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Scalar and Collectional Relationships in Shostakovich's Fugues, Op. 87

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SCALAR AND COLLECTIONAL RELATIONSHIPS IN SHOSTAKOVICH'S FUGUES, OP. 87

by

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SCALAR AND COLLECTIONAL RELATIONSHIPS IN

SHOSTAKOVICH'S FUGUES, OP. 87

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The Preludes and Fugues, Op. 87 of Shostakovich come out of common practice tonality; however, Shostakovich's music departs from the tonal tradition in his lack of functional harmonic progressions and his unexpected chromatic twists. *Chromatic* is a problematic term in Shostakovich's music because it can be difficult to know if a chromatic note is acting inside or outside of the system. Much of the music of Op. 87 is based on diatonic scales or scales that derive from the diatonic set. Because Shostakovich's music does not follow the patterns of common practice tonality and sometimes uses non-diatonic scales, it is more appropriate to discuss scales than keys in this context. This thesis will explore the relationships between scales, including two types of transposition and Hook's signature transformations, and will establish a theory of eight-note collections that add a chromatic note to a diatonic set. In this way, a framework for placing a chromatic note into the context of a scale or a shift in collection will be established. The concepts of scalar and collectional relationships will be applied to whole fugues and especially to the specific context of Shostakovich's expositions and counter-expositions.

Acknowledgements

I owe many thanks to Dr. Stanley Kleppinger for advising this thesis and for acquainting me with Shostakovich's Op. 87 and signature transformations. He has been a trustworthy sounding board and guide as the ideas in this thesis have developed. I am also grateful to Dr. Gretchen Foley for her enthusiastic encouragement and meticulous comments concerning this thesis and many of my other endeavors. Thank you also to Dr. Jeffrey McCray for serving on my thesis committee.

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Chapter 1

Introduction

In 1950, Shostakovich began to write a cycle of twenty-four preludes and fugues in each of the major and minor keys. The Op. 87 cycle has grown to be quite popular since that time, but relatively little theoretical or analytical study of the work has been undertaken. Mark Mazullo's 2010 book claims to be "the first in English devoted to Shostakovich's Twenty-Four Preludes and Fugues for Piano," and Mazullo himself acknowledges that his intended audience is teachers, performers, and listeners.[1] Dmitri Tymoczko cites the cycle several times in *A Geometry of Music*, mostly in the context of scalar study, but according to the nature of his book, the examples are brief.

This thesis aims to investigate relationships between the scales and collections in Shostakovich's Op. 87 fugues. As such, it is both a study of Shostakovich's chromatic usage and an application of transformations to diatonically-derived music.

Much of Shostakovich's music grows visibly out of common practice tonality, but his unique use of extradiatonic notes has attracted much attention. As Ellon Carpenter points out, Russian theorists have sought to expand the concept of diatonicism in order to define new modes that encompass chromatic tones, including twelve tone modes.[2] Similarly, Gabe Fankhauser has developed a harmonic theory to relate seemingly-random chromatic chords to a diatonic framework.[3] In this thesis, I also seek to show how chromatic notes relate to their surroundings, both within single subjects and in larger

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[1] Mark Mazullo, *Shostakovich's Preludes and Fugues: Contexts, Style, Performance* (New Haven, CT: Yale University Press, 2010), xii–xiii.
[2] Ellon D. Carpenter, "Russian Theorists on Modality in Shostakovich's Music," in *Shostakovich Studies*, ed. David Fanning (Cambridge: Cambridge University Press, 1995), 81–86.
[3] Gabe Fankhauser, "Flat Primary Triads, Harmonic *Refraction*, and the Harmonic Idiom of Shostakovich and Prokofiev," in *Musical Currents from the Left Coast*, ed. Jack Boss and Bruce Quaglia (Middlesex: Cambridge Scholars Publishing, 2008), 202–15.

contexts. I do this by developing a theory of eight-note scales formed by a diatonic scale and one added chromatic note. I will also examine the chromatic byproducts of various transformations between scales.

Several theorists have explored transformations in diatonic contexts, either between scales or specific musical passages, including Julian Hook, Dmitri Tymoczko, Ian Bates, and Steven Rings. Fugues are well suited to the study of scalar and collectional relationships because they present the same subjects and countersubjects several times using different scales. Such repetition aids in segmentation and comparison. Shostakovich's Op. 87 fugues provide a particularly rich body of music to study because he uses diatonic modes and eight-note scales and also finds various ways to deviate from expected fugal relationships—both those established by fugal tradition and his own conventions.

All of the fugues in this cycle include expositions with traditional subject-answer relationships. Shostakovich incorporates both real and tonal answers in these expositions and uses countersubjects extensively. One unique aspect of these fugues is Shostakovich's consistent and distinctive use of the counter-exposition. The term *counter-exposition* typically refers to a second presentation of subjects and answers with a tonic-dominant relationship in the original key.[4] In Op. 87, Shostakovich's counter-expositions include one subject and one answer in the relative major or minor key with countersubjects. These relationships are illustrated in Example 1 and are recognizable in twenty-two of the twenty-four fugues. The other two fugues also contain a second exposition with a subject-answer relationship, but these are not in the relative key.

[4] Robert Gauldin, *A Practical Approach to Eighteenth-Century Counterpoint* (Long Grove, IL: Waveland Press Press, 2013), 217.

Exposition (I)		Counter-exposition (vi)	
Subject	I	Subject	i
Answer	V	Answer	v

Example 1. Typical relationships in a Shostakovich major-mode fugue

Chapter 2 of this study introduces the concepts of *scale* and *collection* and their applications to Op. 87. The second half of the chapter looks at transformations that relate two scales, including chromatic transposition (T_n), diatonic transposition (t_n), and signature transformations (s_n and f_n). I also show how combining these operators might be analytically useful, especially in tracking the addition of chromatic notes. These operators are used in an analysis of the B-flat minor fugue.

A theory of eight-note scales is presented in chapter 3. The system includes ten scales. Each consists of a diatonic scale and one additional note that can be related to the original scale by use of the circle of fifths. These scales are classified by how closely the added note is related to the rest of the collection. Chapter 3 includes a formalization of the theory, an intervallic study of all ten scales, and a more detailed look at one pair of scales that is used most often in Shostakovich's fugues. The chapter concludes with analysis of the eight-note scales in the cycle.

Chapter 4 applies the ideas of the previous two chapters to Shostakovich's expositions and counter-expositions specifically. This chapter investigates how Shostakovich uses scale relationships in the most structured part of his fugues. A short study of the implications of centricity in this study is followed by a discussion of macrocollections, which allow the analyst to think about the pitch content of these fugues at a larger level instead of simply scale to scale. The fifth chapter includes conclusions and suggests future avenues of research in this area of study.

This study focuses on scales and collections. These theoretical concepts, once defined in the next chapter, will be useful aids in looking at the pitch organization in Shostakovich's fugues and the relationships between subject statements. Though both terms will be used throughout the thesis, as they are related, relationships between scales will be the focus of the chapter 2. This chapter uses a fairly straightforward approach. The last part of the thesis concentrates on collections and faces some of the aspects that complicate matters but also illuminate larger patterns in the fugues.

Chapter 2

Scales, Collections, and Transformations

Scales

A scale is a group of pitch classes (pcs) used in the composition of part or all of a work.[5] For the purposes of this thesis, a scale must have a center—a pc that is privileged over all the others. The traditional tonic is a type of center that has a special meaning in common practice tonal music, where keys and centers are often established by specific melodic and harmonic patterns. Music written after this period, including the music of Shostakovich, contains remnants of the tonal tradition, but tonal cues are supplemented, and sometimes contradicted by other cues. As Op. 87 is a work based on genres and styles of the common practice period, this work contains more CP (common practice) cues than many of Shostakovich's works, but it is by no means tonal.[6] One key ingredient of tonal music missing from the cycle is functional harmonic progressions. The absence of such a familiar feature sometimes blurs the center of a passage, which will have consequences for analysis later.

Because Shostakovich's Op. 87 fugues do not fall into the category of common practice tonality, it is not entirely appropriate to discuss key relationships in the work. The term *key* carries associations from tonal music. Scales, on the other hand, here refer specifically to pc content and not to usage; therefore, scales encompass more than the

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[5] The term *pitch* refers to a specific note in a specific octave, such as C4. C4 and C5 are distinct pitches. On the other hand, *pitch class* (or pc) refers to a class of pitches that have the same letter name. In this context C4 and C5 are both instances of the same pitch class, which will be used when the specific octave does not matter to the discussion.

[6] For more on CP cues and other types of cueing in pitch-centric music, see Stanley V. Kleppinger, "Reconsidering Pitch Centricity," *Theory and Practice* 36 (2011): 73–76.

traditional major and minor keys. A center is the only requirement for a group of pcs to be considered a scale.

Scales will be presented with all of the pcs in ascending order, beginning with the center, either as quarter notes on a staff or with letter names. Using letter names is often preferable because this method best fits the pc nature of scales, where octave does not matter. It should be assumed that the first letter is the center of the scale, though it will sometimes be analytically advantageous to spell the scale in another order. At these times, a box will be placed around the center.

Abbreviating complex pieces of music as a series of scales can be both helpful and artificial. Expressing certain pitch elements of the music in the concise form of a scale helps to clarify relationships that would be difficult to express with complete quotations of subjects. However, representing music as scales might, at times, oversimplify the intricacies of the music itself, especially when the center is not always obvious—as is sometimes the case in Shostakovich's Op. 87.

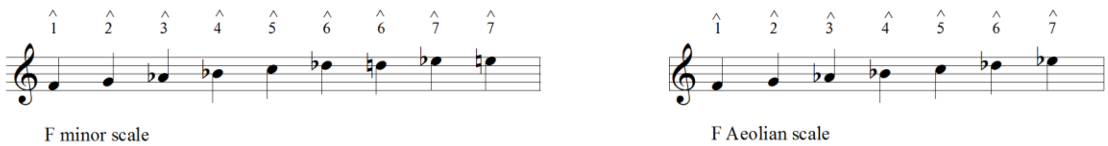
Although the preludes and fugues are labeled by the composer to be in major and minor keys, Shostakovich frequently incorporates modes into this work—both the diatonic modes and others, sometimes created by Shostakovich himself. Modes are scales, as they include a center. I will frequently use the term scale to refer to modes, even in labels such as "the Aeolian scale." This usage is admittedly unconventional, especially because it will group modes with other scales like the major scale, which once again carries tonal connotations. However, the line between tonal and modal is not as clear in twentieth-century music as it might seem. The centers of modes can be established by cues from the common practice period and often are in Op. 87.

I will begin with a discussion of the use of diatonic modes, as they are more common. Perhaps the most prominent appearance of modes in the cycle is Shostakovich's frequent use of the Aeolian mode in subjects of "minor-key" fugues and as the relative minor in counter-expositions. One instance of this usage is the subject from the F minor fugue, shown in Example 2, though examples like this one are plentiful in the cycle.

Example 2. F Aeolian subject

The Aeolian mode is often equated with the natural minor scale, an association that Shostakovich also apparently made as the preludes and fugues that feature the Aeolian mode are named with minor keys. However, for the purposes of this paper, I would like to make a distinction between the minor scale and Aeolian mode. Instead of splitting the minor scale into three forms—natural, harmonic, and melodic—Example 3 shows the minor scale as a unified entity containing nine notes with variable sixth and seventh scale degrees. This hybrid minor scale is the one used in tonal minor music. Neither form of $\hat{6}$ or $\hat{7}$ would be considered foreign to the key. Shostakovich only rarely uses this minor scale in Op. 87.[7] He much more frequently uses the diatonic form of the minor scale, which will hereafter be referred to as the Aeolian mode, also shown in Example 3.

[7] Pertinent examples include the subject of the A minor fugue and the minor version of the E Major subject (mm. 11–19).



Example 3. Minor vs. Aeolian scales

Example 3 highlights one of the weaknesses of using scales. Although both versions of $\hat{7}$ are presented adjacently in the minor scale, it is unlikely that one form of $\hat{7}$ would proceed to the other form in a piece of tonal music. The two forms are placed side-by-side simply because the scale is spelled in ascending order. Scales only portray pc content; they do not suggest common voice leadings within scales or which pcs are used most often—all of the pcs are presented as equals in a specific ordering.[8]

Because scales reflect pc content, it will not be necessary to distinguish between the major scale and Ionian mode as they share the same pcs. The distinction between minor and Aeolian is only necessary because the pc content is different. Shostakovich referred to his works in major keys, so the term “major scale” will be used in this thesis rather than “Ionian mode.”

Shostakovich also writes subject statements in diatonic modes in several of the fugues. Perhaps the most famous example is the C major fugue, which is sometimes called the white-note fugue because it contains no accidentals and presents the subject in all possible diatonic modes. More typically, Shostakovich uses a diatonic mode in one or

[8] One slight twist to the statement that scales show pc content is the fact that several of the subjects, as with much music, do not use all the pcs of the underlying scale, as in Example 2, where no G is present. In such examples, the missing pcs can often be filled in, especially by looking at later countersubjects. The missing pc element could be seen as another weakness of representing music as scales. In most cases, the distinction between which pcs are in an underlying scale and which are actually used in the music is an arbitrary one, but this issue of missing pcs will become important in later analysis.

two isolated statements. Diatonic modes sometimes have an important role in the counter-expositions of these fugues.

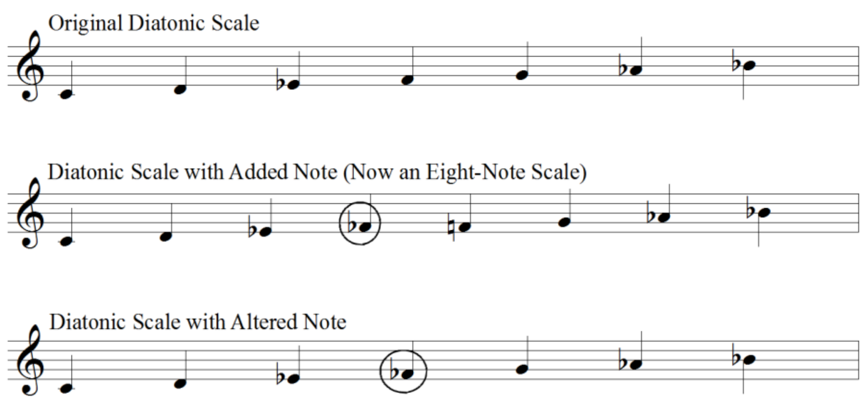
As was mentioned in the first chapter, almost all of Shostakovich’s fugues contain counter-expositions with a subject and answer in the relative scale immediately following the expositions. Sometimes, Shostakovich retains the center of the relative scale but changes the mode. This phenomenon is more common in minor fugues, where he occasionally presents the major version of the subject in Mixolydian. The relative major relationship is recognizable, even though the literal major scale is not used. Mixolydian and Lydian could be recognized as major modes because their $\hat{3}$ is a major third above $\hat{1}$. A similar case could be made to classify Dorian, Aeolian, Phrygian, and Locrian as minor modes. One example of this modal substitution is shown in Example 4 with the subject from the counter-exposition of the E-flat minor fugue.

Example 4. Mixolydian standing in for the relative major

Although diatonic scales are used most often in Op. 87, Shostakovich occasionally makes use of other modes, often derived from the diatonic set. The study of mode in Shostakovich’s music has been one of the most popular areas of research, especially in Russian music theory.[9] The diatonic set can be changed in two ways to create new modes—existing pcs can be altered in ways that do not create other diatonic

[9] Carpenter, “Russian Theorists,” 76.

modes or new pcs can be added to the set to create scales with more than seven members. Shostakovich uses both methods in his cycle of preludes and fugues. Scales with added pcs will be the focus of chapter 3, though many principles of that study can be applied to scales with altered pcs. Example 5 shows a sample diatonic scale that has had one pc added or altered. The concept of added and altered pcs can be expanded to include more than one addition or alteration.



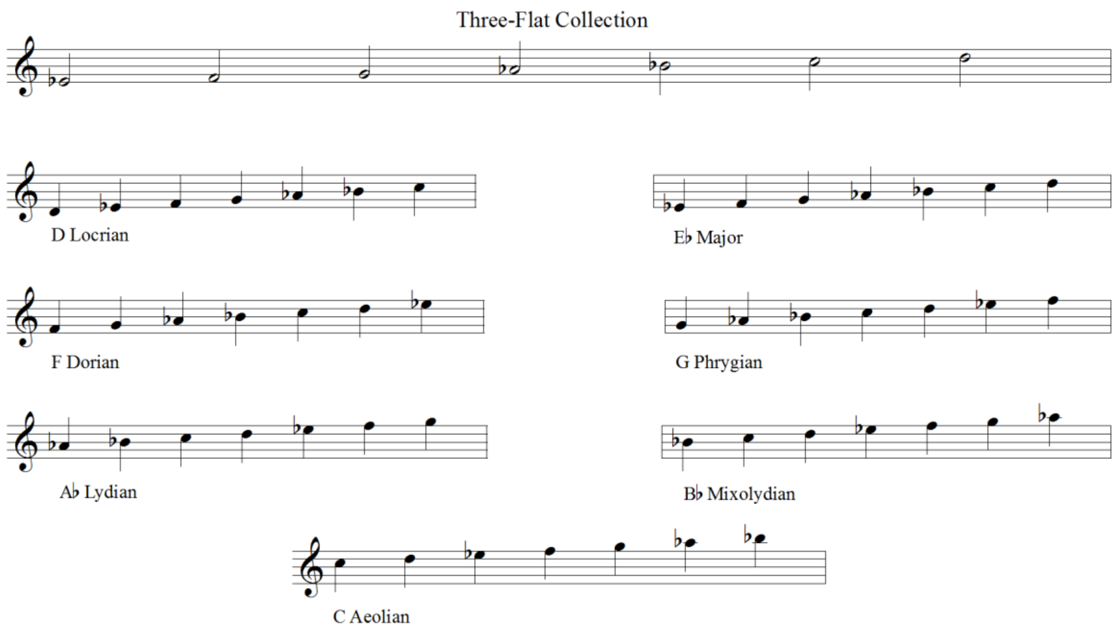
Example 5. Diatonic scales with added and altered notes

Collections

A collection, in contrast to a scale, is a group of pcs in which no pc is privileged more than the others as a center. The three-flat collection, for example, contains A-flat, B-flat, C, D, E-flat, F, and G. As a collection, none of those pcs are emphasized over the others. Any of them could be made into a center, as shown in Example 6.[10] Consequently, several scales use the same collection. This conception of *collection* is similar to Dmitri Tymoczko's conception of *scale*, in the fact that any of the members could be promoted

[10] In this thesis, scales will be represented by quarter notes, with the lowest pc suggesting center. Collections will be represented by half notes. Both will also be represented by letter names. When the center is relevant, that pc will be boxed.

as center, and the ideas of collection and center are separate.[11] Scales, on the other hand, are tied to the concept of a center. F Phrygian cannot exist without the idea of F as $\hat{1}$, and scale degrees are an integral part of a scale.

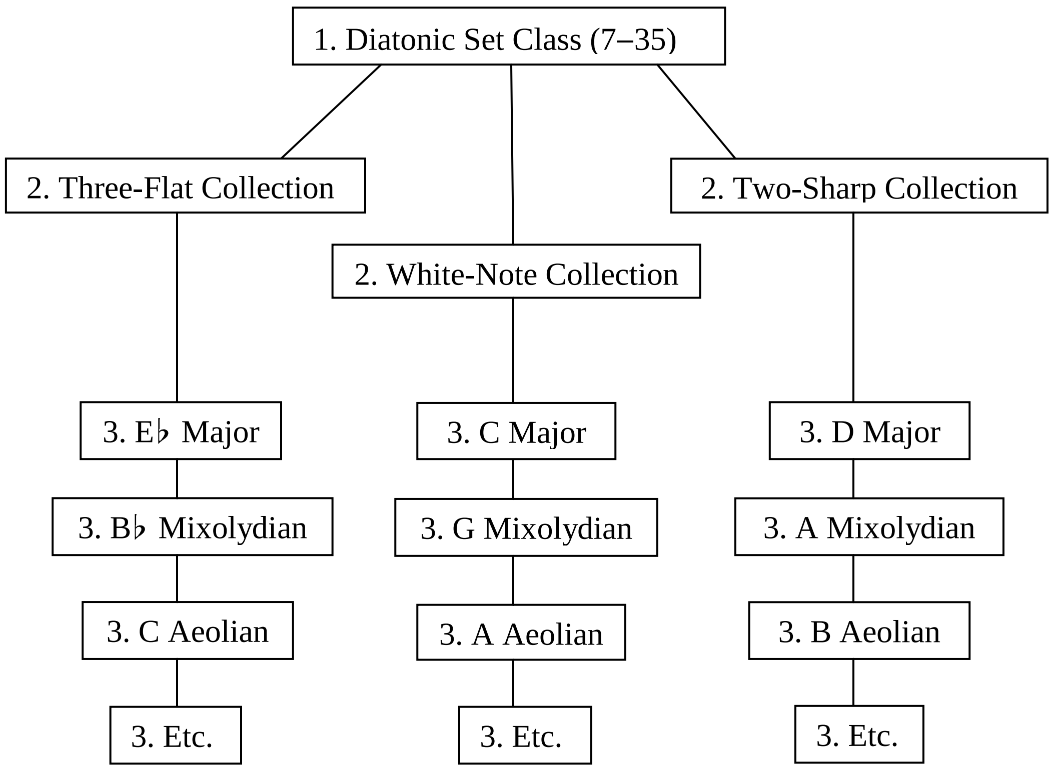


Example 6. Scales of the three-flat collection

Much of this thesis will refer to diatonic scales, which are members of the diatonic set class, or 7–35, according to Forte's labeling system.[12] The "three-flat collection," as an example, will refer to the collection of pcs suggested by a traditional three-flat key signature; the "white-note collection" will refer to the diatonic set containing only white notes on a piano and represented by a key signature with no flats or sharps. Twelve diatonic collections exist, allowing for enharmonic equivalencies. Each

[11] Dmitri Tymoczko, *A Geometry of Music* (New York: Oxford University Press, 2011), 116–20.
[12] A set refers simply to a collection of pitches or pcs. A set class, on the other hand, includes all sets that preserve the same intervallic content under transposition and inversion. All diatonic scales contain the same intervals. For example, they all contain one diminished fifth and six perfect fifths. The diatonic set class encompasses all of the diatonic modes, from B-flat Locrian to D Lydian. The term *diatonic set* will often be used in this thesis as a synonym of collection. It will especially be used to discuss features that would be true of any individual collection.

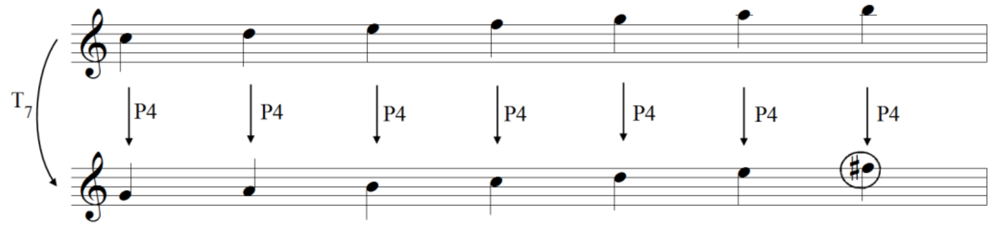
diatonic collection contains seven pcs and therefore can be rotated to form seven different scales. Example 7 illustrates this hierarchy by including three examples at each level.



Example 7. Diatonic set class hierarchy

Transformations

One way scales can be related is through transposition. Two types of transposition will be discussed in this chapter. The first is the most commonly discussed type of transposition, here called chromatic transposition, where all of the pcs are transposed by the same specific interval. For example, in Example 8, all of the pcs in the C major scale are transposed down by a perfect fourth. Since every pitch is transposed by the same interval, the mode remains the same—major. Transposing every note in a section of music by the same interval almost always creates chromatic alterations—a fact reflected in the term *chromatic* transposition.[13] In the case of Example 8, the only new pc created by the chromatic transposition was F-sharp. Chromatic transpositions are frequently represented by the T_n operator, where n represents the number of half steps the transposed scale or passage is above the original in pitch-class space. T_7 represents transposition up seven half steps.



Example 8. Chromatic transposition

Technically, intervals in pitch-class space do not exhibit direction. In Example 8, C and G are shown as pitches, but the T_7 transposition could measure the distance from any (and all) C to G. The G could be a perfect fourth below the C, a perfect fifth above the C, or two octaves and a perfect fourth below the C. The most important factor is that the C came first. T_7 and T_5 both involve transposition of a perfect fourth or fifth. Order is important in this context because the two transpositions produce two different scales, as is shown in Example 9. T_7 is an especially pertinent example in this study: it represents the normative transposition level of a real fugal answer. T_5 returns the subject to the original scale. T_7 and T_5 are inversions of each other.

T_7: C D E F G A B → G A B C D E F♯

T_5: C D E F G A B → F G A B♭ C D E

Example 9. T_7 and T_5 transpositions

[13] Chromatic transposition of a diatonic set will always create new, chromatic pitches as long as all seven members of the set are represented in the music.

The number of common notes between scales related by chromatic transposition can be predicted by the interval-class vector of the diatonic set class, which shows how many of each interval class can be found in any diatonic scale. An interval class (ic), much like a pitch class, contains multiple intervals. These include an interval, its inversion, and compound intervals created by adding one or more octaves to either the interval or its inversion. For example, the inversion of a perfect fourth is a perfect fifth. Both of these intervals are included in the same interval class, which is represented by one slot in the ic vector because the quantity of both intervals in a set will always be the same. Every diatonic scale contains six perfect fourths and also six perfect fifths.

Because direction does not matter with interval classes, the smallest interval often represents the class. In post-tonal theory, this interval is measured in half steps, but traditional diatonic intervals will be used in this thesis. As an example, minor seconds and major sevenths are inversions of each other. In the context of interval classes, I will speak of minor seconds, but anything said about them also applies to major sevenths. Interval-class vectors give the quantity of every interval class in a set class. These are arranged from smallest to largest as read from left to right as is illustrated in Example 10.

<___ ___ ___ ___ ___ ___>
m2 M2 m3 M3 P4 TT

Example 10. Interval-class vector

The interval-class vector for the diatonic set class is unique in that it contains a different number of every type of interval—<254361>.[14] Each number shows how many common tones will be retained if the diatonic set is transposed by that interval class. For

[14] Richmond Browne, "Tonal Implications of the Diatonic Set," *In Theory Only* 5, no. 6–7 (1981): 6.

example, if the diatonic set is transposed up or down a half step, two of the pcs will remain the same. This information is gathered from the number in the first slot of the interval-class vector. Transposing by a perfect fourth or perfect fifth will produce the most common tones, while transposing by the tritone produces only one.[15]

Since diatonic scales contain a different number of each interval class, transposition by each interval class retains a different number of common tones. Each interval class is represented in two chromatic transpositions. Any diatonic scale will share six notes with one pair of same-mode scales, five notes with another pair, and so on. In Example 9, the C major scale was transposed by T_7 and T_5. Both are transpositions of ic 5 (the fifth slot), so both retain six notes. However, the distinction between T_7 and T_5 is important because which notes are shared is different. Transposition of any diatonic scale by T_7 results in the addition of one sharp, while transposition by T_5 adds one flat. Neither the T_n label or the interval-class vector communicates this fact directly.

Example 11 shows all twelve possibilities, organized from most shared pcs to least. The pairs are also arranged in circle of fifths order, though this information cannot be gathered by the interval-class vector directly. The interval-class vector only conveys how many common notes will be shared in each pair of transpositions. Because T_n always retains the mode of the original scale, Ian Bates names the relationship between the two scales a *fixed-scale-type relationship*.[16]

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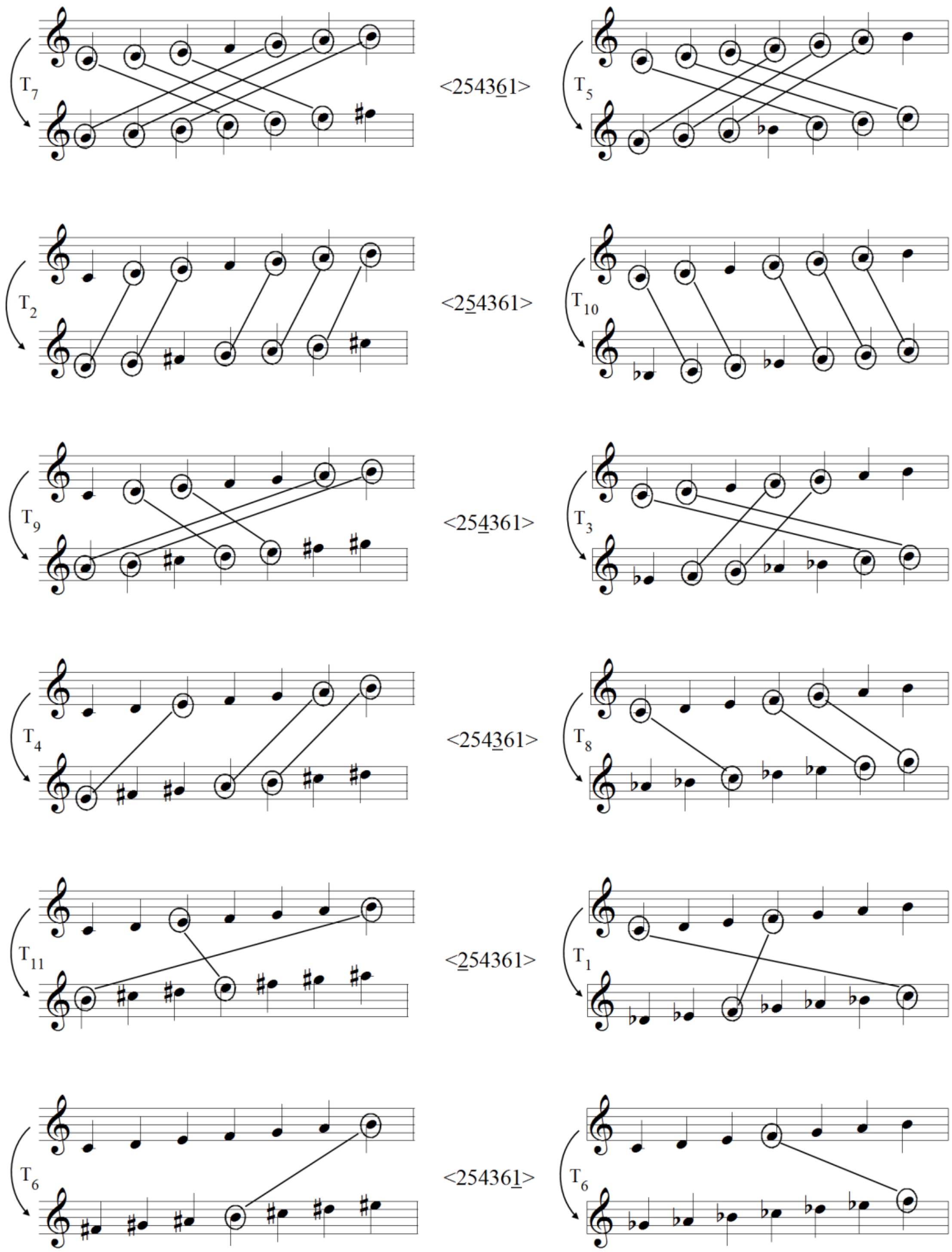
[15] The tritone is the inversion of itself, so each tritone in the interval class vector actually represents two common tones. However, one of these notes is spelled differently in each scale. In post-tonal theory, two enharmonically equivalent notes are considered to be the same, but in a diatonic context, such notes are often considered to be different. C major and F-sharp major, as an example, both contain B. The other "shared" note is the F in C major and the E-sharp in F-sharp major as can be seen in Example 11. Because they are spelled differently, I do not consider these notes to be the same for the purposes of this thesis.

[16] Ian Bates, "Vaughan Williams's *Five Variants of 'Dives and Lazarus'*: A Study of the Composer's Approach to Diatonic Organization," *Music Theory Spectrum* 34, no. 1 (2012): 35–36.



Example 11. Diatonic set pitch invariance

Because Example 11 is arranged by the number of shared pcs, it also shows a spectrum of relatedness, with the closely related keys at the top of the page, and the least related keys—with a relation of a tritone or T_6—at the bottom of the page. The traditional conception of closely related keys requires that they share six or all seven pcs. In tonal music, any key is closely related to five other keys—the relative major or minor key and the major and minor keys one sharp and flat away. For example, C major would be closely related to A minor, which shares its key signature; G major and E minor, the major and minor keys one sharp away; and F major and D minor, which differ by one flat.

This situation is somewhat complicated by considering the modal possibilities of each collection: in this perspective, each scale is actually closely related to twenty other scales. Six modes belong to the same collection as the original and thus share all seven pcs. Seven modes belong to each of the collections related to the original scale by T_5 or T_7. In the case of C major, we would count the other six modes in the white-note collection, the seven modes in the one-sharp collection, and the seven modes in the one-flat collection. The relationships among these twenty scales will be explored in more depth later in this section.

In contrast to chromatic transposition, scales can also be related by diatonic transposition.[17] Diatonic transposition refers to transposition by a generic interval, such as a fifth, while in chromatic transposition, every pc is transposed by the same specific interval, such as a perfect fifth or diminished fifth. Diatonic transposition uses generic intervals so that the transposed music will keep the same key signature as the first. A traditional example of this phenomenon is diatonic sequences, where a motive is moved to different pitch levels within a key, but no chromatic alterations occur. As a result, the

[17] *Diatonic* is a term that will be used in several contexts in this thesis. It will chiefly be used to describe diatonic scales, which are members of 7–35; however, diatonic is sometimes used to refer to music that does not leave the key. It is from this latter definition that the term diatonic transposition takes its name.

motive might contain minor thirds in one version and major thirds in another. The repetition is not exact, but we recognize the units as similar.[18] Diatonic transposition is often recognized as transposition within one scale, especially in common practice music where use of diatonic modes is rare.

Julian Hook represents diatonic transpositions with the operator t_n, which will also be used here. The lower-case t distinguishes it from the T_n chromatic transposition operator, and the n represents the number of ascending scale-steps. Hook agrees that this operator can be used in the context of a single scale (as in the transposition of a motive to a new location in the scale, for example). In addition, he sees that t_n can act within a collection to move between scales that share a key signature, such as C Major and D Dorian.[19] Bates uses this latter perspective in one of his fixed-domain diatonic relationships—*fixed-key-signature relationships*—to relate scales that share a key signature but differ in mode and center.[20] For the most part, this thesis will consider t_n to be transposition to other scales within a collection, but this view will be challenged in later chapters. Deciding whether a diatonic transposition occurs within one scale or within a broader collection depends on whether the center changes or not.

Example 12 shows the subject for the B-flat minor fugue. The subject is diatonic to the five-flat collection and centered around B-flat, so it is in the Aeolian mode. An expected real answer would be a chromatic transposition of the subject at T_7—either transposed up a perfect fifth or down a perfect fourth—to F Aeolian, as shown in the

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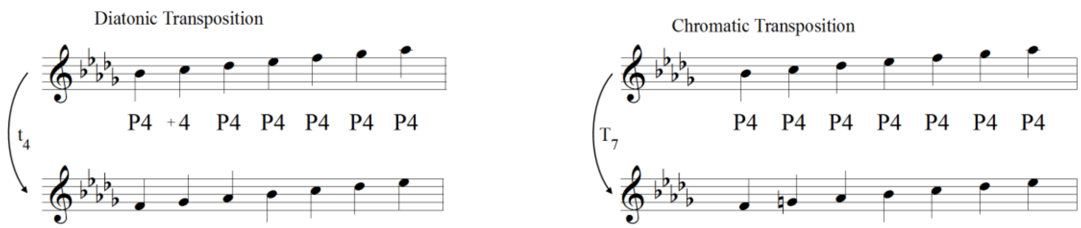
[18] Tymoczko, *Geometry of Music*, 116–20.
[19] Julian Hook, “Signature Transformations,” in *Music Theory and Mathematics: Chords, Collections, and Transformations*, ed. Jack Douthett, Martha Hyde, and Charles J. Smith. (Rochester, NY: University of Rochester Press, 2008), 137–40 and 147–48.
[20] Bates, “*Five Variants*,” 35–36.

second line of the example. Such a transposition would introduce the chromatic pc G-natural. The answer that Shostakovich actually gives, reproduced in the third line of Example 12, is a diatonic transposition of the subject. Instead of introducing a G-natural, Shostakovich's answer retains the G-flat, so both the subject and the answer share the same five-flat collection.

Example 12. B-flat minor fugue subject with chromatic and diatonic answers

Each pitch of this answer is still a fourth below the corresponding pitch of the subject, but as no chromatic alterations are introduced, not every fourth is of the same quality. As Example 13 illustrates, most of the fourths between pitches of the scales are perfect fourths, but the fourth between C and G-flat is an augmented fourth. This is the difference between the diatonic transposition on the left side of Example 13 and the chromatic transposition on the right side, where every member of the scale is transposed by the exact same interval.

Retaining the G-flat in the answer also changes the relationship between intervals and scale degrees. In the subject, a whole step exists between $\hat{1}$ and $\hat{2}$, followed by a half step between $\hat{2}$ and $\hat{3}$; in the answer, the interval between $\hat{1}$ and $\hat{2}$ is a half step, and the $\hat{3}$ is a whole step above $\hat{2}$. Even though the collection stays the same, the change in



Example 13. Diatonic vs. chromatic transposition

intervals between scale degrees causes a change of mode. The answer has a lowered $\hat{2}$ and therefore is now in F Phrygian.

Modes that are related by diatonic transposition can also be regarded as rotations of the same collection. Because none of the pcs in the scale change, the motion between scales which share a key signature can be visualized in a horizontal direction, where the collection is shifted a number of slots to the left to begin on a new center. This process is illustrated in Example 14. For t_1, each of the pcs moves one slot to the left, and the C is moved to the right edge. A similar process happens with t_6.

$\boxed{t_1}$

$\xleftarrow{\text{one slot}}$

C D E F G A B = D E F G A B C

$\boxed{t_6}$

$\xleftarrow{\text{six slots}}$

C D E F G A B = B C D E F G A

Example 14. Rotation of a diatonic collection

The B-flat minor fugue continues to present statements of the subject at different pitch levels while remaining true to the same five-flat collection. As a result, no chromatic pitches appear in the first thirty-six measures of the fugue, and the subject eventually appears in each of the diatonic modes. The exposition introduces B-flat

Aeolian. The rotation which produces F Phrygian in the answer is an unexpected substitute for the normative real answer. Another diatonic transposition of t_3 cancels out the original motion of t_4 to take us back to where we started—B-flat Aeolian—for the second presentation of the subject. At m. 20, the counter-exposition, as expected, begins in the relative major, D-flat, followed by its answer in A-flat Mixolydian. These scales are related, once again, by t_4, which continues to keep the fugue in the same collection. At m. 32, Shostakovich rotates the collection back one slot to G-flat Lydian. At this point, Shostakovich has presented the subject with five scales from the five-flat collection, with no chromatic pitches. All of the relationships between subject entrances can be explained by t_n transpositions, as shown in Example 15.

m. 1 B♭ Aeolian
↓ t_4
m. 5 F Phrygian
↓ t_3
m. 11 B♭ Aeolian
↓ t_2
m. 20 D♭ Major
↓ t_4
m. 24 A♭ Mixolydian
↓ t_6
m. 32 G♭ Lydian

Example 15. Diatonic transpositions in first half of B-flat minor fugue

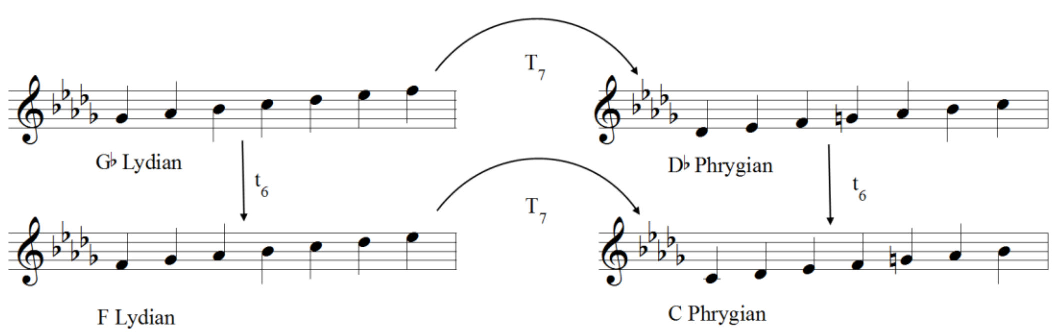
The expectation for a completely diatonic fugue is thwarted in m. 37, when a G-natural suddenly appears. The new chromatic pitch does not emerge in the subject itself until near the end of the second measure of the statement, but it is presented in the countersubjects four times before then, including one appearance on the first sixteenth note of m. 37 in the soprano voice. What consequences does this chromatic pitch have for the scalar structure? The subject in these measures is presented beginning on C. In the five-flat collection, the scale centered on C is the C Locrian scale—the one diatonic scale with a tritone between $\hat{1}$ and $\hat{5}$, which often makes the center of a Locrian scale more difficult to establish through tonal means.[21] The interval between $\hat{1}$ and $\hat{5}$ features prominently in the subject of the B-flat minor fugue and in its interactions with the countersubjects. These are boxed in Example 16, which reproduces the subject and countersubjects as they appear in m. 37.

Example 16. Use of G-natural in B-flat minor fugue

[21] The relationship between $\hat{5}$ and $\hat{1}$ is one of the cues for center remaining from common practice tonality. In the absence of other information, the upper note of a fourth or the lower note of a fifth is likely to be center because of this crucial relationship. Piet G. Vos, "Key Implications of Ascending Fourth and Descending Fifth Openings," *Psychology of Music* 27, no. 1 (1999): 4–17. Centers of modes can also be established with common practice cues, but the cue of the ascending fourth/descending fifth is weak in the Locrian mode because the interval between the first and fifth scale degrees is a diminished fifth in this mode. For this reason, "the use of ic5" is a better summary of the cue as used in pitch-centric music. Kleppinger, "Pitch Centricity," 76–78.

Shostakovich raises the diatonic G-flat to G-natural, which creates a perfect fifth between $\hat{1}$ and $\hat{5}$. This alteration also changes the collection and scale used in these measures. Instead of the anticipated C Locrian of the five-flat collection, this presentation of the subject uses C Phrygian. The relationship between C Phrygian and the G-flat Lydian which preceded it cannot be explained with the methods presented so far. The change of scale includes a shift in mode and a shift in collection. The relationship can, however, be explained by a *combination* of chromatic and diatonic transpositions.

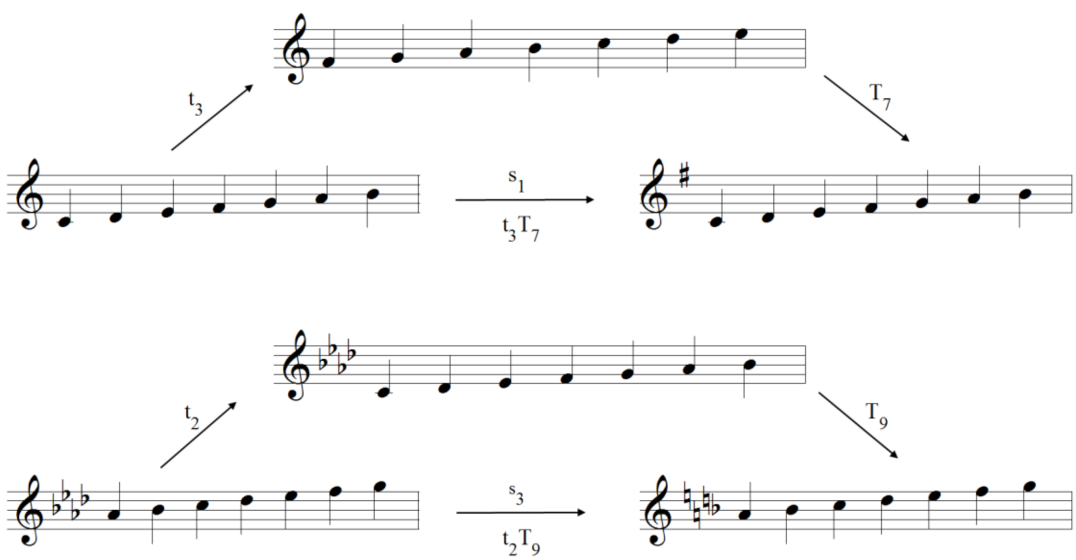
Tymoczko calls this combination of transpositions an interscalar transposition.[22] Example 17 shows the relationship between G-flat Lydian and C Phrygian according to his method. In order to get to C Phrygian, a compound transposition must be undertaken—first a chromatic transposition up seven half steps and then a diatonic transposition up six diatonic steps. Example 15 also shows that these operations are commutative, which means that if G-flat Lydian were transposed up six diatonic steps first and then underwent the chromatic transposition second, C Phrygian would still be the result. In other words, it does not matter in which order these operations are applied—horizontal then vertical or vertical then horizontal. Interscalar transposition explains the relationship between the two scales, but the method is somewhat unwieldy and does not convey the seemingly simple addition of one new chromatic pitch.



Example 17. Interscalar transposition after Tymoczko

[22] Tymoczko, *Geometry*, 140–42.

Hook includes an example that shows similar scalar relationships in the beginning of his article on signature transformations. In this article, Hook introduces two new operators—s_n adds n sharps (or takes away n flats) and f_n adds n flats (or takes away n sharps). Sharps and flats are added or taken away according to the order of the circle of fifths, so s_1 adds the next sharp in the key signature and f_1 takes the last sharp away.[23] Example 18 shows two examples of signature transformations and how they are related to the T and t operators. Notice that the s_3 transformation takes three flats away from a four-flat key signature. Also, the scale beginning on A-flat changes into a scale beginning on A, showing that the center can be changed by signature transformations, as long as an A of some type is preserved.[24]

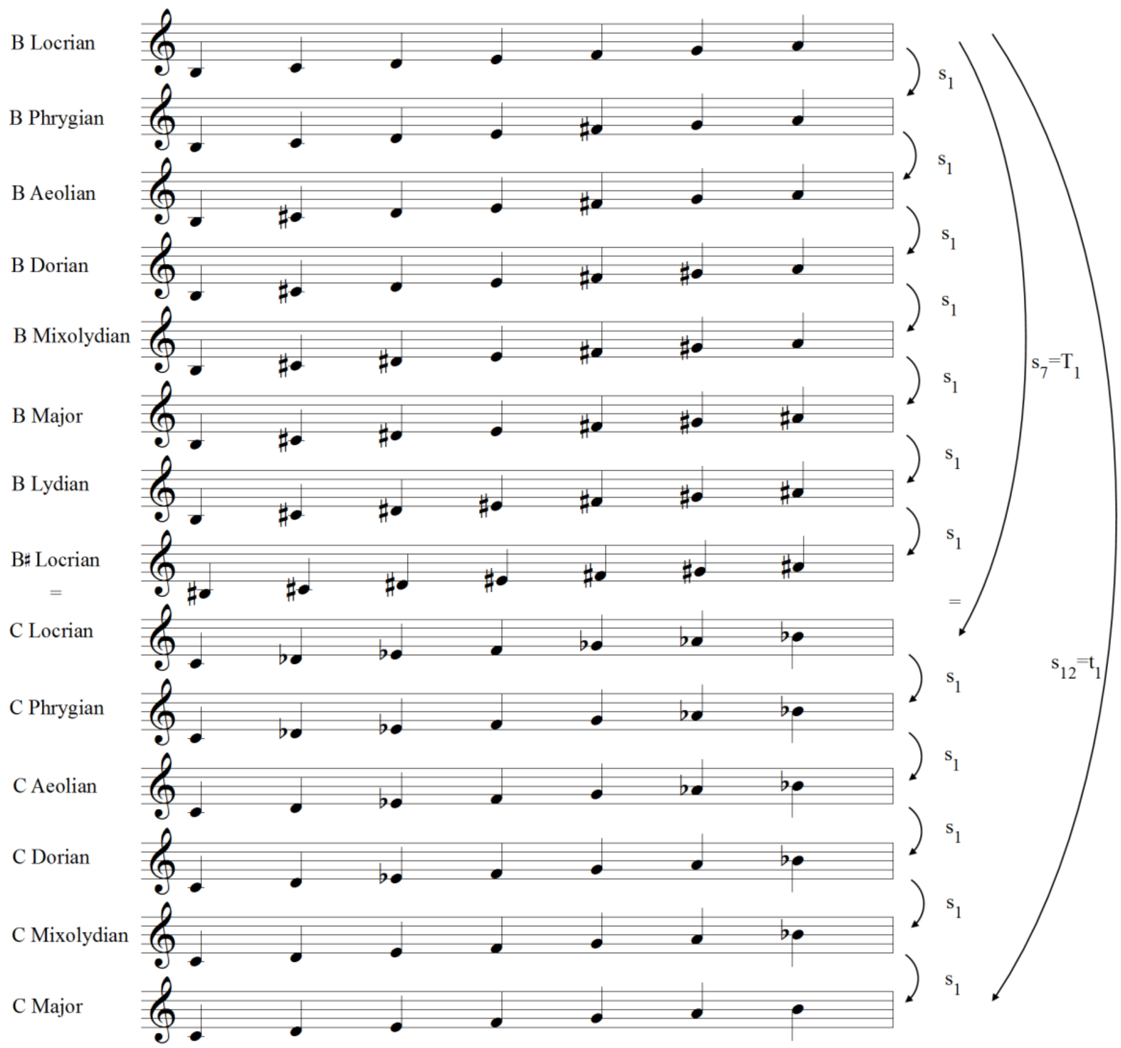


Example 18. Signature transformations and interscalar transpositions

Signature transformations can show the relationship between any two of the diatonic modes (assuming enharmonic equivalency). The modes form a cycle with eighty-four members—the product of twelve chromatic pcs with seven diatonic modes

[23] Hook, "Signature Transformations," 137–38.
[24] Ibid., 140.

starting on each pc. Example 19 shows part of this cycle, much like Hook's Figure 6.10.[25] This example shows more clearly how signature transformations are related to chromatic and diatonic transpositions. Adding seven sharps is the same as applying T_1. Similarly, adding twelve sharps gives the same result as t_1.[26] One adjustment for enharmonic equivalency (B-sharp Locrian = C Locrian) is included in this example.



Example 19. Portion of signature transformation cycle after Hook

[25] Ibid., "Signature Transformations," 148. In contrast to Hook, Example 19 names each of the scales. Hook makes it clear that signature transformations act on fixed diatonic forms and do not inherently imply centers.
[26] Ibid., 143–44.

Notice that the seven modes that share the same center are adjacent in the signature transformation cycle. This grouping shows another type of relationship between modes—what Bates calls *fixed-tonic relationships*—where the tonic is retained with a gradual accumulation of sharps along a continuum between the Locrian and Lydian modes. A similar perspective is often used to describe modes as based on the major and minor scales: Mixolydian is the same as major but with a lowered $\hat{7}$; Dorian is a minor scale with a raised $\hat{6}$. Bates uses the concept of fixed-tonic relationships in his analysis of Vaughan Williams, who frequently made use of these relationships, especially as part of a trajectory that gradually adds sharps to the scale.[27]

This perspective of tonic-preserving modes is distinctive from the idea of modes related by a rotation of the same collection (t_n). In Op. 87, Shostakovich is much more likely to use modes that share the same collection. Unlike Vaughan Williams, he seldom uses modes that share the same tonic. For this reason, the application of signature transformations to show tonic-preserving relationships is not useful in the analysis of this work. Fortunately, the concept of signature transformations is much larger—because the cycles of T_n and t_n are embedded within the cycle of signature transformations, the relationships between the three operators, expressed in Example 19, can be used to determine how any two diatonic modes are related.

Example 20 illustrates two examples of signature transformations. The first is an example of a simple s_1. One sharp is added to the scale, and the center is retained. In the second, the center is changed from C to G, so the two scales are much further apart in the cycle of eighty-four scales. The relationship between the two scales is T_7, and as was

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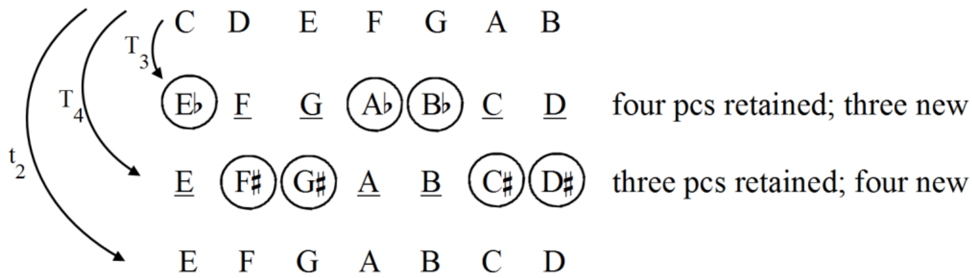
[27] Bates, "*Five Variants*," 36–38, 46.

pointed out earlier, each T_1 is the same as adding seven sharps, so applying T_7 is the same as adding forty-nine sharps, adjusting for enharmonic equivalency where necessary. If the F-sharp were not present in the third line (making the scale G Mixolydian), the relationship between the first and third lines would be s_{48}—a result which could also be reached by realizing that the scales are related by t_4, and each t_1 is the same as applying twelve sharps (4 * 12 = 48).

Example 20. Two signature transformations

This number of sharps is large and impractical. Using a transformation such as s_{49} in an analysis is as cumbersome as an interscalar relationship, such as T_7t_6. The number of sharps separating the two scales tells us little about the music and, once again, does not reflect the seemingly simple addition of one chromatic pc. However, the addition of signature transformations provides new flexibility in the analysis of compound transformations.

Example 21 shows the relationships between C major and three scales centered on a version of E—E-flat major, E major, and E Phrygian.



$T_3 = s_{21}$	$s_{21} = t_2 f_3$	(2)(12)-3=21
$T_4 = s_{28}$	$s_{28} = t_2 s_4$	(2)(12)+4=28
$t_2 = s_{24}$	$s_{24} = T_3 s_3$ or $T_4 f_4$	(3)(7)+3=24 or (4)(7)-4=24

Example 21. Transformation equivalencies

We know from the earlier discussion on transposition that the transposition between C major and E-flat major is T_3 or up a minor third. This transposition retains four common tones. The chromatic transposition to E major is similar because it is also a chromatic transposition and produces a fixed-scale-type relationship, but this time, the transposition is up a major third and retains three common tones. The last transposition, C major to E Phrygian, is a diatonic transposition: the white-note collection is moved up two scale steps to begin on E. This example allows us to see that the white notes retained by the E-flat and E major scales correspond to the pcs that make up E Phrygian. This correspondence happens because a diatonic transposition transposes each pc by a generic third—both major and minor thirds are used. In the chromatic transpositions, each pc is transposed by either a minor third or a major third, so the pcs of the white-note collection are divided between T_3 and T_4.

The second half of Example 21 shows how each of these transpositions can be expressed in different ways, thanks to signature transformations. Every T_n can be expressed as a combination of t_n and s_n, and every t_n can be shown with a T_n and an s_n. In either case, f_n might be used in place of s_n because it is often advantageous to keep the number of flats and sharps as small as possible. The first column shows how each transposition is translated into sharps, using the information from Example 19.[28] The second column changes that number into the other type of transposition plus sharps or flats, and the third column demonstrates arithmetically how the two transformations are equivalent. For example, with s_{21} in the top row, 21 is quite close to 24, which is a multiple of 12 (12 sharps=t_1), so s_{21} can be expressed as t_2 with the addition of f_3 to take the three extra sharps away (24-3=21).

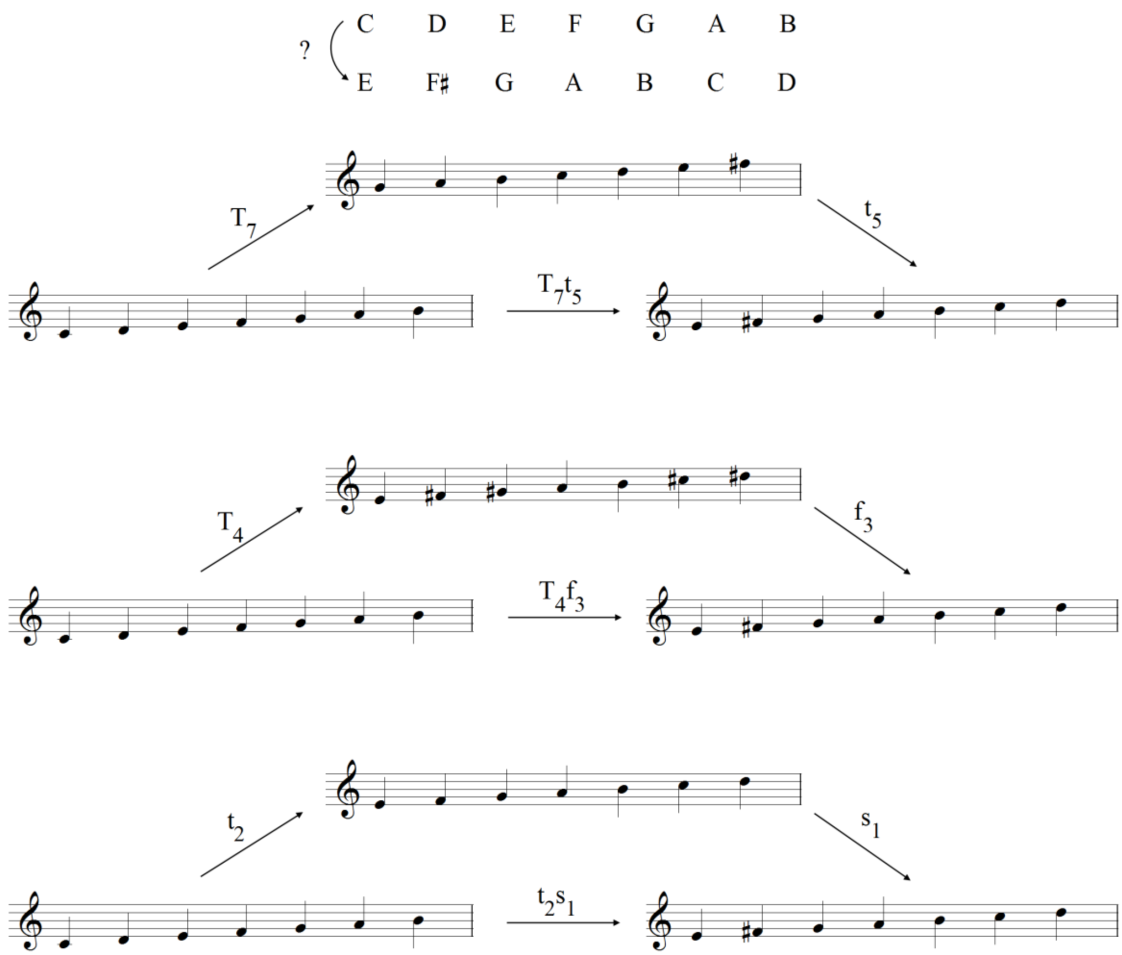
Expressing each transposition in terms of the other provides potentially useful information to the analyst. T_n transpositions add chromatic pitches, but no information about the added accidentals is provided by the T_n label. It is impossible to know how many and which accidentals are involved—T_n simply communicates how many ascending half steps are used in the transposition. Changing the chromatic transpositions to a form of $t_n s_n$ allows us easily see how many sharps or flats were added in the transposition. The transposition from C major to E-flat major is the same as rotating the C major scale to begin on E and adding three flats. A similar process occurs in the transposition from C major to E major, but this time, four sharps were added after the rotation. The information provided by a $t_n s_n$ label is the opposite of the information

[28] From this information, we can see that E Phrygian would fall between E-flat major and E major in the signature transformations cycle.

gathered from the interval-class vector—the interval-class vector tells us how many pcs will be retained in a chromatic transposition, and a $t_n s_n$ label shows how many and which accidentals will be added. This type of information will become useful in the upcoming analyses, where one of the aims will be to keep track of how many chromatic pcs are added or taken away.

Writing a diatonic transformation as a $T_n s_n$ is, in some ways, not as useful because less new information is provided. Any t_n will keep the same key signature because diatonic transpositions move within one collection. The sharps and flats in $T_n s_n$ labels in the third row are misleading—no sharps or flats were added to the original scale. Instead of giving us information about the diatonic transposition itself, this label is relational. It allows us to see how the diatonic transposition compares to nearby chromatic transpositions. This type of information would be useful analytically when a chromatic transposition is the norm, so that it would be advantageous to show a diatonic transposition in terms of that chromatic transposition. For example, in the beginning of the B-flat minor fugue, Shostakovich uses a diatonic answer, which is unexpected. Most fugue answers are related to the subjects by T_7, a transformation that adds one sharp in diatonic contexts. To show exactly how the answer in the B-flat minor fugue deviates from what is expected, a $T_7 f_1$ label could be used.

Similar combinations of T_n, t_n, and s_n can be used to describe the relationship between scales that cannot be expressed by T_n or t_n alone. The distance between such scales can be measured by a $T_n s_n$ or $t_n s_n$ label as demonstrated above, but they can also be shown with a $T_n t_n$ compound transposition, such as Tymoczko's interscalar transpositions.[29] Example 22 will illustrate this claim.



Example 22. Compound transformations that cannot be explained by a single operator

Of these three, the bottom example showing $t_2 s_1$ will be most helpful in the study of Shostakovich's fugues because, once again, it emphasizes how the collection changes. Using a $t_n s_n$ label shows which accidentals are added, but also how the collection must be rotated to begin on the new center. A change in both domains is necessary to get from the

[29] Hook, "Signature Transformations," 144.

first scale to the second. In this way, we will be able to make use of signature transformations with music that does not feature fixed-tonic relationships.

Especially at the beginnings of his fugues, Shostakovich often uses scales which are closely related, so it will be helpful to show how only one sharp or flat was added to the collection. The relationship between half of closely-related scales cannot be explained by T_n, t_n, or signature transformations alone. As such, these scales differ in all of Bates's domains—they do not share a center, key signature, or tonic. However, as closely-related scales, they have more in common than not, at least in terms of pc content. Using a combination of operators helps us to visualize this relationship as well as the two-step process necessary to get from one scale to another.

Example 23 is a table which includes all twenty of the scales that are closely related to C major. The first column consists of the scales in the one-flat collection. The scales that share a collection with C major are in the second column, and the third column includes the scales from the one-sharp collection. As the table shows, moving from C major to any scale in the one-flat collection adds one flat; the only part that changes is the t_n which shows how the collection is rotated to stop on different centers. The same type of process applies to modes in the one-sharp collection, while moving to scales in the white-note collection uses the expected diatonic transpositions.[30]

[30] In some ways, this table resembles Bates's Table of Diatonic Relations. I arranged the scales within a collection by their letter names instead of by fifths. For this reason, the signature transformations move from left to right in the rows of the table. Bates, "*Five Variants*," 39.

Scales from 1♭ Collection			Scales from White-Note Collection			Scales from 1♯ Collection	
	C Major			C Major			C Major
C Mixolydian	f_1		(C Major)			C Lydian	s_1
D Aeolian	t_1f_1		D Dorian	t_1		D Mixolydian	t_1s_1
E Locrian	t_2f_1		E Phrygian	t_2		E Aeolian	t_2s_1
F Major	T_5 or t_3f_1		F Lydian	t_3		F♯ Locrian	t_3s_1
G Dorian	t_4f_1		G Mixolydian	t_4		G Major	T_7 or t_4s_1
A Phrygian	t_5f_1		A Aeolian	t_5		A Dorian	t_5s_1
B♭ Lydian	t_6 f_1		B Locrian	t_6		B Phrygian	t_6s_1

Example 23. Closely related scales table

Up to this point, all scales have been shown beginning with the center. As such, all scalar relationships have been shown as the motion from one scale beginning on its $\hat{1}$ up to another scale also beginning on $\hat{1}$, as in the relationship between C major and E Aeolian in Example 22. Example 24 illustrates a contrasting perspective used by Tymoczko to highlight voice leading between scales.[31] In this view, the same letter names appear in each column to emphasize the change in collection, and the shift of center is present but secondary. This example resembles a simple s_1 signature transformation, but the center also changes, reinforcing once again the idea of a dual motion between scales like these.

C D E F G A B

C D $\boxed{E}$ F♯ G A B

Example 24. Voice leading between C major and E Aeolian

Keeping these ideas in mind, we can continue our analysis of the B-flat minor fugue. Much of the fugue uses t_n transpositions, which we now see could be expressed in

[31] Tymoczko, *Geometry*, 129–31.

other ways. Using the t_n operator is still the best option in this analysis as it shows that each statement of the subject remains within the collection, and as such, no chromatic pitches are added until m. 37. We now have the tools to describe the relationship between the G-flat Lydian and C Phrygian statements in the middle of the fugue. The operation t_3s_1 concisely captures the idea that the G-flat was raised one half step, and this new, closely-related collection was rotated three scale-steps.

Example 25 shows these concepts in action. The first line provides the pitch-class content of the first part of the fugue. Because the collection does not change for five statements of the fugue subject, the box denoting center moves back-and-forth across the collection horizontally—first from B-flat to F, then to D-flat, A-flat, and G-flat. The dashed box around the G-flat indicates that the G-flat was center immediately before the collection change. When the collection changes, the center also changes to C, the pc boxed in the second row of Example 25. We now see that the shift between G-flat Lydian and C Phrygian is a simple two-part motion like the one described theoretically in the previous pages.

B♭ C D♭ E♭ F G♭ A♭

B♭ $\boxed{\text{C}}$ D♭ E♭ F G♮ A♭

Example 25. Voice leading between first two collections of B-flat minor fugue

In this fugue, that two-part shift between G-flat Lydian and C Phrygian is not gradual—the collection does not change before the center or vice versa, as sometimes happens in the time between scales.[32] Instead, both the collection and center change together, quite abruptly, on the downbeat of m. 37, right when the statement in C Phrygian begins. This sudden shift emphasizes the change in collection and helps to make the new pc quite audible in a fugue that, up to this point, glided along quite smoothly and slowly in a collection that evaded change.

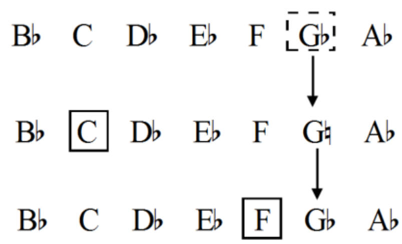
The move back to the five-flat collection is more subtle. Example 26 reproduces Example 16 with an extension to show the transition out of C Phrygian. The next scale used is F Phrygian, so this time, Shostakovich retains the mode by using the Phrygian scale that is a part of the five-flat collection. As a result, the shift back to the original collection can be expressed using a single operator—T_5. Even though these two scales share a mode, they still differ in center and collection. Unlike the move into C Phrygian, these two factors do not change simultaneously in the music. The transition is much more gradual because the shift in collection, with the reintroduction of G-flat, occurs five measures before the center changes in m. 46.

At first, it appears that a subject will be presented in F Phrygian, but the long F and triplets repeat, and the countersubject is discontinued, leading to the recognition of this passage as a false entry of the subject. We now see that Shostakovich provides this F Phrygian "bridge" to facilitate the transition back into the previous collection. The *poco rit.* indication in m. 48 leads into the true arrival of the subject, which is not only in the five-flat collection but uses the original scale of B-flat Aeolian.

[32] Ibid.

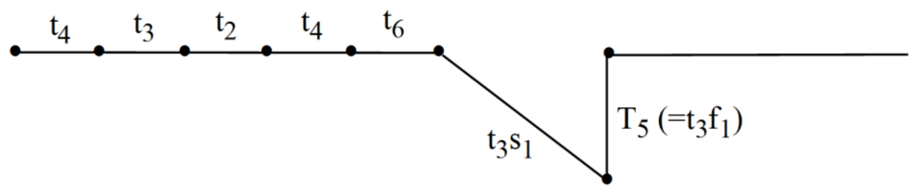
Example 26. Fugue in B-flat Minor, mm. 38–49

Looking at the voice leading between scales helped us to visualize how Shostakovich moves from G-flat Lydian to C Phrygian. Example 27 extends this representation to include the move back to F Phrygian and the five-flat collection. Showing the relationships between scales in this way highlights the change of collection and center; on the other hand, it does not emphasize graphically that the second and third scales use the same mode.



Example 27. Voice leading between collections in B-flat minor fugue

Example 28 provides another visualization of the analysis of the B-flat minor fugue up to this point. The line in the figure moves according to the chronological ordering of subject entries, left to right. Diatonic transpositions are shown by a horizontal line to reflect the conception of rotation as a ‘horizontal’ phenomenon used thus far. This fact is most clearly seen by looking at the top row of Example 27 where the box could be moved left and right to show a change of center, but none of the pcs change. The other lines of Example 28 show motion that is not represented by Example 27. The vertical line represents a T_n operation that provided a direct motion back to the original collection. The t_3s_1 transformation is represented by a diagonal line as its departure includes a shift in collection and a change of mode. After the T_5 transposition, the fugue continues on in the five-flat collection, as shown by the horizontal line continuing after the dip.



Example 28. Visual representation of transformations in B-flat minor fugue before m. 49

The section of the fugue after the collection shift is more difficult to describe using operations because it makes use of stretto. Overlapping statements at different pitch levels makes identifying a clear center tricky. The rhythms in the subject also change slightly in the stretti, complicating pitch and rhythmic aspects as the fugue nears its close. Because it would be difficult to hear two or three centers at once, describing scalar relationships in any stretto is problematic. For this analysis, the most important detail is that the collection remains constant. A representation of the pitch levels used in this section is shown in Example 29.

	m. 49	m. 50		m. 56		m. 57
soprano:	B♭ ————————					C————————
alto:		D♭———————			G♭————————	
bass:				E♭————————		

Example 29. Stretti near conclusion of B-flat minor fugue

One other noteworthy aspect of this section of the fugue is Shostakovich’s use of the subject at the C-level, beginning in m. 57. The only other time the subject began on C was with the detour from the collection in the form of C Phrygian. Using the G-natural in that instance created a perfect fifth in prominent places between $\hat{1}$ and $\hat{5}$, where the only diminished fifth in the five-flat collection would have appeared. In m. 57, the G-flat is retained as a part of the stretto in the five-flat collection. Shostakovich also layers subjects beginning on E-flat, G-flat, and C. As a result of both of these choices, horizontal diminished fifths and emphasized vertical diminished triads occur frequently in these measures—in contrast to their avoidance earlier in the fugue. The diminished fifth and diminished triad exist in only one location in a diatonic set. The exploitation of these sonorities raises tension as the fugue reaches for its final moments.

After a second *poco rit.*, Shostakovich again changes the collection for the last seven measures of the fugue, shown in Example 30. Here, the new center is asserted a brief moment before the change in collection is made known. Following in the footsteps of Bach, all of Shostakovich's minor-mode fugues end with a Picardy third. In this fugue, Shostakovich not only ends with the parallel major triad, he includes a rather lengthy (though incomplete) statement of the subject based on the parallel major scale, including G-naturals and A-naturals in addition to the expected D-natural. Especially because centricity in the proceeding section is uncertain due to the stretto, it might be most useful to relate this final B-flat major scale to the original scale—B-flat Aeolian. The relationship between these scales is s_3, one of the few simple signature transformations that can be spotted in this cycle, even though it is indirect.

Example 30. Fugue in B-flat Minor: mm. 66–72 and transformation between first and last scales of the fugue

This study of the B-flat minor fugue has shown how the combination of chromatic transposition, diatonic transposition, and signature transformations can be useful in the analysis of Shostakovich’s fugues. The next chapters will explore scalar relationships that do not fall under these categories and will survey some of the problems that come up in this type of analysis.

Chapter 3

Theory of Eight-Note Scales and Collections

Introduction

The theory described in the last chapter is useful in the study of relationships between scales, centers, and collections, but it is insufficient in the study of the Op. 87 fugues because it presumes the use of strictly diatonic collections. Scales with altered or added scale degrees cannot be addressed by these systems.

In this chapter, I will focus on scales with added notes. Many of the observations made in the study of scales with added notes can be applied to scales with altered notes, but in the latter case, the altered pc *replaces* one of the original pcs instead of being presented *alongside* it. Adding a note to a traditional seven-note scale might suggest adding an eighth scale degree. In fact, a scale with more than seven pitch classes often contains two versions of one scale degree. An example of this concept can be found in traditional tonality with the minor scale. As was explained earlier, I believe, for the purposes of this thesis, the best way to think of the minor scale is as a nine-note scale which contains two versions of $\hat{6}$ and two versions of $\hat{7}$. Even if one version is used more often in a particular passage, or even in an entire style, the other should not be considered a chromatic alteration. Both versions of the scale degrees should be considered to be part of the scale. The same concept can be used with other scale degrees in other scales and will become extremely useful in the analysis of Shostakovich's music.

The present exploration of scales with added pcs will begin by looking at eight-note scales which only contain one variable scale degree. Later, the theoretical framework surrounding eight-note scales will be expanded to include scales with more than one variable scale degree. The idea of scales with added pcs can be applied to small sections of music, like a fugue subject, or to larger spans, like an exposition or even an entire work. In the end, I will show how the addition of a theory about eight-note scales affects our perspective on the relationships between scales.

Example 31 reproduces the subject of the A-flat major fugue. The subject is mostly diatonic to the key of A-flat major, but it also contains D-natural. Although this D-natural could be seen as a chromatic alteration, we could also envision the D-natural to be an equal member of an eight-note scale. According to the latter option, the fugue subject is based on an eight-note scale with a variable fourth scale degree; this scale is reproduced in Example 31. Each instance of $\hat{4}$ participates in a different perfect fourth. D-flat is a perfect fourth above A-flat or $\hat{1}$, while D-natural is a perfect fourth below G or $\hat{7}$. Shostakovich uses this quality of the scale in the subject by using both fourths in the first two measures.

Example 31. Eight-note scale used in the A-flat major fugue

When the subject appears in the Aeolian mode as the subject of the counter-exposition, only one $\hat{4}$ is used, as shown in Example 32. The Aeolian mode naturally contains a perfect fourth above and below $\hat{4}$, so only one $\hat{4}$ is needed to give the minor version of the subject the same attributes as the major version. Looking at the Aeolian version also reinforces our interpretation of the original subject because both versions of $\hat{4}$ from the major subject are merged into one $\hat{4}$ in its minor counterpart. In the A-flat major fugue, seven-note scales interact with eight-note scales. The relationships between these scales and the consequences of their differences will be explored in a more detailed analysis of the fugue later in the document.

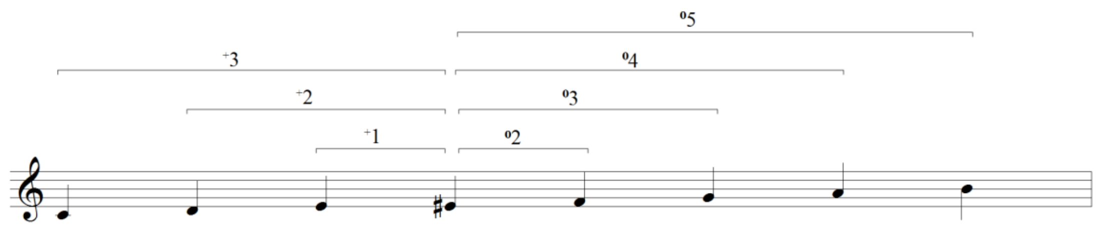
Example 32. A-flat major fugue: subject of counter-exposition

Formalization of a System of Eight-Note Scales

Before we can pursue further analysis, the concept of eight-note scales must be formalized. There are several ways to produce eight-note scales. One is to derive them from already existing scales, such as the diatonic scale. This method is perhaps most useful with the fugues of Op. 87 because they are nominally in major and minor keys; therefore, the scales used often derive from the diatonic set. Example 33 shows the ten possible eight-note scales created by adding an extra note to a major scale (one scale of a diatonic collection). Any one of five raised scale degrees or five lowered scale degrees can be added.

Example 33. Ten eight-note scales

It might seem that, as a major scale contains seven scale degrees, fourteen options for eight-note scales should exist. However, as is also shown in Example 34, because of the distribution of whole steps and half steps in the major scale, when two of the scale degrees are altered, a pitch enharmonically equivalent to another scale degree in the set is created. For example, raising the third degree of C major creates E-sharp, which is enharmonically equivalent to F, the fourth scale degree. In tonal or quasi-tonal contexts, E-sharp and F are quite different pitches. The bigger problem here is that the inflected note, E-sharp, is not able to resolve, in a traditional sense, to any of the other pitches in the set. All of the intervals formed with the E-sharp are either augmented or diminished, as outlined in Example 34. While Shostakovich certainly uses augmented and diminished intervals in some of his works, usually the inflected pitch is able to at least resolve by half step in the direction of the inflection. This fact is true in each of the ten scales outlined in Example 33.[33]



Example 34. Adding E-sharp to the white-note collection

The following section will explore these ten scales further. I will often refer to them as scales, but they are in fact eight-note collections, as each of them can be rotated to begin on any of the scale degrees, and any of the eight notes could serve as center. Rotations of the collection that form an Aeolian mode with an added scale degree are especially common in Op. 87. Example 35 shows the rotations of the white-note collection with an added D-flat. Rotations that include two $\hat{1}$s are also valid in this system because Shostakovich did use modes with two different forms of tonic, especially in the context of $\hat{1}$ being different from $\hat{8}$.[34] One valid question is whether these two scales, shown on the top row of Example 35, are in fact the same scale or not. For the purposes of this thesis, they will be considered to be the same scale simply written in two different ways. The second way, which distinguishes between one form as $\hat{1}$ and the other as $\hat{8}$, is preferable in this unusual case where pitch space is a crucial component of the scale.

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[33] Adding an E♯ would however be possible in a nine-note scale if an F♯ were also added—C D E E♯ F F♯ G A B. The E♯ would then be able to move to the F♯, which would resolve to the G, much like the raised sixth and seventh scales degrees in the minor scale.

[34] Stephen Brown, "ic1/ic5 Interaction in the Music of Shostakovich," *Music Analysis* 28, no. 2–3 (2009): 193–97.

Example 35. Rotations of white-note collection with added D-flat

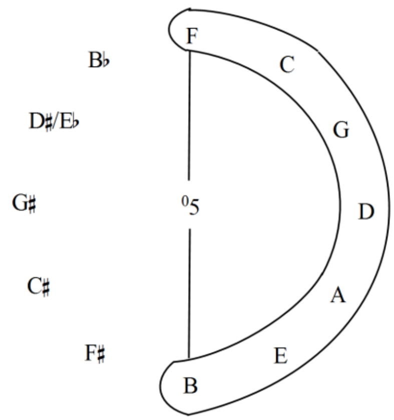
The ten eight-note scales can be paired in a couple of different ways. First, notice that each scale on the sharp side has an enharmonic twin on the flat side. These pairs are presented side-by-side in Example 33. Enharmonically equivalent scales have the same intervallic sequence, when counted in half steps. In much post-tonal theory, C-sharp and D-flat are considered to be the same pitch class and the otherwise identical sets that include them would be considered equivalent. However, for the purposes of this theory, the scales derive from a tonal background, where C-sharp and D-flat have different implications. The C-sharp is a $\hat{1}$, while the D-flat is a $\hat{2}$. Also in tonal contexts, the type of inflection matters: C-sharp and D-flat are both unstable pitches in C major, but since one is raised and one is lowered, they tend to gravitate toward different stable pitches. C-sharp would resolve to D by half step, while the D-flat would resolve to C.

The ten scales can also be paired according to key signature order. The explanation for this pairing reveals another reason to distinguish between enharmonically equivalent scales.

The top pair of scales in Example 36 are related because the added note is the next sharp or flat in the key signature. The scales in the second row both skip over the first sharp or flat—in this case F-sharp and B-flat—to include the next sharp or flat in the sequence (C-sharp and E-flat respectively). All of the scales are paired in this way and are named by how many sharps or flats are skipped.

Example 36. Eight-note scales in circle of fifths order

Another way to visualize this pairing is to write out the scales as chains of fifths because key signatures, and the diatonic set itself, are based on the circle of fifths. The diatonic set is a slice of the circle of fifths, as is illustrated by Example 37. (Here and in the rest of this section of the paper, the members of the circle of fifths refer to individual pitch classes, not keys.) The slice shown in Example 37 contains the pitch classes of the C major scale, but also the pitch classes used for any of the other scales in the white-note collection. The pcs at the edge of the slice break the pattern and form the only diminished fifth in the set.



Example 37. Circle of fifths with white-note collection circled

Traditional key signatures also work with fifths. The first sharp added to the key signature is F-sharp, which is the next fifth in the circle of fifths. When F-sharp is added to the key signature, it replaces F-natural. C-sharp is the next sharp in the line of fifths; it replaces C-natural to form the two-sharp collection. Example 38 illustrates this phenomenon by writing out a portion of a circle of fifths linearly. The white-note collection, one-sharp collection, and two-sharp collection are each underlined to show how with each new sharp, the line inches over one slot to the right. The lines also show the overlap of pcs between these closely related keys. The overlap between the white-note collection and the one-sharp collection includes six pcs, while the overlap between the white-note collection and the two-sharp collection consists of five pcs.

F–C–G–D–A–E–B–F♯–C♯

Example 38. Demonstrating diatonic collections with a chain of fifths

This process could be continued until we reach E-sharp, which is the last sharp in the circle which can exist in a diatonic scale with a member of the white-note collection

(B). The same method could be applied to the flat side of the circle by continuing the line of descending perfect fifths.

The pitch classes in the eight-note scales can also be arranged in a chain of fifths. In the first line of Example 39, the first sharp/flat of the key signature was added, but the natural version was retained. To demonstrate this concept and those that follow, I will focus on the sharp half of the example, but the same principles are true for the flat side. Both will be shown in the examples. When F-sharp is added to the collection, it does not replace F-natural as it would with a change of key signatures. The new scale simply adds a fifth to one end of the chain. Since the added pc is adjacent to the pcs in the original white-note collection, few dissonances are created by this new pc. In fact, the new scale is composed of two overlapping diatonic sets, as shown by the underlining in Example 39. The quality of overlapping sets present in this pair of scales will become important later.

B♭–F–C–G–D–A–E–B	F–C–G–D–A–E–B–F♯

Example 39. Skip-0 scales

The scales in the second line of Example 36 skip over one sharp/flat in the key signature and add the second one. This fact can be illustrated by our chain of fifths, where one of the fifths is also skipped, as is shown by the dash in Example 40. In this case, the eight-note scale is not comprised of two diatonic sets, as diatonic sets containing C-sharp must contain either F-sharp, the skipped pc, or six sharps in the rightward direction.

E♭–__–F–C–G–D–A–E–B	F–C–G–D–A–E–B–__–C♯

Example 40. Skip-1 scales

The fifth line of Example 36 adds the pc that is most removed from the original set without adding an enharmonically equivalent pitch. The added A-sharp is five slots away from the white-note collection. The large distance can be understood by considering the four sharps of the key signature that were skipped to reach this fifth sharp, but the distance is much more visually apparent when it is illustrated by a chain of fifths, as it is in Example 41.

G♭–__–__–__–__–F–C–G–D–A–E–B	F–C–G–D–A–E–B–__–__–__–__–A♯

Example 41. Skip-4 scales

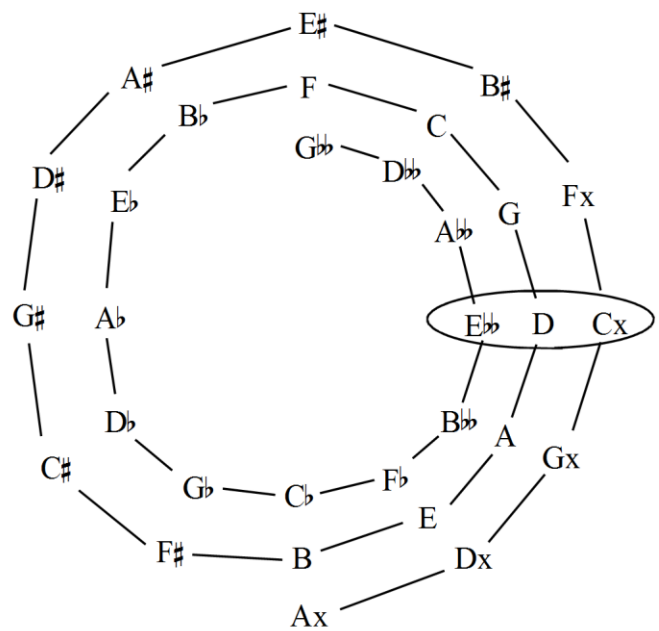
With this example, the problem of enharmonic equivalency again comes up. The A-sharp looks much removed from the set, but according to enharmonic equivalency, A-sharp is the same as B-flat, which is adjacent to the white-note collection on the flat side. It is important to reinforce that A-sharp and B-flat are not equivalent in this system, for several reasons. The point just raised adds another reason. A-sharp and B-flat sit in different locations in relation to the white-note collection. B-flat is as close to the collection as an added pitch can be, while A-sharp is the furthest away. The difference in distance will have ramifications for the characteristics of the two scales, chiefly in their intervallic makeup, which will be explored in the next section.

Example 42 shows all ten eight-note scales, as organized in chains of fifths.

B♭–F–C–G–D–A–E–B	F–C–G–D–A–E–B–F♯
E♭–__–F–C–G–D–A–E–B	F–C–G–D–A–E–B–__–C♯
A♭–__–__–F–C–G–D–A–E–B	F–C–G–D–A–E–B–__–__–G♯
D♭–__–__–__–F–C–G–D–A–E–B	F–C–G–D–A–E–B–__–__–__–D♯
G♭–__–__–__–__–F–C–G–D–A–E–B	F–C–G–D–A–E–B–__–__–__–__–A♯

Example 42. Ten eight-note scales, organized in chains of fifths

Distinguishing between enharmonically equivalent notes is crucial in this system, so using the circle of fifths is not useful, as without enharmonic equivalency, the circle will never get back to its starting point. However, past a certain point, chains of fifths become quite lengthy, and we lose sight of relationships that are illuminated by the circle of fifths. As a solution, the circle of fifths can be turned into a spiral, as shown in Example 43. Every note in the spiral is distinct, but enharmonically equivalent notes from each level form a line, as shown by the three circled notes in Example 43.



Example 43. Circle of fifths as a spiral

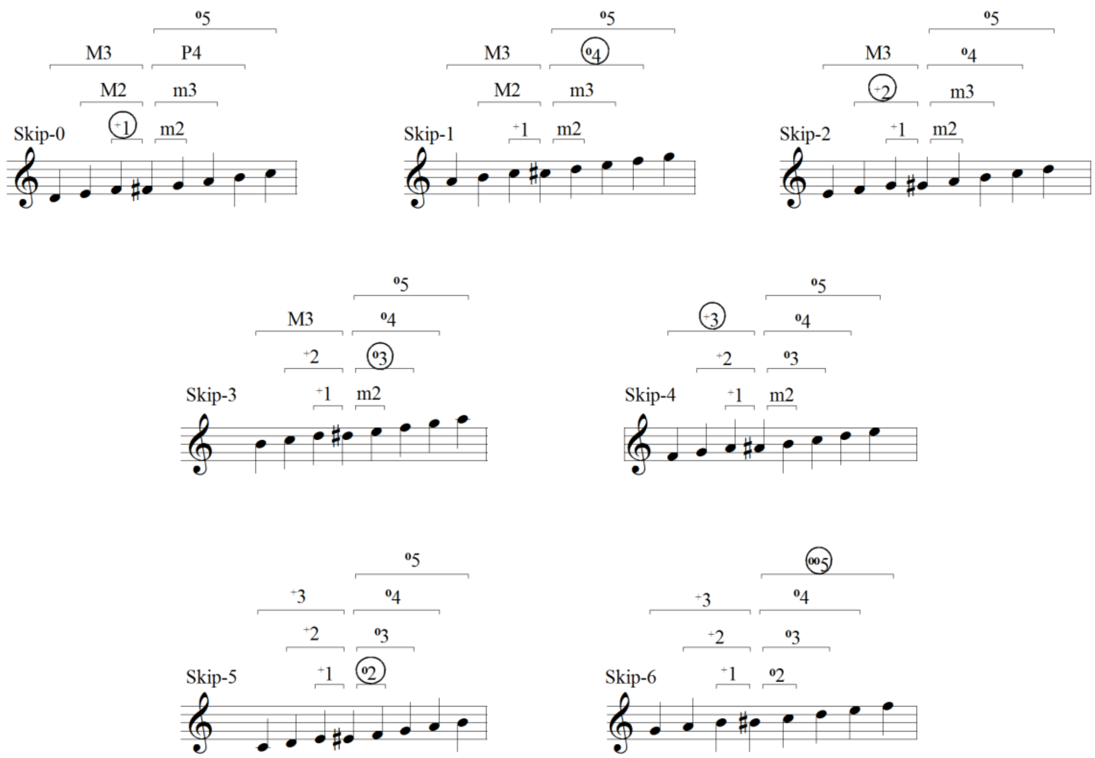
Just as with signature transformations, at a certain point, the number of sharps or flats becomes impractical. Enharmonic equivalency between entire scales, as was shown with the signature transformation cycle, is perfectly acceptable according to this system. For example, the eight-note scale “B♯ C× D× E♯ F× G× A♯ A×” is enharmonically equivalent to “C D E F G A B♭ B.” The entire collection simply jumps to a different level of the spiral. However, changing only one note of an eight-note scale—for example A♯ to B♭—and leaving the other notes the same creates two different scales. In this system, the rules for individual notes are not the same as for entire scales.

This spiral will be used in the next section to help us see the relationship between a diatonic collection and an added note and how the intervals change as the added note moves further and further from the collection.

Study of Intervallic Relationships in Eight-Note Collections

When considering the intervallic makeup of these eight-note scales, it is only necessary to consider the interval classes formed by the added pitch, as the relationships between the pcs in the original diatonic set will always remain the same—<254361>. In this eight-note scale system, the further the added pitch is from the rest of the scale, the more dissonant intervals are created. Recall that when E-sharp was added to the white-note collection, only dissonant intervals were formed. The only interval formed with the E-sharp which can also be found in the diatonic set was the diminished fifth, the rarest and most dissonant interval in the diatonic set. In Example 44, the intervallic makeup of this scale is reproduced, along with similar diagrams of the other six possibilities of eight-note scales.[35] The scales that add E-sharp and B-sharp are included, even though they were rejected as scales, because they follow the same pattern. In this example, the collections have been rotated so that the added pitch falls in the same location in the scale to help with comparison. Only the collections with added sharps are represented, and they are presented in key signature order, with the two rejected collections in the third row.

[35] Example 44 shows only one example of each interval class. This practice will be continued for the remainder of the chapter.

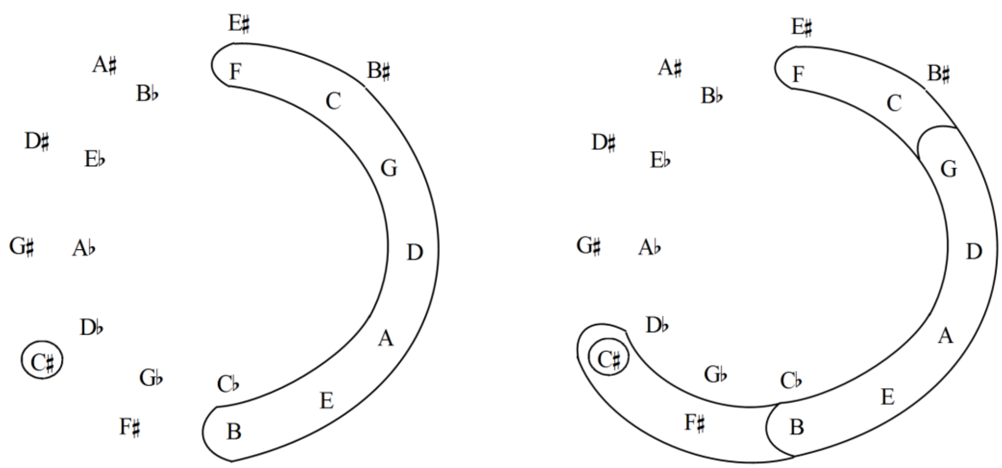


Example 44. Intervallic makeup of eight-note scales

The skip-0 scale adds the most new consonant intervals, here defined as the intervals that occur in any diatonic scale: the minor second, major second, minor third, major third, perfect fourth, and tritone. This fact should not be surprising, as it was already noted that this particular eight-note scale contains two overlapping diatonic sets. Therefore, the intervals formed by the added note will all be diatonic intervals. The only exception is the augmented unison created between the two Fs. This augmented unison will always appear between two versions of the same scale degree in this system.

Gradually, as the added pc moves further and further away from the white-note collection, more and more dissonant intervals, foreign to the diatonic set, are created and replace the diatonic intervals. In Example 44, each scale in the sequence gains a new dissonant interval, which is circled.

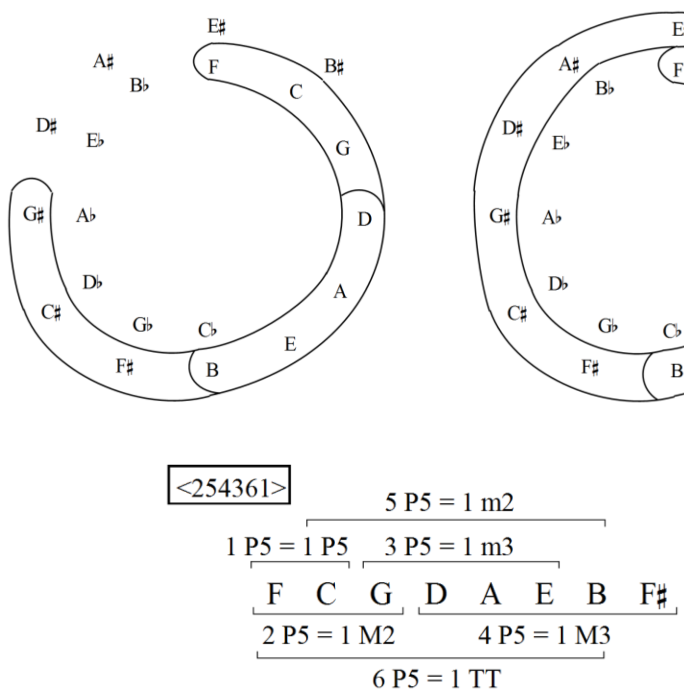
It might seem strange that the perfect fourth is lost first, as ic5 is the most common interval class in the diatonic set, but remember how this scale looked in a chain of fifths. Since the C-sharp is one slot removed from the white-note collection, it is an island in the chain of fifths—neither of the perfect fifths on either side of it belong in the eight-note scale, as is pictured by the left spiral in Example 45. However, all of the other diatonic intervals are retained. C-sharp participates in seven different diatonic collections. By circling the collection that places C-sharp furthest to the left in the chain of fifths, the second spiral in Example 45 helps us to see how many pcs in the white-note collection overlap with a collection that contains C-sharp and therefore form diatonic intervals with the C-sharp.



Example 45. Relationship of C-sharp to white-note collection

As the added note moves one more slot away from the rest of the scale, the next most common interval of the diatonic set disappears. This phenomenon can again be explained by the chain of fifths. The cumulative interval of two perfect fifths is the same as a major second, the second most common interval in the diatonic set, and the second to disappear when the added note moves another slot away. The spiral on the left side of

Example 46 illustrates these facts and shows that four pcs are part of the maximum overlap between the white-note collection and a collection containing G-sharp. The right spiral shows the overlap between the white-note collection and a collection containing E-sharp. Only one pc is shared—the B which forms a tritone with the E-sharp. Beginning with B-sharp, the overlap disappears. No collection that contains B-sharp contains F, C, G, D, A, E, or B, so none of the consonant intervals in the diatonic set are formed between B-sharp and these pcs. The bottom of Example 46 illustrates how many perfect fifths equal other diatonic intervals classes. The least common intervals classes in the diatonic set are the last to disappear because they are furthest apart in a chain of fifths.



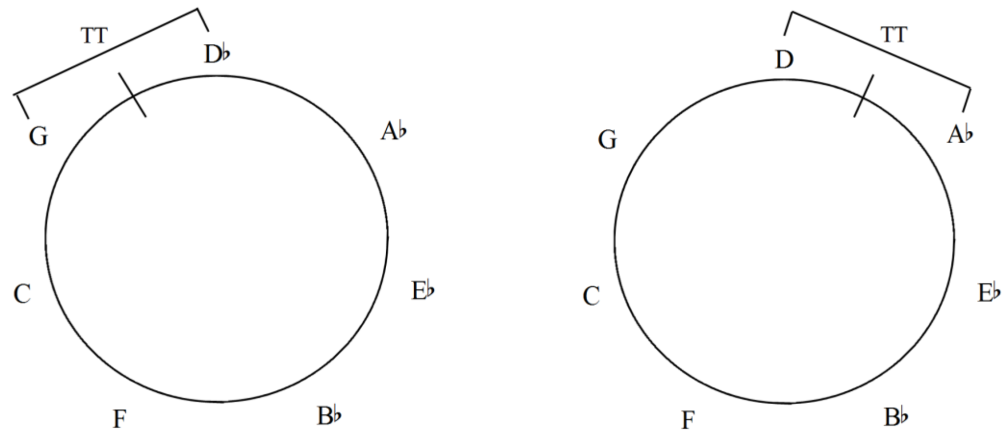
Example 46. Interval classes lost as the added note moves further away

Of all of the eight-note scales, the pair that is most commonly encountered in Shostakovich’s fugues, perhaps unsurprisingly, is the pair that adds the next sharp or flat in the key signature. Example 47 shows the scales that add the next sharp or flat to a major or Aeolian scale. Notice that the first scale is the one used in the A-flat major fugue—a major scale with two fourth scale degrees. In the following section, the characteristics of skip-0 will be examined further, as well as the voice leading between such scales. The chapter will conclude with a study of pertinent examples in which Shostakovich uses eight-note scales.

Example 47. Skip-0 scales based on major and Aeolian scales

Characteristics of Skip-0 Scales

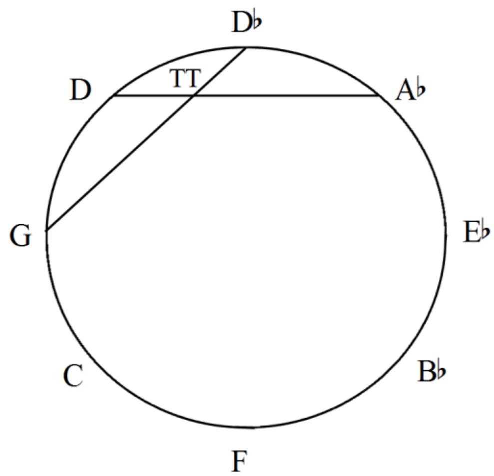
As was previously mentioned, by adding the next sharp or flat in the key signature, skip-0 adds the pc which is most closely related to the original diatonic set. The resulting eight-note scale therefore contains two overlapping diatonic sets. In the case of the A-flat major fugue, the scale used in the subject contains the A-flat major and A-flat Lydian scales—two scales that differ only in their fourth scale degree. The change of collection affects the location of the tritone, as shown in Example 48. In the four-flat collection, the tritone falls between G and D-flat, or $\hat{7}$ and $\hat{4}$ when A-flat is the center. When the D-flat is changed to D, the tritone between $\hat{7}$ and $\hat{4}$ disappears, but now, a tritone between D and A-flat is created.



Four-Flat Diatonic Circle Three-Flat Diatonic Circle

Example 48. Four-flat and three-flat diatonic circles

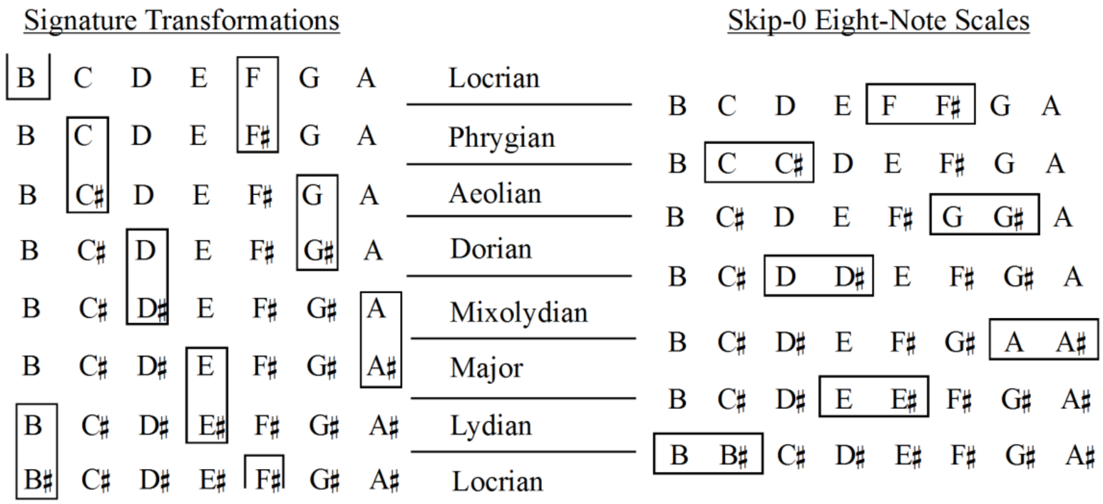
Example 49 shows the circle of fifths formed by the skip-0 eight-note scale, which combines A-flat major and A-flat Lydian. Although both tritones still exist, as indicated by lines within the circle, they may be avoided to generate a “perfect” circle of fifths. If we see both Ds as forms of the same scale degree, this circle of fifths is as perfect as possible without including all twelve pcs. The skip-0 scale is the only eight-note scale that can be used to mask the tritone in this way. Notice that in Example 47, the dual scale degrees are those that form the tritone in major and minor scales—$\hat{4}$ and $\hat{7}$ in major scales and $\hat{2}$ and $\hat{6}$ in minor scales. Each skip-0 scale adds a pc that can make the tritone perfect. This characteristic of masking the tritone is the one most used by Shostakovich in Op. 87, as was seen in the A-flat major fugue, where each $\hat{4}$ participated in a perfect fourth with a different scale degree.



Example 49. Skip-0 circle of fifths

Because skip-0 scales contain two overlapping diatonic sets, they fall in the cracks of Hook's signature transformations, as shown in Example 50. The first scale on the right contains the F from B Locrian and the F-sharp from B Phrygian. The second contains the $\hat{2}$ from the Phrygian mode and the Aeolian mode, and each scale continues this pattern.

In signature transformations, one form of a letter name replaces another. Even though it has been said up to this point that s_1 adds a sharp, it actually replaces one version of a letter with the version that is one half step higher. F-sharp replaces F with an s_1 transformation. If we literally added a sharp to a B Phrygian scale, we would get the hybrid Locrian-Phrygian scale on the right because the F would be retained. Instead, with signature transformations, pcs are constantly cycled in and out of the collection. This process is highlighted by the boxes around the changed pcs on the left side of Example 50. F-sharp replaces F, C-sharp replaces C, and so on.



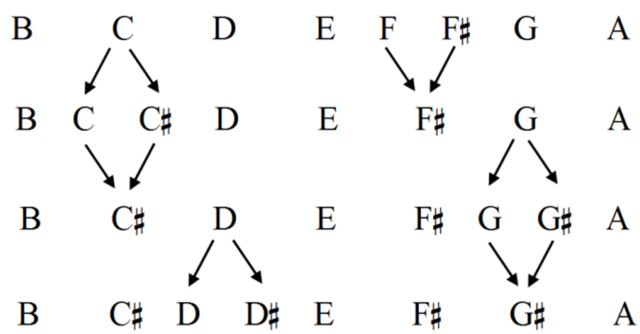
Example 50. How skip-0 scales relate to signature transformations

Skip-0 scales form a similar cycle, but when a sharp is added to a skip-0 scale, the natural version is retained. This fact is highlighted by the horizontal boxes in the right side of Example 50. When a sharp is added to one of these scales, it replaces the natural version of the doubled scale degree from the previous scale. This process sounds complicated, but it can be visualized quite easily with chains of fifths. In both cases, adding a sharp moves the collection over one space to the right. The only thing that changes between signature transformations and transformations between skip-0 scales is the cardinality or number of pcs in the collection. On the left side of Example 51, the C-sharp replaces C, as in a signature transformation, but on the right side, the collection is one pc bigger, so C-sharp replaces F in the same shift.

C–G–D–A–E–B–F♯–C♯	F–C–G–D–A–E–B–F♯–C♯

Example 51. Collection shift when a sharp is added

Because the new pc does not share a letter name with the old pc, showing the voice leading between skip-0 scales is slightly more complicated than the voice leading between diatonic scales was. Example 52 shows a solution, where pcs split into two forms for one scale, and then converge for the next. It took two scales for the C to be replaced by C-sharp. In the intervening scale, both forms coexisted.



Example 52. Voice leading between skip-0 collections

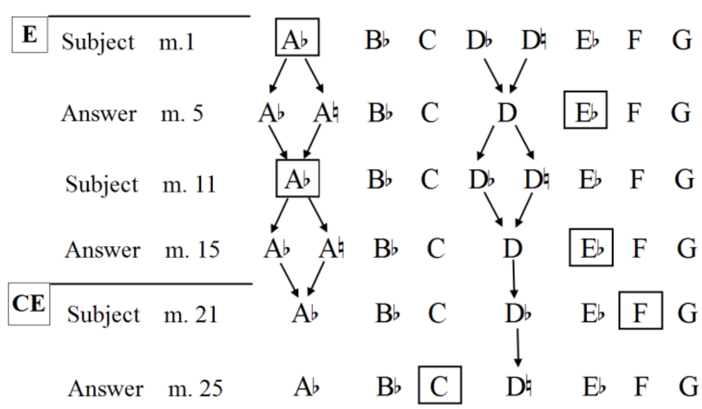
Use of Eight-Note Scales in Op. 87 Fugues

Example 53 shows one practical application of this investigation into the voice leading between skip-0 scales. In the A-flat major fugue, with the exception of the first three notes, which were adjusted to form a tonal answer, the subject underwent a T_7 transformation to produce the answer in m. 5. As we saw in chapter 2, the voice leading

Example 53. Voice leading between subject and answer of the A-flat major fugue

between collections produced by T_7 and s_1 is the same, but in T_7, the center also shifts. Example 53 shows that in the voice leading between the subject and answer, the two Ds converge into the raised version, and the A-flat split into A-flat and A-natural. The voice leading produced by T_7 between two eight-note scales is the same as the voice leading between scales in the cycle on the right side of Example 50 (compare Example 53 to Example 52). In this way, T_7 has the same relationship to signature transformations whether it is added to seven-note scales or eight-note scales.

When T_7 is applied to a diatonic set, the next sharp in the key signature is added. However, because this subject uses the eight-note scale that already includes that sharp, the next sharp in the sequence—A-natural—is added in the answer. As might be expected, Shostakovich alternates between these scales with presentations of subjects and answers in the exposition. A continuation of the voice leading graph follows in Example 54, where the E represents the exposition and CE the counter-exposition. Because the cardinality of the scales changes between the exposition and counter-exposition, the pattern also changes. The A-flat and A converge as before, but the D does not split.

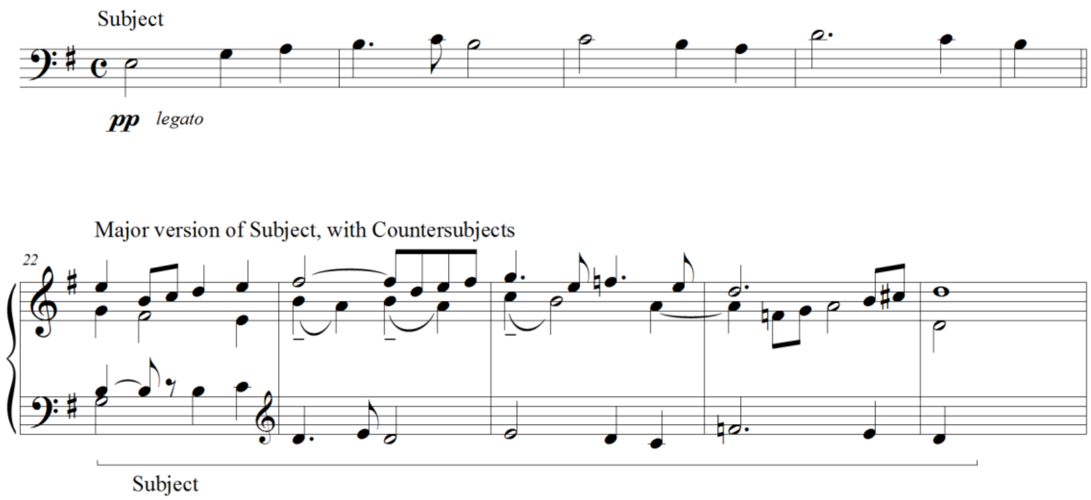


Example 54. Voice leading and scale relationships in the exposition and counter-exposition of the A-flat major fugue

Instead, the D changes into D-flat, the expected form of D in F Aeolian. The counter-exposition uses a standard diatonic scale, so one sharp is added under T_7, changing the D-flat back to D-natural. Extraordinarily, throughout this whole process, only two letter names changed—A and D, and this pattern could be predicted by knowing how various operations, such as T_7, affect specific scales.

So far, this study has almost completely ignored the musical contexts in which subject statements take place. Outside of the original subject, statements never appear in isolation. Sometimes pcs that are missing from the subject itself are present in one or more of the countersubjects or other accompaniment material. For example, in the case of the C major fugue, the answer does not contain an F, making the scale and the transformation required to get there ambiguous. When all of the parts are taken into consideration, however, the F-natural in the countersubject fills in that gap, as it does not make sense to think of the subject and countersubject as being in two different scales. A similar case can be found in the E minor fugue with a skip-0 scale.

Example 55 reproduces the subject of the E minor fugue, which uses a seven-note Aeolian scale. One distinctive feature of this subject is the leap of a perfect fourth between $\hat{4}$ and the lower version of $\hat{7}$. Such a leap would rarely occur in non-modulating tonal music. When the subject is presented in major form in the counter-exposition, also shown in Example 55, the lowered $\hat{7}$ is retained, keeping the perfect fourth leap and creating another case where Shostakovich substitutes Mixolydian for the relative major scale.[36] The use of Mixolydian here is surprising, partly because the $\flat\hat{7}$ appears so late in the subject, but also because of what happens in the countersubjects. In mm. 22 and 23, both countersubjects in the soprano and alto voices contain F-sharp, which sets up the expectation for G major. The modal twist at the end of the subject is foreshadowed just before by the F-natural in the soprano voice.



Example 55. Minor and major versions of the E minor fugue subject

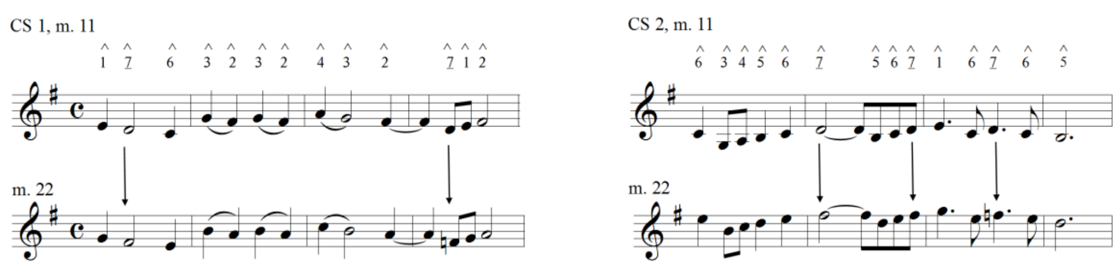
Here we have a subject that, when its musical setting is taken into consideration, begins in major and ends in Mixolydian.[37] It therefore is based on the skip-0 scale that lies in the crack between these two scales. One form of $\hat{7}$ was needed to form a perfect fourth between $\hat{4}$ and $\hat{7}$ in the subject. The other $\hat{7}$ emphasizes the expected major quality in a counter-exposition subject.

---

[36] The major scale is the only one that contains a tritone between the fourth and seventh scale degrees, so any of the other modes could be used to make this fourth perfect. Lydian and Mixolydian are neighbors to major in the signature transformations cycle, and they are the options that preserve the necessary major quality for the counter-exposition. Interestingly, Mixolydian is used here, while Lydian appears later in the fugue.

[37] Note that the C-sharp in the soprano voice in m. 25 should not be considered in the scale of this subject as it is leading into the answer statement of the counter-exposition in D major/Mixolydian, which would begin in the last measure of this example.

It might be wondered why bother with an eight-note scale here at all—why not conceive of the statement as beginning in G major and then moving to G Mixolydian. Perhaps this fugue includes a rare fixed-tonic relationship. While this interpretation is entirely possible, the statement as a whole could be based on one skip-0 scale—a major scale with two $\hat{7}$s. This interpretation takes away the confusion of changing the scale mid-subject. As a result, it is easier to show the correlation between the subject of the exposition and the subject of the counter-exposition. The two versions of $\hat{7}$ present in the counter-exposition were represented by a single $\hat{7}$ (D) in the minor form of the subject with its countersubjects, much like the two $\hat{4}$s were represented by a single $\hat{4}$ in the counter-exposition of the A-flat major fugue. Example 56 compares the countersubjects as they are found in the exposition and counter-exposition of the E minor fugue.



Example 56. Mappings of $\hat{7}$s

Once again, the relationship between the scales can be shown in two ways, both included in Example 57. The relationship could be illustrated by a voice-leading chart, much like the one used in the analysis of the A-flat major fugue. This possibility is given on the left side of Example 57. The right side focuses on how the scale degrees are affected. In this diagram, the center appears at the far left of both scales, as it did in most of chapter 2. Arranging the scales this way allows us to see how one $\hat{7}$ divided into two $\hat{7}$s in the counter-exposition.

E	F♯		G	A	B	C	D
E	F	F♯	G	A	B	C	$\boxed{D}$

$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	
E	F♯	G	A	B	C	D	
G	A	B	C	D	E	F	F♯

Example 57. Two different mappings between scales in E minor fugue

The eight-note scale theory can be expanded to include sections of music with more than one chromatic alteration. For example, the minor scale, with its doubled $\hat{6}$ and $\hat{7}$, could be seen as a combination of a skip-0 scale and a skip-2 scale, as shown in Example 58. The concept of the skip-0 scale itself can be expanded to refer to an unbroken chain of nine pcs or even more. Such a situation is presented in the G-sharp minor fugue.

A B C D E F F♯ G G♯ =

F–C–G–D–A–E–B–F♯ + F–C–G–D–A–E–B–__–__–G♯?

Example 58. Minor scale as a combination of two eight-note scales

Example 59 presents the subject of the G-sharp minor fugue. The subject contains two added chromatic pitches, D-natural and A-natural, which cancel the last two sharps of the key signature. The box below the subject shows how the nine pcs used in this subject form an unbroken chain of fifths. The original seven pcs of the G-sharp Aeolian scale are underlined, emphasizing which pcs were added.

D–A–<u>E–B–F♯–C♯–G♯–D♯–A♯</u>

Example 59. Subject of the G-sharp minor fugue and its nine-note scale

The added pcs once again participate in a perfect fourth, but unlike the other examples, both the D and the A are added pcs. Notice that if D alone were added, it would not be able to form a perfect fourth or perfect fifth with the original collection, as the D would form a skip-1 scale with G-sharp Aeolian. However, inserting A-natural on the next beat fills in the gap in the chain of fifths. The D-natural is no longer isolated from the rest of the collection, and the perfect fourth is made possible.

The same scale degrees are doubled in the answer, when the subject is transposed up a perfect fifth. Notice what happens in the counter-exposition, shown in Example 60, however. When Shostakovich presents the subject in the "relative major," he adds different chromatic pcs. Where in the minor version, the first two flats were added, in the major version, the first sharp, E-sharp, is added, as well as G-natural. Adding the G-natural skips over two other flats. The resulting nine-note scale is presented at the bottom of Example 60 as a chain of fifths, with the five-sharp collection—now a B major scale—once again underlined.

It might seem odd that the subjects of the exposition and counter-exposition add such different chromatic pcs, especially the G-natural, which is so far away from the rest of the collection. Could Shostakovich have used the D-natural and A-natural again in the counter-exposition? Here it is important to remember that while G-sharp Aeolian and B major use the same collection—and therefore contain the same pcs and intervals—they

G–__–__–<u>E–B–F♯–C♯–G♯–D♯–A♯</u>–E♯

Example 60. Counter-exposition subject of G-sharp minor fugue and its nine-note scale

begin on different centers. The distinction between collection and scale is significant. Even though all scales in the five-sharp collection have the same intervallic makeup, the placement of center within this web of intervals is significant and is what makes each scale distinct. The tritone between A-sharp and E is the same in both G-sharp Aeolian and B major, but its relationship to center changes. In G-sharp Aeolian, the tritone lies between $\hat{6}$ and $\hat{2}$, while in B major, it is between $\hat{7}$ and $\hat{4}$. The added pcs D-natural and A-natural also fall on different scale degrees in B major than they do in G-sharp Aeolian.

The fugue subjects use the same collection with scale degrees assigned to different pcs, so the intervallic distribution in each subject is distinct, as both begin on $\hat{1}$. For example, the E-sharp is added in the counter-exposition to mask the tritone, which falls in a different place in the major subject than it did in the minor subject because the tritone falls in a different location in the major scale.

Because the subject is full of perfect fourths and fifths, these intervals exist between many different scale degrees, making it possible for us to imagine what the subject would look like if the D and A were altered instead of the E. This reworking is shown as the top line of Example 61 with a bracket over the altered pcs. The reworking makes the collection of the major version closer to the original, but it sacrifices some of the important qualities of the major scale—mainly, it lowers $\hat{3}$, an important major-defining scale degree.

The G-natural in the counter-exposition is necessary because it fills in the whole step between $\hat{6}$ and $\hat{5}$ where only a half step exists in the minor version. The other option is to keep the whole step between $\hat{6}$ and $\hat{5}$ and to add the lowered $\hat{5}$ as in the original. This possibility is given as the second line of Example 61. Notice how many pcs are altered in the process, however. Lowering so many scale degrees causes the subject to wander much too far from the original collection and severely compromises the major quality of the subject statement.

Example 61. Possible recompositions of the G-sharp minor counter-exposition subject

All of the examples in this chapter use slightly different collections in the exposition and counter-exposition. Although Shostakovich sets up the expectation for the counter-exposition to be in the relative scale, using the same qualities in both subjects while retaining the unique properties of each scale is given precedence over using the same collection.

Global Use of Eight-Note Scales

To see how eight-note scales can be used over larger musical distances, let us return to the B-flat minor fugue. In the first chapter, we saw that this fugue is constructed almost completely from the five-flat collection. Recall that before the Picardy passage at the end of the fugue, the collection was changed only once—in the move from G-flat Lydian to C Phrygian, which introduced a G-natural. After the statement ended, the collection shifted back to its original state with five flats. Example 62 summarizes the scalar events leading up to m. 49.

Scale		transformation to get here
B-flat Aeolian	B♭ C D♭ E♭ F G♭ A♭	
F Phrygian	F G♭ A♭ B♭ C D♭ E♭	t_4
B-flat Aeolian	B♭ C D♭ E♭ F G♭ A♭	t_3
D-flat major	D♭ E♭ F G♭ A♭ B♭ C	t_2
A-flat Mixolydian	A♭ B♭ C D♭ E♭ F G♭	t_4
G-flat Lydian	G♭ A♭ B♭ C D♭ E♭ F	t_6
C Phrygian	C D♭ E♭ F **G** A♭ B♭	t_3s_1
F Phrygian	F G♭ A♭ B♭ C D♭ E♭	T_5
B-flat Aeolian	B♭ C D♭ E♭ F G♭ A♭	t_3

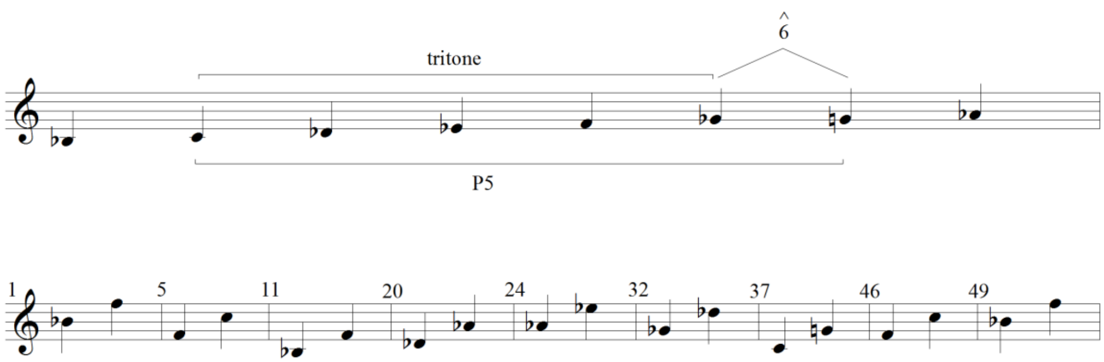
Example 62. Summary of B-flat minor fugue scalar events before m. 37

Although it is accurate and helpful to describe this fugue in terms of changing scales and a shift in collection, it is also possible to see, on another level, how the entire fugue is based on one eight-note scale, or at least an eight-note collection. Up to this point, we have been assuming that diatonic transposition in this fugue takes place within one collection, though the scales themselves change. Now we will consider the other definition of diatonic transposition—that of transposition within one *scale*. Instead of seeing each statement of the fugue subject as a separate entity with its own scale, we will now imagine them all as a part of an all-encompassing B-flat Aeolian scale. With this view, the answer in m. 5 is not based on F Phrygian, but instead focuses on $\hat{5}$ of the original scale. Likewise, the subject of the counter-exposition surveys the region around $\hat{3}$. The entire work is like a slowly moving kaleidoscope that carefully explores every corner of the B-flat Aeolian scale's landscape.

When the fugue turns to explore $\hat{2}$, a shift occurs. As we have already noted, the interval between $\hat{1}$ and $\hat{5}$ is a prominent melodic interval in this fugue subject and also a key harmonic interval formed between the subject and its countersubjects. This interval is always a perfect fifth in the B-flat Aeolian scale, except between $\hat{2}$ and $\hat{6}$. This location marks the gap of the tritone present in any diatonic scale. The addition of the G-natural masks this gap, though somewhat awkwardly given its abrupt introduction. When this chromatic addition is no longer needed to form a perfect fifth, the G-flat is reintroduced. In this sense, Shostakovich is doing the same thing, over a longer span of time, that he did in the A-flat major fugue—he added the next sharp in the key signature when it was needed make a tritone perfect.[38]

[38] It should be noted that the same effect could be accomplished by changing the C to C-flat, which would add the next flat to the key signature and double the second scale degree of B-flat Aeolian, which is the other participant in the tritone of this scale. The resulting Lydian scale would form a closer link to what came before. Because this statement of the fugue is based on the second scale degree, however, changing that scale degree might seem odd in this context.

Example 63 shows the eight-note scale from which the slowly-revolving perspective of the B-flat minor fugue is drawn. It also summarizes the events of the fugue up to m. 49 by outlining the perfect fifths highlighted in each subject. Measure numbers showing where each subject begins are provided above the staff. Much as in the A-flat major fugue, the added pc is isolated and only used when it is needed to perfect a tritone. It serves a specific function, much like the two forms of $\hat{7}$ serve different functions in the minor scale. The variable scale degrees do not appear as equals and are not mixed freely in Shostakovich's fugues. Therefore, it is much easier to separate these eight-note scales into smaller scales, or in the case of the D-natural in the A-flat major fugue, to dismiss it altogether as a one-time chromatic alteration.



Example 63. Eight-note scale and outline of B-flat minor fugue

Eight-Note Collections: Conclusions

The theory behind eight-note collections might be more helpful in explaining passages where both versions of a scale degree are used more-or-less equally. Examples such as this simply do not appear in Shostakovich's fugues, but the concept of eight-note scales is still useful in this study because of its relational aspect. Even if we decide the D-natural in the A-flat major fugue is simply a fleeting chromatic alteration, it is useful to know how that chromatic pc is related to the rest of the collection because, as we have seen, this relation has consequences for how the chromatic pc can interact with the rest of the scale. This theory allows for the chromatic note to be considered simultaneously an “inside” and “outside” member. It can be classified as a chromatic alteration without being dismissed as extra-scalar.

Exploring this theory allows us to see the possibilities for forming eight-note scales in one specific way—adding one pc to a diatonic scale. By distinguishing between them, we are able to see that Shostakovich favored scales that add the most closely-related note. With this knowledge, we can see similarities between passages of music that might otherwise seem unrelated, as in the A-flat major and B-flat minor fugues.

More work could be done in the study and application of eight-note scales. How do chromatic pcs relate to diatonic scales in Shostakovich’s other works? Can the conclusions drawn in this chapter be applied to a larger body of his works? The theory could also be used in the study of other twentieth-century composers, whose music is based on diatonic collections but contains chromatic twists or uses two versions of the same scale degree equally.

Chromatic alterations might be considered in three classes. They might be used functionally to move into a new key or scale in the process of modulation or tonicization. They might be inserted as passing tones and neighbor tones added for decoration to the fundamental members of the scale. Alternatively, they might be introduced as part of a greater shift that still does not change the scale. Chromatic alterations of this type might participate on a deeper level with members of the scale—for example, in leaps. The goal of the theory of eight-note collections is to illuminate the workings of this third group.

Chapter 4

Application: Macrocollections, Centricity, and Shifting Collections

Scalar Relationships in Expositions and Counter-Expositions

In the B-flat minor fugue, no changes were made to the collection for thirty-six measures, making the collection change in m. 37 even more conspicuous. This fugue is atypical for Shostakovich—although some fugues, such as those in B-flat minor and C major, use a relatively small number of pcs, many of the other fugues move to distant keys, and a quick glance at them reveals a myriad of accidentals. However, pursuit of this idea of shifts in collection and the accumulation of accidentals is worthwhile and will be accomplished not by looking at entire fugues, but by looking at a specific section of the fugues—the exposition and counter-exposition.

Limiting analysis to the first part of the fugues will be helpful for a couple reasons. It gives a precise limit to how much music will be studied, and since the exposition and the counter-exposition, as Shostakovich uses them, are the most structured parts of the fugue, comparison between fugues will be facilitated.

It will be helpful to first establish a baseline for how a Shostakovich exposition/counter-exposition might be expected to go. A diagram for such a norm is established in Example 64. A typical exposition would include a subject based on a diatonic scale—either major or Aeolian—and an answer related by T_7, which would add one sharp to the collection. The number of subjects and answers in the fugue is variable, but they would repeat this pattern of scale relationships. A typical counter-exposition would present the subject with the relative scale, so it would therefore use the original collection.[39] The answer in the counter-exposition would, once again, be a chromatic transposition of T_7, adding the same sharp that was added in the exposition.

These relationships are expected because T_7 is the traditional transposition between fugal subjects and answers, even though we have seen examples where Shostakovich does not use this relationship. The relative-scale relationship between exposition and counter-exposition is typical of Shostakovich's personal fugal model. The relationship is used even with non-diatonic subjects, as was shown in chapter 3.

$$\begin{tikzcd}
\underline{\text{Exposition}} & \underline{\text{Counter-Exposition}} \\
\text{Subject} \arrow[r, "t_5", "(\text{ or } t_2\text{ })"'] \arrow[d, "T_7\,(t_4s_1)"'] & \text{Subject} \arrow[d, "T_7\,(t_4s_1)"] \\
\text{Answer} & \text{Answer}
\end{tikzcd}$$

Example 64. Typical scalar relationships in Shostakovich's Op. 87 fugues

Even though these relationships are the ones that are expected, only four of the twenty-four fugues follow this pattern exactly. The other twenty fugues deviate from this model in some respect. Ten of the fugues add a sharp with a typical T_7 transposition in either the exposition or counter-exposition, but another relationship is used in the other section. Example 65 lists which fugues use the expected T_7 relationship between subject and answer. This table is slightly misleading because it only includes the T_7 transpositions that add one sharp, which happens whenever T_7 is applied to a diatonic scale. However, two fugues use the minor scale, which is not a diatonic scale, in a

[39] The exact transformation between the subject of the exposition and the subject of the counter-exposition varies, depending on the mode of the fugue. Moving from a major scale to its relative minor is a t_5 transposition, while motion from an Aeolian scale to the relative major creates t_2.

subject, so T_7 adds more than one sharp in the answer. Also, we know from chapter 3 that a sharp was added in both the exposition and counter-exposition of the A-flat major fugue, but since the exposition used an eight-note scale, a different sharp was added than in the counter-exposition.

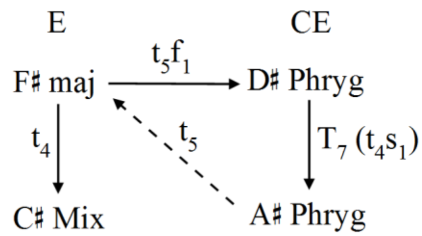
T_7 in both E and CE	**T_7 in E only**	**T_7 in CE only**
B minor	G major	A minor
C♯ minor	E minor	D major
B major	E♭ minor	E major
G minor	C minor	A♭ major
		B♭ major
		F minor

Example 65. Fugues that add a sharp via T_7

We have already encountered the other possibilities for relationships in expositions and counter-expositions. The chapter on eight-note scales covered the change of collection when a chromatic pc is found within the subject itself. The C major and B-flat minor fugues contain only diatonic transpositions within their expositions and counter-expositions. The A major fugue is the only other fugue that uses a single collection for both the exposition and counter-exposition, but in other fugues, Shostakovich might use diatonic transposition for one and not the other. Already we see the variety of combinations available within the strictest section of the fugue.

The final option is to add a flat to the collection, which Shostakovich does consistently in his counter-expositions but never in his expositions.

Example 66 shows a diagram for the exposition and counter-exposition of the F-sharp major fugue. The exposition uses diatonic transpositions, much like the B-flat minor fugue. Not a single accidental appears until the counter-exposition, where Shostakovich uses one of the relationships described in chapter 1. The subject of the counter-exposition is centered on D-sharp, which would be the expected center of the relative scale. However, instead of using D-sharp Aeolian, Shostakovich here adds a flat to present the subject in D-sharp Phrygian. The subjects do not share the same collection, therefore, but they are still closely-related scales. The t_5f_1 label captures this relationship and the addition of the next flat to the overall collection.



Example 66. Fugue in F-sharp Major: exposition/counter-exposition scalar network

Because the subject of the counter-exposition uses f_1, when the scale is transposed by T_7, it produces a scale in the original collection. The sharp, which is a byproduct of T_7, cancels out the flat added between the exposition and counter-exposition. This fact is highlighted in Example 66 by the translation of T_7 into a t_ns_n compound transformation. The dashed arrow reminds us that the answer of the counter-exposition shares a collection with the original subject—a useful relationship, though these scales are never heard back-to-back.

As was pointed out in chapter 2, altering the relative scale by adding a flat to the collection is much more common in Shostakovich's minor fugues. Example 67 includes the subjects and answers from the F minor fugue, as well as a diagram of scale relationships, as an example. The subject contains no $\hat{2}$, and $\hat{2}$ corresponds to the pc that would be raised in a T_7 transposition of an Aeolian scale; therefore, it is impossible to know if the relationship between the subject and answer of the exposition is T_7 or t_4 by looking at the subject and answer alone. However, because the countersubject that accompanies the answer contains G-flat, we can say that the answer remains in the original collection and is based on a C Phrygian scale. The subject of the counter-exposition is centered around A-flat, but G-flat is added, so the subject is in A-flat Mixolydian instead of the exact relative scale. Much like the F-sharp major fugue, a T_7 relationship between the subject and answer of the counter-exposition returns us to the original four-flat collection. It might be helpful to compare this diagram to our norm from Example 64 and the major version in Example 66.

$$
\begin{tikzcd}
\text{E} & \text{CE} \\
\text{F Aeol} \arrow[r, "t_2f_1"] \arrow[d, "t_4"'] & A\flat\text{ Mix} \arrow[d, "T_7\ (t_4s_1)"] \\
\text{C Phryg} & E\flat\text{ Mix} \arrow[ul, dashed, "t_1"']
\end{tikzcd}
$$

Example 67. Fugue in F Minor: exposition/counter-exposition scalar relationships

Two unique cases of scale relationships in the exposition and counter-exposition are the G major fugue and the B-flat major fugue. Both depart from the original model in interesting ways. The G major fugue begins as expected, with a subject in G major and its answer in D major. Example 68 shows these statements and the subject and answer of the counter-exposition. The subject of the counter-exposition is a rare case because it does not use G major's relative scale or even the center of the relative scale. A presentation in A Aeolian is extremely unexpected, based on the norm Shostakovich establishes in his other fugues.

$$\begin{tikzcd} \text{E} & & \text{CE} \\ \text{G maj} \arrow[rr, "t_1f_1"] \arrow[d, "T_7\ (t_4s_1)"'] & & \text{A Aeol} \arrow[d, "T_7\ (t_4s_1)"] \\ \text{D maj} & & \text{E Aeol} \arrow[ull, dashed, "t_2\ (\text{REL})"'] \end{tikzcd}$$

Example 68. Fugue in G Major: exposition/counter-exposition scalar relationships

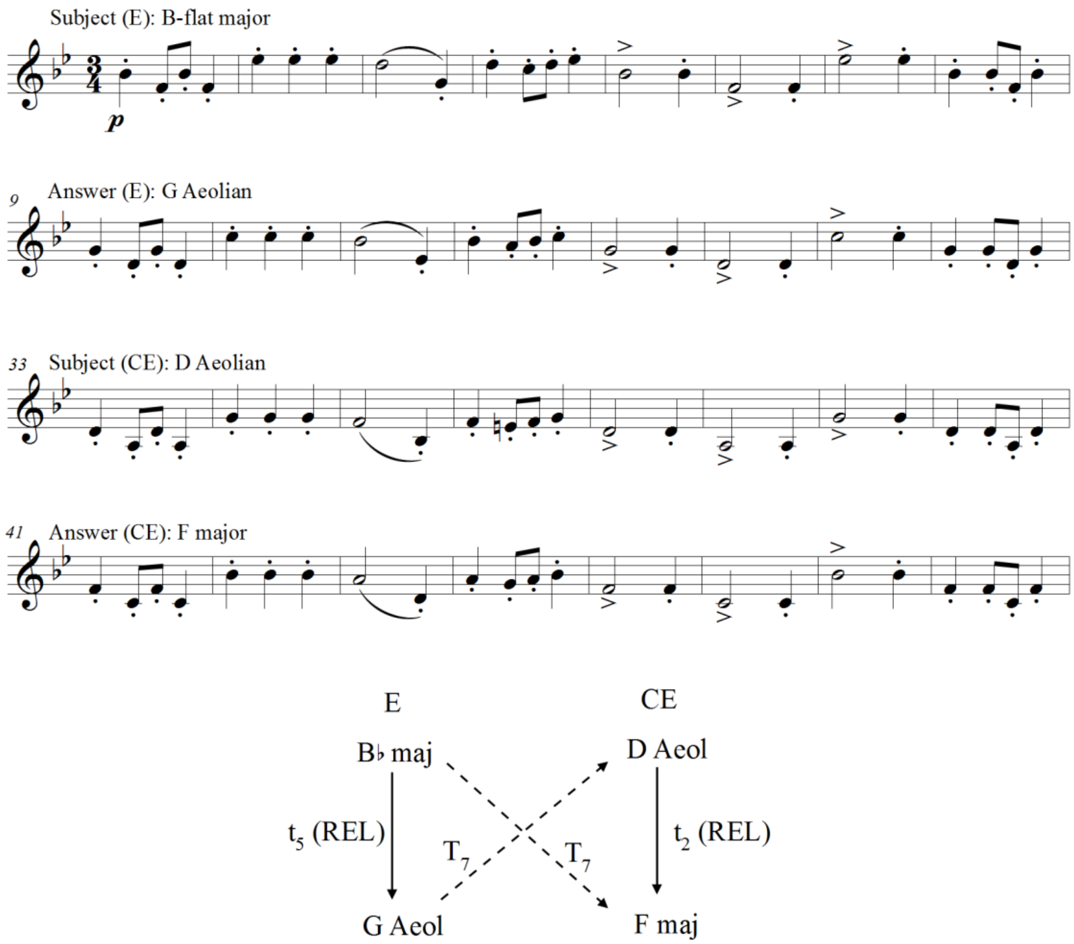
However, when this A Aeolian scale is transposed by T_7, G major's relative scale becomes the answer. In the case of this fugue, the new flat was introduced in such a way that the answer not only returns to the original collection but forms an important relationship with the original subject.

The B-flat major fugue is worthy of note because it uses all of the expected scalar relationships but not in the expected order. As can be seen from Example 69, the answer of the exposition is presented in the relative scale. The subject and answer of the counter-exposition are related to the scales of the exposition by T_7—a relationship typically reserved for within an exposition or counter-exposition. The chromatic transposition of the original subject is used in the answer of the counter-exposition, and the chromatic transposition of the original answer is used for the subject of the counter-exposition.

Because chromatic transpositions retain mode, this fugue is also unique because it uses both major and "minor" scales in both the exposition and counter-exposition. Typically, T_7 is the relationship between the subject and answer of the same section so the exposition contains statements in one mode, and the counter-exposition uses the other mode. In the B-flat major fugue, T_7 is the relationship between sections, so both modes appear in each section. Because the subjects are related to the answers in this fugue, the modes are reversed in the counter-exposition. The exposition presents the subject with a major scale and answers it in Aeolian, while the counter-exposition begins in Aeolian and ends in major.

All of these complex relationships are shown succinctly by the network in Example 69. In the case of this fugue, the same collection is used for the entire exposition, and one sharp is added for the entire counter-exposition. Although there is no

$\hat{7}$ in the subject, and therefore no E-naturals appear in the answer of the counter-exposition, this scale member is implied by the counter-subjects.



Example 69. Fugue in B-flat Major: exposition/counter-exposition scalar relationships

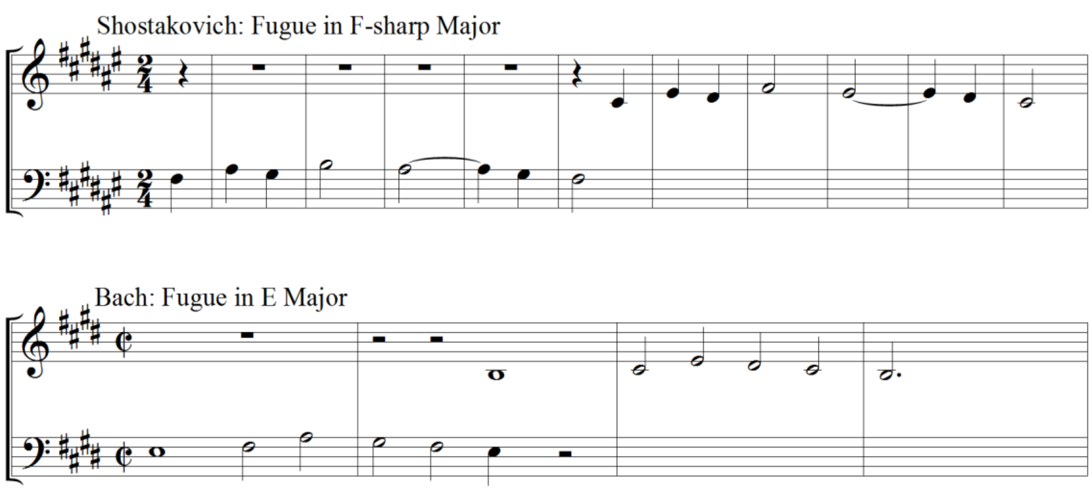
Although Shostakovich uses a variety of scalar relationships within his expositions and counter-expositions, he almost always uses closely-related scales. The exception is when the subject uses more than the seven pcs of a diatonic scale. When the subject uses a diatonic scale, only one or possibly two chromatic pcs will be added for the entire stretch of the exposition and counter-exposition. After this part of the fugue, Shostakovich might take it in any direction, but in this first section, he uses a limited number of pcs. The consequences of this fact will be discussed after a more detailed study of one of the fugues.

Fugue in F-sharp Major

We will now take a closer look at the fugue in F-sharp major. The first three statements of the exposition are given in Example 70. A relatively short subject is used for this five-voice fugue, and because only diatonic transpositions appear in the exposition, no accidentals are used until m. 38. It seems unlikely that we experience a change of scale every five measures when we listen to the exposition of this fugue, especially because the collection does not change. However, with the exception of the B-flat minor fugue analysis in chapter 3, a change of scale between each statement has been assumed. If we do hear the exposition in one scale, the answer would be based on $\hat{5}$ of F-sharp major instead of its own scale of C-sharp Mixolydian, but the transformation between subject and answer would be the same.

Example 70. Fugue in F-sharp Major, mm. 1–19

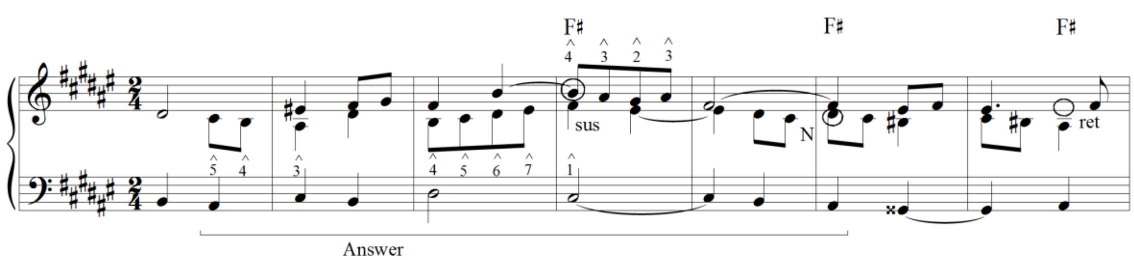
Shostakovich's Fugue in F-sharp Major shares many qualities with Bach's Fugue in E Major from the second book of the *Well-Tempered Clavier*. The subjects and answers of these fugues are shown side-by-side in Example 71. In his study of Bach's fugue, Steven Rings presents the same choice of how to hear the answer— as $\hat{5}$ to $\hat{1}$ in the old key or as $\hat{1}$ to $\hat{4}$ in the new key. Rings decides that the first option is not viable in this case because the counter-subject contains an accidental, and the use of that accidental in a common practice context leads the listener to reinterpret the scale degrees of the answer accordingly.[40] However, in the case of the Shostakovich fugue, the countersubject does not supply any new accidentals. As voices are added into the texture, it becomes clear that while Shostakovich often uses triads and lines which resemble tonal patterns, his avoidance of functional progressions weakens the strength of any center and allows room for multiple interpretations. The choice facing the listener is whether to assign the same scale degrees to each entrance of the subject or to stay in the original scale as long as possible.



Example 71. Subject and answer of two fugues

[40] Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), 155–56.

Shostakovich adds a flat in the subject of the counter-exposition. An addition of an accidental is often a cue to hear a passage in a new key with a new center. The subject begins over a D-sharp minor triad, seemingly centering the statement firmly around D-sharp. We will return to this subject in a moment. Example 72 shows the answer of the counter-exposition with countersubjects. Although the answer itself, when interpreted with the same scale degrees as the original subject, appears to be in A-sharp Phrygian, this reading is not the only possibility. The answer could be heard as beginning on $\hat{3}$ of the original F-sharp major. This reading could be gathered from the answer itself, especially as it belongs to the same collection as the original F-sharp major; however, the countersubjects reinforce this hearing by forming vertical F-sharp major triads. Each countersubject also contains figures that could be interpreted in F-sharp major, as shown by the scale degree numbers in Example 72. These figures follow tonal melodic patterns, especially with motion down to $\hat{3}$ and up to $\hat{1}$.



Example 72. Suggestions of F-sharp major in the answer of the counter-exposition

Even though a strong arrival on a D-sharp minor triad occurs just before the subject of the counter-exposition, this center is not reinforced through the statement itself. The subject could be heard in B major, in a parallel manner to the answer. Even with a center of D-sharp, this subject can be enveloped in a larger picture—the F-sharp major scale, or at least the six-sharp collection.

Tymoczko's discussion of rootedness and tonicity is relevant in this situation. He distinguishes between a root—a center of a chord that only applies to short spans of music, and a tonic—a center that lasts much longer and retains that position even when it is not sounding.[41] This phenomenon is true in tonal music, where $\hat{5}$ is the root of a V chord, even though $\hat{1}$ is still the tonic. The concepts of rootedness and tonicity are more easily conflated in centric music written after 1900, especially when they are applied to longer spans of time. Here, we can imagine the D-sharp subject as rooted in D-sharp or B while floating in a larger sea of F-sharp major.

Whether we conceive of the exposition and counter-exposition as being in several different scales or in the context of one macro-scale, the diagram in Example 66 still accurately describes the relationships between statements. Operators can act on an object, in this case the fugue subject, and its environment, which here is the scale. In this situation, the pcs themselves are moved and so is the context in which we read them. Alternatively, operators can act on objects without affecting their environment. We have already seen that both are possible with diatonic transposition. Diatonic transposition can move objects within a collection, which would change the pcs themselves and the scale, or they can move objects within a scale, which would change the notes but not the scale.

We generally would not think to extend such a double reading to chromatic transposition and signature transformations, as adding an accidental implies a change of key signature, and by extension, a change of scale.[42] However, in Shostakovich's music,

[41] Tymoczko, *Geometry*, 169.
[42] Hook uses this argument to distinguish between T_n in post-tonal contexts and T_n in diatonic contexts. In a diatonic context, "each T_n is understood to transpose the underlying diatonic scale (and therefore the key signature) along with the notes." Hook, "Signature Transformations," 140.

it can be difficult to know which pitches are truly chromatic and outside the system and which ones are not. In the case of the F-sharp major fugue, it is possible to justify the E-natural by saying that the exposition and counter-exposition are based on a macro-eight-note scale that includes two $\hat{7}$s.

This interpretation of the F-sharp major fugue may seem unlikely, but it raises an important point. It can be too easy simply to look at subjects and to define their scale by using a parallel interpretation of scale degrees—for example, each statement starts on $\hat{1}$—a method that is especially relevant in the context of post-common practice music because it provides a stronghold in the absence of familiar tonal harmonic patterns. However, studying scales in this way ignores how the context created by the countersubjects colors the subject. Scales need centers, so scale studies should not ignore the complexities of centricity, especially in non-fugal settings where a parallel interpretation of a melody or theme might not be an option.

Macrocollections

Because centers are so tenuous in this music, perhaps the most salient perceptual feature of the fugues is the addition of chromatic pcs and the shifting of collections instead of a change in scale. This fact may be most true in situations like the B-flat minor fugue or the beginning of the F-sharp major fugue because the pc content is extremely limited. Most of the music in these examples uses the same collection, so a change of collection with one new pc might be seen as a major event.

Most of the exposition and counter-exposition pairings in Op. 87 contain a relatively small number of pcs because they are comprised of closely related diatonic scales.[43] All of the pcs together form a *macrocollection* or a collection that acts over a long period of time and is often made up of more than one smaller collection. The macrocollection for the beginning of the F-sharp major fugue would be F♯ G♯ A♯ B C♯ D♯ E E♯, though each E also participates in its own collection at unique points of the exposition or counter-exposition.

The term macrocollection is similar to Tymoczko's conception of *macroharmony*, which he describes as "the total collection of notes used over small stretches of time."[44] Although he usually uses the term to describe musical lengths between a chord and a key, Tymoczko uses pitch-class circulation graphs to show how many pcs are used in varying window sizes. Each window size is a number of note attacks, so they are not determined by musical events. When Tymoczko applies the concept of macroharmonies to works of music, they are often synonymous with scales.[45] My use of the term *macrocollection* is slightly different because they always occur over long periods of time defined for musical reasons. For example, the specific musical window I am using in this study is the pitch-class content of the expositions and counter-expositions of Shostakovich's fugues.

Example 73 includes a table of the macrocollections used in this specific window as well as explanations for how they were formed. These ways, such as adding a flat in the counter-exposition, were discussed earlier in the chapter. The parentheses indicate which pcs are added after the original fugue subject.

[43] The exceptions to this are the fugues that use non-diatonic scales in their subjects.
[44] Tymoczko, *Geometry*, 15.
[45] Ibid., 154–69.

1–adds one sharp in E and CE
1a–adds one sharp in E only
1b–adds one sharp in CE only
2–adds one flat in CE
3–completely diatonic
4–subject is not based on diatonic set

Fugue	Macrocollection of E and CE	
C major	C D E F G A (B)	3
A minor	A B C (C♯) D (D♯) E F F♯ G G♯	4
G major	G A B C (C♯) D E (F) F♯	1a, 2
E minor	E (F) F♯ G A B C (C♯) D	1a, 2
D major	D E F♯ G (G♯) A B C♯	1b
B minor	B C♯ D E F♯ G (G♯) A	1
A major	A B C♯ D E F♯ G♯	3
F♯ minor	F♯ (G) A (B♭) B (C) C♯ D E♭ (E)	4
E major	E (E♯) F♯ (F𝄪) G♯ A (A♯) B (B♯) C♯ D♯	1a, 4
C♯ minor	C♯ D♯ E F♯ G♯ A (A♯) B	1
B major	B C♯ D♯ E (E♯) F♯ G♯ A♯	1
G♯ minor	G♯ A A♯ B C♯ D D♯ E (E♯) F♯ (G)	4
F♯ major	F♯ G♯ A♯ B C♯ D♯ (E) E♯	2
E♭ minor	E♭ (F♭) F G♭ A♭ B♭ C♭ (C) D♭	1a, 2
D♭ major	complete chromatic scale	4
B♭ minor	B♭ C D♭ E♭ F G♭ A♭	3
A♭ major	A♭ (A) B♭ C D♭ D E♭ F G	4, 1b
F minor	F (G♭) G A♭ B♭ C D♭ E♭	2
E♭ major	complete chromatic scale	4
C minor	C (D♭) D E♭ F G A♭ (A) B♭	1a, 2
B♭ major	B♭ C D E♭ (E) F G A	1b
G minor	G A B♭ C D E♭ (E) F	1
F major	F G A B♭ (B) C D E	1b
D minor	D (E♭) E F G A B♭ C	2

Example 73. Macrocollections in exposition/counter-exposition window

Some of these macrocollections are quite chromatic because non-diatonic scales were used in their original subjects. Most of these chromatic fugues use the same relationships between scales as the fugues with diatonic subjects, but because a different set was used in the original subject, these relationships do not have the same influence on the macrocollection. For example, the subject for the D-flat major fugue contains eleven of the twelve chromatic pcs, as can be seen in Example 74. It still follows the typical pattern of a Shostakovich exposition/counter-exposition sequence: the subject begins on D-flat, the answer on A-flat; the subject of the counter-exposition begins on B-flat, the center of D-flat's relative minor, and the answer begins on F. However, these relationships have little impact on the pc content because the original subject is so chromatic.

Example 74. Fugue in D-flat Major: subjects and answers

Most of the fugues, as shown in Example 73, do use diatonic scales for their subjects, and their macrocollections resemble skip-0 eight or nine-note scales. This fact is not surprising because with the use of traditional diatonic scales, sharps and flats are added in key signature order. It would not make sense for the macrocollection to skip over a sharp or flat because one of its constituent collections would no longer be diatonic. These macrocollections contain only eight or nine pcs because Shostakovich only adds the next sharp or flat or sometimes both. Thinking about these scale relationships in this way allows us to see how the addition of chromatic pcs between statements resembles the addition of chromatic pcs within statements.

These ideas will be illustrated with the fugue in G major. Example 75 shows the collections and macrocollection of the G major fugue's exposition and counter-exposition. All of the pcs together form the macrocollection, and the three individual collections are underlined. The middle collection is the one originally used in the subject. In the answer of the exposition, the collection is shifted one slot to the right. The collection shifts back to its starting place for the second subject of the exposition (this is a three-voice fugue). In the counter-exposition, one flat is added, so the collection on the left is used. The answer, as was discussed earlier in the chapter, returns the fugue to its original collection once again. Because the pc content is so small and the collections are clearly isolated, the shifts in collection are more audible than they would be in the D-flat major fugue.

F–C–G–D–A–E–B–F♯–C♯

Example 75. Fugue in G Major: collections and macrocollection of the E/CE

It might be beneficial to think of the macrocollection as a playing field where the pitch action is taking place. The collections of a macrocollection are in close orbit and function together as a unit. A larger shift indicates a boundary between macrocollections. These shifts are especially distinct when a large number of pcs are changed. Example 76 shows such a shift after the counter-exposition of the E-flat minor fugue. The box encloses the macrocollection and shows the shifts between individual collections. After the answer of the counter-exposition, the new collection introduces four new pcs, assuming enharmonic equivalency.

m. 1: S (E)	E♭	F	G♭	A♭	B♭	C	D♭
m. 14: A (E)	E♭	F	G♭	A♭	B♭	C♭	D♭
m. 33: S (E)	E♭	F	G♭	A♭	B♭	C	D♭
m. 59: S (CE)	E♭	F♭	G♭	A♭	B♭	C♭	D♭
m. 72: A (CE)	E♭	F	G♭	A♭	B♭	C♭	D♭
m. 102	D	E	F♯	G	A	B	C♯

Example 76. Collectional shifts in the E-flat minor fugue, mm. 1–102

This chapter showed how pitch content interacts with form in Shostakovich's fugues. It also provided an introduction to the concept of macrocollection, which allows analysis of Shostakovich's fugues at a higher level instead of merely statement to statement. The chapter also brought up a few of the complications that arise in a scale and collection study. More of these will be addressed in the final chapter.

Chapter 5

Conclusions

This thesis began by talking about transformations between scales: the transposition between C major and E major is T_4; the transformation between E Phrygian and E Dorian is s_2. As the paper progressed, the focus shifted instead to collections. Because centers are sometimes difficult to capture, focusing on non-centric collections is a viable alternative approach. Scale relationships should not be ignored, however. It is simply necessary to take several factors into account.

In many of the Op. 87 fugue subjects, the center is fairly obvious because the subjects are influenced by common practice tonality. In such cases, any subject contains three components: its center, its collection, and its scale degree assignment.[46] These components are not completely independent but interact with each other in complex ways. As we have seen in this study, scale relationships can be studied in terms of position in the fugue subject, change in collection, and change of position in the collection. A diatonic transposition might change the center and the mode, including the relationships between pcs and scale degrees, if it acts within a collection and changes scales. However, a diatonic transposition might not change either of these components but simply change the position specific pcs occupy within the subject.

Now we see that transformations such as T_n, t_n, and s_n actually say nothing about centers or scale degrees. They simply predict pitch content. As such, the transformation t_n is not able to distinguish between motion within a scale and motion within a collection. The pc content remains the same in either case. In the future, it will be necessary to

[46] Bates also studies the relationship between these components in his article. Bates, "*Five Variants*," 35.

develop transformations that include a reference to the center because a study of scales is not possible without the center component.

Rings has solved part of this problem by including a scale degree component in his study of transformations in tonal music. As such, he is able to show that diatonic transposition, in his study, involves transposition within one scale because the relationship between scale degrees and pcs remains unchanged. Rings is also able to distinguish between chromatic transposition that acts within the original key and modulatory chromatic transposition (which he terms *real transposition*). Rings's system would have to be expanded to be applied to post-common practice centric music, especially to include the diatonic modes.[47]

The research of collections and macrocollections begun in this thesis can also be continued in several ways. This study focused on the exposition and counter-exposition of the fugues. More work could be done with the remainder of the fugues. Naming the first macrocollection of each fugue is relatively easy because they are limited by a formal boundary. What kind of musical cues could contribute to the formation of subsequent macrocollections? How would these macrocollections interact and do any patterns form? Is the concept of macrocollection helpful outside the first section of the fugues?

Even inside the exposition and counter-exposition, some chromatic pcs deserve a closer look. Although the pitch-class content in the exposition and counter-exposition is often minimal, Shostakovich sometimes adds chromatic notes into the links between statements, such as those in the Fugue in B-flat major. One such link is shown in Example 77. I did not include these inserted chromatic pcs in the macrocollection as they take place between statements and appear to be transitional. It would be worthwhile to

[47] Rings, *Tonality and Transformation*, 81–82.

study how these chromatic insertions relate to the macrocollection itself, and what their function might be. Do they distract the listener from the underlying collection shift?

Example 77. B-flat major fugue: chromatic link between answer and subject

Questions like these pertain to chromatic alterations outside of or between statements, but chromatic notes within individual subject statements could also be studied further. Shostakovich's distinction between "major" and "minor" versions of the same subject is fascinating, especially when the subjects are extremely chromatic. Even in the D-flat major fugue, where eleven chromatic pcs are used in the original subject, Shostakovich still adjusts a few intervals in the "minor" version of the statement. This study is perhaps easiest with his fugues because major and minor versions of every subject are presented and could easily be compared, but Shostakovich presents highly chromatic "major" and "minor" versions of melodies in some of his other works, such as his string quartets. It might be instructive to investigate what makes "major" melodies major and "minor" melodies minor when the scales used are not the literal major and minor scales.

Many of the concepts in this thesis could be applied to other works by Shostakovich and to works by other composers, especially those whose music derives from common practice tonality. For example, how could the ideas of macrocollections and eight-note scales apply to the Op. 87 preludes? Perhaps the relationships explored in this thesis could be useful in explaining the chromatic usage of Prokofiev, Stravinsky, or Hindemith. A similar study of Hindemith's fugues might be especially fascinating. His approach to the fugue differs from Shostakovich's, especially in his use of scalar relationships in the beginnings of fugues. However, the ideas of scale and collection pursued here would likely apply well to Hindemith's fugues. Such a study would allow for the investigation of different patterns in twentieth-century fugal writing.

Shostakovich's Op. 87 fugues provide a stimulating environment in which to study scalar and collectional relationships. This study has aimed to provide a context for chromatic notes in a diatonic framework at various levels. Several relationships were discussed, but many areas of investigation into how Shostakovich carefully manipulates a complicated pitch system still remain to be explored.

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