The dimensionality of an empirically reconstructed attractor indicates the minimum number of state variables required in theoretical modeling to reproduce the observed long-term dynamics. We prefer to reconstruct an attractor in an embedding dimension as small as possible as long as it produces topologically correct results for reduced computational cost and simplicity of interpretation. Many methods have been proposed to estimate *m* but none is yet widely accepted. Two of the more prominent proposed methods are false nearest neighbors (Kennel, Brown, and Abarbanel 1992; Liebert, Pawelzik, and Schuster 1991) and correlation dimension (Grassberger and Procaccia 1983). In the latter method, one embeds the data for many values of *m* and computes the correlation dimension, then selects the *m* such that the value of the correlation dimension settles down. We will be using the false nearest neighbors (FNN) method for estimating the embedding dimension *m* as used in Huffaker (2015); Huffaker, Canavari, and Muñoz-Carpena (2018); Berg and Huffaker (2015); and Huffaker and Bittelli (2015). A false neighbor is a point in the state space that is a neighbor[12] just because we are viewing the attractor in a very small embedding space. In other words, false neighbors are the points that appear closer in lower dimension but start to move apart as the embedding dimension increases. Once the number of false neighbors reaches zero, it ensures that the correct embedding dimension has been selected, resulting in a

[12] Neighbor refers to a data point in the reconstructed phase space that is considered to be close or nearby to a reference point.

successful unfolding of the attractor and an accurate representation of its underlying dynamics (Kennel, Brown, and Abarbanel 1992). In the FNN method, we start by embedding the data in two dimensions ($m = 2$) and measure the distance between points in the phase space. This is repeated for higher dimensions ($m = 3, 4, 5, \ldots$), and each point is identified as either a false neighbor (distance from central point increases) or a true neighbor (distance from central point remains constant), and the percentage of false nearest neighbors computed. The appropriate embedding dimension is the dimension for which the percentage of false nearest neighbors decreases approximately to zero (or some given threshold). The idea is that if some nearby points in the k^{th} embedding dimension are no longer nearby in the $(k+1)^{th}$ embedding dimension (false neighbors), the dynamics were not properly unfolded at the k^{th} embedding dimension.

A new parameter called Theiler window (Theiler 1986) is required to run the false nearest neighbor method. The Theiler window is a technique used in time series analysis to account for temporal correlations that can create fortuitous geometric structure in the reconstructed attractor. When neighboring points in phase space are close, it could be due to their proximity in time rather than a true geometric relationship. FNN test excludes points on an attractor within the Theiler window as potential nearest neighbors. By doing so, the Theiler window helps to avoid mistaking temporal correlation for a true attractor's geometric structure and allows for more accurate analysis of time series data (Kantz and Schreiber 2004). The Theiler window was estimated using the space-time separation plot method as introduced by Provenzale et al. (1992). The space-time separation plots are the scatter plots of spatial distance and elapsed time between each pair of points in shadow (reconstructed) phase space. Intuitively, the space-time separation plot shows how the

dependence (or distance) between points in reconstructed phase space changes over time. The plot is visually inspected, and the time lag at which the points in the reconstructed phase space begin to significantly spread out is identified. This is observed in the plot as a saturation point where the contours stop increasing. This time lag corresponds to the optimal Theiler window, representing the minimum time separation required to account for temporal correlations in the data. This means that the Theiler window is chosen to be the duration at which the neighboring points in the phase space stop showing significant temporal correlations and stabilize.

4.1.2.2 Nonlinear stationarity

It is important for the application of NLTS methods that the time-series data is stationary. Stationary signals guarantee that the data is long enough to adequately sample the dominant low-frequency cycles isolated by singular spectral analysis. Linear time series analysis requires only weak stationarity in which the second-order moments (mean and variance) do not fluctuate over time. However, weak stationarity is not sufficient for nonlinear time series analysis because the dynamic properties of a system can change even though these measures (mean and variances) remains fairly constant (Huffaker, Bittelli, and Rosa 2017; Kantz and Schreiber 2004). We will be using space-time separation plots (Provenzale et al. 1992) and nonlinear cross prediction (Schreiber 1997) to test for stationarity in our analysis for the purpose of application of NLTS.

The space-time separation plots are not only used to estimate the Theiler window but also used to detect whether the time series is stationary. For a nonstationary time series, the space-time separation plot may never cease to increase and there might be no Theiler window sufficiently large to control for temporal correlation in the time series. The

failure of the space-time separation plot to decorrelate indicates that we don't have sufficient data to study cycle lengths that dominate the time series (Morgan et al. 2022; Huffaker et al. 2021).

Nonlinear cross prediction is a method that seeks to identify similarities in nonlinear patterns within a time series, rather than simply comparing statistical parameters like mean and variance between different segments of the series. This is done by dividing the series into non-overlapping segments, and using nonlinear prediction techniques to evaluate how well each segment can predict the others. If the predictive skill decreases significantly as the segments become more distant in time, then the time series is considered nonstationary for the purposes of this method (Schreiber 1997; Sugihara and May 1990; Sugihara et al. 1997).

Particularly, this method involves dividing time series into equal segments, with each segment being used as a training set to predict the other segments of the series which serve as test sets. The size of each segment is a trade-off between allowing the system dynamics to evolve over time with longer segments versus achieving a more precise test resolution by using shorter segments (Schreiber 1997). For our analysis, we will divide the time series into 5 segments and use each segment as a learning test to predict the remaining test sets. Once we specify a learning and a test segment for each iteration, we will use time delay embedding to embed the learning set and estimate embedding parameters which will then be used to embed test segments. Then one-step ahead nonlinear prediction is used to predict the test set using learning set. The cross-prediction skill is measured using Nash-Sutcliffe Coefficient of Efficiency (NSE). Stationarity will

be concluded if the predictive skill (NSE value) does not decrease with the increase in distance between learning and test segments.

4.1.3 Surrogate data testing

We can find preliminary empirical evidence of the signal being generated by deterministic dynamics through the successful reconstruction of the shadow attractor. However, it is not reasonable to expect that the signal processing can remove all noise from the signal, and since it is difficult to visually distinguish between noisy linear behavior and nonlinear behavior, there is a possibility that the systematic geometric appearance (deterministic structure) of reconstructed shadow attractor may have been due to mimicking linear stochastic process (Kantz and Schreiber 2004; Schreiber and Schmitz 2000; Theiler et al. 1992). When we do not have strong evidence of some property, we must rely on statistical hypothesis testing. One can never prove the underlying dynamics with finite time series data, but can still calculate the probabilities of finding certain properties under the null hypothesis and provide evidence about the dynamics which is more plausible for the data (Bradley and Kantz 2015). Observed regularity of shadow attractor coupled with strong statistical rejection of fortuitous regularity increases the probability that the observed data are generated by deterministic dynamics (Huffaker, Bittelli, and Rosa 2017).

The discriminating statistics Lyapunov exponents and correlation dimension are the most widely used to detect deterministic structure in time series. However, their probability distributions are rarely known for finite data sets, and they may not even follow the normal probability distribution (Schreiber and Schmitz 2000), which makes it difficult for employing standard methods of hypothesis testing. Traditional bootstrap methods require

model equations to be extracted from data and employ Monte Carlo simulation to generate samples (typical realization). However, it is very difficult to extract the model equations and estimate parameters which may preclude us from using these traditional bootstrap methods (Schreiber and Schmitz 2000). Surrogate data testing was developed to statistically test the null hypothesis that the noticeable regularity in reconstructed attractors was generated by a linear stochastic process. These tests are based on resampling methods without replacement i.e., the original time series is resampled to destroy intertemporal patterns in observed data while preserving the statistical properties. Kantz and Schreiber (1997) strongly recommend using other discriminating statistics such as nonlinear prediction skill and permutation entropy instead of the conventional Lyapunov exponents and Correlation dimension that are difficult to be estimated reliably from short and noisy data series. We will follow this recommendation. The steps followed in surrogate data testing in our analysis are described below.

First, we specify the null hypothesis, and we use the detrended price signal to generate a set of surrogate vectors. For example, a set of *N* surrogates generated from a detrended price signal, P_t is:

$$P_t = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ P_T \end{bmatrix} \Rightarrow P_t^{S_1} = \begin{bmatrix} {P_1}^{S_1} \\ {P_2}^{S_1} \\ \cdot \\ \cdot \\ {P_T}^{S_1} \end{bmatrix}, P_t^{S_2} = \begin{bmatrix} {P_1}^{S_2} \\ {P_2}^{S_2} \\ \cdot \\ \cdot \\ {P_T}^{S_2} \end{bmatrix}, \ldots . P_t^{S_N} = \begin{bmatrix} {P_1}^{S_N} \\ {P_2}^{S_N} \\ \cdot \\ \cdot \\ {P_T}^{S_N} \end{bmatrix}$$

$where\ T\ is\ the\ Length\ of\ time\ series\ and\ P_t^{S_1}, P_t^{S_2} \ldots P_t^{S_N}\ are\ surrogate\ price\ vectors.$

We use two conventional types of surrogate vectors: amplitude-adjusted Fourier transform (AAFT) surrogates and pseudo-periodic (PPS) surrogates. AAFT surrogates are the most common surrogates used to test the null hypothesis of stochastic linear dynamics

against the alternative of deterministic dynamics. They are generated as “static monotonic nonlinear transformation of linearly filtered noise” that preserves both the probability distribution and Fourier power spectrum of the signal (Theiler et al. 1992; Small and Tse 2002; Lancaster et al. 2018). PPS surrogates can be generated for data characterized by aperiodic oscillations using the algorithm developed by Small and Tse (2002; 2003). PPS surrogates test the null hypothesis that the aperiodic oscillations are generated by randomly shifting limit cycle characteristics of noisy stochastic linear dynamics. This is done by generating the surrogate vectors such that the coarse[13] deterministic features (periodic trends) are preserved while fine structures (deterministic chaos) are destroyed. The rejection of the null hypothesis points towards the oscillations being generated by chaotic dynamics (Small and Tse 2003).

We then reconstruct the shadow phase space using AAFT and PPS surrogates and estimate the discriminating statistics (nonlinear prediction skill and permutation entropy) for each surrogate vector. We perform one-tailed hypothesis tests based on these discriminating statistics as suggested by Huffaker, Bittelli, and Rosa (2017) and used by Huffaker and Hartmann (2021), Medina et al. (2019) and Morgan et al. (2022). The null hypothesis is that the observed geometric regularity in the reconstructed attractor is fortuitously generated by linear stochastic processes. Rejection of the null hypothesis would mean that the detected regularity in shadow attractor is not due to stochastic behavior but rather to nonlinear determinism, which can be interpreted as possible

[13] Coarse structure refers to larger-scale patterns or trends driven by deterministic processes, exhibiting slower and predictable variations, such as long-term trends, seasonal cycles, or regular oscillations. Fine structure encompasses smaller-scale patterns driven by deterministic processes, capturing intricate and detailed dynamics, including smaller fluctuations, irregularities, or chaotic behavior, characterized by rapid and less predictable variations over a shorter time span.

evidence of the existence of chaos (Gotoda et al. 2012). On the other hand, if the null hypothesis cannot be rejected, then the observed time series data is more likely to be generated by a linear stochastic process and linear stochastic modeling remains a viable option.

4.1.3.1 Discriminating statistics: nonlinear predictive skill

A system is deterministic if its future behavior is causally set by past events. The ability to use a shadow attractor to make short-term predictions is a key characteristic of deterministic behavior (Kaplan and Glass 1995; Small and Tse 2002; Theiler et al. 1992). The nonlinear predictive skill provides evidence of the attractor's deterministic structure because nonlinear systems are expected to be more predictable than stochastic ones (Lancaster et al. 2018). A system with perfect predictions can be considered completely deterministic, while good but not perfect predictions suggest the presence of a deterministic component. Conversely, a system with poor predictions is not deterministic at all (Kaplan and Glass 1995).

A nonlinear prediction algorithm involves several steps. First, it reconstructs a shadow attractor by using time-delay embedding from the time series. Then, the points on the shadow attractor are divided into two sets: a learning set, and a testing set. The final point in the learning set is selected as a reference point, and its nearest neighboring points are identified. The neighboring points are then advanced one period ahead, and the prediction is calculated as the weighted average of these advanced neighboring points. Weights are determined according to the simplex averaging algorithm of Sugihara et al. (2012), which is based on the distance of the reference point to the nearest neighbor. The weight given for a nearest neighbor is its distance from the reference point relative to the total distance

of all nearest neighbors from the reference point. Then, the learning set is expanded by adding the first point in the testing set, and the same one-step ahead prediction method is used to predict the next point in the testing set. This process is repeated until all points in the testing set are predicted. Finally, the algorithm assesses its predictive accuracy by calculating a goodness-of-fit measure. For more information, readers can refer to Kaplan and Glass (1995) and Huffaker, Bittelli, and Rosa (2017).

We use the Nash-Sutcliffe Coefficient of Efficiency (NSE) as a goodness of fit measure to compare actual and predicted time series values (Nash and Sutcliffe (1970)).

$$nse = 1 - \frac{\sum_{i=1}^{n}(x_i - p_i)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = 1 - \left(\frac{RMSE}{\sigma}\right)^2$$

where i denotes the period in an n-period test set, x_i is the test value in period i, p_i is the predicted value in period i, $\bar{x}$ is the mean of the test series, RMSE is the Root-Mean Square error, and σ is the standard deviation of the observed test series. The NSE (Nash-Sutcliffe Efficiency) value varies between negative infinity and 1. A perfect match between predicted and test values is represented by an NSE value of 1. This indicates that the system can be accurately predicted using nonlinear prediction methods, suggesting that the underlying dynamics of the system are nonlinear and deterministic. When the NSE value is 0, it means that the time series average performs just as well as the non-linear prediction method, indicating the poor performance of the latter. Negative values of NSE indicate that the time series average predicts better than the non-linear prediction algorithm. This suggests that the non-linear prediction method is not effective in capturing the underlying dynamics of the system, indicating that the system's behavior may not be governed by nonlinear deterministic processes. A value of NSE greater than

0.65 is often considered a good quality threshold for the performance of the nonlinear prediction algorithm (Ritter and Muñoz-Carpena 2013).

We specify an upper-tailed test for nonlinear predictive skill with null hypothesis that NSE calculated for the signal is equal to the NSE calculated from the surrogates. This is because higher NSE values indicate an increase in the deterministic structure, and hence a nonlinear prediction method is expected to perform better for predicting nonlinear deterministic systems compared to linear stochastic surrogates (Morgan et al. 2022).

4.1.3.2 Discriminating statistics: permutation entropy

Permutation entropy measures the information content of the time series and is a popular probability-based complexity measure that is used as discriminating statistics in surrogate data analysis (Gotoda et al. 2012; Gotoda, Kobayashi, and Hayashi 2017; Huffaker et al. 2021). It was introduced by Bandt and Pompe (2002) and is a modification to Kolmogorov-Sinai (KS) entropy for use with finite noisy data. Bandt and Pompe claim that this is a simple and robust invariant measure that can be calculated for any type of time series and helps to distinguish between regular, chaotic, and random behavior just like (or even better in the presence of noise) other invariants such as correlation dimensions and Lyapunov exponents.

Given a time series $\{x_t\}_{t=1 \ldots T}$ with embedding dimension m, we study all possible $m\,!$ permutations as π of order $m \geq 2$ that are considered the possible order types of m different numbers. Then, we calculate the relative frequency for each π as:

$$p(\pi) = \frac{\#\{t | t \leq T - m, (x_{t+1}, \ldots, x_{t+m}) has\ type\ \pi\}}{T - m + 1}$$

The permutation entropy is defined as:

$$H(m) = -\sum p(\pi) \log p(\pi)$$

where the sum runs over all $m!$ permutations π of order m.

The value of $H(m)$ varies between 0 and $\log m\,!$. The lower bound (zero) corresponds to a monotonically increasing or decreasing sequence of values, and the upper bound (log m!) indicates a completely random system (i.i.d sequence) where all $m!$ possible permutations occur with the same probability. In other words, $H = 0$ implies that the time series is perfectly predictable from its past values and is highly structured whereas higher values of H indicate independently and identically distributed observations or *IID* noise. We construct a lower-tailed test for permutation entropy with null hypothesis that the permutation entropy calculated for the signal is equal to the permutation entropy calculated from the surrogates. This is because higher values indicate highly complex random behavior and smaller values indicate increasing low-dimensional nonlinear deterministic structure.

4.1.3.3 Hypothesis testing

Now that we have identified the discriminating statistics that will be used for hypothesis testing, we need to discuss how to actually perform the tests. Previous literature commonly utilizes two methods of hypothesis testing in surrogate data analysis to assess the statistical difference between a discriminating statistic calculated from observed data and that obtained from surrogate data. The first one is the one sample t-statistics which assumes that the distribution of the discriminating statistics is Gaussian and thus sets the significance level as the number of standard deviations above or below the mean (Berg

and Huffaker 2015; Huffaker 2015; Huffaker and Bittelli 2015)[14]. However, the distribution of these statistics may not be normal (Lancaster et al. 2018; Schreiber and Schmitz 2000). Alternatively, there are nonparametric rank-order statistics that determine the significance level as $(1-\alpha)$ percentile of the surrogates. This procedure is more robust for the statistical test compared to the parametric method and thus will be used in this study as suggested by Lancaster et al. (2018), Schreiber and Schmitz (2000), and Theiler et al. (1992).

We generated a group of $S=\frac{K}{\alpha}-1$ surrogates where α is the probability of false rejection and *K* is the parameter that controls the sensitivity of tests and the number of surrogates. The null hypothesis is rejected if the discriminating statistics calculated from the signal fall within the extreme ranges of surrogate measurements ranked in descending order. Specifically, for an upper-tailed test, the statistic from signal should be among the k largest surrogate statistics, while for a lower-tailed test, it should be among the k smallest surrogate statistics.

We set the value of $\alpha=0.05$ and $K=5$ and generated $S=99$ AAFT and PPS Surrogates each and rejected the null hypothesis if the statistics computed from signal falls within 5 largest (for Nonlinear prediction) or 5 smallest (for permutation entropy) of these values computed from surrogates.

To recap, we used nonlinear time series analysis approaches to investigate nonlinear deterministic dynamics in the data. Our methodology comprises several key steps. Firstly, we used singular spectrum analysis to separate signal from noise and tested for nonlinear

[14] Subsequent works such as Morgan et al. (2022), Huffaker and Hartmann (2021), and Huffaker et al. (2021) uses nonparametric rank-order statistics instead of t-test for hypothesis testing for the reason outlined earlier.

stationarity to ensure that the behavior of isolated signal components is consistent over time. Then we use phase space reconstruction to visualize the underlying dynamics of the system. Lastly, we conducted a surrogate data test to validate the presence of low-dimensional nonlinear deterministic dynamics.

4.2 Data

We use daily closing prices of nearby futures contracts for corn (CME Group), soybeans (CME Group), wheat (CME Group), cotton (ICE), coffee (ICE), sugar (ICE), live cattle (CME Group), feeder cattle (CME Group), hogs (CME Group), and orange juice (ICE). The number of observations and the interval of study for each commodity are shown in Table 4.2.1. To obtain a continuous price series for each commodity, the nearby delivery month was always used, and contracts were rolled over 15 days before expiration. These commodities are selected because they represent different types of commodities, and they are or have been the most commonly traded agricultural commodities in futures markets.

Table 4.2.1: Details of commodities selected for this study.

Commodity	Sample period	Number of Observations
Corn	1 July 1959 – 12 October 2022	15,938
Soybeans	1 July 1959 – 12 October 2022	15,933
Wheat	1 July 1959 – 13 October 2022	15,938
Coffee	1 September 1972 – 12 October 2022	12,553
Sugar	4 January 1961 – 12 October 2022	15,363
Cotton	1 July 1959 – 13 October 2022	15,855
Hogs	28 February 1966 – 12 October 2022	14,261

Live Cattle	30 November 1964 – 12 October 2022	14,564
Feeder Cattle	30 November 1971 – 11 October 2022	12,810
Orange Juice	1 February 1967 – 12 October 2022	13,952

The price series may have some linear trend which may cause the problem of non-stationarity. Since the linear trend is not part of nonlinear dynamics, it can be removed from the data. The traditional approach to deal with non-stationarity in the data is to differentiate the series; however, this would amplify the noise (Su et al. 2014; Medio and Gallo 1995) and hence hinder the application of nonlinear time series analysis methods. Medio and Gallo (1995) suggested that the non-stationarity caused by linear trends can be handled by regressing the series against time and detrend it. So, we focus our analysis on the detrended series obtained by regressing the original series against time.

5. RESULTS

5.1 Preliminary statistical analysis

The analysis starts with visual representations of price series for each commodity over their study periods. These visual representations serve as a tool to easily interpret and comprehend the fluctuations and trends in agricultural commodity prices during the study period. It is important to note that the term price series used henceforth specifically refers to the detrended price series. This denotes that the original data underwent a detrending process where linear trends were removed by regressing the original series with time. All the series included in the analysis underwent this detrending procedure. Consequently, it is possible for certain prices within the detrended series to display negative values. These negative prices arise as a direct outcome of removing the linear trends through the detrending process.

Figure 5.1.1 (a) shows the daily futures prices of corn from July 1, 1959 to October 12, 2022 (15,938 trading days). The series does not show an obvious trend since we already removed the linear trend present in the data. However, we observe that the price series is consistently volatile over time, and it exhibits persistent, aperiodic, random-looking oscillations. Figure 5.1.1 (b) demonstrates the empirical distribution of daily futures prices of corn over the study period, which is centered approximately around zero, but is positively skewed because the upper tail of the distribution is thicker than the lower tail. It also shows that the price series is not normally distributed.

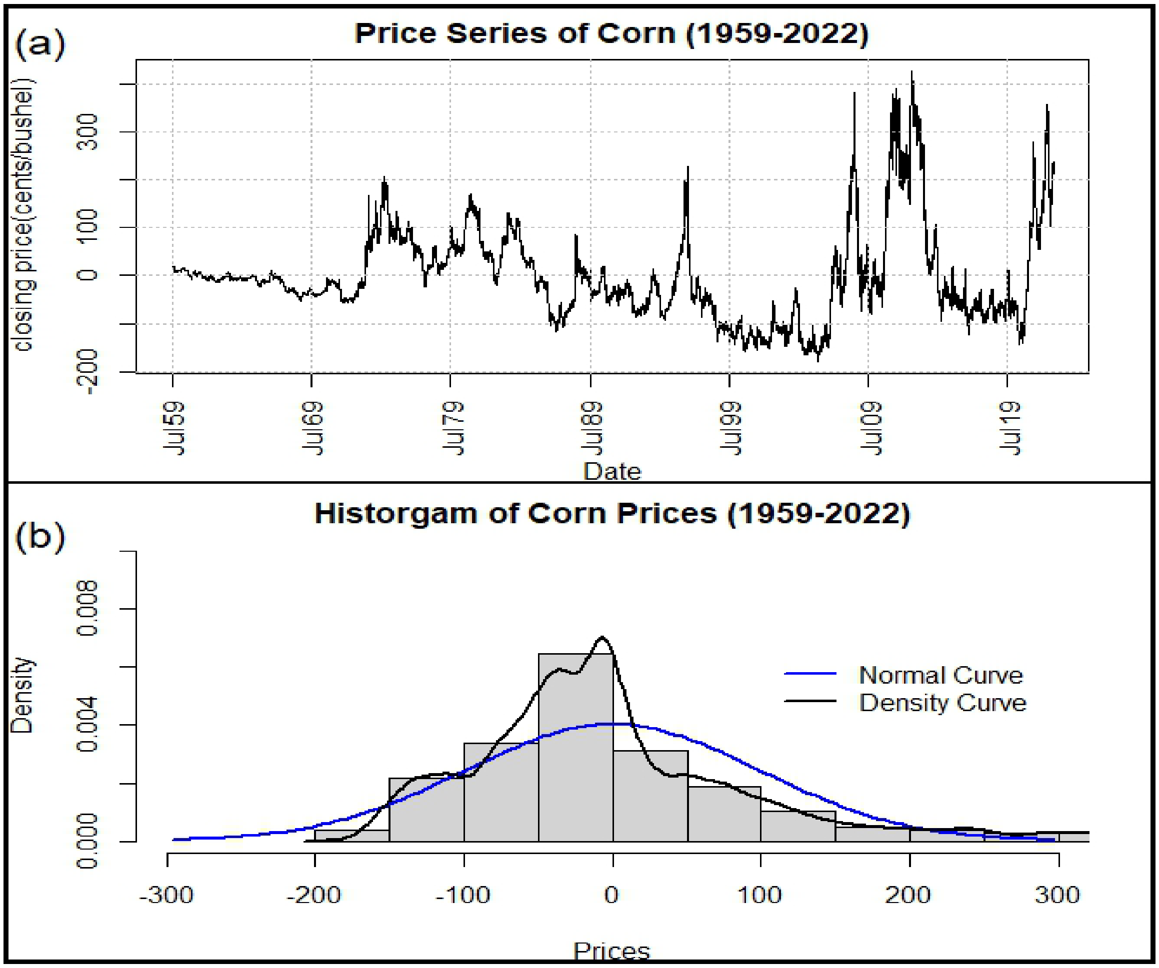

Figure 5.1.1: Daily price series and histogram of corn.

Figure 5.1.2(a) shows the daily futures prices of wheat from July 1, 1959 to October 13, 2022 (15,938 trading days). This demonstrates similar features to that of corn prices. Figure 5.1.2 (b) shows the empirical distribution of daily futures prices of wheat. This demonstrates that the wheat price is positively skewed, leptokurtic, and nonnormal. It also clearly shows the presence of multiple modes that suggests multimodal distribution to be appropriate. A distribution can be unimodal (having one peak), bimodal (having two peaks), trimodal (having three peaks), and so on. A multimodal distribution means that

the dataset has more than one peak or cluster of values. The histogram of wheat prices shows three distinct peaks suggesting the presence of multiple modes.

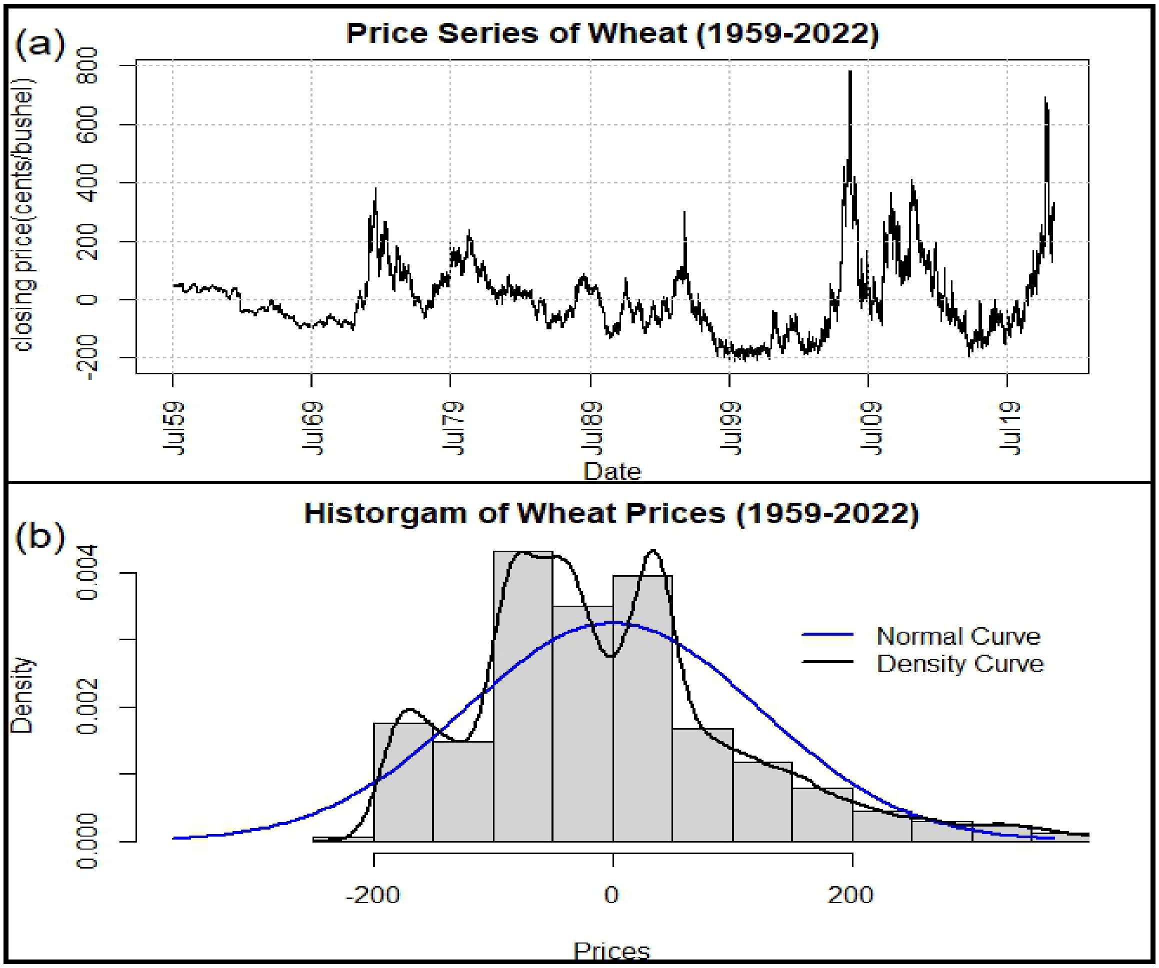

Figure 5.1.2: Daily price series and histogram of wheat.

Figure A.0.1 - Figure A.0.8 in Appendix A show daily futures prices and histograms for soybeans, sugar, coffee, cotton, feeder cattle, live cattle, hogs, and orange juice respectively. They all demonstrate similar features of high volatility, skewness, kurtosis, and nonnormal distributions. Most of these commodities also show the presence of multiple modes like wheat, thus suggesting multimodal distribution.

To confirm the results obtained from visual inspection, descriptive statistics of the detrended price series for the ten commodities were calculated and are shown in Table 5.1.1. The results demonstrate that commodity prices are highly skewed and leptokurtic (positive excess kurtosis), except for orange juice which is platykurtic (negative excess kurtosis). This skewness and kurtosis have resulted in the non-normal distribution of commodity prices since these measures should have been zero for a normal distribution. The normality assumption was formally tested using Jarque-Berra test and we were able to reject the null hypothesis of normal distribution. The results also demonstrate that the price of soybeans has the most variability (standard deviation = 205.16) and the price of sugar has the least variability (standard deviation = 6.16) among the commodities studied. The result from the Augmented Dickey-Fuller (ADF) test for stationarity rejected the null hypothesis of unit root in favor of stationarity in the price series of all commodities except feeder cattle. Note that the mean values of all commodity prices are zero, which can be attributed to the removal of the trend component in the price series. This was achieved by conducting a regression analysis with time as the independent variable. As a result of this detrending process, the mean values of the commodity prices were adjusted to zero.

Table 5.1.1: Summary statistics of detrended commodity prices

Commodity	Mean	Std. dev	skewness	Excess Kurtosis	JB-test	ADF
Corn	0	98.86	1.29	2.23	7681.3***	-3.41**
Soybeans	0	205.16	0.7	0.57	1525.5***	-3.98 ***
Wheat	0	122.7	1.26	2.88	9721.6***	-4.19 ***
Coffee	0	49.31	0.86	0.85	1934.1***	-3.29 *
Sugar	0	6.16	2.76	11.69	107023***	-4.33 ***
Cotton	0	18.68	1.55	6.26	32275***	-4.49 ***
Hogs	0	11.88	0.56	1.65	2369.5***	-6.56 ***
Live Cattle	0	12.85	0.88	2.11	4562.9***	-3.94 ***
Feeder Cattle	0	20.51	1.62	5.11	19518***	-2.92
Orange Juice	0	34.6	0.63	-0.28	965.1***	-3.66 **

Note: *** Significant at 1 % level, ** Significant at 5 % level, and * Significant at 10 % level.

The overall results suggest that the price series exhibit non-normality, skewness, and leptokurtosis. Leptokurtosis and the significant deviation from normality can be the signature of nonlinear dynamics, as suggested by Fang, Lai, and Lai (1994). Additionally, the presence of multiple modes in the price series suggests the occurrence of several different trajectories occurring frequently enough, which may signify the possibility of chaotic behavior, as proposed by Cromwell (2004). The next sections present the results of the NLTS analysis that will help answer the research question of whether the observed volatility in price series is caused by exogenous shocks affecting an otherwise stable market or by inherent nonlinear deterministic behavior of an unstable market.

5.2 Computational procedures

The analysis was conducted using the R programming language, leveraging several R packages to implement the proposed Nonlinear Time Series (NLTS) methods. Specifically, the following R packages were utilized: ‘RSSA’ (Golyandina, Korobeynikov, and Zhigljavsky 2018) for singular spectrum analysis; ‘tseriesChaos’ (Antonio 2019), ‘nonlinearTseries’ (Garcia 2022), and ‘pdc’ (Brandmaier 2015) for phase space reconstruction and surrogate data analysis. To facilitate the implementation of these packages and gain a comprehensive understanding of their usage, we referred to the R codes provided by Huffaker, Bittelli, and Rosa (2017).

It is important to note that surrogate data analysis, a computationally intensive task, required substantial computational resources. To address this, we utilized the high-performance clusters provided by the Holland Computing Center at the University of Nebraska. Parallel computing techniques were employed using the ‘parallel’ package in R to optimize the computation time.

On average, conducting surrogate data testing for each commodity consumed approximately 20 hours of computing time. The utilization of high-performance clusters and parallel computing methodologies enabled efficient execution of the analysis, ensuring timely completion of the extensive computational tasks involved.

5.3 Signal processing

We run singular spectral analysis to isolate cyclical components in the detrended price series of the commodities under study. SSA helps us distinguish potentially deterministic signals from unstructured noise in each dataset. Figure 5.3.1 shows the results of signal processing for price series of corn. The top row (a) displays individual price signal (depicted by blue curve) plotted against the corresponding detrended price series (depicted by black curve). The result indicates that the price signal follows the detrended price closely, implying that the structured variation makes up a larger proportion of the total variation in the detrended series of corn. The bottom row (b) showcases the unstructured variation, which represents the noise isolated in detrended price series of corn. The noise is calculated by finding the difference between the observed detrended price series and the corresponding signal for each day. Similar findings can be seen in Figure *B*.0.1 - Figure *B*.0.9 of Appendix B that shows the result of singular spectral analysis for other commodities considered in the study.

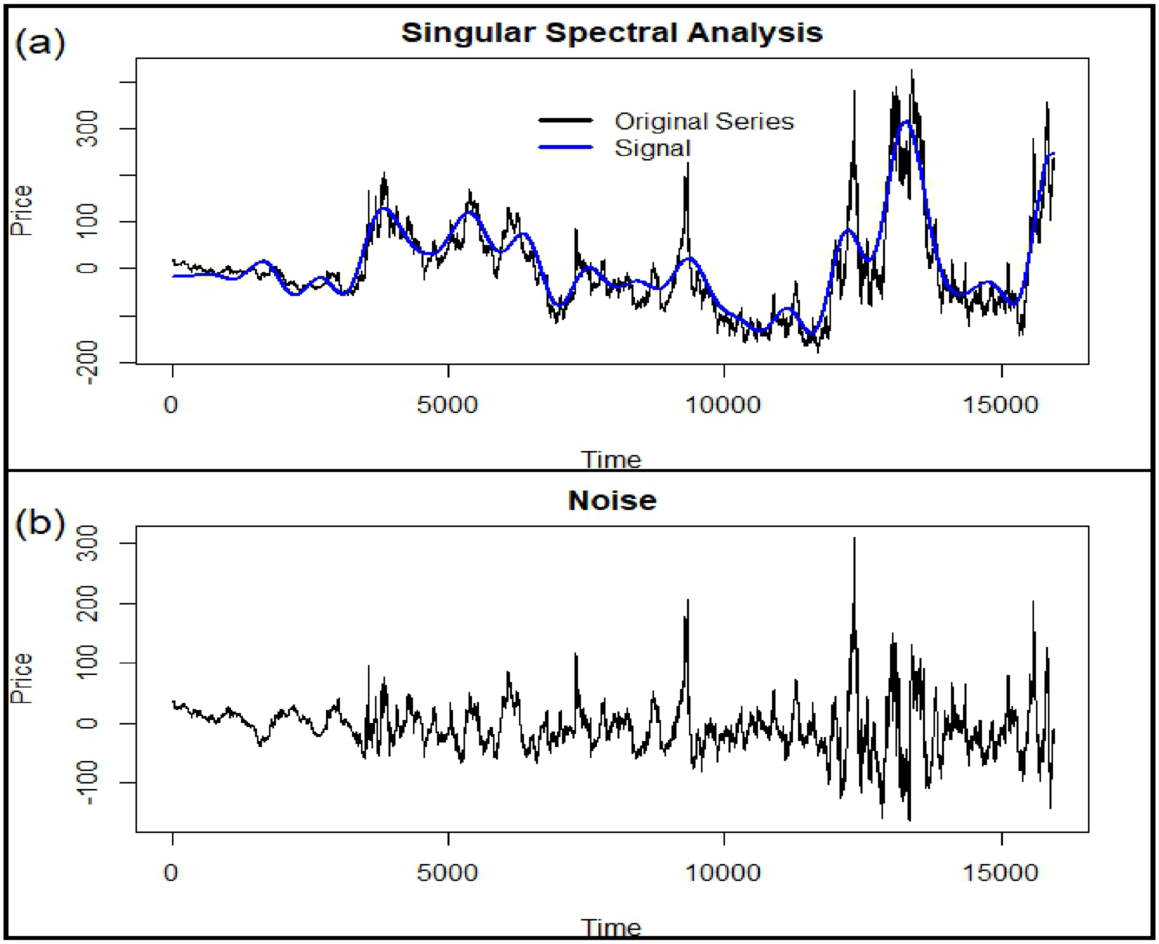

Figure 5.3.1: Signal processing for price series of corn.

Table 5.3.1 shows the relative strength of isolated signal and noise as a percentage of the total variability in detrended price series attributed to these components. The results indicate that a significant portion of the total variation, ranging from 76.24% for Hogs to 92.48% for live cattle, can be explained by the isolated signals. This suggests that the price signals account for the majority of variation in each detrended price series, supporting the use of the NLTS approach.

Table 5.3.1: Singular spectral analysis (SSA) relative strengths

Commodity	Signal Strength	Noise
Corn	83.16	16.84
Soybeans	83.47	16.53
Wheat	84.41	15.59
Coffee	84.00	16.00
Sugar	82.35	17.65
Cotton	79.79	20.21
Hogs	76.24	23.76
Live Cattle	92.48	7.52
Feeder Cattle	87.02	12.98
Orange Juice	76.95	23.05

5.4 Phase space reconstruction

SSA results showed large structured variation in each price series. Now it will be tested whether it comes from stable linear stochastic dynamics or endogenously unstable nonlinear deterministic market dynamics. We reconstruct the phase space depicting the dynamics of all commodities using time delay embedding method. We were able to successfully reconstruct a shadow attractor from the signal separated from the price series.

First, we used reconstructed attractors to test for stationarity of corresponding price signals using space time separation plots. Figure 5.4.1 (a) shows the estimation of embedding delay for hogs prices by identifying the first minimum of average mutual

information. This embedding delay is then used to generate Figure 5.4.1 (b) that depicts the space time separation plots for price series of hogs. The plot shows that the contours stop increasing and reach a saturation point within the first 630 days. Given that the analysis included 14,261 observations, this relatively short duration indicates that we have enough data to study dominant cycles in the series. In other words, the length of the time series is long enough to adequately capture dominant oscillatory patterns. This observation suggests that the price series of hogs is stationary because it spans a period much longer than the maximum time period over which significant changes or patterns occur in the system.

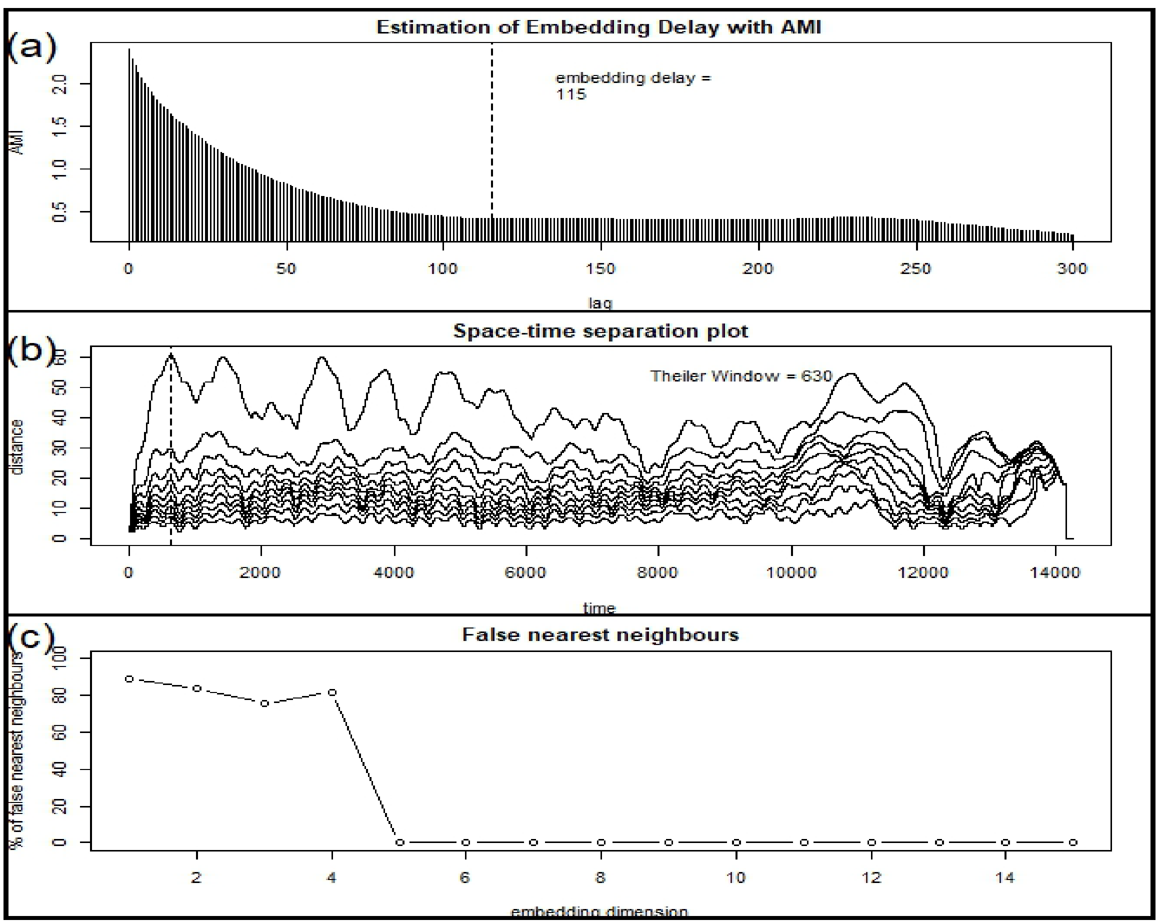

Figure 5.4.1: Estimation of embedding parameters for hogs.

Similarly, Figure 5.4.2 (a) depicts the determination of the embedding delay for corn prices as the first minimum on average mutual information graph. This delay is then used to construct Figure 5.4.2 (b), which illustrates the space time separation plots for the price series of corn. The plot shows that the contours stop increasing and reach a saturation point within the first 1,750 days. Given the availability of 15,938 observations for analysis, this relatively short duration indicates that we have enough data to investigate the dominant cycles within the series. As a result, this observation suggests that the price series of corn exhibits stationarity. Similar results can be suggested for other commodities studied by looking at their respective space time separation plots depicted in Figure C.0.1 - Figure C.0.8 of Appendix C The nonlinear stationarity required for the application of NLTS was satisfactorily satisfied.

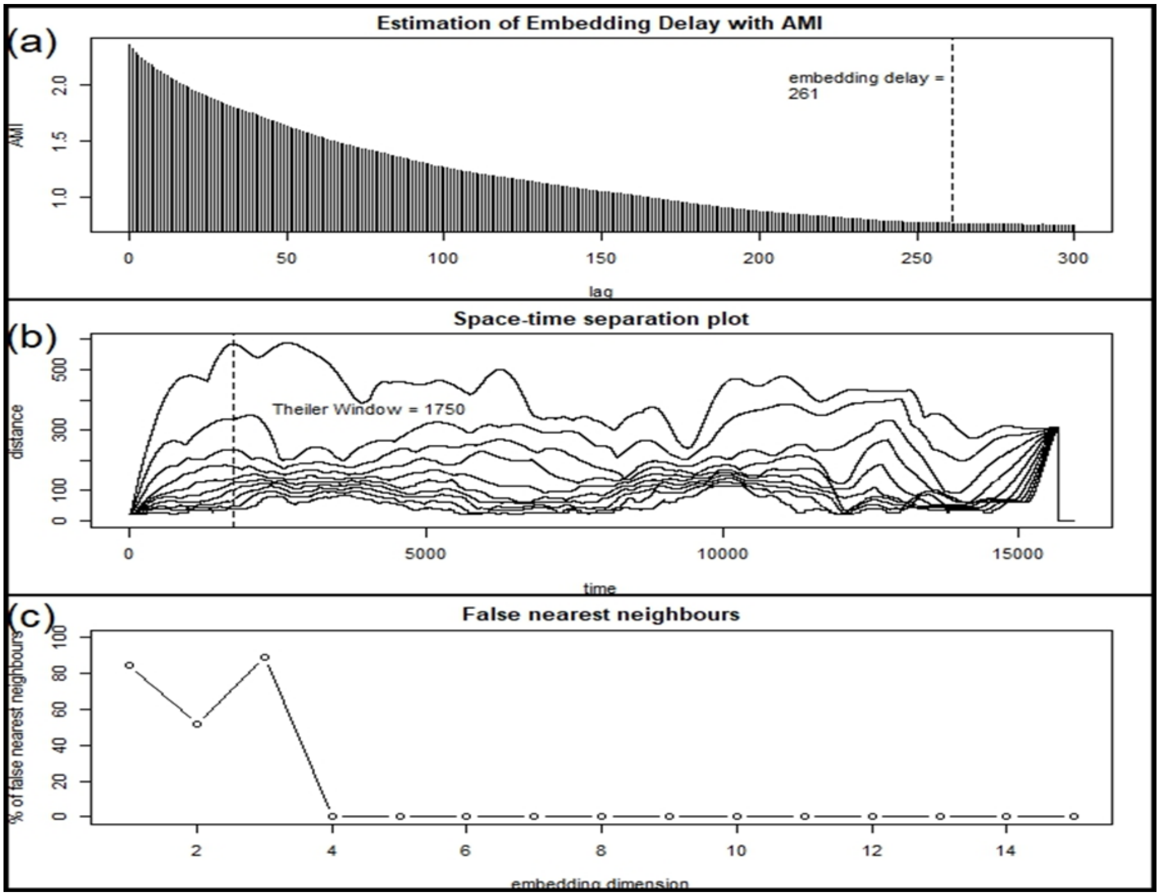

Figure 5.4.2: Estimation of embedding parameters for corn.

In order to enhance the reliability of our findings and complement the results obtained from the visual inspection of space-time separation plot for assessing nonstationarity, we integrated the use of the nonlinear cross prediction method. The comprehensive results of the nonlinear cross prediction are presented in Table D.0.1 of the Appendix D. This table includes the NSE (Nash-Sutcliffe efficiency) values of cross prediction for each commodity (1 → 2 means 1st segment was used as a learning segment to predict 2nd segment and so on). Using the information from this table, we constructed Table 5.4.1, which specifically highlights the lowest NSE values of cross prediction for each commodity. Notably, the table demonstrates that each segment cross predicts the other

with a high skill, as even the lowest prediction skill exceeds the threshold NSE value of 0.65. Additionally, it is observed that the NSE values do not deteriorate with the increase in distance between learning and testing segments, across all the commodities studied. This observation confirms that the price series exhibit stationarity within the context of Nonlinear Time Series Analysis (NLTS).

Table 5.4.1: Lowest NSE values of cross prediction

Commodities	Lowest NSE
Corn	0.93651
Soybeans	0.97174
Wheat	0.98808
Coffee	0.99012
Sugar	0.99238
Cotton	0.98598
Hogs	0.99638
Live Cattle	0.97219
Feeder Cattle	0.97892
Orange Juice	0.98219

After the nonlinear stationarity assumption was satisfied, we proceeded to investigate the geometry of phase space reconstructed from the price signal of each commodity. The estimates of embedding delay (Figure 5.4.1 (a) and Figure 5.4.2 (a)) and Theiler window (Figure 5.4.1 (b)) and Figure 5.4.2 (b)) were used to generate Figure 5.4.1 (c) and Figure

5.4.2 (c) , which illustrates the false nearest neighbor method of estimating embedding dimensions for hogs and corn respectively. Similar approaches were used to estimate the embedding parameters for other commodities which are shown in Figure C.0.1 to Figure C.0.8 of Appendix C. The estimated embedding delay, Theiler window, and embedding dimensions for all the commodities are summarized in Table 5.4.2. All reconstructed shadow attractors required 3-6 embedding dimensions for phase space reconstruction, indicating the presence of low-dimensional attractor. The embedding dimensions, which depend on the complexity of the underlying system, determine the number of variables or dimensions used to represent the reconstructed space. A low-dimensional attractor indicates that the system's dynamics can be effectively captured and represented by a small number of essential variables. Consequently, the future states of the system can be reasonably predicted using a relatively small set of variables.

Table 5.4.2: Embedding Parameters for Phase Space Reconstruction

Commodity	Time Delay (d)	Theiler Window (tw)	Embedding Dimension (m)
Corn	261	1,750	4
Soybeans	292	2,650	4
Wheat	403	1,900	4
Coffee	220	930	5
Sugar	245	1,020	6
Cotton	263	500	5
Hogs	115	630	5
Live Cattle	312	1,330	4
Feeder Cattle	283	1,360	4
Orange Juice	246	2,230	3

We used these embedding parameters to reconstruct the phase space of the price signal for each commodity. The top and bottom views of the 3-d projection of the shadow price attractor for corn are shown in Figure 5.4.3 (a) and Figure 5.4.3 (b) respectively. The attractor exhibits noticeable geometric regularity (cyclical appearance) with aperiodic oscillations. The outer orbits are due to a lower frequency cycle or nonlinear trend cycles and the inner tighter cycles are due to higher frequency oscillations isolated by SSA.

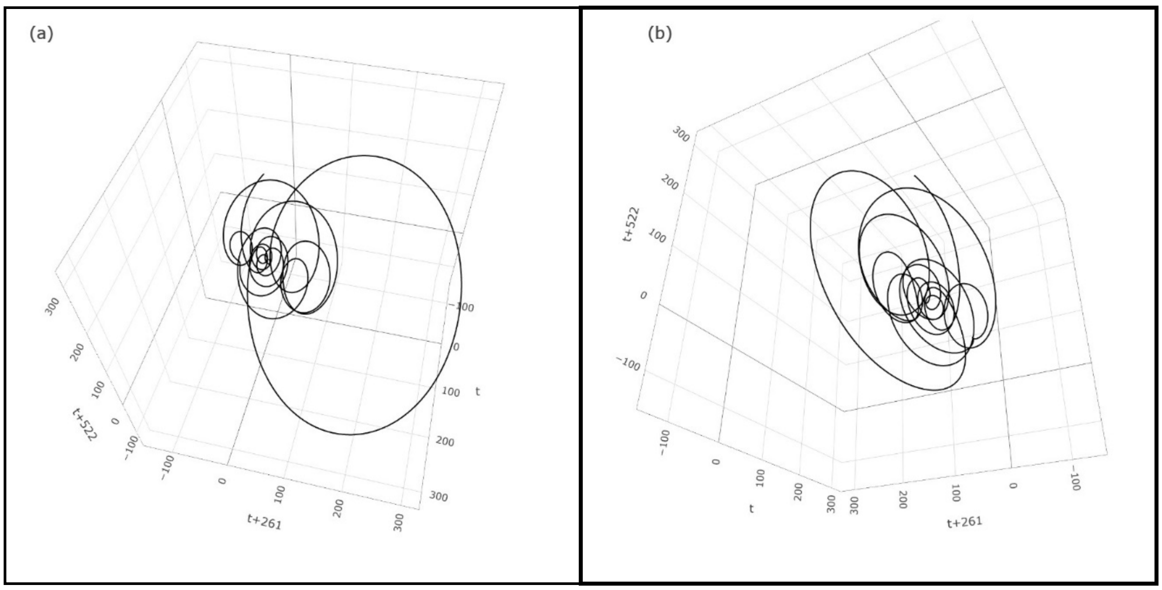

Figure 5.4.3: Shadow phase space attractor reconstructed from the price signal of corn – top view (a) and bottom view (b).

Figure 5.4.4 (a) and Figure 5.4.4 (b) shows the top and bottom views of the 3-d projection of shadow price attractor for hogs. This shadow attractor also exhibits wide-swinging outer oscillations reflecting lower frequency cycles and tighter interior cycles showing higher-frequency oscillations. The 3-d projection of shadow price attractor for other

commodities are depicted in Figure D.0.1 - Figure D.0.8 of Appendix D. All of them show noticeable geometric regularity with aperiodic oscillations.

If we can reconstruct an attractor with visual geometric regularity in a subset of phase space, this is the preliminary evidence of the signal being generated by deterministic nonlinear dynamics. Signals not generated by nonlinear deterministic dynamics would have a shadow phase space with randomly scattered points all over the phase space.

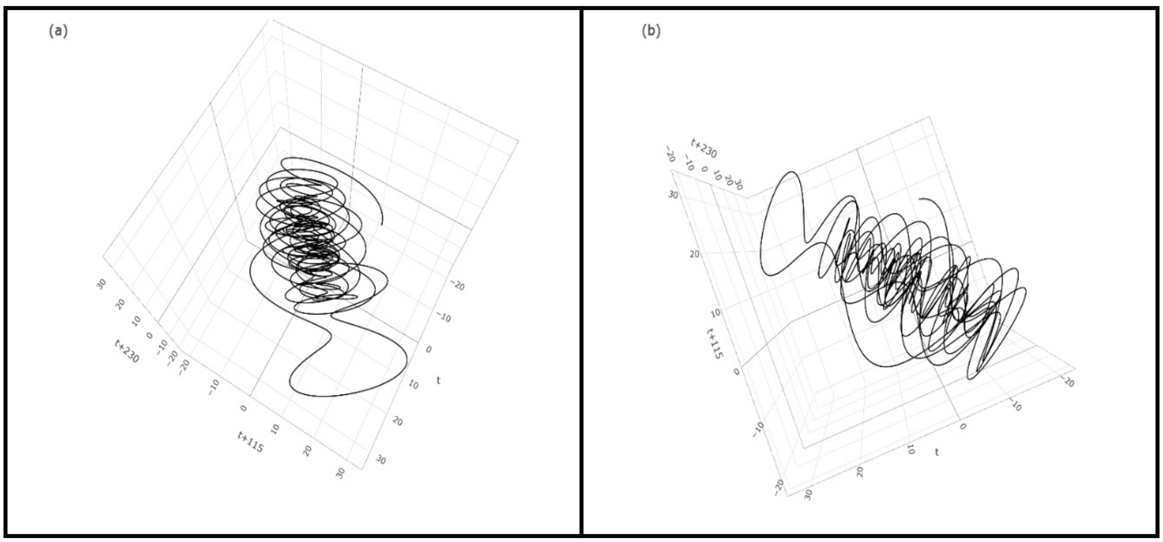

Figure 5.4.4: Shadow phase space attractor reconstructed from the price signal of hogs – top view (a) and bottom view (b).

Additionally, the nonlinear predictive skills of all the attractors are almost perfect since the value Nash-Sutcliffe Coefficient of Efficiency are very close to 1 (Table 5.4.3). This suggests that the price series is highly deterministic. Moreover, the permutation entropies calculated for the attractors of the commodities ranges from 0.23141 for live cattle to

0.29569 for hogs (Table 5.4.3). These entropies values are very low[15] that implies the price series is highly structured and predictable.

Table 5.4.3: NSE and Permutation entropy computed from price signal.

Commodity	Nash-Sutcliffe Coefficient of Efficiency (NSE)	Permutation entropy (PE)
Corn	0.99998	0.24620
Soybeans	0.99998	0.24709
Wheat	0.99998	0.23490
Coffee	0.99996	0.24413
Sugar	0.99998	0.25207
Cotton	0.99996	0.24841
Hogs	0.99977	0.29569
Live cattle	0.99998	0.23141
Feeder cattle	0.99998	0.24700
Orange juice	0.99997	0.24942

These results coupled with the low dimensionality of the attractors offer the preliminary evidence of the signal being generated by low-dimensional nonlinear deterministic dynamics.

[15] Note that the permutation entropy values range from 0 to log (m!). The values closer to zero represents perfectly predictable and highly structured system, whereas higher values indicate IID noise.

5.5 Surrogate data testing

Even though the phase space reconstruction provided initial indication of low-dimensional nonlinear deterministic dynamics, there remains a possibility that the apparent visual structure might have been falsely generated by random dynamic. We will provide rigorous statistical test using surrogate data analysis to rule out this possibility. We first tested the null hypothesis that the attractor's noticeable regularity is due to a linear stochastic market dynamics consistent with exogenous price volatility using AAFT surrogates. A confidence level of 95 % was specified that generated 99 AAFT surrogates for each price series.

The results of AAFT surrogate test are summarized in Table 5.5.1. When nonlinear predictive skill was used as discriminating statistics, the NSE obtained for signals isolated from each price series exceeded the corresponding upper-threshold value obtained from surrogate attractors, which suggests that the price signals are more predictable than the surrogates. The only exception was hogs, whose NSE (0.99977) for the price signal was below the upper-threshold value of 0.99987 obtained from surrogate attractors.

Table 5.5.1: AAFT surrogate tests.

	Signal	Surrogate (Low)	Surrogate (High)	Surrogate (Mean)	H_0
Corn					
NSE	0.99998	0.99849	0.99928	0.99897 (0.00023)	Reject
PE	0.2462	0.71730	0.77280	0.74900 (0.01650)	Reject
Soybeans					
NSE	0.99998	0.99960	0.99970	0.9996 (0.00010)	Reject
PE	0.24709	0.63014	0.70393	0.6636 (0.01982)	Reject
Wheat					
NSE	0.99998	0.99871	0.99936	0.99912 (0.00019)	Reject
PE	0.23490	0.69941	0.75958	0.72647 (0.01664)	Reject
Coffee					
NSE	0.99996	0.99823	0.99925	0.99887 (0.00030)	Reject
PE	0.24413	0.67066	0.73382	0.70430 (0.01811)	Reject
Sugar					
NSE	0.99998	0.99990	0.99994	0.99993 (0.00001)	Reject
PE	0.25207	0.36914	0.43775	0.40206 (0.02233)	Reject
Cotton					
NSE	0.99996	0.99945	0.99976	0.99965 (0.00010)	Reject
PE	0.24841	0.55769	0.60705	0.58042 (0.01664)	Reject
Hogs					
NSE	0.99977	0.99968	0.99987	0.99977 (0.00006)	Fail to Reject
PE	0.29569	0.20712	0.22191	0.21401 (0.00458)	Fail to Reject
Live Cattle					
NSE	0.99998	0.99899	0.99960	0.99940 (0.00019)	Reject
PE	0.23141	0.67394	0.73808	0.70708 (0.01796)	Reject
Feeder Cattle					
NSE	0.99998	0.99922	0.99971	0.99953 (0.00018)	Reject
PE	0.24700	0.61421	0.68780	0.64808 (0.02181)	Reject
Orange Juice					
NSE	0.99997	0.99987	0.99993	0.99991 (0.00002)	Reject
PE	0.24942	0.45509	0.50941	0.48055 (0.01783)	Reject

Similarly, when permutation entropy was used as discriminating statistics, the H-values (entropy) were below the lower-threshold value obtained from the surrogate attractors for price signals of each commodity, which suggests that the price signal are statistically more predictable and structured compared to the surrogates. The only exception was

again hogs, whose H-value (0.29569) obtained for the shadow attractor reconstructed from the price signal was higher than the lower-threshold value of 0.20712 obtained from surrogate attractors.

Given the AAFT surrogate test results, we were able to reject the null hypothesis that the linear stochastic dynamics are most likely the source of observed geometric regularity in shadow attractor constructed from the price signal for all commodities except hog. This supported the alternative hypothesis that the observed regularity was indeed due to the low-dimensional nonlinear deterministic dynamics. It provided evidence that the observed volatility in the price series is not due to exogenous shocks but is rather endogenously generated due to underlying nonlinear deterministic dynamics.

Further, we used PPS surrogates to test the null hypothesis that the aperiodic oscillations observed in the shadow attractor were likely generated by random shifting of periodic orbits, which are typical characteristics of noisy linear dynamics. A confidence level of 95 % was specified that generated 99 PPS surrogates for each price series.

The results of test using PPS surrogates are presented in Table 5.5.2. The NSE computed from the price signal of all the commodities are higher than the corresponding upper-threshold value obtained from the surrogate data indicating that the price signals are statistically more predictable than the surrogates. Similarly, the values of permutation entropies (H-value) calculated for each price signals are lower than the corresponding lower-threshold value calculated from their surrogates suggesting that the price signals are statistically more predictable and structured than the surrogates. These results reject the null hypothesis that the aperiodic oscillations observed in the shadow attractors are randomly generated. It corroborates the evidence from AAFT surrogate tests that the

price series are generated by low-dimensional nonlinear deterministic dynamics. Moreover, this suggests the presence of determinism in the price series beyond the periodic behavior and provides support for chaotic dynamics.

Table 5.5.2: PPS surrogate tests.

	Signal	Surrogate (Low)	Surrogate (High)	Surrogate (Mean)	H_0
Corn					
NSE	0.99998	-0.25971	-0.22046	-0.24048 (0.01182)	Reject
PE	0.24620	0.98326	0.98371	0.98350 (0.00014)	Reject
Soybeans					
NSE	0.99998	-0.25462	-0.21914	-0.23797(0.01125)	Reject
PE	0.24701	0.98320	0.98331	0.98344(0.00013)	Reject
Wheat					
NSE	0.99998	-0.25794	-0.22153	-0.24016 (0.01078)	Reject
PE	0.23490	0.98296	0.98343	0.98323 (0.00014)	Reject
Coffee					
NSE	0.99996	-0.20261	-0.17122	-0.18646 (0.00992)	Reject
PE	0.24413	0.98237	0.98272	0.98263 (0.00014)	Reject
Sugar					
NSE	0.99998	-0.16792	-0.14026	-0.15464 (0.00786)	Reject
PE	0.25207	0.98311	0.98355	0.98337 (0.00013)	Reject
Cotton					
NSE	0.99996	-0.20095	-0.16868	-0.18576 (0.01004)	Reject
PE	0.24841	0.98326	0.98370	0.98351 (0.00013)	Reject
Hogs					
NSE	0.99977	-0.19974	-0.17259	-0.18597 (0.00843)	Reject
PE	0.29569	0.98301	0.98349	0.98328 (0.00014)	Reject
Live Cattle					
NSE	0.99998	-0.25994	-0.22385	-0.24096 (0.01191)	Reject
PE	0.23141	0.98269	0.98319	0.98297 (0.00014)	Reject
Feeder Cattle					
NSE	0.99998	-0.26607	-0.21220	-0.23666 (0.01454)	Reject
PE	0.24700	0.98236	0.98280	0.98261 (0.00014)	Reject
Orange Juice					
NSE	0.99997	-0.34812	-0.29713	-0.32497 (0.01491)	Reject
PE	0.24942	0.98267	0.98311	0.98291 (0.00013)	Reject

Overall, surrogate data testing soundly rejected the null hypothesis of linear stochastic dynamics in favor of low-dimensional nonlinear deterministic dynamics for all commodities prices except hogs. The result for price series of hogs was inconclusive because we were able to reject the null hypothesis of linear stochastic dynamics using PPS surrogates but failed to reject the null hypothesis when AAFT surrogates was used. In sum, results initially showed evidence of low-dimensional nonlinear deterministic dynamics based on signal processing and phase space reconstruction. This initial evidence was further supported by the results of surrogate tests for all commodities but hogs. Therefore, it can be claimed that the observed variation in commodity prices is most likely driven by low-dimensional nonlinear deterministic dynamics (chaotic dynamics)[16].

[16] It is important to highlight the distinction between nonlinear deterministic dynamics and chaos, particularly in theoretical contexts. While chaotic systems are inherently nonlinear and deterministic, not all nonlinear deterministic systems are chaotic. For example, as explained in the background section, the logistic map is a nonlinear deterministic system that exhibits chaotic behavior only within a specific range of parameters ($r > 3.57$). However, in empirical application of NLTS methods aimed at detecting the presence of low-dimensional nonlinear determinism, the terms chaos and nonlinear determinism are frequently used interchangeably.

6. CONCLUSION

Whether the observed price volatility is due to exogenous random shock to the otherwise stable market or endogenously created due to an inherently unstable market is an import issue. Understanding the underlying market dynamics driving commodity prices is important in policymaking, forecasting, production, storage, investment, risk management, and hedging decisions.

The nonlinear time series analysis method is useful to investigate if the underlying dynamics is linear stochastic consistent with exogenous volatility or low-dimensional nonlinear deterministic consistent with endogenous volatility. The method primarily relies on phase space reconstruction techniques, but these techniques may not provide a clear representation of the time series when it contains significant noise, which is a norm in economic time series. Singular spectral analysis was therefore applied first because it has the capability to construct a noise-free series while preserving the deterministic dynamics of the original series. This allows for the separation of structured variations, which are expected to be well-approximated by real-world models, from unstructured variations in the observed price series. Phase space reconstruction offers a geometric representation of real-world dynamics that should be reproduced by theoretical models. The estimated embedding dimensions indicate the minimum system dimensionality, i.e. the minimum number of interacting variables in the system that are needed to replicate the diagnosed market dynamics.

We employ nonlinear time series analysis approaches to test endogenous market dynamics in the price of agricultural commodities. We used the daily futures prices for corn, soybeans, wheat, cotton, coffee, sugar, live cattle, feeder cattle, hogs, and orange

juice in our analysis. The empirical results confirmed low-dimensional nonlinear deterministic dynamical behavior in the price series of corn, soybeans, wheat, cotton, coffee, sugar, live cattle, feeder cattle, and orange juice. The result was inconclusive for the price series of hogs.

Singular spectral analysis was able to identify strong structured variation in the observed price series. The signal extracted using the signal processing method was able to explain a significant amount of the variation in the observed price series. A shadow phase space was reconstructed using the signal, providing the geometric picture of the real-world market attractor. The reconstructed phase space for all commodities showed geometric regularity with aperiodic oscillations. The estimated embedding dimensions for all commodities were between 3 – 6, indicating a low-dimensional attractor. This provided preliminary evidence that the price series are generated by low-dimensional nonlinear deterministic dynamics.

Surrogate data testing was employed to test whether the preliminary evidence of low-dimensional nonlinear deterministic dynamics depicted in the phase space reconstruction is fortuitously generated by a linear stochastic process. AAFT and PPS surrogates were generated to test the null hypothesis of a linear stochastic process. The surrogate data testing soundly rejected the null hypothesis that the noticeable geometric regularity in the attractors is mimicked by linear stochastic dynamics for all commodities except hogs. In the case of hog prices, the null hypothesis was rejected while using PPS surrogates, but we failed to reject it using AAFT surrogates, leading to inconclusive results.

Thus, our empirical diagnostic provides strong evidence that the observed volatility in agricultural commodity prices is due to an inherently unstable market governed by low-

dimensional nonlinear deterministic dynamics. The evidence of deterministic structure in commodity prices raises questions about the relevance of the Efficient Market Hypothesis. It also challenges the assumption that all price systems lead to a state of equilibrium. Our findings lend support to Mandelbrot's view that the financial system is not a linear, continuous, and rational machine. Instead, the markets are highly volatile and carry greater risks than what traditional theories assume. Furthermore, markets are inherently unstable, and the occurrence of bubbles is an inevitable part of market dynamics (Mandelbrot and Hudson 2005).

These results have several practical implications. First, the nature of the attractor precludes medium- and long-term forecasts. The sensitive dependence to initial conditions is an important property of low-dimensional nonlinear deterministic dynamics (or chaos), which makes long-term forecasts unreliable. However, the deterministic nature of the system allows us to improve short-term forecasts using nonlinear or chaotic prediction algorithms such as neural networks. This also explains the empirical findings of deteriorating forecasting ability with increasing forecasting horizon (Xu 2020; Ouyang, Wei, and Wu 2019; Irwin and Good 2015a; 2015b) as well as the superior performance of machine learning algorithms over traditional linear and nonlinear stochastic algorithms (Kohzadi et al. 1996; Jha and Sinha 2013; Ouyang, Wei, and Wu 2019).

Second, these results support the idea of government intervention to stabilize the market. The evidence of low-dimensional nonlinear deterministic dynamics in commodity prices suggests that the market is not self-correcting. Instead, it suggests that the market is endogenously unstable and thus we cannot rely on it to stabilize prices and reduce

volatility. Therefore, the government should opt for policy measures to stabilize volatility due to an inherently unstable market. Moreover, due to the nature of nonlinear deterministic dynamics, policy measures intended to stabilize prices might have unprecedented consequences. The sensitivity to the initial condition property of low-dimensional nonlinear deterministic systems may cause very similar policies to have drastically different effects. Thus, policies must be chosen very cautiously.

Third, findings from this research allow for data-driven modeling. It provides a benchmark for future model-driven price modeling since it includes a geometric picture of price dynamics that conceptual models should reproduce. Additionally, it provides an estimation of the minimum model dimensionality required to accurately capture these dynamics. We can create mathematical models that can mimic real price movements. These mathematical models may not exactly replicate real price movements, but still behave statistically in a similar manner and can help better understand the complexity of the real world. The knowledge of the existence of nonlinear deterministic dynamics in market prices, as Mandelbrot puts it, may not be used to predict prices with absolute precision; however, the ability to imitate reality is a form of understanding that can provide immediate insights into how markets work. While it may not guarantee wealth, this type of understanding could help people avoid losing money as much as they do due to their underestimation of risk (Mandelbrot and Hudson 2005).

In conclusion, it is important to acknowledge that the nonlinear time series (NLTS) analysis method used in this study may face limitations. For instance, price data may not be governed by a low-dimensional attractor, noisy data could hinder the detection of an existing attractor, or limited data may not sufficiently sample a real-world attractor. In

cases where NLTS fails, stochastic approaches can be a viable alternative. However, it is crucial to test for low-dimensional nonlinear deterministic dynamics in the observed data before assuming a stochastic structure, as this can provide insights into the complexity of the real-world system.

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APPENDICES

Appendix A: Price series and histogram of commodities studied.

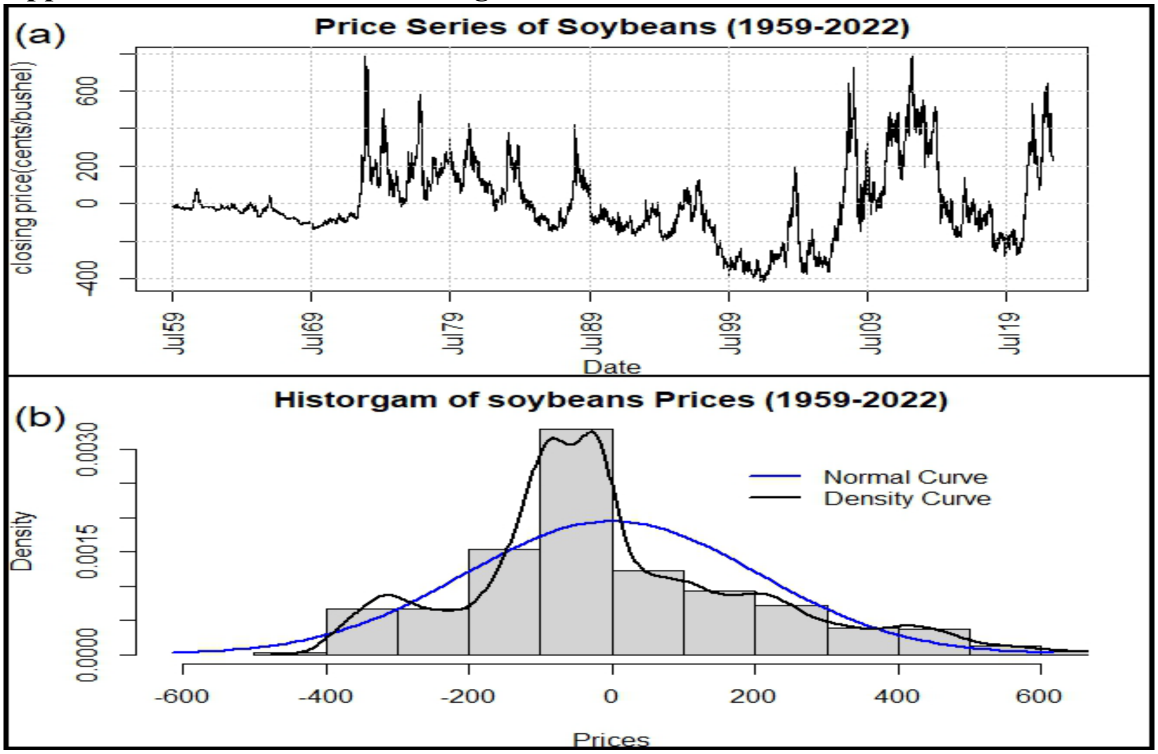

Figure A.0.1: Daily price series and histogram of soybeans.

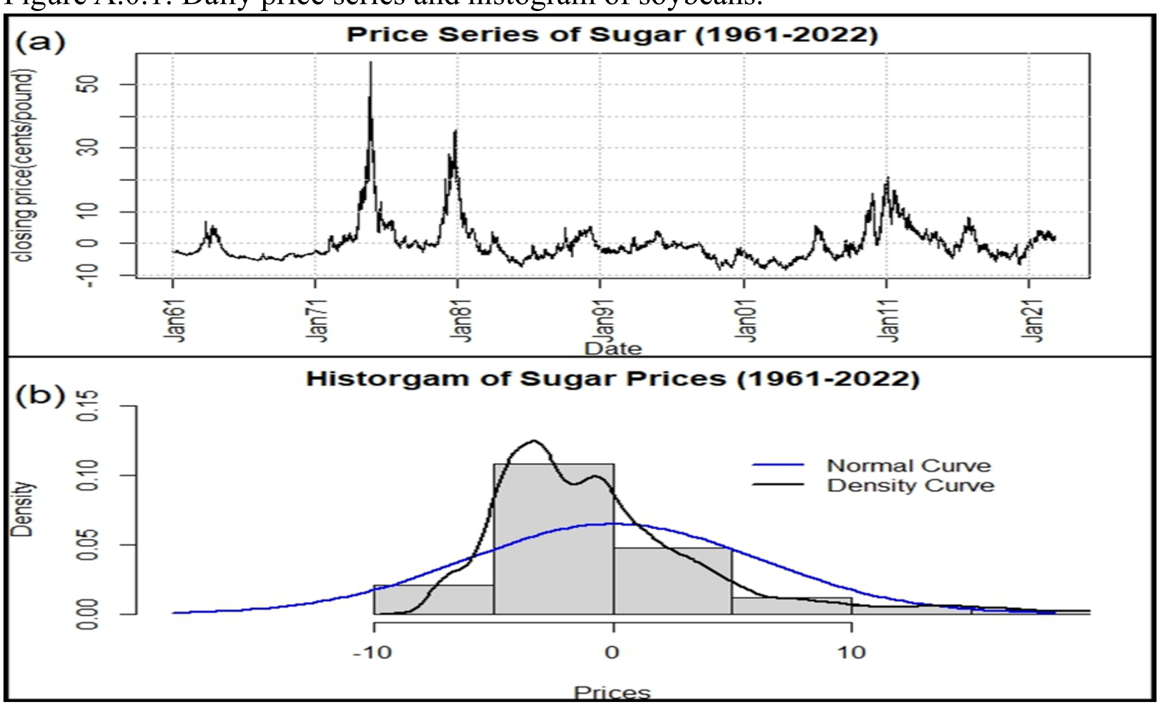

Figure A.0.2: Daily price series and histogram of sugar.

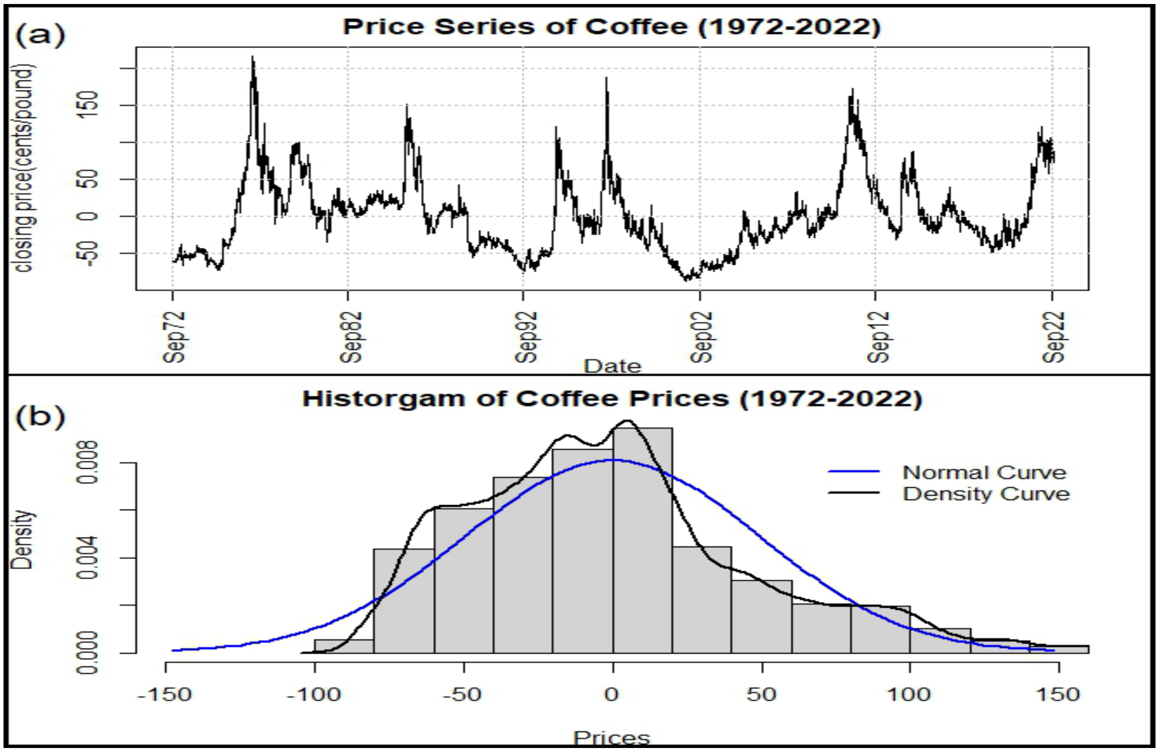

Figure A.0.3: Daily price series and histogram of coffee.

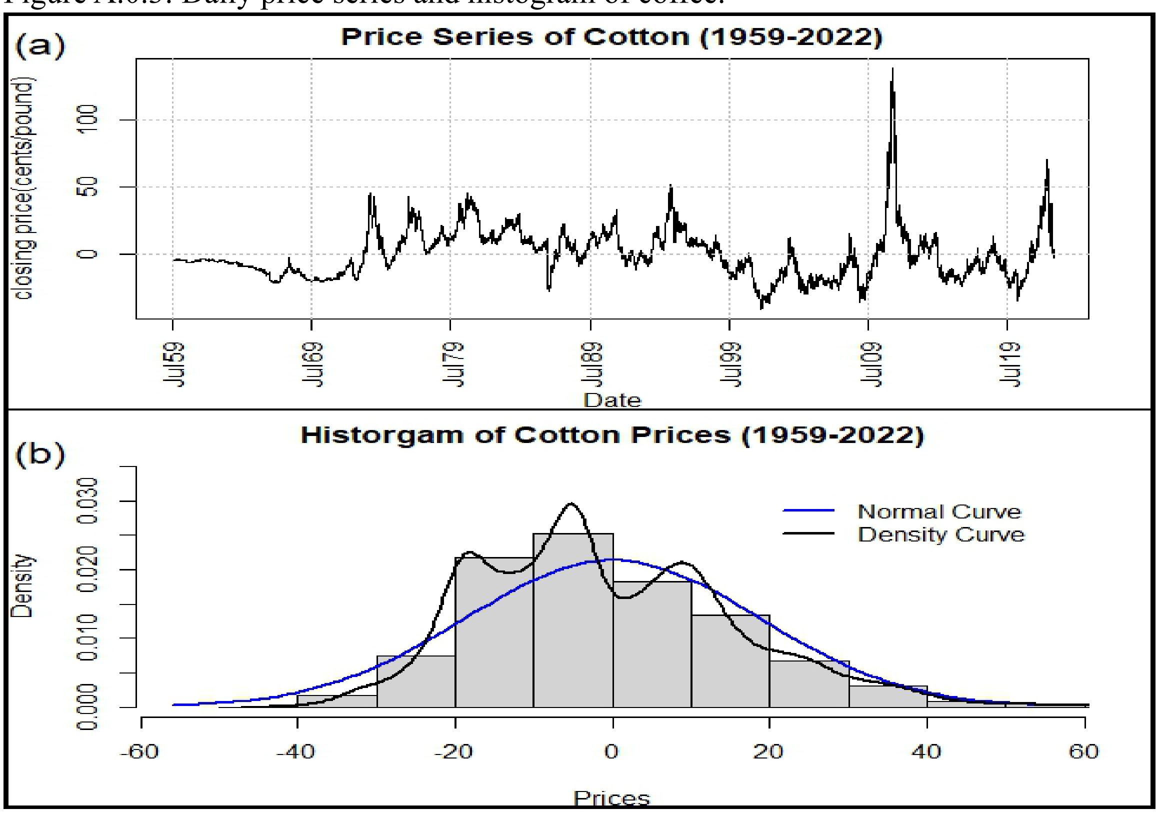

Figure A.0.4: Daily price series and histogram of cotton.

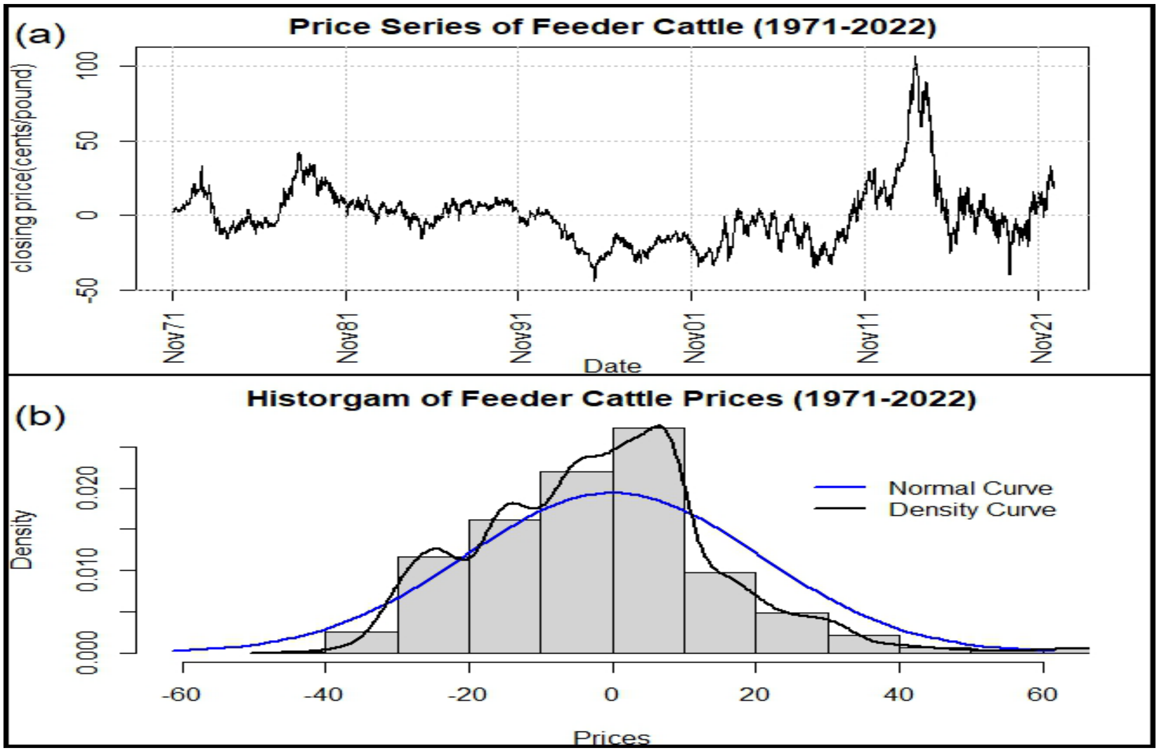

Figure A.0.5: Daily price series and histogram of feeder cattle.

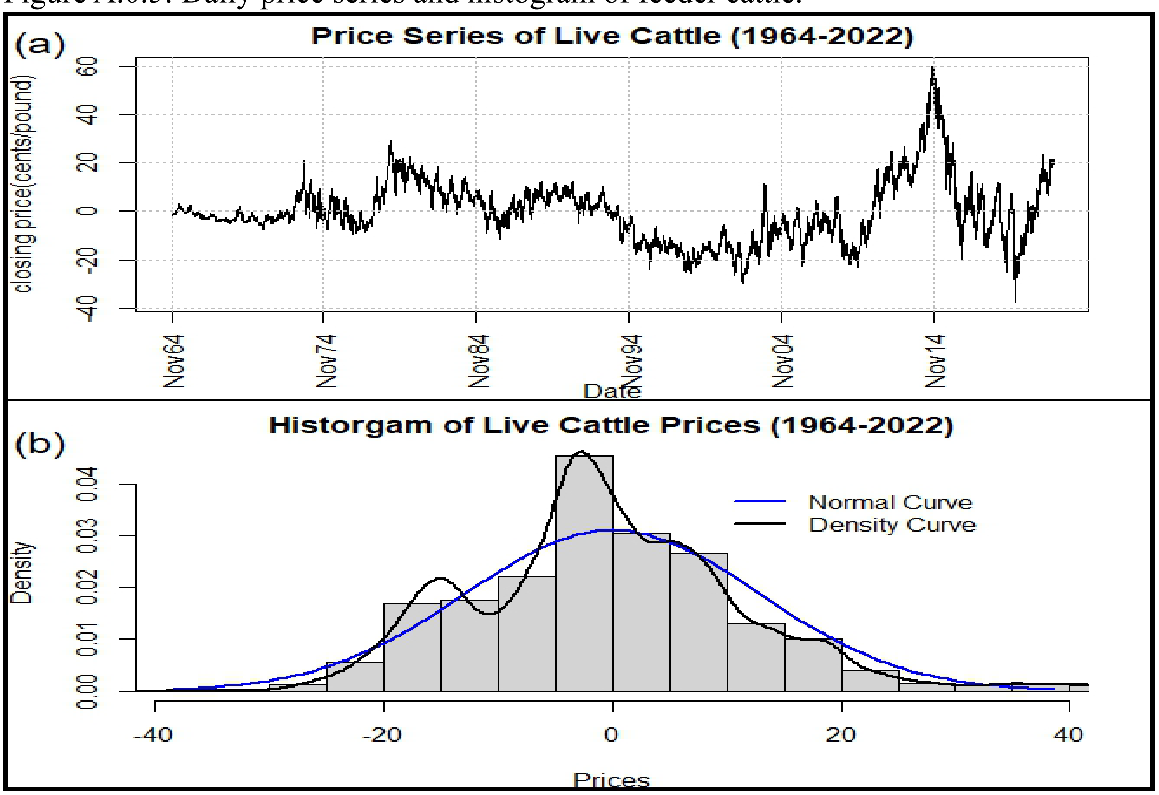

Figure A.0.6: Daily price series and histogram of live cattle.

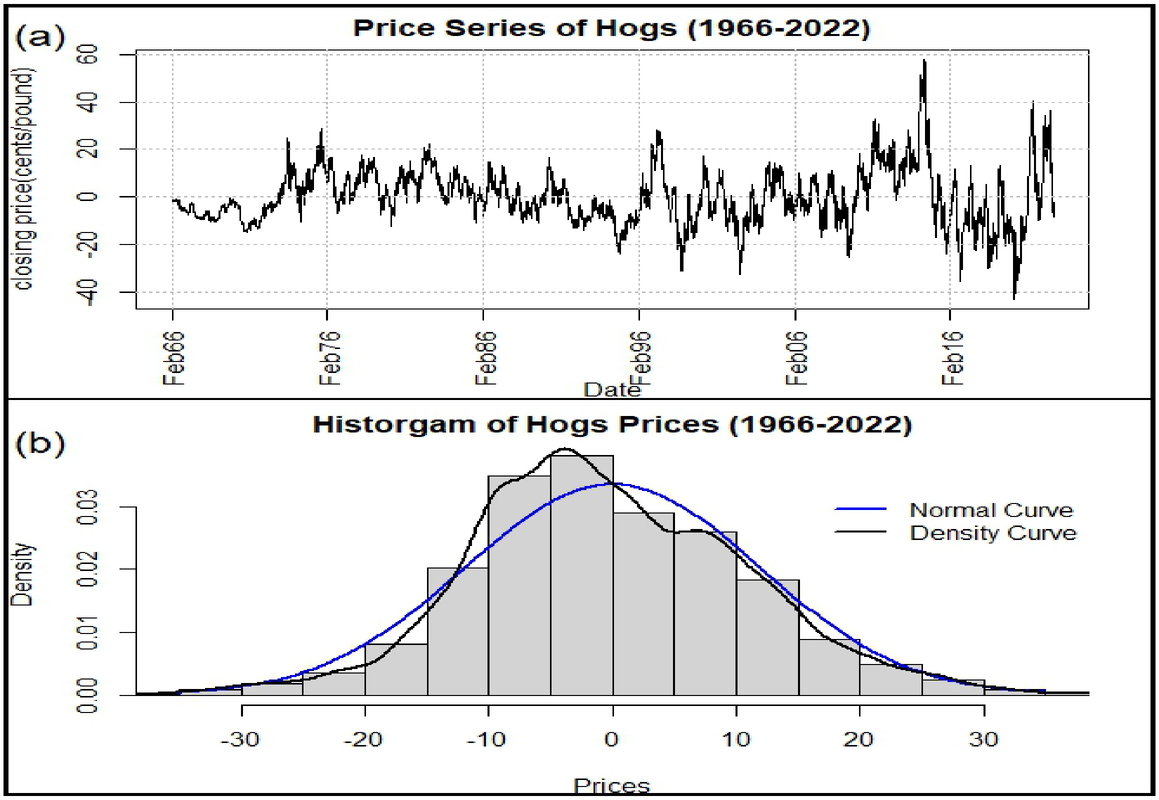

Figure A.0.7: Daily price series and histogram of hogs.

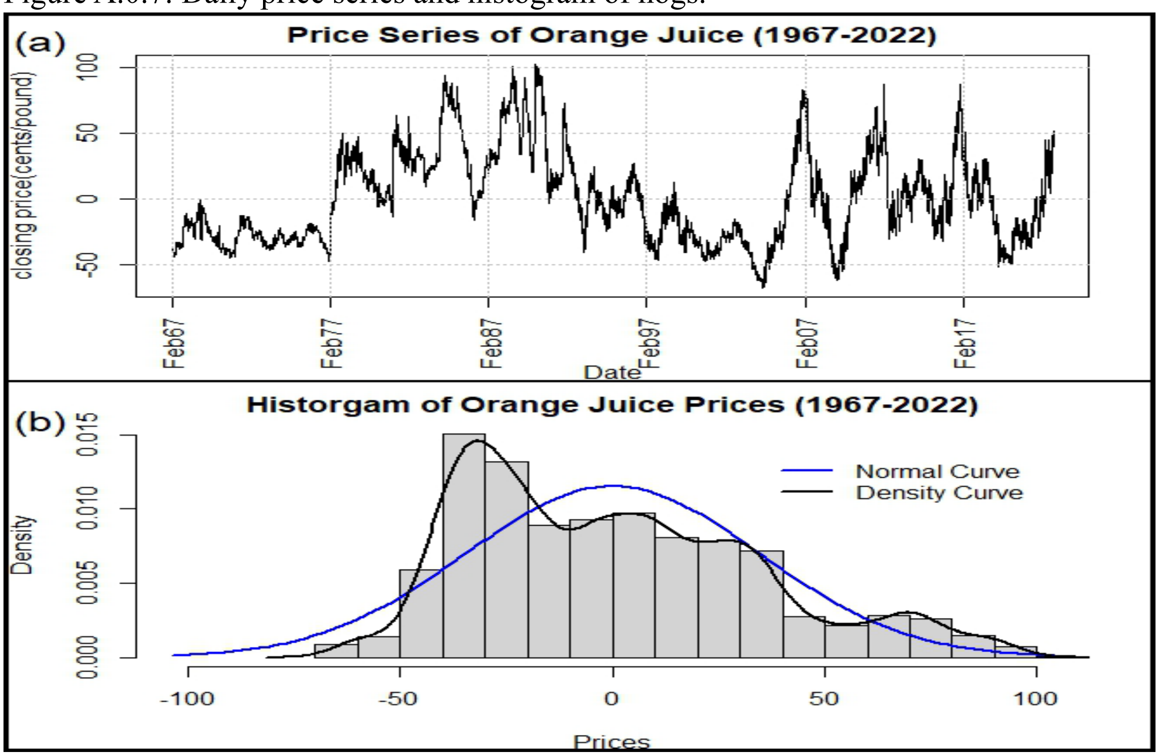

Figure A.0.8: Daily price series and histogram of orange juice.

Appendix B: Signal processing of detrended price series.

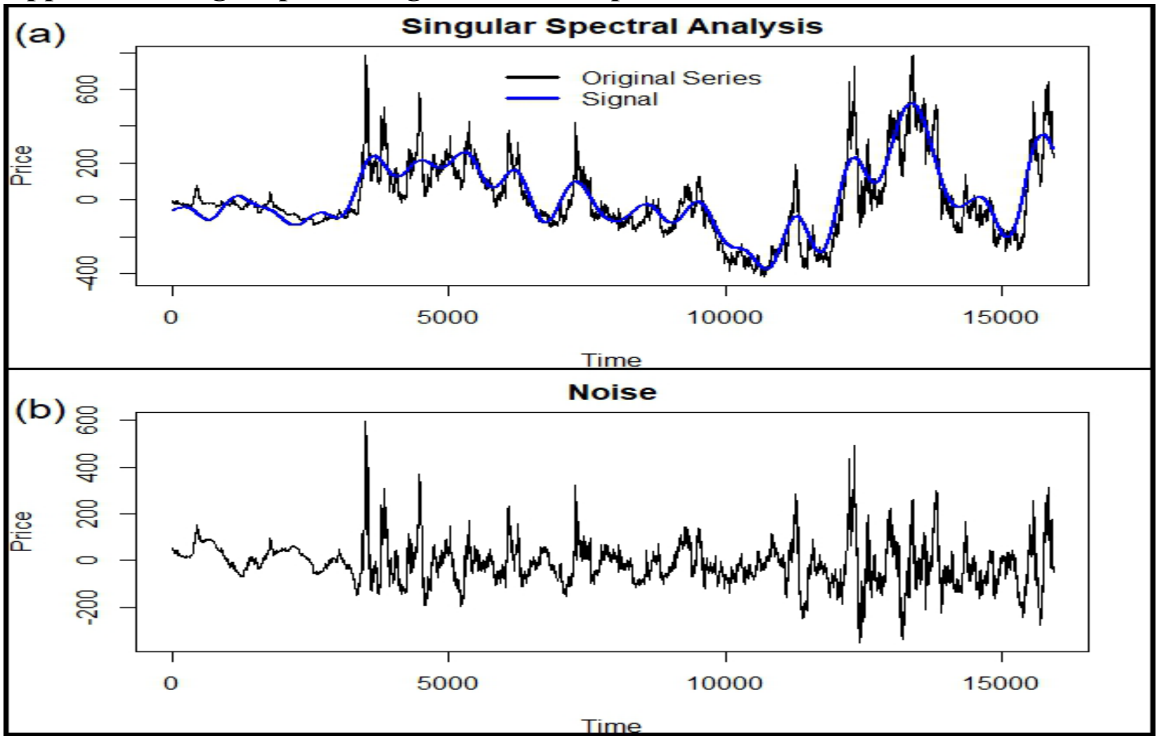

Figure B.0.1: Signal processing for price series of soybeans.

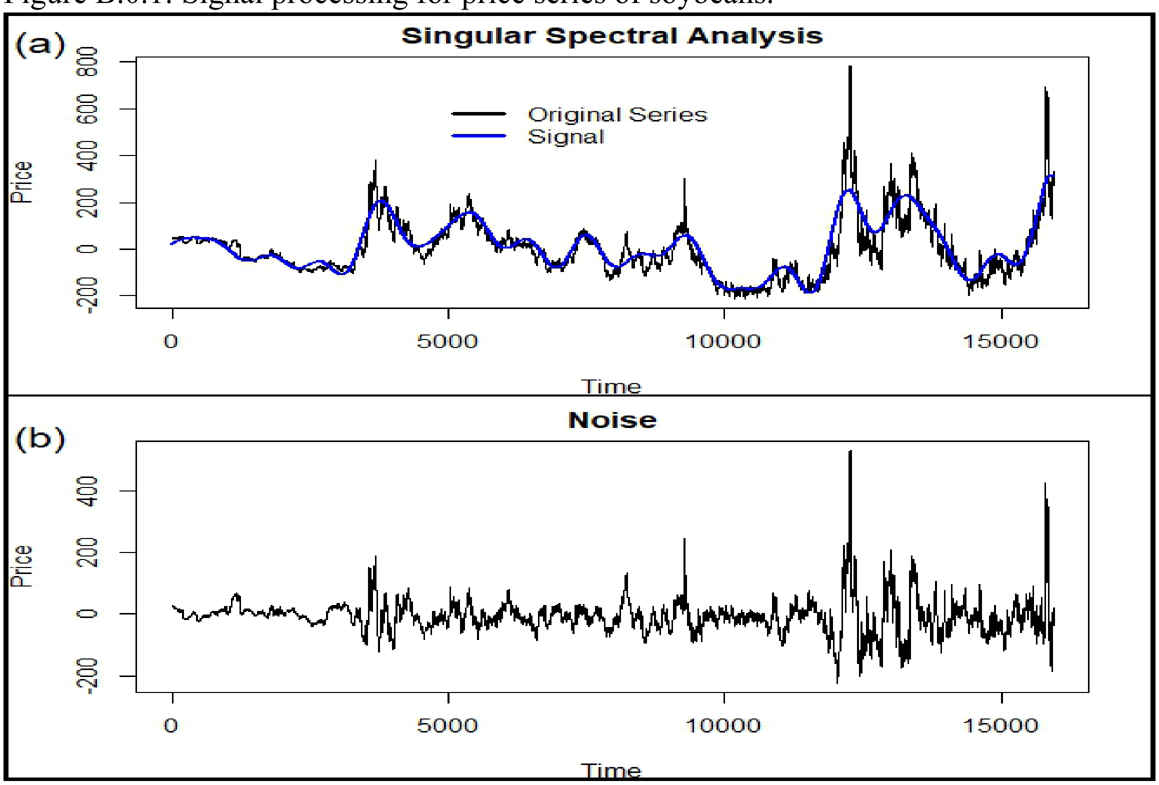

Figure B.0.2: Signal Processing for price series of wheat.

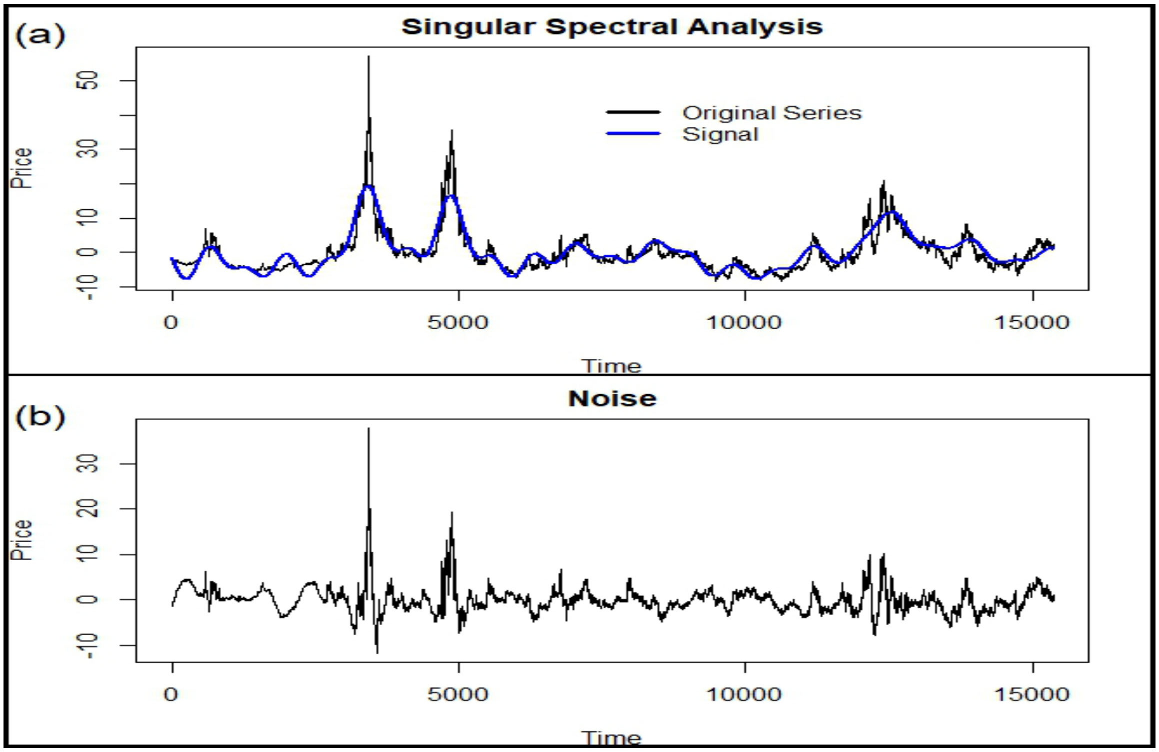

Figure B.0.3: Signal processing for price series of sugar.

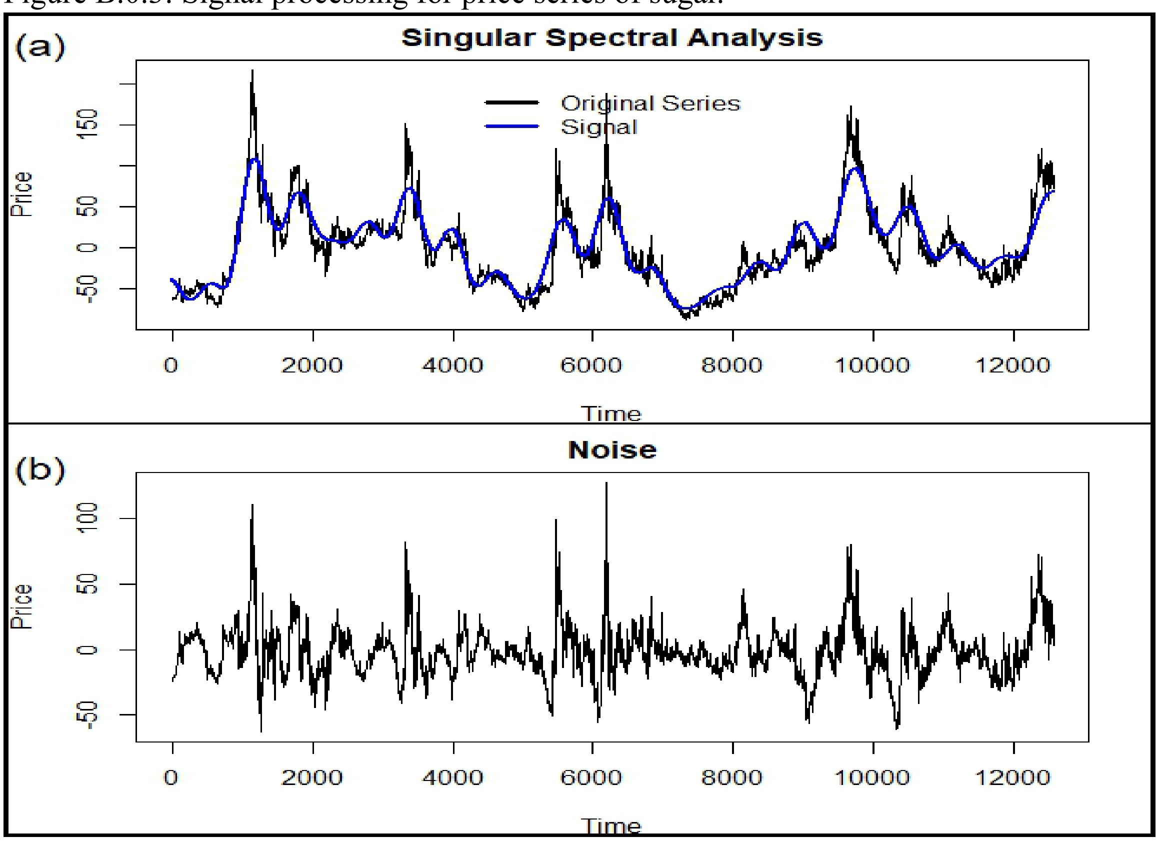

Figure B.0.4: Signal processing for price series of coffee.

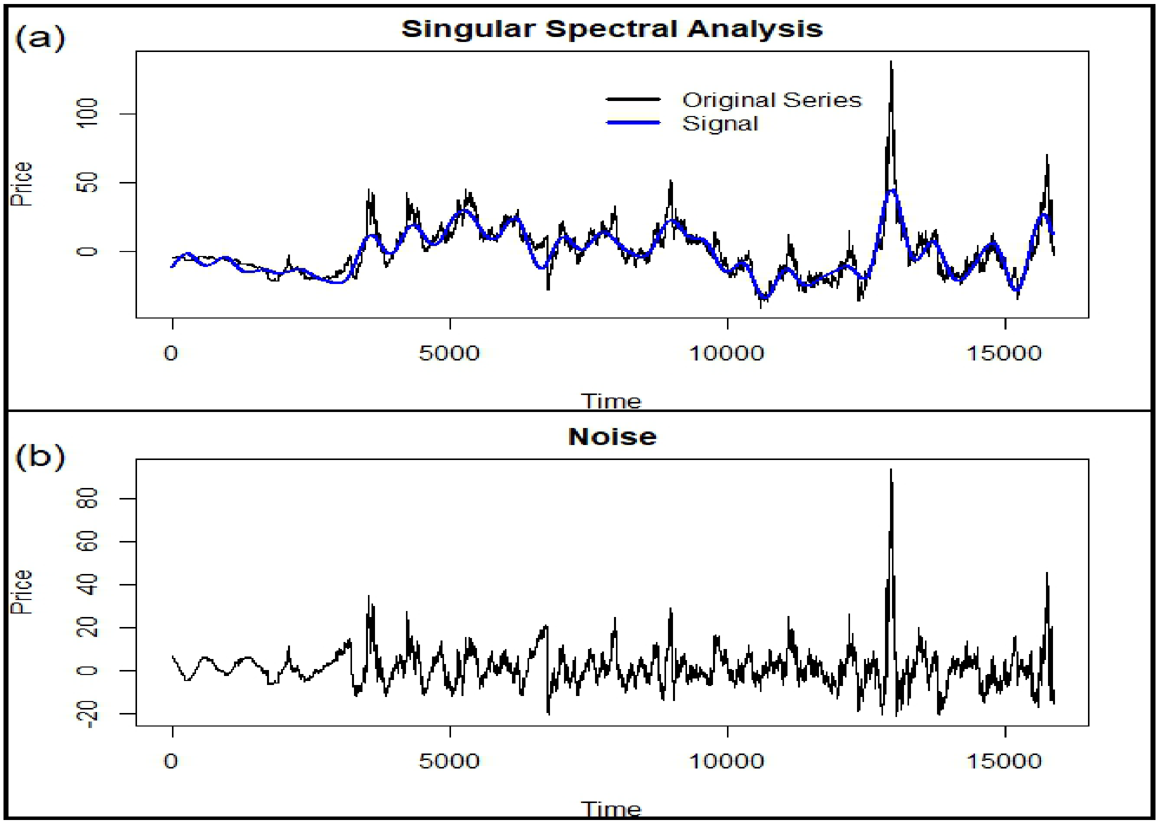

Figure B.0.5: Signal processing for price series of cotton.

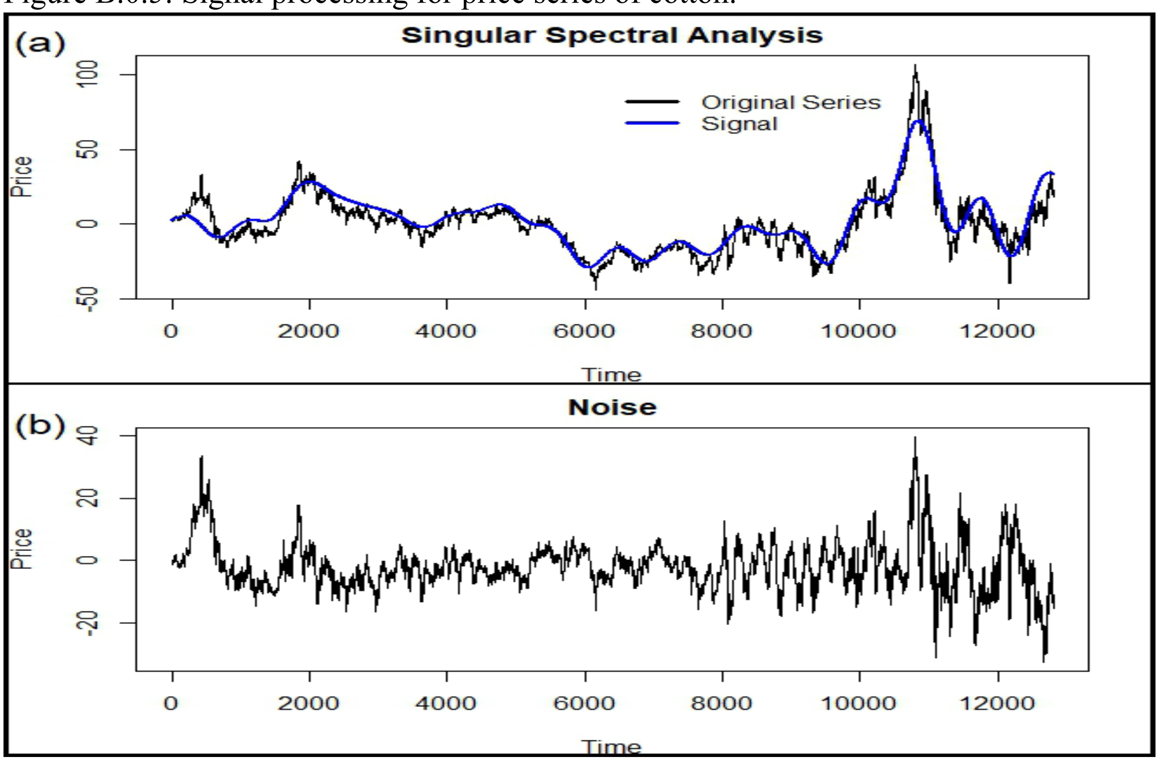

Figure B.0.6: Signal processing for price series of feeder cattle.

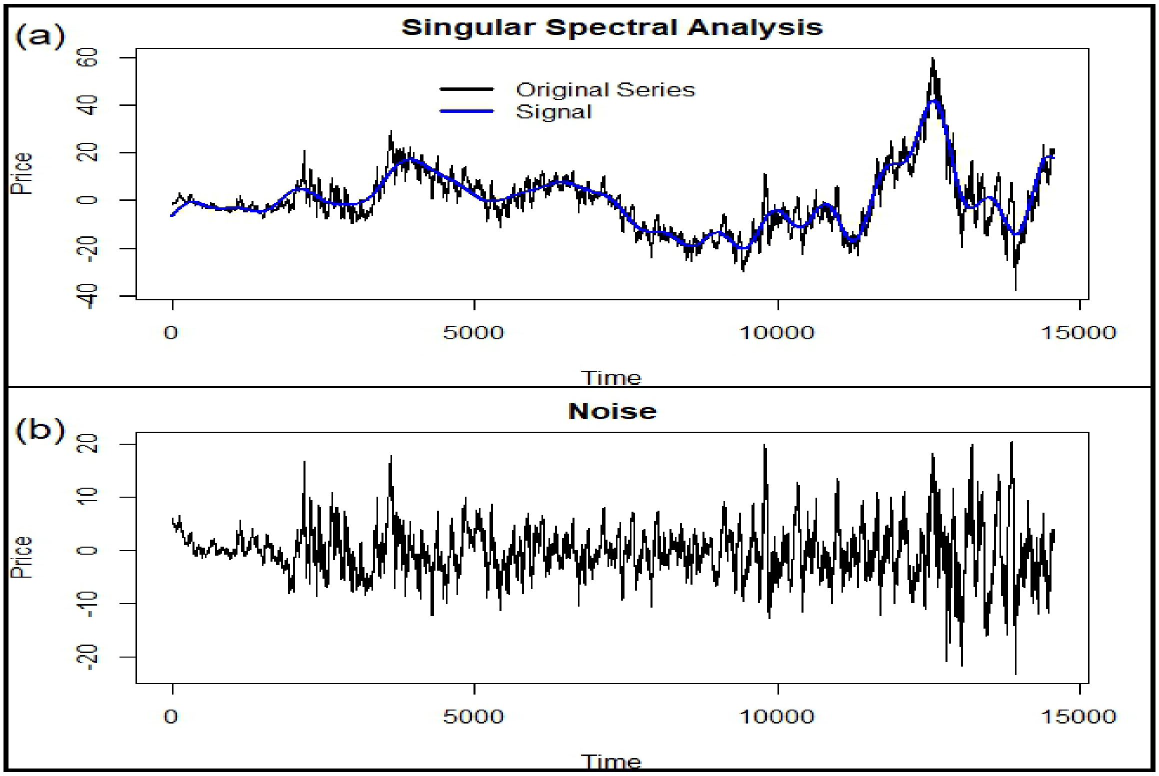

Figure B.0.7: Signal processing for price series of Live Cattle

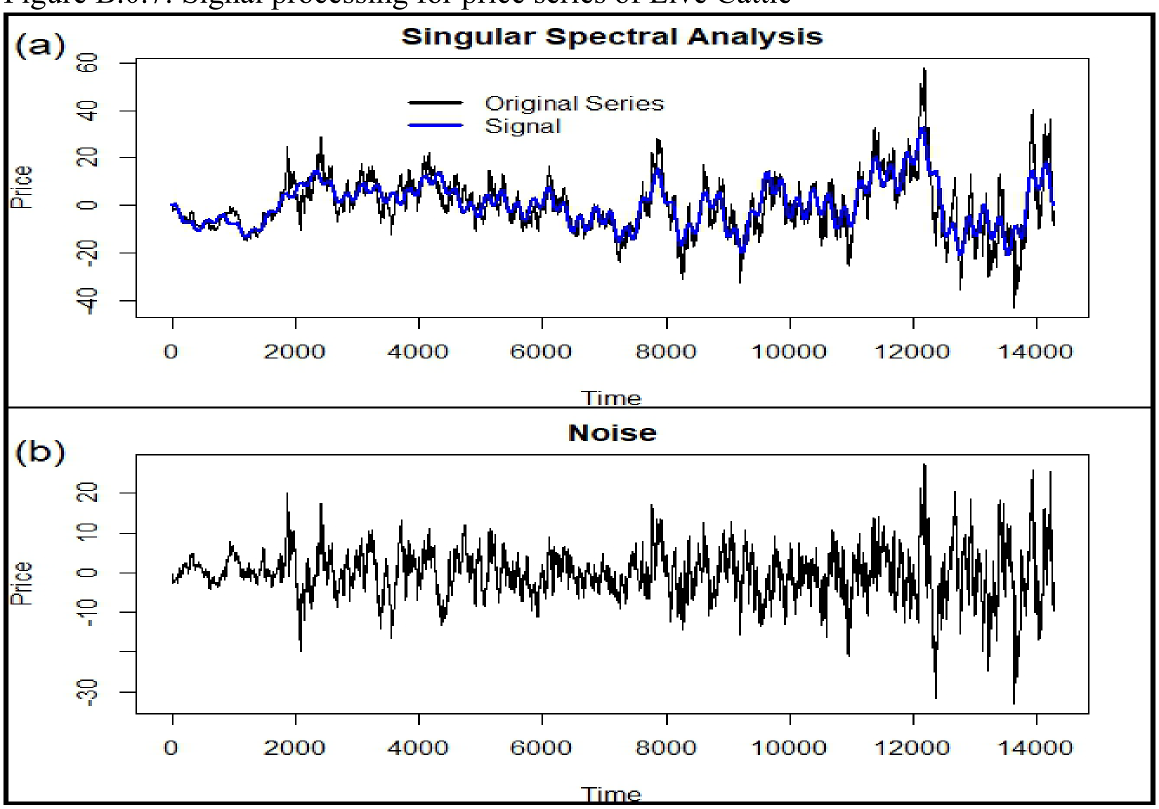

Figure B.0.8: Signal processing for price series of hogs.

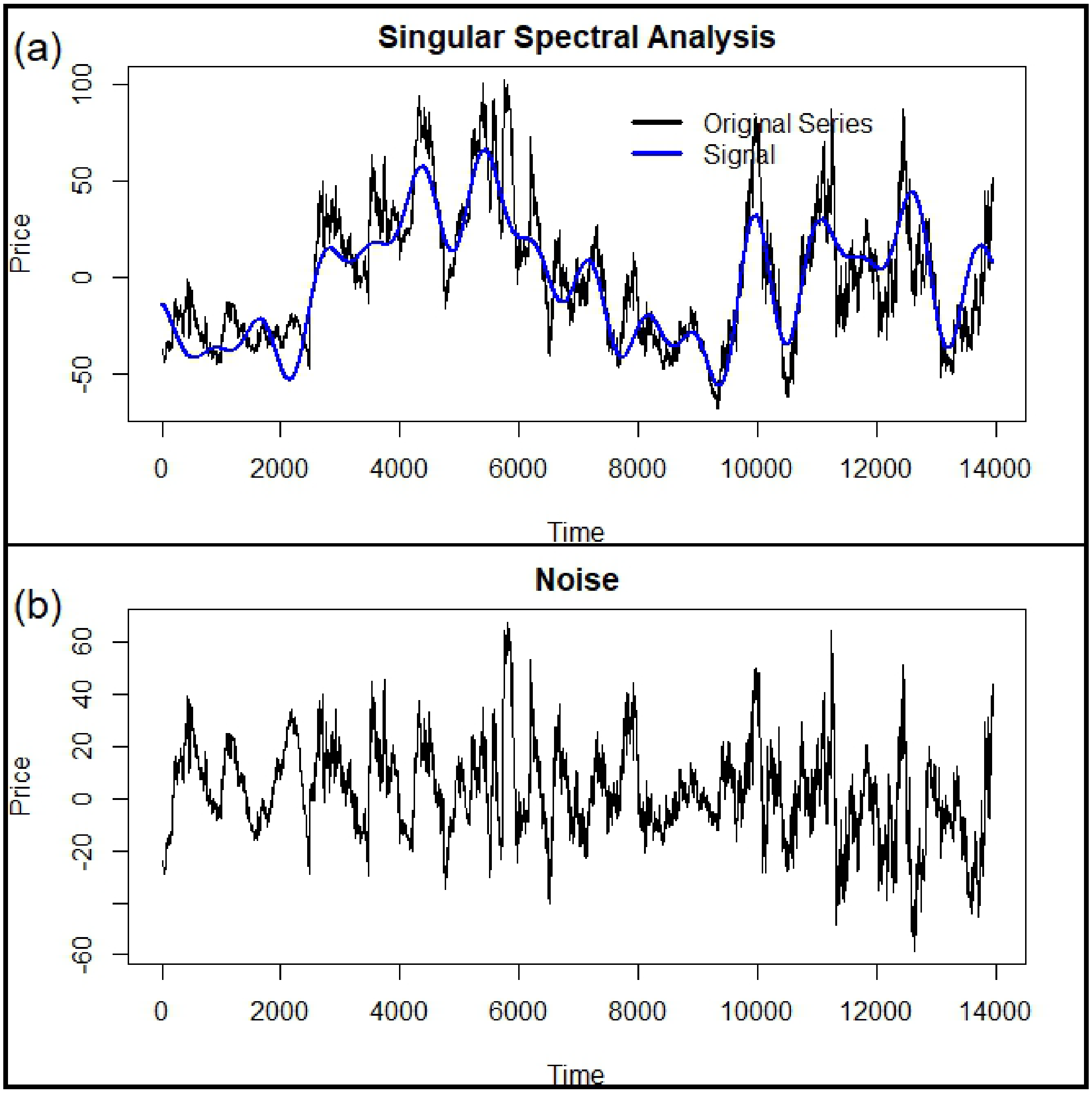

Figure B.0.9: Signal processing for price series of orange juice.

Appendix C: Embedding parameters for phase space reconstruction

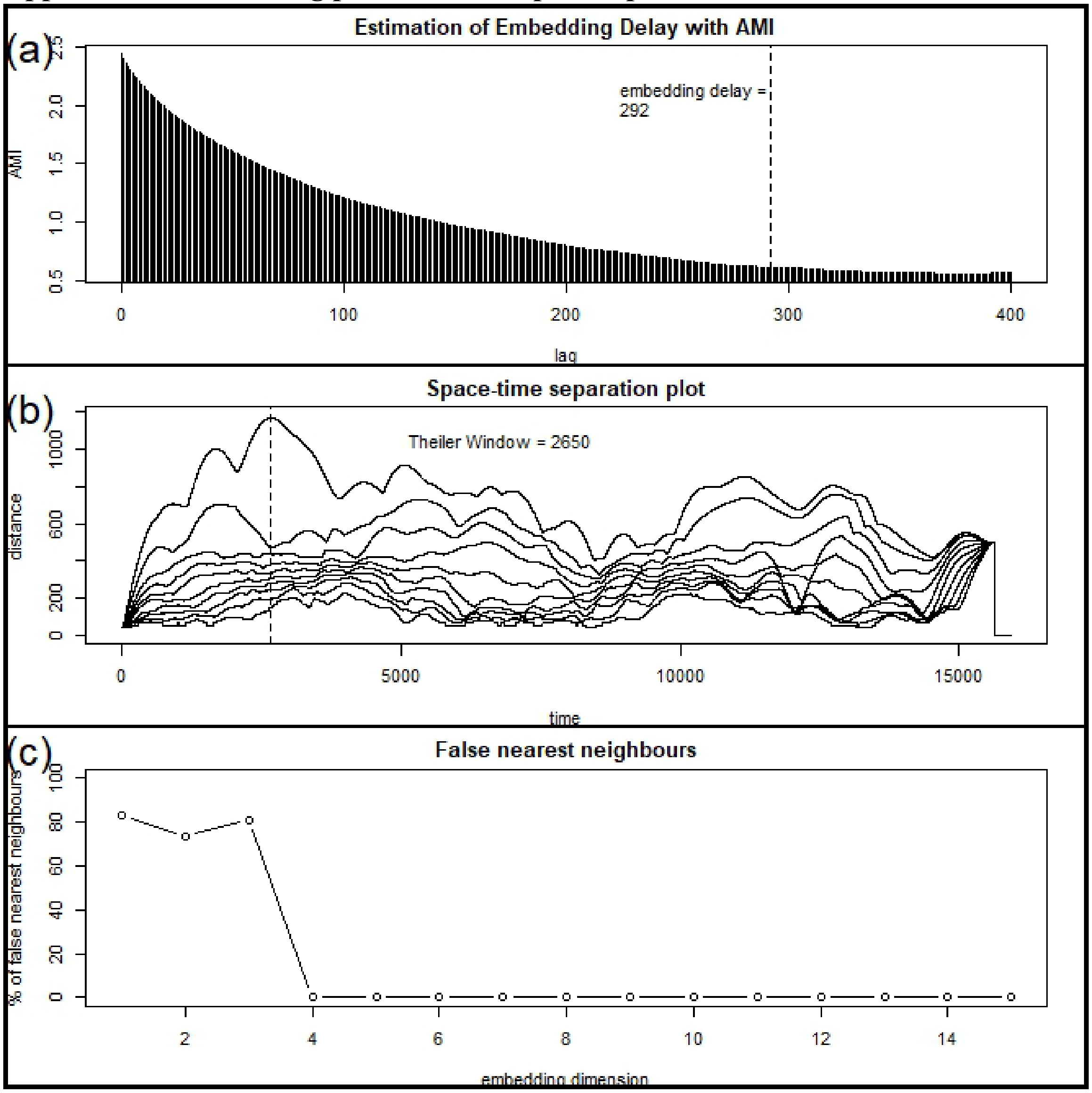

Figure C.0.1:Estimation of embedding parameters for soybeans price series.

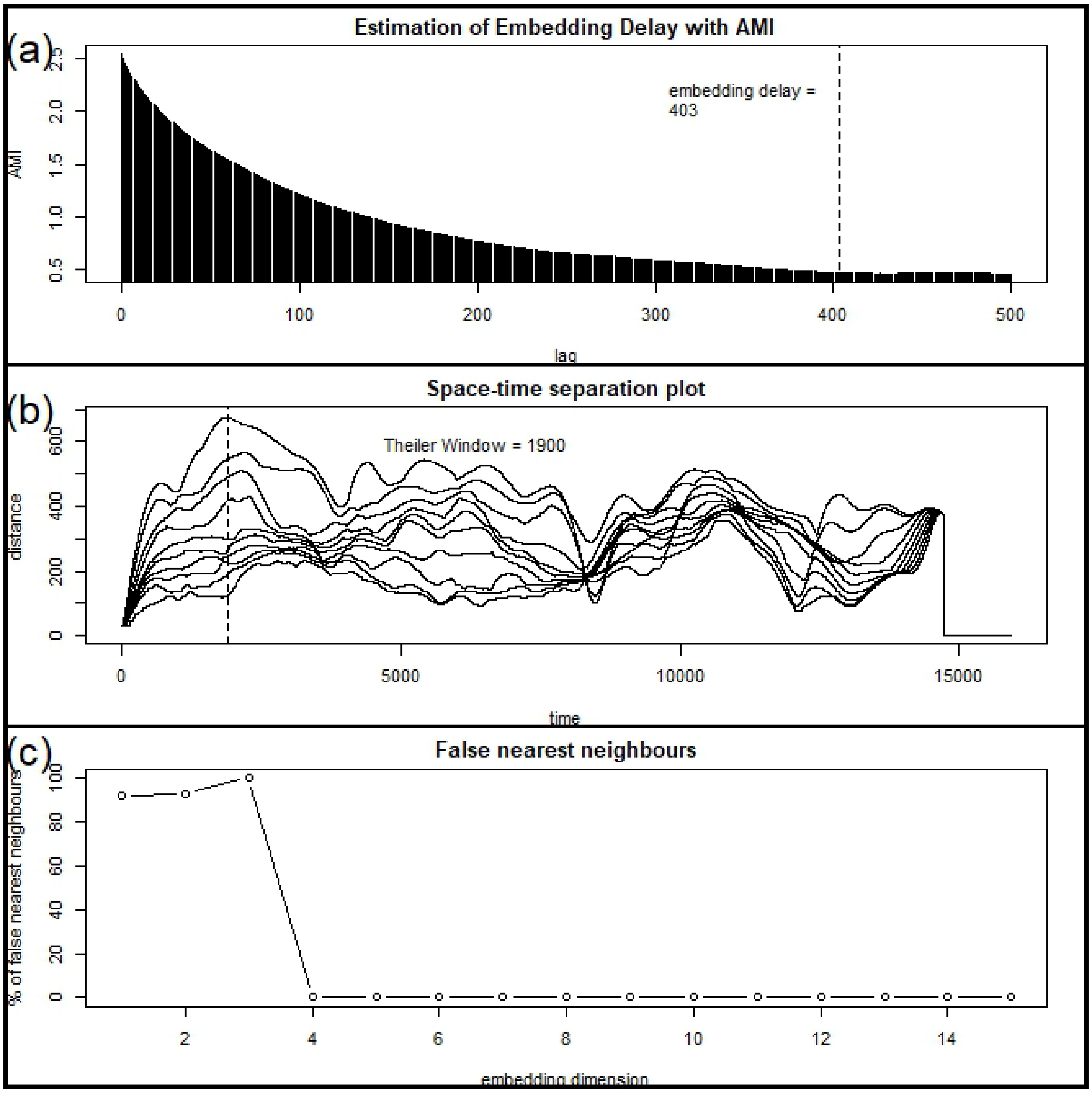

Figure C.0.2: Estimation of embedding parameters for wheat price series.

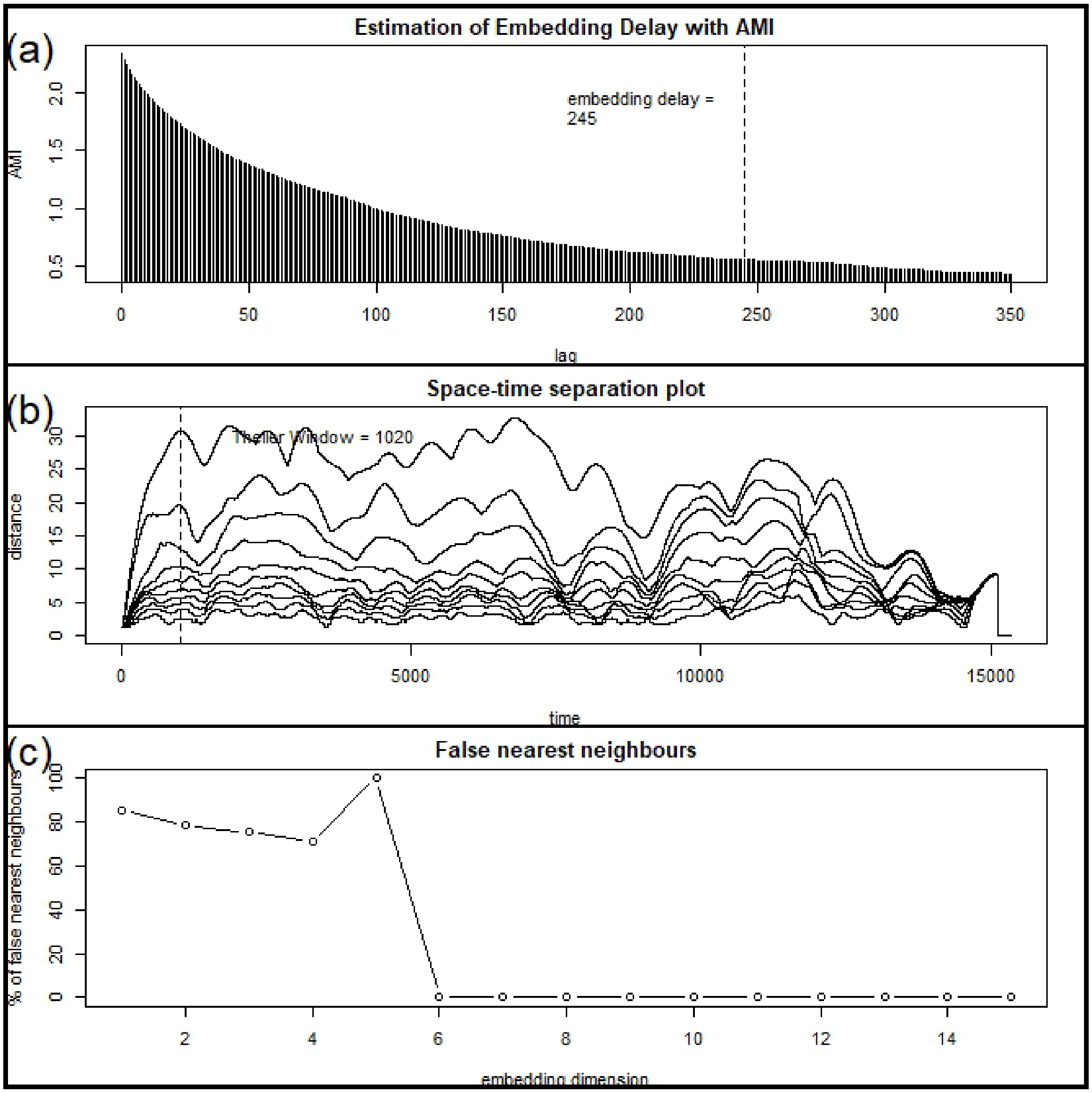

Figure C.0.3: Estimation of embedding parameters for sugar price series.

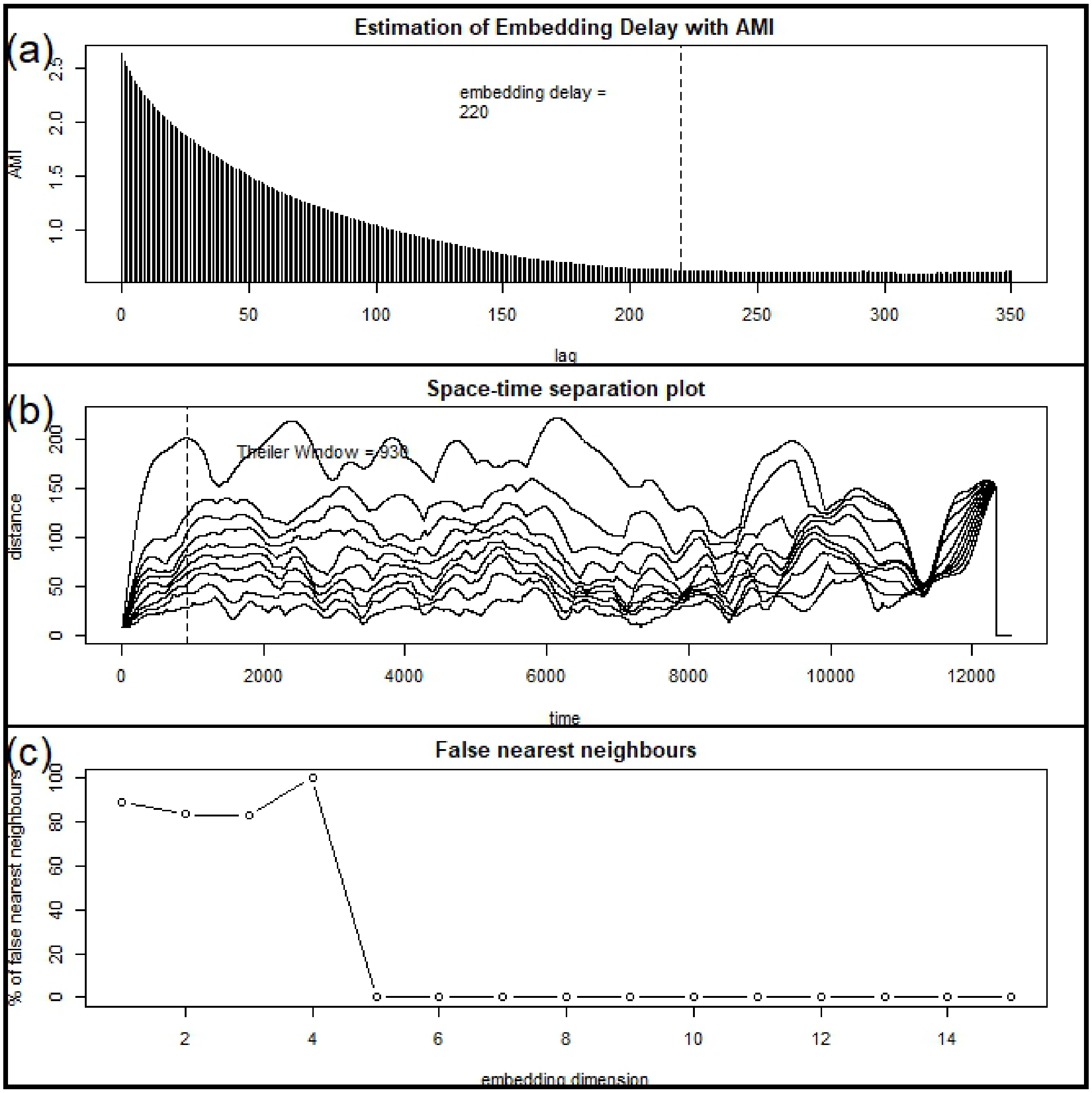

Figure C.0.4: Estimation of embedding parameters for coffee price series.

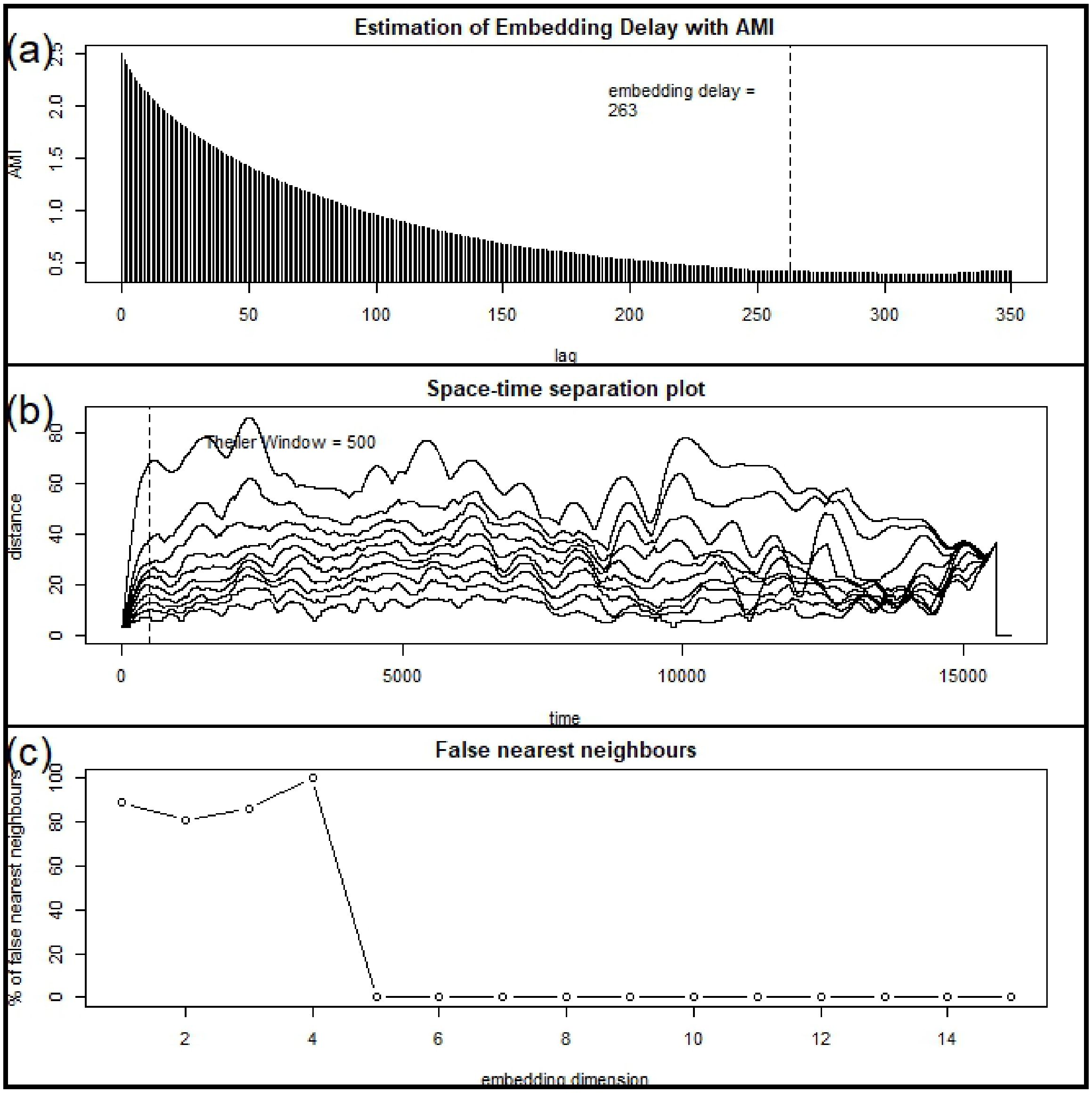

Figure C.0.5: Estimation of embedding parameters for cotton price series.

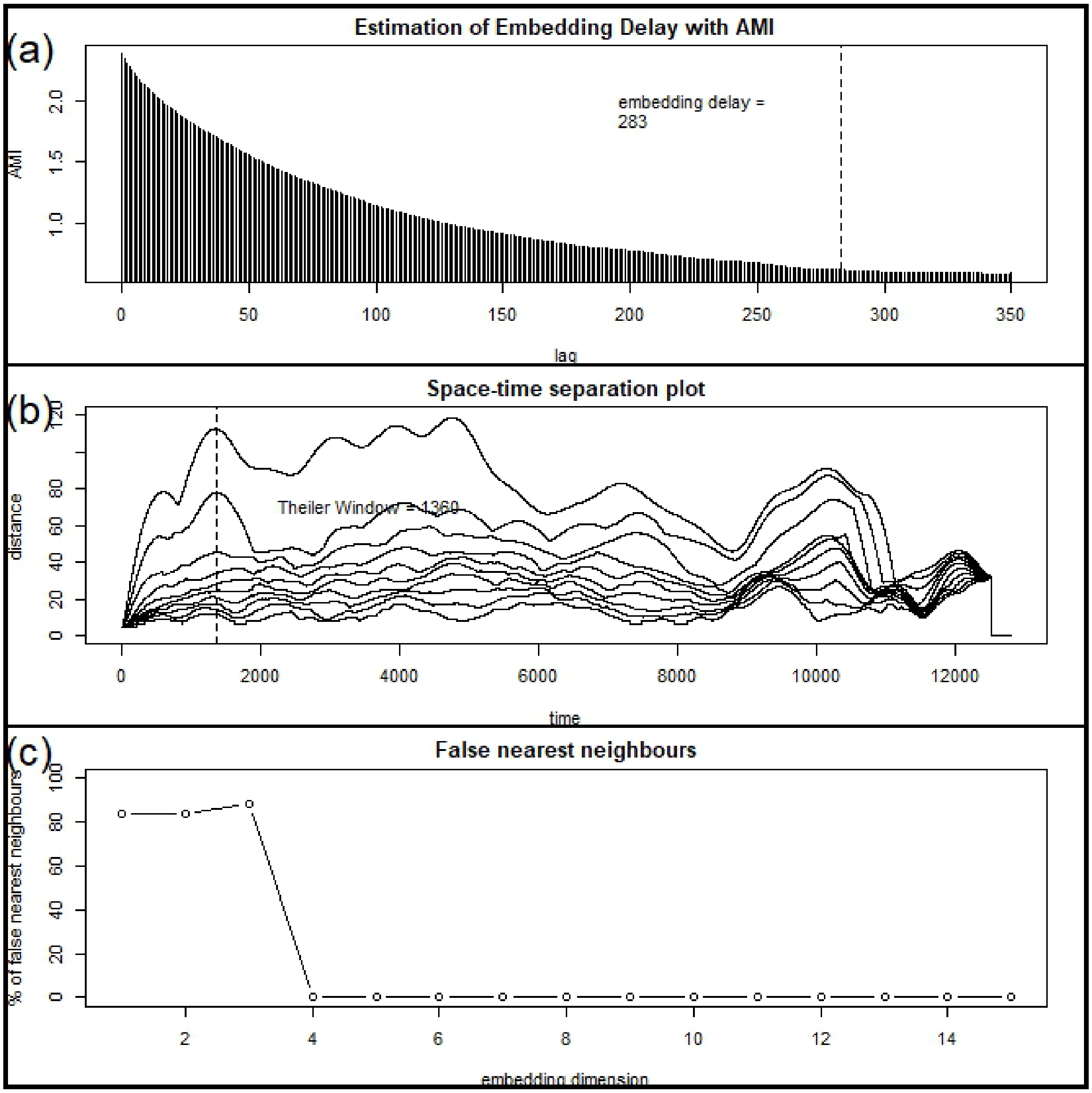

Figure C.0.6: Estimation of embedding parameters for feeder cattle price series.

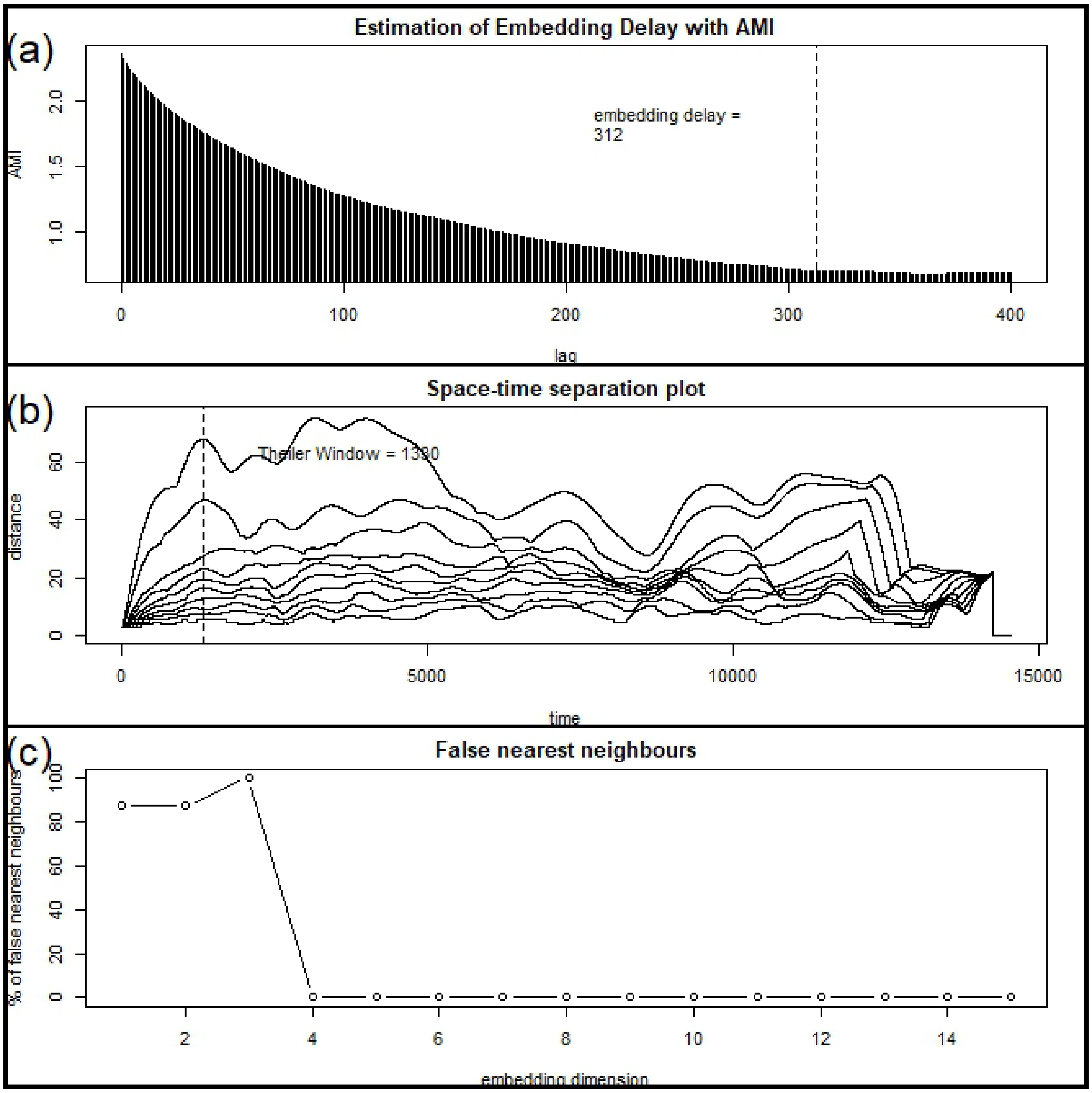

Figure C.0.7: Estimation of embedding parameters for live cattle price series.

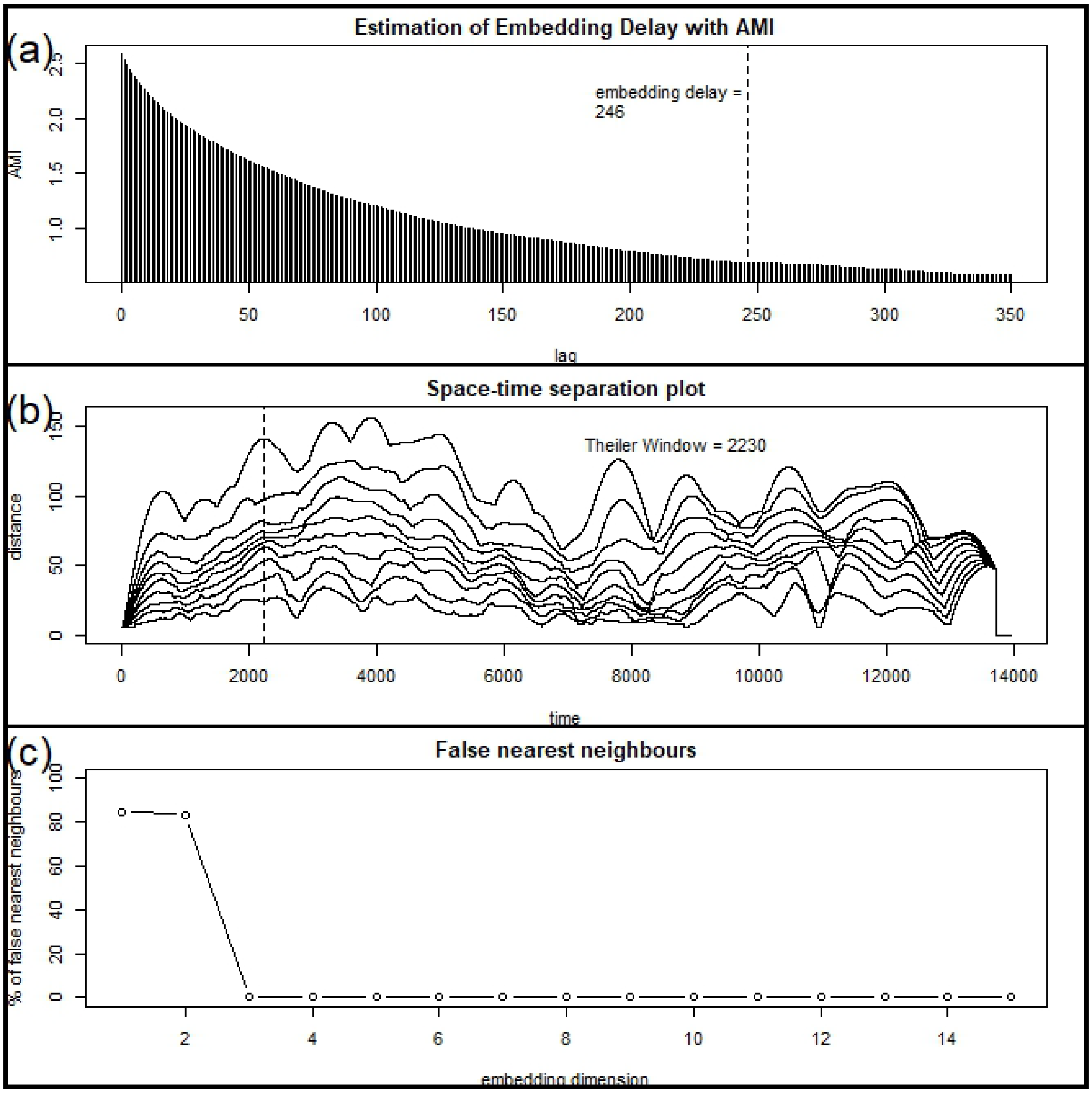

Figure C.0.8: Estimation of embedding parameters for orange juice price series.

Appendix D: Reconstructed state space dynamics from price signals.

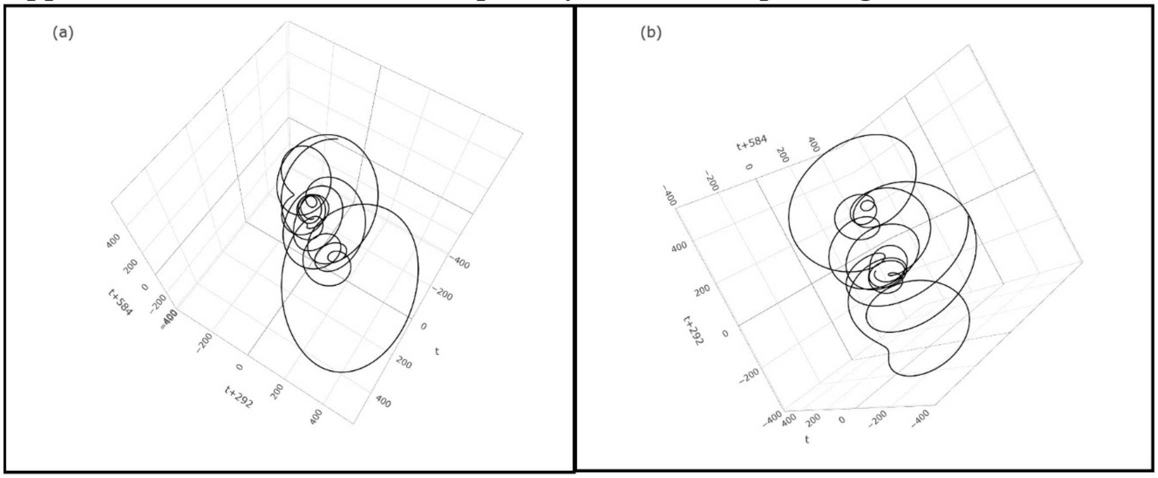

Figure D.0.1:Shadow phase space attractor reconstructed from the price signal of soybeans.

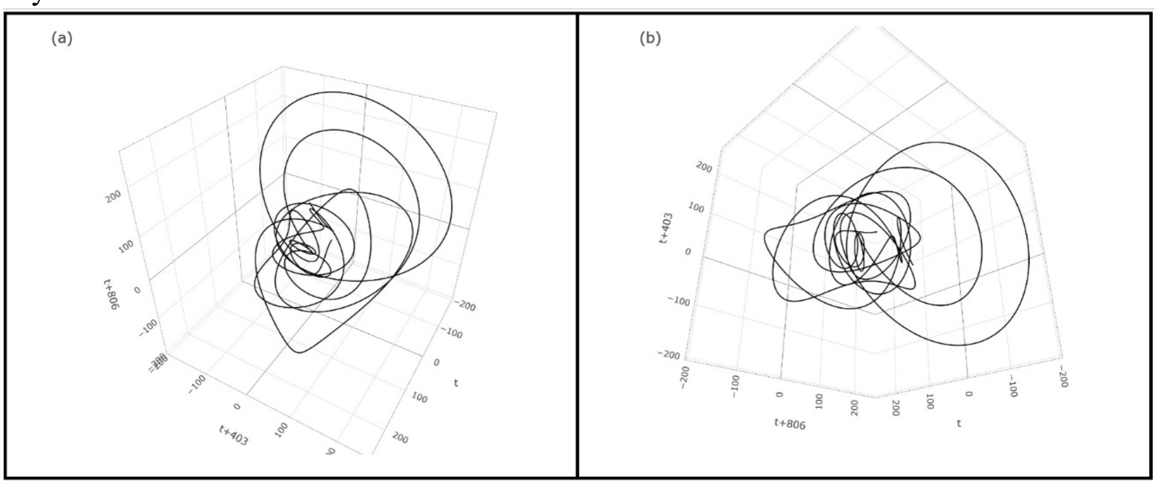

Figure D.0.2: Shadow phase space attractor reconstructed from the price signal of wheat.

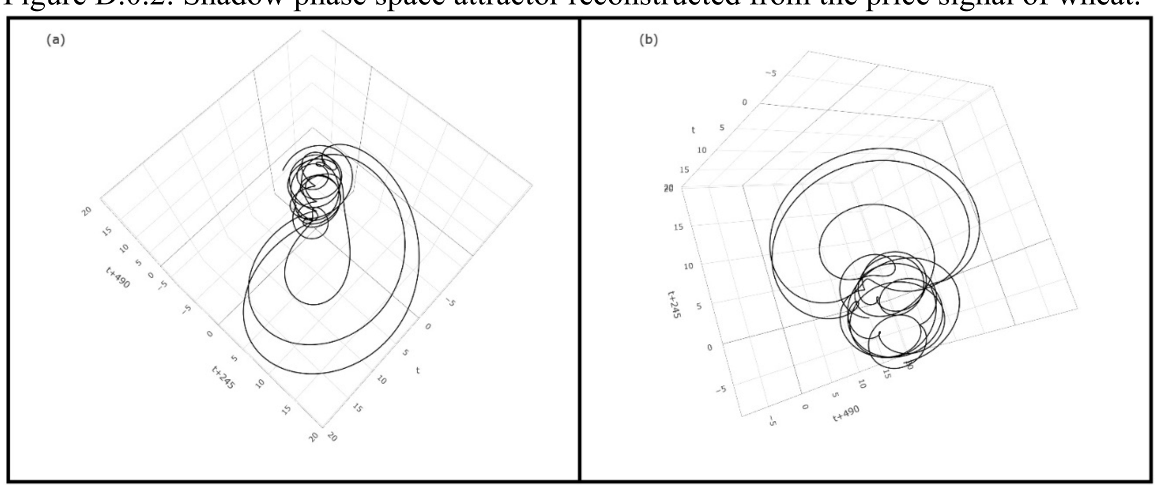

Figure D.0.3: Shadow phase space attractor reconstructed from the price signal of sugar.

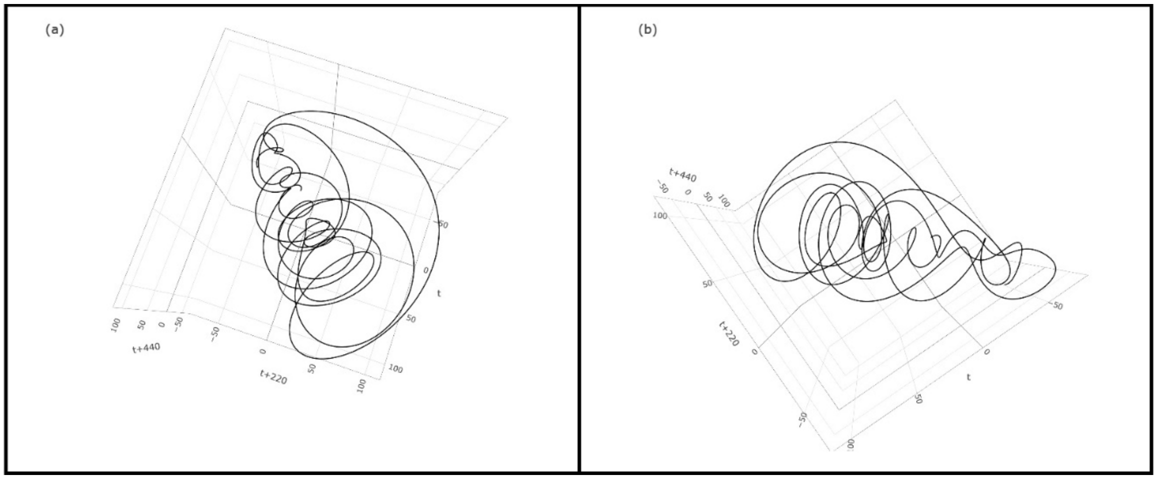

Figure D.0.4: Shadow phase space attractor reconstructed from the price signal of coffee.

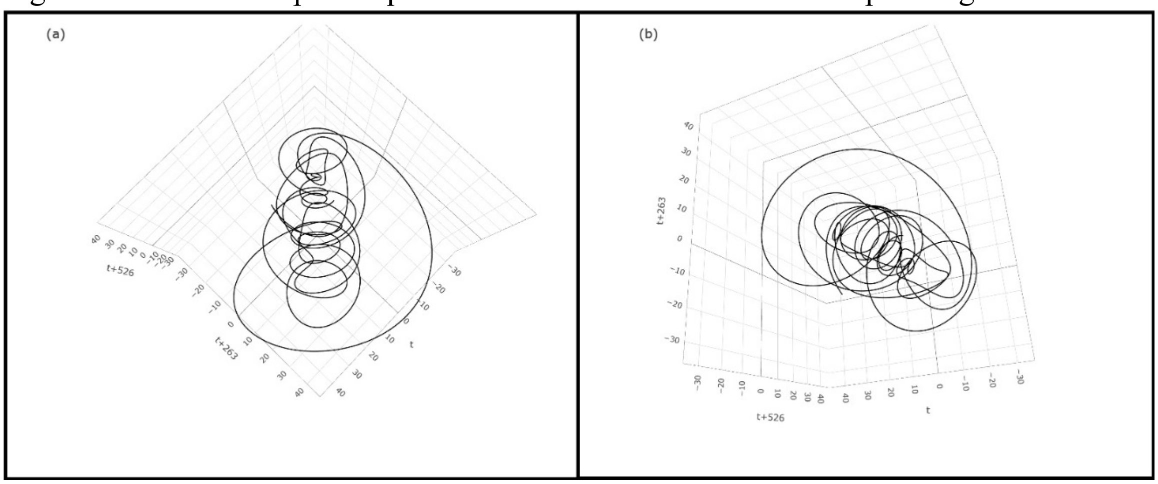

Figure D.0.5: Shadow phase space attractor reconstructed from the price signal of cotton.

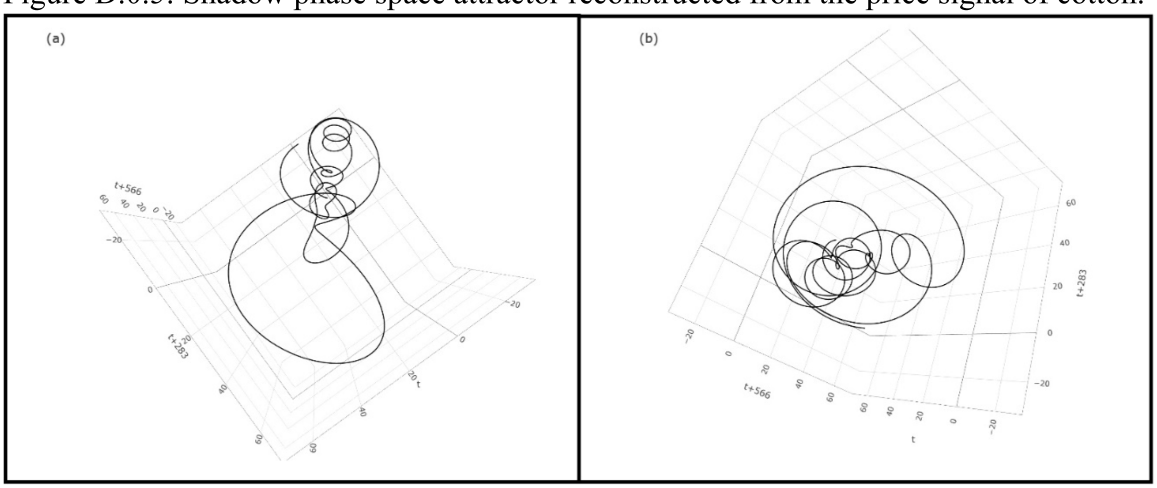

Figure D.0.6: Shadow phase space attractor reconstructed from the price signal of feeder cattle.

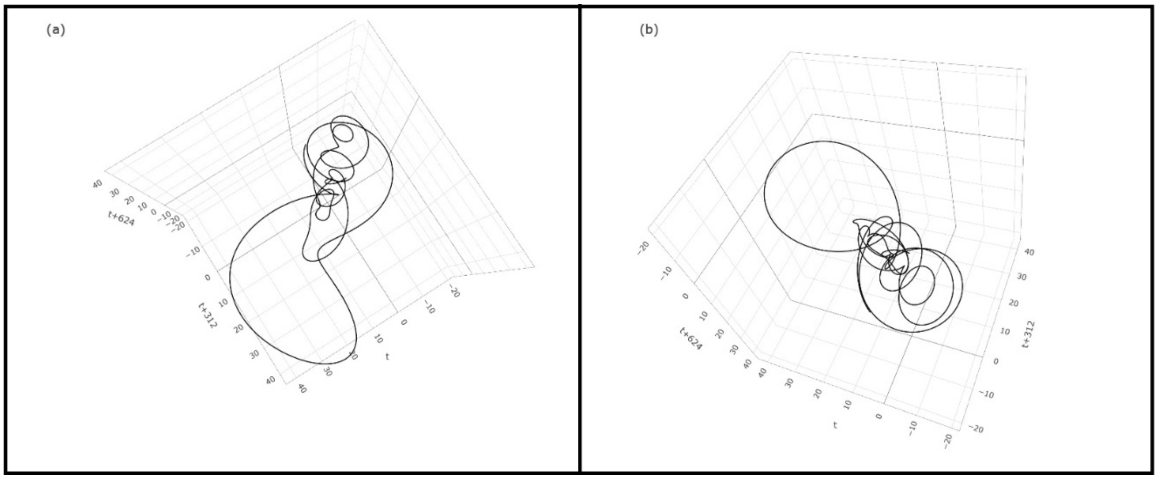

Figure D.0.7: Shadow phase space attractor reconstructed from the price signal of live cattle.

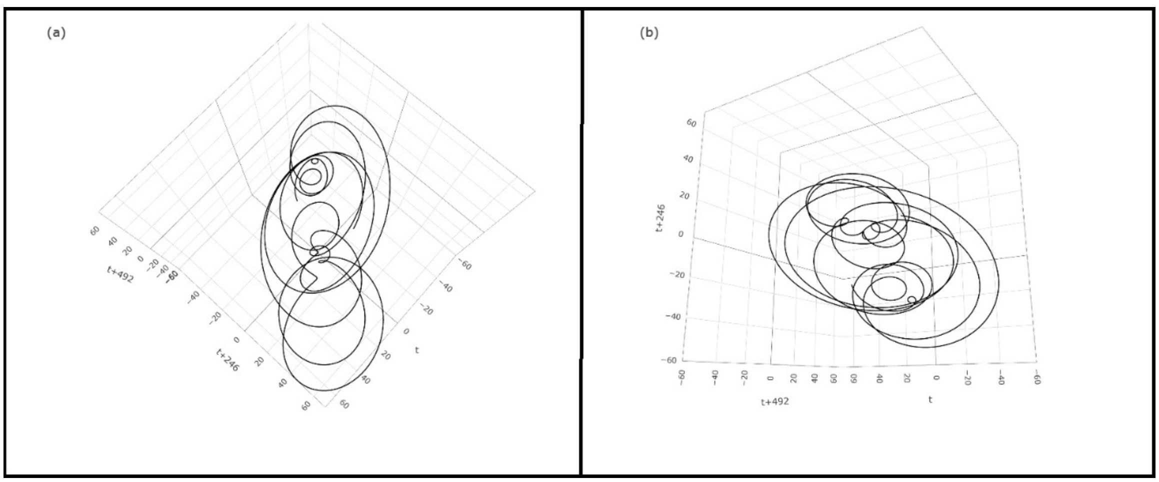

Figure D.0.8: Shadow phase space attractor reconstructed from the price signal of orange juice.

Table D.0.1: Test of nonlinear stationarity using nonlinear cross prediction.

	Corn	Soybeans	Wheat	Coffee	Sugar	Cotton	Hogs	L. Cattle	F. Cattle	O. Juice
1 → 2	0.99995	0.99988	0.99994	0.99996	0.99993	0.99989	0.99955	0.99998	0.99994	0.99911
1 → 3	0.99411	0.99736	0.99643	0.99878	0.99815	0.99272	0.99926	0.99988	0.99894	0.99585
1 → 4	0.9998	0.99994	0.99984	0.99897	0.99946	0.99548	0.99947	0.99466	0.99368	0.99962
1 → 5	0.99977	0.99976	0.99933	0.99957	0.99856	0.99926	0.99925	0.99987	0.99995	0.99845
2 → 1	0.99208	0.99094	0.99996	0.99993	0.99936	0.98598	0.9999	0.99479	0.99998	0.98219
2 → 3	0.99996	0.99963	0.99994	0.99988	0.99986	0.99938	0.99955	0.99999	0.99996	0.99993
2 → 4	0.99957	0.99962	0.99991	0.99988	0.99997	0.99896	0.99955	0.99227	0.99723	0.99452
2 → 5	0.99996	0.99994	0.99995	0.99433	0.99722	0.99947	0.99883	0.99996	0.99989	0.9987
3 → 1	0.99801	0.99976	0.99998	0.99976	0.99975	0.99106	0.99958	0.99399	0.99889	0.99966
3 → 2	0.99896	0.99988	0.99914	0.99733	0.99988	0.99598	0.99638	0.99752	0.97892	0.9984
3 → 4	0.99993	0.99997	0.99997	0.99996	0.99929	0.9998	0.99941	0.99992	0.99992	0.99995
3 → 5	0.99995	0.99987	0.99989	0.99012	0.99812	0.99979	0.9984	0.99911	0.99925	0.99769
4 → 1	0.99793	0.99295	0.99959	0.99868	0.99722	0.99926	0.99928	0.99881	0.99941	0.99461
4 → 2	0.99876	0.99715	0.99814	0.99923	0.99989	0.99994	0.99892	0.99901	0.99996	0.99949
4 → 3	0.99935	0.9998	0.99972	0.99651	0.99238	0.99454	0.99759	0.99865	0.99893	0.99888
4 → 5	0.99998	0.99995	0.99998	0.99957	0.99973	0.99937	0.99961	0.99934	0.99995	0.99994
5 → 1	0.93651	0.97174	0.9883	0.99837	0.99976	0.99019	0.99924	0.97219	0.99804	0.99864
5 → 2	0.98278	0.98813	0.98918	0.99886	0.99971	0.99518	0.99879	0.99629	0.99789	0.99996
5 → 3	0.99133	0.9959	0.98808	0.99546	0.99864	0.99976	0.99755	0.99907	0.99703	0.99856
5 → 4	0.99588	0.99873	0.99826	0.99636	0.99944	0.99937	0.99855	0.98428	0.99001	0.99918