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Abstract†

Wilkie's stochastic investment model and its variants have been increasingly applied by actuaries around the world to actuarial modeling and simulation. This paper performs time series outlier analysis on retail price inflation, which is the driving force of Wilkie's composite model. The data come from four developed countries: the United Kingdom, the United States, Canada, and Australia. The fit of the model is significantly improved after the adjustment of outliers. The analysis also identifies exogenous events that have intervened in the inflation dynamics. An example is given to demonstrate the importance of outlier analysis on stochastic simulation. Finally, inflation trends for these four countries are examined. The results suggest caution in the use of the 4 percent inflation assumption of some U.K. and Australian actuaries.

Key words and phrases: *economic assumptions, inflation trend, stochastic model, time series outlier, Wilkie model*

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1 Introduction

The original Wilkie (1984, 1986) model is a composite stochastic investment model that attempts to capture the interdependence of four key variables: the retail price index, share yield, share dividends, and consols yield.[1] The model has been extensively used by U.K. actuaries for various purposes, ranging from assessing the solvency of life offices (Limb et al., 1986) to modeling uncertainty in general insurance companies (Daykin and Hey, 1990). Other actuarial applications of the Wilkie model in the U.K. include Wilkie (1987), Purchase et al., (1989), Ross (1989), and Hardy (1993). Wilkie (1995) extends the original model to add five particular variables, plus a family of variables (i.e., currency exchange rates). A comprehensive review of Wilkie's model is provided by Huber (1997).

Following Wilkie's footsteps stochastic investment models have been developed for other countries. They include Metz and Ort (1993) for Switzerland; Deaves (1993) for Canada; Daykin et al., (1994) for Finland; Thomson (1996) for South Africa; Frees et al., (1997) for the U.S.; and Sherris et al., (1997) for Australia. Unfortunately, these models, as well as the original Wilkie model (see, for examples, Wilkie (1995), Kitts (1990) and Clarkson (1991)), usually produce non-normal and nonlinear residuals, which could be due to the existence of outliers in the data series.

Retaining outliers in the time series could lead to erroneous model specification and biased predictions (Chan, 1995). Chan and Wang (1998) apply the time series outlier detection technique, developed by Chen and Liu (1993), to U.K. price inflation. The results show that the residuals are significantly closer to a normal distribution. Foster (1997) also detects outliers and level shifts in U.S. real wage series. In a similar study, Balke and Fomby (1994) examine 15 U.S. macroeconomic time series and they conclude that outliers appear to be present in all series. Also, after controlling for outliers, much of the evidence of nonnormality and nonlinearity of the residuals is eliminated.

In this article we extend Chan and Wang's (1998) work to perform time series outlier analysis on price inflation, which is the most important driving force of Wilkie's composite model, for four developed countries. The price inflation series is defined by

$$I_t = \ln P_t - \ln P_{t-1} \tag{1}$$

[1] They are called the *consumer price index*, *stock return*, *stock dividends* and *long-term interest rate*, respectively, in the U.S. and Canada.

where P_t is the price index at time t. A first order autoregressive (AR(1)) model is often employed to describe the inflation dynamic:

$$I_t = \mu + \phi(I_{t-1} - \mu) + \varepsilon_t, \tag{2}$$

where for $t = 1, 2, 3, \ldots$, the ε_t (called *stochastic disturbance terms*) are independent and identically distributed (i.i.d.) normal random variables with mean 0 and variance σ^2; ϕ is the autoregressive parameter ($|\phi| < 1$); and μ is the mean rate of the inflation process.

This model is widely accepted by actuaries for pension simulations and other actuarial applications (see, for example, Knox (1993) and Wilkie (1995)). The model can be interpreted as follows: each year the force of inflation (I_t) is equal to its mean rate (μ), plus a proportion (ϕ) of last year's excess inflation ($I_{t-1} - \mu$), plus a random disturbance (ε_t) which has zero average and variance σ^2.

The main objectives of this paper are:

- To show actuaries the importance of outlier analysis in building a stochastic model;
- To identify global exogenous events that might have significant impact on the inflation dynamic of different countries;
- To study the inflation trend for each country under examination.

2 Outlier Analysis

2.1 Time Series Outlier Models

Time series observations are often influenced by interruptive events such as strikes, outbreaks of wars, sudden political or economic crises, or even unnoticed errors of typing and recording. The consequences of these interruptive events create aberrant observations, which are inconsistent with the rest of the series. Such observations are usually referred to as *outliers*. Most outliers are not simply spurious observations (e.g., recording or typing errors). They may contain important information about the external interruptive events affecting the series. In general, outliers in time series can be viewed as the result of non-repetitive interventions. Thus, a contaminated inflation series I_t^* consists of an outlier-free inflation series I_t plus an exogenous intervention effect $\Delta_t(T, \omega)$, i.e.,

$$I_t^* = I_t + \Delta_t(T, \omega) \tag{3}$$

where I_t follows the model equation (2), T is the location of the outlier, and ω is the magnitude of the outlier.

Four commonly used types of outliers (see Tsay, 1988) and two newly proposed types of outliers (De Jong and Penzer, 1998) are considered in this paper: additive outlier (AO), innovational outlier (IO), level shift (LS), temporary change (TC), switch outlier (SO), and linear increase outlier (LIO).

- An additive outlier affects only the level of the given observation.
- An innovational outlier affects all observations beyond the given time through the memory of the underlying outlier-free process.
- A level shift is an event that affects a time series at a particular time point whose effect becomes permanent.
- A temporary change is an event having an initial impact and whose effect decreases exponentially according to a fixed dampening parameter, say, δ. In practice the value of δ often lies between 0.6 and 0.8 (Liu and Hudak, 1994, page 76). We employ $\delta = 0.7$ in this article as recommended by Chen and Liu (1993).
- A switch outlier is where there are consecutive extreme values on either side of the current level of the series. An SO would occur when the economy has a dramatic opposing change, such as from a response to unanticipated inflation or severe government intervention.
- A linear increase outlier occurs where the average level of the series ramps up to a higher level. A LIO reveals periods of regime changes within an economy such as technology improvements or again government intervention. The length of the linear increase period in a LIO is denoted by q, and we assume $q = 4$ in this study.

The form of $\Delta_t(T, \omega)$ for each type of outlier is given as:

$$\begin{aligned}
AO: \quad \Delta_t(T,\omega) &= \omega D_t^{(T)} \\
IO: \quad \Delta_t(T,\omega) &= \frac{\omega}{1-\phi B} D_t^{(T)} \\
LS: \quad \Delta_t(T,\omega) &= \frac{\omega}{1-B} D_t^{(T)} \\
TC: \quad \Delta_t(T,\omega) &= \frac{\omega}{1-\delta B} D_t^{(T)} \\
SO: \quad \Delta_t(T,\omega) &= \omega \times \left(D_t^{(T)} - D_t^{(T+1)}\right) \\
LIO: \Delta_t(T,\omega) &= \omega \times \left[\sum_{k=0}^{q-1} \left(\frac{k+1}{q+1}\right) D_t^{(T+k)} + \frac{1}{1-B} D_t^{(T+q)}\right]
\end{aligned}$$

where B is the backward shift operator such that $B^s D_t^{(T)} = D_{t-s}^{(T)}$, and

$$D_t^{(T)} = \begin{cases} 1 & \text{if } t = T \\ 0 & \text{if } t \neq T \end{cases} \tag{4}$$

is the indicator variable representing the presence or absence of an outlier at time T. Graphical examples of the $\Delta_t(T,\omega)$ function for various types of outlier are given in Figure 1.

More generally, a time series may contain m outliers of different types, and we have the following general time series outlier model:

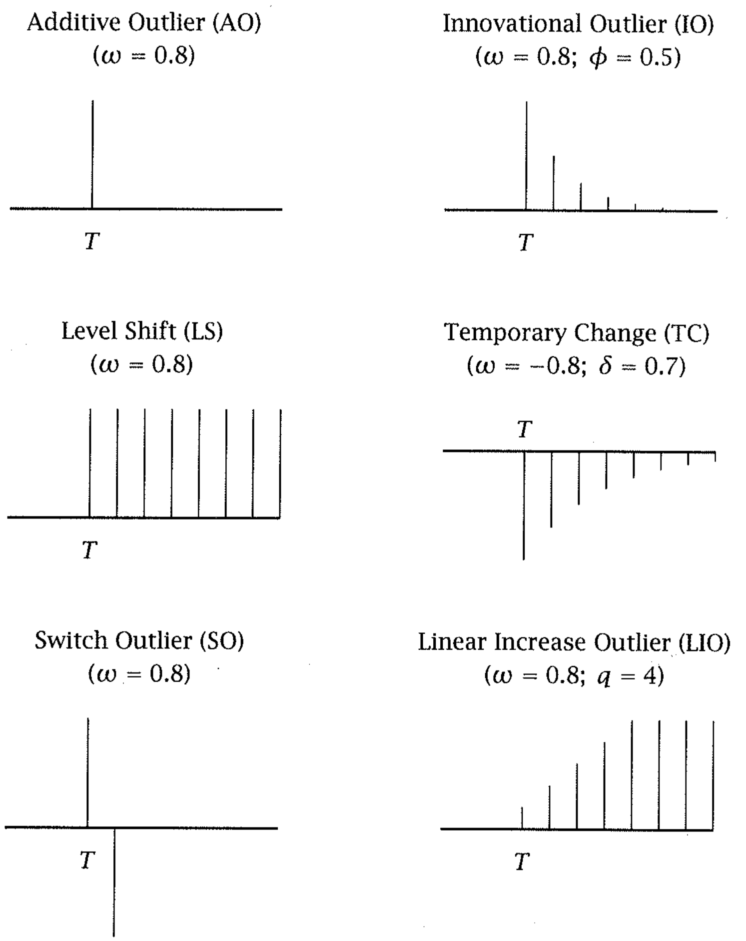
$$I_t^* = I_t + \sum_{j=1}^{m} \Delta_t(T_j, \omega_j). \tag{5}$$

In this article we assume that the underlying outlier-free process for I_t is AR(1). For other time series outlier models with underlying outlier-free process following a general autoregressive moving average model, see Tsay (1988); or for a general state-space model, see De Jong and Penzer (1998).

2.2 Outlier Detection and Adjustment

The search for the location and type of an outlier in a contaminated time series is known as an *outlier detection problem* in time series literature. It was first studied by Fox (1972), who employed the likelihood ratio test. Chang, Tiao, and Chen (1988) extend Fox's idea and propose an iterative procedure to detect multiple outliers. Chen and Liu (1993) further develop a simultaneous estimation and outlier detection procedure. Their approach consists of three-stage iterative cycle based on detection, estimation, and adjustment.

Figure 1
Effects of Time Series Outliers



Chen and Liu's (1993) method is used in this paper. For the detection stage, the fitted residuals $\hat{\varepsilon}_t^* = (I_t^* - \hat{I}_t)$ from model equation (2) are first obtained. The outlier effects in model equation (3) will be transmitted from the contaminated time series to the fitted residuals. Therefore, time series regressions can be written as

$$\hat{\varepsilon}_t^* = \omega\, d(s,t) + \varepsilon_t \quad for\ s \in S, \tag{6}$$

where $S = \{\mathrm{AO},\ IO,\ LS\ ,TC,\ SO,\ LIO\}$ is the set of residual types;

$$
\begin{aligned}
d(s,t) &= \begin{cases} 0 & \text{for all } s \text{ and } t < T, \text{ and} \\ 1 & \text{for all } s \text{ except, } s = \mathrm{L}IO, \text{ and } t = T; \end{cases} \\
d(\mathrm{L}IO,T) &= \frac{1}{q+1}; \\
d(\mathrm{A}O,t) &= \begin{cases} -\hat{\phi} & \text{for } j = 1 \\ 0 & \text{for } j \geq 2; \end{cases} \\
d(\mathrm{I}O,t) &= 0 \quad \text{for all } j; \\
d(\mathrm{L}S,t) &= 1 - \hat{\phi} \quad \text{for all } j; \\
d(\mathrm{T}C,t) &= \delta^{j-1}(\delta - \hat{\phi}); \\
d(SO,t) &= \begin{cases} -(1+\hat{\phi}) & \text{for } j = 1, \\ \hat{\phi} & \text{for } j = 2, \text{ and} \\ 0 & \text{for } j \geq 3 \end{cases} \\
d(\mathrm{L}IO,t) &= \begin{cases} \frac{j+1-j\hat{\phi}}{q+1} & \text{for } j = 1, \ldots, q \\ 1 - \hat{\phi} & \text{for } j > q \end{cases}
\end{aligned}
$$

for $t = T + j$ $(j = 1, 2, \ldots)$. They are used for detecting outliers. For given T (suspected location of the outlier) and s (suspected type of outlier), the usual regression t-statistic, $\tau(s,T)$, for the slope parameter ω in the regression model equation (5) can be computed. The final test statistic is the maximum value of this statistic searching all possible locations (T) and types (s), i.e.,

$$\mathcal{T} = \max_{1 \leq T \leq n} \max_{s \in S} \{\tau(s,T)\}\,. \tag{7}$$

For a given location, the test statistic follow a normal distribution approximately. An outlier is detected if $\mathcal{T}$ is greater than a critical value C. Following Liu and Hudak (1994), we employ $C = 3.5$ in this paper.

With the type and the location of an outlier, we can jointly re-estimate the model parameter and the outlier effects. After the estimation, one can adjust the outlier effects on the observations by the model equation (3). The detection-estimation-adjustment cycle is repeated for the adjusted series until no new outliers are found. Finally, the model is re-estimated for the autoregressive parameter and all outlier effects simultaneously.

The detection procedures can be easily implemented in many time series and regression computer packages. The SCA (Liu and Hudak, 1994) programming package provides outlier analysis as one of its standard features. It can automatically process four commonly used types of outliers (AO, IO, LS, and TC) in the data series. Additional macro statements can be incorporated into the system to deal with SO and LIO.

3 The Data

In this paper we apply outlier analysis to annual inflation series from 1900 to 1995 of four developed countries. They are United Kingdom, United States, Canada, and Australia.

3.1 Data Sources

Following Wilkie (1995) the U.K. Retail Prices Index for June (in each year) is taken from the following periods:

- 1900–1914: Board of Trade Wholesale Price Indices (Total Index), Table Prices 5 of Mitchell and Deane (1962);
- 1914–1947: "All Items" Cost of Living Index, Table 84 of Central Statistical Office (1991);
- 1947–1990: "All Items" Retail Prices Index, Table 1 of Central Statistical Office (1991);
- 1990–1993: "All Items" General Index of Retail Prices, Table 18.7 of Central Statistical Office (1994); and
- 1993–1995: "All Items" Retail Prices Index, Table 18.7 of Office of National Statistics (1997).

For the U.S. annual average consumer price index, two series have been combined:

- 1900–1970: Consumer Price Index, U.S. Department of Commerce (1973).
- 1970–1995: Consumer Price Index Number, International Monetary Fund (1998).

Note that Consumer Price Index Number has variable name AL64 in the International Financial Statistics Database.

For Canada, two annual average consumer price index series have been connected:

- 1900–1914: Price Indexes of Selected Retail Services, Statistics Canada (1965).
- 1914–1995: "All Items" Consumer Price Index, Statistics Canada, CANSIM Database.

Note that the CANSIM Database can be accessed through the internet at: *http://www.statcan.ca/english/CANSIM/index.html.* The price data are stored in matrix 9957 and it costs Cdn$3.00 per series.

For Australia, annual average retail price index numbers from 1900 to 1995 are recorded in Australian Bureau of Statistics (1997, p. 660).

3.2 Data Description

We use 1923 as the common base year for all index series. They are shown in Figure 2 using a vertical logarithmic scale. The graphs have similar shape. Their corresponding inflation series, as defined in equation (1), are plotted in Figure 3.

Descriptive statistics for the inflation series are summarized in Table 1. The U.K. and Australia, on the average over the past 96 years, have one percent inflation rate per year higher than the U.S. and Canada. The standard deviation in U.K. inflation is higher than that of the other countries investigated. The distributions of inflation for Canada and Australia are negatively skewed. The inflation distributions appear to have thick right tails, while Australian inflation is closer to a normal distribution.

Skewness (κ_1) and excess kurtosis (κ_2) of a random variable X are:

$$\kappa_1 = \frac{\mu_3}{\mu_2^{3/2}} \quad \text{and} \quad \kappa_2 = \frac{\mu_4}{\mu_2^2} - 3$$

where $\mu_k = E[(X - E[X])^k]$.

Table 1
Summary Statistics for Inflation Series

	U.K.	U.S.	Canada	Australia
Mean	0.0409	0.0307	0.0313	0.0408
S.D.	0.0678	0.0491	0.0481	0.0532
Skewness	0.2814	0.1057	-0.1822	-0.1577
Kurtosis	1.7796	1.4538	1.7748	0.9324
r_1	0.5200	0.6200	0.5800	0.5500
r_2	0.3200	0.2500	0.3100	0.2900
		Correlation		
U.K.	1.00			
U.S.	0.67	1.00		
Canada	0.76	0.91	1.00	
Australia	0.66	0.64	0.74	1.00

Note: S.D. = Standard deviation; r_1 = First-lag autocorrelation; r_2 = Second-lag autocorrelation.

The evidence of highly significant first-lag autocorrelation coefficients plus the fact that all r_2 are approximately equal to the square of their corresponding r_1 in Table 1 support the use of the AR(1) model for the inflation dynamic for each country. The correlations among inflation series are high. Due to geographic, political, and economic reasons, it is not a surprise to observe that the correlation between U.S. inflation and Canadian inflation is 0.91, the highest among all other combinations.

4 Empirical Results

4.1 Model Fitting

A first-order autoregressive model is fitted to each data series without considering the possibility of outlier effects. Based on the outlier analysis described in Section 2, AR(1) models are also fitted to the outlier-adjusted series. Table 2 presents fitting results under both situations.

Figure 2
Retail (Consumer) Price Indexes, 1990–1995

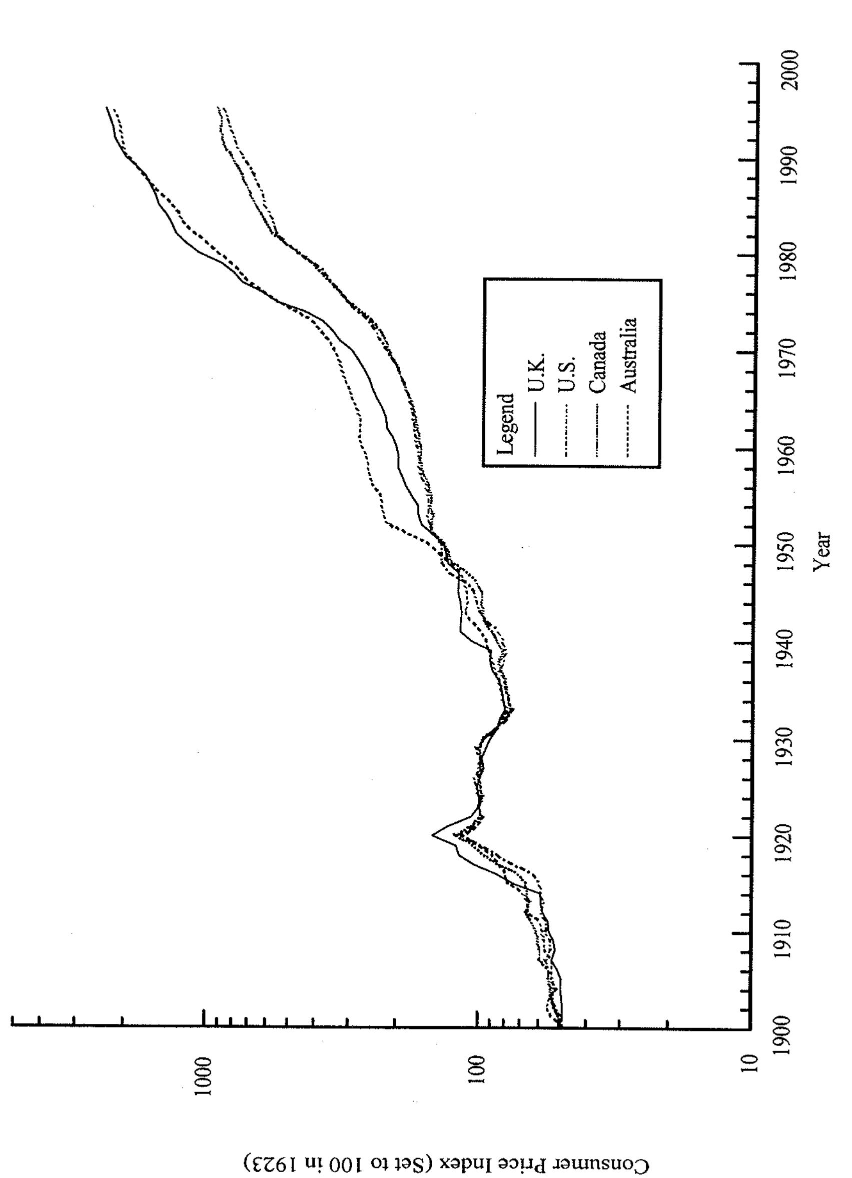


Figure 3
Price Inflation Series, 1990–1995

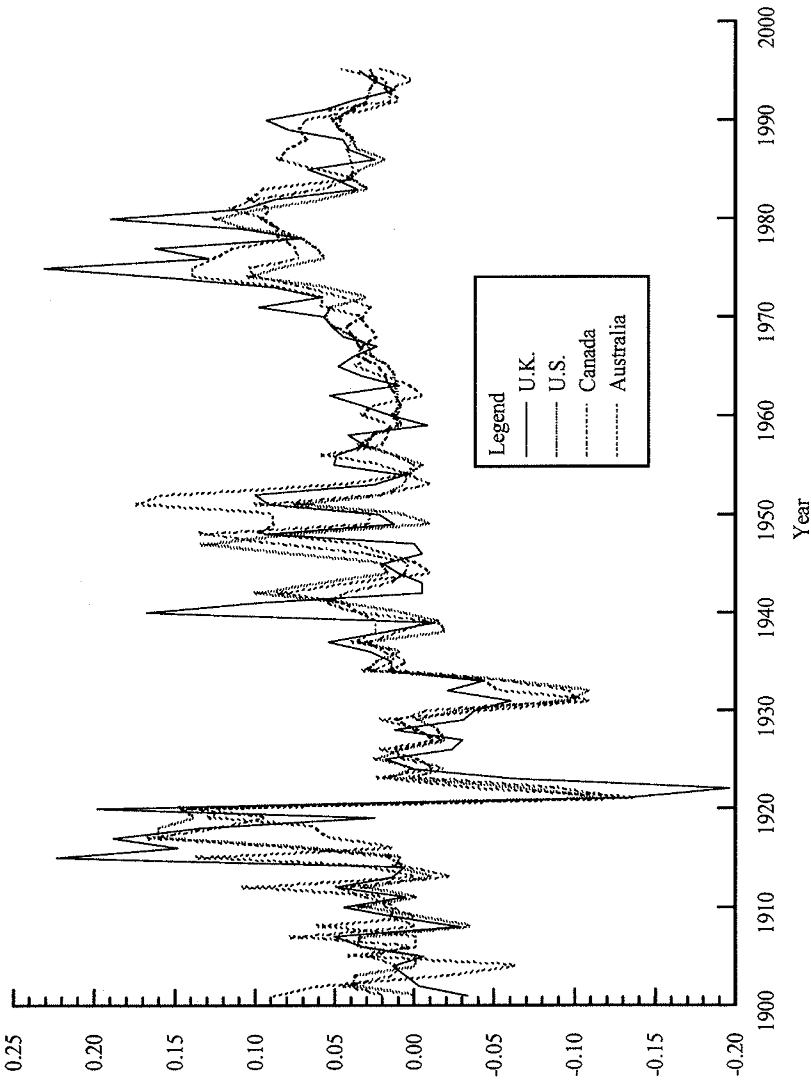


Table 2
Model Fitting Results for the Inflation AR(1) Process of Equation (1) For Various Countries (Before and After the Outlier Adjustments)

	United Kingdom		United States		Canada		Australia	
	Before	After	Before	After	Before	After	Before	After
$\hat{\mu}$	0.042	0.029	0.032	0.040	0.031	0.040	0.040	0.046
$\hat{\phi}$	0.519	0.534	0.617	0.752	0.575	0.547	0.550	0.692
$\hat{\sigma}$	0.058	0.035	0.039	0.031	0.039	0.027	0.044	0.035
AIC[1]	-529.3	-612.3	-603.0	-644.9	-601.2	-665.1	-577.7	-618.3
AIC(2)	-520.8	-607.4	-596.6	-641.7	-590.8	-664.9	-564.0	-609.4
JB[2]	103.6	3.1	354.8	2.3	463.5	3.6	141.9	2.7
NLT[3]	0.012	0.447	0.325	0.987	0.604	0.789	0.114	0.310
Q[4]	15.7	10.6	10.6	12.3	8.1	4.7	11.7	4.2

[1] AIC $= n \ln \hat{\sigma}^2 + 2M$, where n is the number of effective observations, and M is the number of parameters in the model. Under this criterion, we should choose the model with *smaller* AIC. AIC(2) is the AIC value for an alternative AR(2) model for the series.

[2] JB is the Jarque and Bera's (1981) test statistic for normality of the residuals. Under the null (normality) hypothesis, the critical value of the test is 5.99 at the 5 percent level.

[3] NLT is the p-value of Tsay's (1989) F test for linearity of the residuals. We will reject the null (linearity) hypothesis if the p-value is less than the significance level, say, 5 percent.

[4] Q is the Ljung and Box's (1978) Portmanteau statistic (with 10 lags) for testing serial correlation of the residuals. Under the null (independence) hypothesis, the critical value of the test is 15.507 at the 5 percent level.

The residual standard deviation ($\hat{\sigma}$) is significantly reduced for each country after adjusting for outliers. Because we have introduced additional parameters into the outlier model, it is not appropriate to focus only on the improvement of the fit. Akaike (1974) proposes an information criterion to compare alternative models fitted to a dataset with different number of parameters. The criterion has been called AIC (Akaike Information Criterion) in the literature and is defined as

$$AIC(M) = n\ln(\hat{\sigma}^2) + 2M \tag{8}$$

where n is the number of effective observations and M is the number of parameters in the model. The criterion considers both the model fitting ($\hat{\sigma}^2$) and the model parsimony (M). Under this criterion, one should choose the model with the smallest AIC. The results in Table 2 indicate that the outlier model is preferred to the original model in every country. They all give a smaller value of AIC. To guard against model misspecification, the AIC value for an alternative AR(2) process is computed for each country. The results justify our choice of the AR(1) model for the inflation series.

The problem of nonnormality and nonlinearity of residuals from the original AR(1) model for U.K. inflation has caused some concerns for many authors (see, Kitts, 1990; Geoghegan et al., 1992; Huber, 1997; and Chan and Wang, 1998). In this paper we examine the normality and linearity of the residuals using the Jarque and Bera (1981) test and the Tsay (1989) test, respectively. The results in Table 2 show that normality of the residuals has been remarkably improved after controlling for outliers. Furthermore, degree of nonlinearity in the residuals could also be alleviated by the outlier model in each country. Finally, Portmanteau Q statistics (Ljung and Box, 1978) are computed for testing serial correlation of the residuals. The results do not indicate any inadequacy of the fitted models.

4.2 Detected Outliers

Table 3 displays the outliers found for each country. It describes the type, size, and t-ratio of the outlier as well as the year in which it occurred. In addition, we also try to link the year of each outlier to an economic event that occurred in or near that year.

An examination of Table 3 reveals that British inflation is more vulnerable to external shocks compared to other countries. Five outliers are identified. On the other hand, the U.S. inflation is more robust to economic disturbances. Only one outlier is detected in 1921.

Table 3
Detected Outliers in Chronological Order

		United Kingdom			United States		
Year	Event	Type	Size	*t*-ratio	Type	Size	*t*-ratio
1915	World War I	TC	0.208	6.30			
1917	World War I						
1920	Post-WWI	SO	0.160	4.77			
1921	Post-WWI				TC	-0.233	-7.54
1922	Post-WWI	AO	-0.188	7.22			
1931	Recession (Canada)						
1940	World War II	IO	0.163	4.83			
1975	Oil Crisis Shock	TC	0.139	4.25			

		Canada			Australia		
Year	Event	Type	Size	*t*-ratio	Type	Size	*t*-ratio
1915	World War I				AO	0.112	3.87
1917	World War I	TC	0.109	4.09			
1920	Post-WWI						
1921	Post-WWI	TC	-0.229	-8.57	IO	-0.235	-6.64
1922	Post-WWI						
1931	Recession (Canada)	TC	-0.119	-4.46			
1940	World War II						
1975	Oil Crisis Shock						

In addition to the global events, the Canadian inflation dynamic was interrupted by the internal severe economic depression in early 1930s (*cf.* Dominion Bureau of Statistics, 1938, p. 813). Historically, the inflation dynamic in Australia was disturbed twice (1915 and 1921), possibly due to the effects of World War I.

5 An Example

The importance of outlier analysis in stochastic (actuarial) simulation will now be demonstrated.

Consider an insurance company that is interested in selling index-linked policies.[2] Before offering such policies, the company's actuaries would need to consider the characteristics of index-linked assets to match the resulting index-linked liabilities.[3]

In this example we consider the U.K. government indexed bonds, which are commonly called *index-linked gilts.*[4] For simplicity, we assume that the bond makes annual coupon payments that are based on the inflation-adjusted face value of the bond over time. The initial face amount is 1,000, coupon interest rate $c = 5$ percent per annum, and time to maturity $N = 20$ years. The adjustment for inflation is made using the annual U.K. Retail Price Index (RPI) with a one year lag. At maturity, the redemption value also is adjusted for the realized inflation between the initial indexation year and one year prior to the maturity. We further assume that the bond is currently selling at par (i.e., $P = 1,000$).

Let y be the yield rate for the bond; y can be obtained by solving the following bond pricing equation:

$$P = \sum_{t=1}^{N} \frac{cF_t}{(1+y)^t} + \frac{F_N}{(1+y)^N} \tag{9}$$

where F_t is the inflation-adjusted face value at time t. The current value of F_t is the last year's face value (F_{t-1}) adjusted by the lagged inflation, i.e.,

[2]Wilkie (1981) presents arguments in favor of inflation-indexed life insurance contracts.

[3]Recently, several countries have started issuing inflation-indexed government bonds, that is, securities with yields that rise and fall with inflation. Such bonds provide tools for matching index-linked liabilities. For more details of inflation-indexed bonds, see Huh (1996).

[4]The U.S. version of government indexed bonds is called *Treasury Inflation-Protection Securities* (TIPS), see Roll (1996) and Madsen (1998) for details.

$$F_t = F_{t-1} e^{I_{t-1}} \tag{10}$$

with $F_0 = 1,000$.

If the force of inflation is assumed to be static at the 4 percent level (i.e., $I_t = I = 0.04$), the yield rate (y) can be solved analytically as

$$\begin{aligned} y &= (1+c)e^I - 1 \\ &= (1+0.05)e^{0.04} - 1 = 9.285\%. \end{aligned}$$

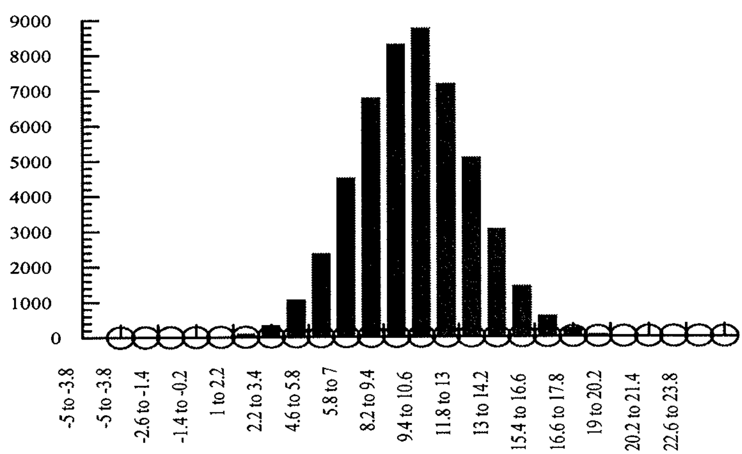
If I_t follows an AR(1) process, the distribution of y can be studied through simulation. Twenty years of inflation rates are generated using the fitted AR(1) process in Table 2 (without outlier adjustments). Given the inflation rates, the value of y can be solved from equation (8). The experiment is repeated 50,000 times. The simulation study is also carried out using the fitted AR(1) model after adjusting the outliers in Table 2. The empirical distributions of y, under both situations, are plotted in Figure 4. After controlling the outliers, actuaries are able to obtain a more precise distribution of y.

Finally, we should emphasize that the sole objective of this example is to demonstrate the reduction of the volatility of the yield of an inflation-protected bond under traditional pricing methods. We do not mean that one should price these bonds using non-risk neutral expected pricing techniques. Discussions on pricing considerations of these index-linked gilts are, however, beyond the scope of this paper.

6 Inflation Trends

Following Chan and Wang (1998), we study the inflation trend for each country. Periods from starting year (SY) through 1995 are considered. The starting year is rolled from 1900 to 1971. It creates 72 periods. The last period 1971–1995 has 25 observations which is the minimum for computing reasonable AR(1) estimates. In each period we calculate the mean rate of the inflation process ($\hat{\mu}$) after controlling the outliers. We repeat the computation for each country. The results are plotted in Figure 5. There is an upward trend in the long-term mean of the U.K. inflation process: it climbs from 4 percent to 8 percent. There is also an upward trend in the long-term mean of the Australia inflation process.

Figure 4
Simulated Distribution of y (Yield Rate) Hypothetical U.K. Inflation-Indexed Bond



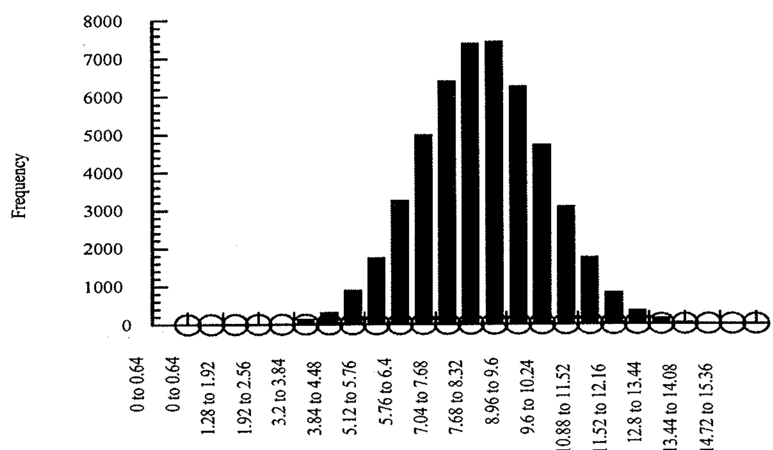
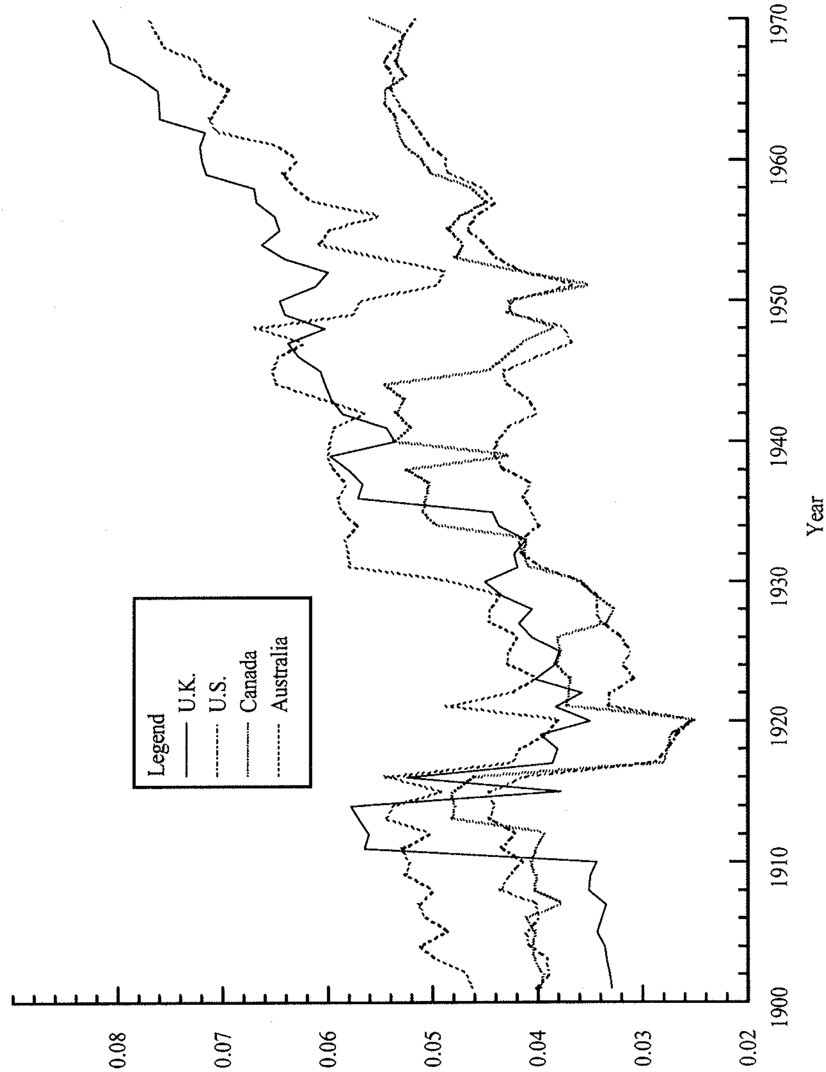


Figure 5
Inflation Trends



The results urge caution the use of 4 percent inflation assumption made by some U.K. and Australian actuaries (see, for example, Cooper, 1997, p. 18; and Knox, 1993, p. 54). On the other hand, inflation trends for the U.S. and Canada seem to fluctuate around the 4 percent level.

7 Conclusion

This research highlights the importance of outlier analysis for actuaries wishing to construct and use stochastic investment models. We perform time series outlier analysis on price inflation, which is a driving force of most existing actuarial investment models. Several exogenous events that have intervened in the inflation dynamics are identified.

The U.K. outlier-adjusted inflation model is applied to examine the distribution of the yield rate of an index-linked gilt. Finally, inflation trends are studied. The results question the use of 4 percent inflation assumption by some U.K. and Australian actuaries.

In addition to price inflation, stochastic modeling of interest rates, investment yields and other component variables in the Wilkie model are also important in a life office (see, for examples, Bragg, 1984; Greeley and Leff, 1984; Panjer and Bellhouse, 1980; and Smart, 1977). Actuaries are reminded to include outlier analysis as an initial step for modeling these variables.

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