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Irrigation-as-a-Service for Smallholder Farmers

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IRRIGATION-AS-A-SERVICE FOR SMALLHOLDER FARMERS

by

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IRRIGATION-AS-A-SERVICE FOR SMALLHOLDER FARMERS

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Irrigation is a crucial management practice that can help increase food security among smallholders globally while mitigating climate change impacts. High efficiency irrigation technologies such as drip kits and sprinkler systems are relatively expensive and smallholder farmers cannot afford them to buffer crop yields against low precipitation. In many developing countries, farmers participate in robust informal markets for renting and sharing of irrigation equipment. Such services may be operated by farmers or via a third party such as irrigation start-ups, water user associations, non-governmental organizations, or even government agencies. These services are referred to collectively as Irrigation-as-a-Service (IaaS).

The objective of this study is to develop and analyze a decision-making model for IaaS. For purposes of this study, a decision maker is defined as someone who must choose an optimal strategy to provide occasional and mobile irrigation service across multiple possible fields when there are constraints that prevent all fields from being irrigated fully. Analytical and simulation methods will be used for solving the objective functions.

Here, a crop water production function was used to model the crop yield resulting from a given irrigation application. This establishes a mathematical relationship between crop yield and variable irrigation water inputs for a given set of climate conditions and farm management practices. Optimal irrigation service strategy across landscape was analyzed as it varies as a function of field-level parameters (soil type, crop type, and field size), regional parameters

(weather), physical parameters (pump and pipe capacities), and economic parameters (fuel cost, labor cost, and crop prices).

Results reveal that decision-making under IaaS is complex, with solutions ranging from irrigating all, some, or only one field depending on key parameters. Analyzing the crop water production function and understanding the value of marginal irrigation is crucial to determining the optimal irrigation strategy for each field to maximize yield and profit. Asset utilization rates of irrigation equipment may increase or remain the same in IaaS as compared to a fixed irrigation system. The research also found there are situations where not using the equipment to its full potential is actually more profitable.

The results have policy implications regarding the cost-effectiveness of donor funds: indicating that IaaS can increase water-use efficiency, returns on invested funds, and asset utilization rates in most cases. In some countries pump sharing and pump rental markets are discouraged by government policies so a strong implication from these findings is that government would achieve its goals much better through encouraging water entrepreneurs and pump sharing than trying to suppress it.

In memory of my beloved parents,

Vibha Baxi and Anupam Kumar

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1.0 Overview and motivation

Irrigation is important for smallholder farmers. It has the potential to buffer crop yields against the detrimental effects of low precipitation and heat, which can lead to improved agricultural productivity (Burney et al., 2013). A significant increase in crop yields leads to an increase in income, contributes to food security and poverty reduction, and enhances resilience to climate change (Lowder et al., 2021; Lowder et al., 2016; Molden et al., 2007). Therefore, smallholder irrigation is a key tool for sustainable agriculture and rural development, particularly in regions where farmers rely heavily on rainfed agriculture. Despite these benefits, smallholder farmers often face constraints in accessing irrigation technology, including lack of access to finance, technical assistance, and infrastructure (Bjornlund et al., 2017; Lipton et al., 2003).

There is an ongoing policy debate regarding how best to increase irrigation access for smallholders, with a range of potential mechanisms being discussed. Some argue that large government- and donor-funded schemes are necessary to provide financial and technical support for smallholder irrigation (de Fraiture & Giordano, 2014; Giordano et al., 2012). However, others have suggested that farmer-led initiatives, such as the use of solar pumps, may be more effective in reaching smallholders and increasing their access to irrigation (Izzi et al., 2021; Burney et al., 2013). To date researchers have generally assessed the suitability of business models for smallholder irrigation through analyses focused on individual pump ownership and by estimating benefit-cost ratios and returns on investment for individual pumps (e.g. Otto et al., 2018). However, these studies provide only a limited perspective. When making decisions, it is important to consider how to maximize the benefits from a fixed number of irrigation pumps for a group of farmers in a specific geographical region. In such areas, the primary objective is to enhance irrigation access for smallholders. Few studies mention entrepreneurial irrigation service provision as a potential approach to increasing smallholder access to irrigation (Namara et al., 2019). This approach involves private-sector actors providing irrigation services to smallholders, often using innovative business models and technology. One of the prominent business models for providing irrigation services to smallholder farmers is the use of mobile irrigation equipment, which can easily be transported between fields.

Although there is no one-size-fits-all approach to increasing smallholder irrigation access, it is clear that policy and institutional reforms are necessary to promote sustainable and equitable access to irrigation technology. For example, Woodhouse et al. (2017) suggested that public financing of smallholder irrigation may need to be complemented with private sector financing to achieve financial sustainability for smallholder farmers. In addition, regulatory frameworks may need to be adapted to support farmer-led initiatives and the provision of entrepreneurial irrigation services (Namara et al., 2011).

Irrigation-as-a-Service (IaaS) has emerged as a potential solution for increasing smallholder access to irrigation technologies. IaaS involves private sector actors providing irrigation services to smallholders, often using innovative business models and technologies (Akaliza et al., 2023).

In this thesis, Irrigation-as-a-Service (IaaS) is defined as a *for-profit service provided by non-farming entrepreneurs who rent irrigation equipment and technology to farmers.*

While there have been numerous studies on farmer-to-farmer sharing and renting of agricultural equipment (Shah, 2010; Koppen et al., 2007; Shah et al., 2006), the research gap is that there has not been any economic study on entrepreneurial approaches, where a non-farmer businessperson rents irrigation equipment to multiple farmers.

There is mixed evidence on the equity impacts of water markets. Some studies show that informal water markets have increased equity in access to water and can be 'pro-poor' (Buisson, 2021; Manjunatha, 2011; Mukherjee et al., 2009). Other studies show that informal water markets and pump rental systems are unregulated markets where large farmers share majority of gains, and poor and women farmers are often excluded because of the perceived risks of non-payment (Lefore et al., 2019). This raises concerns regarding the equity and inadequacy with respect to increasing irrigation access of such systems. Informal approaches can enhance irrigation access, but their unregulated nature often hinders scalability. Additionally, these unregulated systems can contribute to the overexploitation of water resources as they lack appropriate oversight and management. Therefore, there is a need to have a better understanding of informal water markets to address these limitations.

Irrigation-as-a-service is a type of informal water market with several potential advantages. It is structured and is managed as a business. It has the potential to be regulated in comparison with informal pump rentals among farmers. Informal groundwater markets (IGMs), while increasing water access to those who cannot afford to purchase their own pumps, could also lead to increased water abstraction beyond the permissible limit (Balasubramanya and Buisson, 2022); however, in the case of IaaS, water abstraction might be kept in check by imposing regulations on irrigation service providers. IaaS is a technological and market innovation in which businesses are exploring different ways to disrupt conventional irrigation methods. Irrigation service providers often use mobile irrigation technologies, such as hose-reel or travel guns, small diesel pumps that can be transported using a bike, and even mobile solar irrigation pumps. The mobility of these devices enables them to be used across different fields in a geographical region. It makes irrigation affordable and accessible to a large group of farmers who do not own irrigation equipment and are otherwise adversely affected by drought-related crop losses.

Policy makers have focused on enhancing food security and alleviating poverty by expanding irrigation access in smallholder settings. While this strategy is generally effective, its applicability may vary when considering mobile irrigation rentals. To better capture the complexities of real-world situations, in this thesis I introduce and analyze two key elements in an economic model of irrigation as a service:

- Mobile irrigation equipment that allows sharing among multiple sites: This introduces decision making regarding where and how much to irrigate. This allows for trade-offs in the distribution of benefits among multiple users.
- An upper bound on asset utilization: This reflects the constraints often found in practical landscapes where resources are constrained. In a landscape with an unlimited number of pumps, the decision for each field is the same, if the fields are identical. With limited irrigation infrastructure resources or assets, tradeoffs must be considered in terms of who benefits and to what extent in the decision-making process. This leads to trade-offs such as deciding whether to fully irrigate certain fields or provide supplemental irrigation to a

greater number of fields. These decisions have implications for both individual and aggregate profit.

The aim of this thesis is to explore the conditions under which mobile irrigation equipment may be preferred over fixed irrigation equipment. Fixed irrigation systems are machines that are permanently installed in agricultural fields, such as center pivot systems and drip systems. This study is the first to develop and present an economic model and analysis of Irrigation-as-a-Service to explore the conditions under which equipment rental might be a practical and profitable solution for smallholders, and be preferred to a fixed irrigation system. This study explores the hypothesis regarding the optimal utilization of mobile equipment by assessing the marginal value of water across multiple locations in the landscape simultaneously.

I use analytical and simulation modeling approaches to create an economic model that examines the optimal allocation of water across multiple fields under IaaS, with the aim of finding yield maximizing and profit maximizing solutions.

1.1 Relevant previous literature

De Fraiture and Giordano (2014) highlighted the significance of small private irrigation providers in meeting the water needs of smallholder farmers and emphasized the need for further research and attention on the role of the private sector in irrigation development.

The Research Report by the International Water Management Institute (Otoo et al., 2018) reported evidence-based information for development actors and investors by mapping business model scenarios for smallholder solar pump-based irrigation in Ethiopia. They shed light on the suitability and sustainability of solar pump irrigation along with recommendations for facilitating smallholders' investment in individually owned photovoltaics (PV). The analysis excluded larger pumps and clusters of high-power, solar-based pumps that can be used for communal irrigation schemes.

Even in the absence of formal regulations or subsidies, informal markets exist in which both equipment owners and users benefit economically (Parry et al., 2020). In the context of Indian irrigation, this study showed that farmers access water by renting irrigation equipment through informal agreements. The evidence highlighted the existence of informal markets that provide benefits to both parties involved in transactions. Hence, entrepreneurial approaches to irrigation equipment rental might be of interest because they have the potential to address the limitations of informal markets and farmer-to-farmer rental systems.

Another report conducted in Uttar Pradesh, India, showed that a significant proportion of marginal farmers (approximately 60%) relied on purchasing water or renting pumps for irrigation, which is often the most expensive option (Jain et al., 2018). Estimates for the number of diesel irrigation pumps in the country range from six to nine million; however, as of 2012, nearly 30 million farmers reported utilizing them, indicating widespread sharing of these diesel pumps. The practice of renting farm equipment, including pumps, has become more common among farmers, subsequently increasing access to technology. While only 14% of farmers owned a tractor, nearly 99% reported using a tractor (Jain et al., 2018). The study also suggested that current farm equipment owners, primarily medium and large farmers, are more likely to invest in emerging technologies in the future with the intention of renting them out. Nevertheless, mobility constraints may limit the potential of sharing or renting solar pumps (Jain et al., 2018).

The response of pump rental prices to diesel price increases is particularly significant for the economically disadvantaged, as the poorest segment of India's agricultural population relies on water markets for their irrigation needs (Shah, 1993). However, recent studies on informal groundwater markets (IGMs) in India argue that irrigation services can be competitive, monopolistic, or oligopolistic (Balasubramanya and Buisson, 2022; Dubash, 2002; Easter et al.,1999). Wilson (2002) noted that the expansion of irrigation through renting mobile diesel pump sets in Bihar has been a major factor in increasing the irrigated area of cultivated land, reaching around 73% in 1995-96. Predominantly, small and marginal cultivators hire pump sets and cultivate an average of 1.35 acres, compared to the 3.89-acre average cultivated by pump set owners. Mukherji (2007) further highlighted the numerous advantages of water markets for the agricultural poor in West Bengal through extensive study. Both water markets and groundwater irrigation have provided considerable support to impoverished agricultural communities.

In areas with active informal groundwater markets (IGMs), incentive programs may not effectively reduce groundwater extraction and could negatively impact the welfare of water buyers (Balasubramanya and Buisson, 2022). Such programs might lead pump owners to sell electricity units back, affecting the livelihoods of smaller farmers and potentially increasing the cost of irrigation services, especially in oligopolistic or monopolistic IGMs. These programs may fail to reduce groundwater use and worsen existing inequalities in water access by altering the welfare distribution between service providers and buyers within the IGMs (Fishman et al., 2016).

The agricultural industry, especially in Sub-Saharan Africa, faces a significant challenge with millions of acres of highly fragmented, non-irrigated cropland and poor irrigation infrastructure (Chandra et al., 2023; Xie et al., 2014). Consequently, farmers regularly experience crop losses resulting from droughts despite an abundant water supply. Furthermore, if a smallholder farmer has multiple distributed and non-contiguous parcels of land, it requires him to invest in multiple pumps, which is costly. In these situations, installing permanent irrigation equipment may not be profitable. However, offering temporary and occasional irrigation services might provide a profitable opportunity to mitigate crop losses from droughts and irrigate multiple fields belonging to a single operator cost-effectively. This approach can enable farmers to achieve higher yields and better returns on investments.

Beekman, Veldwisch, and Bolding (2014) found that informal irrigation activities in Mozambique might double the officially recognized area of irrigation. Similarly, in Tanzania, official datasets such as the Living Standards Measurement Survey (LSMS) and agricultural censuses significantly underestimate rice irrigation practices, suggesting that the actual irrigated rice area could be 10-20 times larger than reported. This underreporting extends to other crops such as vegetables, maize, and beans. This discrepancy highlights that farmers are more likely to be involved in irrigation development than official records indicate. Namara et al. (2014) discuss a market-based network in Ghana, where pumps can be rented on a daily, seasonal, yearly, or even hourly basis.

1.2 Private businesses providing IaaS

The concept of Irrigation-as-a-Service represents an emerging business opportunity, with only a handful of operational companies currently in existence. In this section, I introduce two such companies and briefly explain their service.

AgriRain is a small enterprise offering data-driven precision irrigation-as-a-service on a mobile, on-demand, pay-per-use, and operator-run model. This service is designed to increase yields while reducing irrigation expenses. AgriRain's products are tailored to small farms and can be operated by anyone, even those without prior irrigation knowledge. This system is compatible with both electric and diesel pumps.

Over the past four years, AgriRain has been implemented in India, Rwanda, and the Ivory Coast. In India, it has been applied across 20 crops and five states, with soybean and sugarcane farmers experiencing an increased net income (www.agrirain.com). AgriRain has partnered with 12 Farmer Producer Organizations (FPOs) and multiple cooperatives and serves as a trusted irrigation partner for the National Seeds Corporation. The organization created more than 33 full-time water entrepreneurs.

In Rwanda and Ivory Coast, AgriRain operates within the corn and cotton value chains. Their work encompasses cotton, corn, sugarcane, and potato value chains, and they charge irrigation fees based on the volume of water applied.

Agriworks is a Ugandan company that provides irrigation services for smallholder farmers. When farmers require irrigation during dry spells, they can request assistance from Agriworks, who then dispatch water pumps to their fields. Charges are based on the service time and fuel consumption, which are calculated using a custom smartphone app. The Agriworks Mobile Irrigation System is designed for 1–5-acre plots and is both mobile and modular, allowing multiple farmers to share the system and upgrade it incrementally.

Agriworks currently operate in the Elgon Region of Eastern Uganda, with customers throughout Eastern and Northern Uganda. Since 2019, more than 1,500 farmers have signed up, covering more than 780 acres.

The majority of Agriworks clients are smallholder farmers, with 47% living below $2/day based on livelihood measures and 69% earning less than $1.5/day according to self-reported income.

In later sections, I discuss the methods and economic model developed to understand irrigation decision-making behavior using mobile irrigation equipment and a limited water supply under IaaS. This is followed by the discussion of our findings from the model. Following the results, a discussion of the policy implications and conclusions is presented.

2.0 Methodology

I developed an economic model to understand the irrigation decision-making behavior of how much to irrigate and how many farms can be irrigated using mobile irrigation equipment and limited water supply. I considered a system with multiple identical and adjacent farms, and a single set of mobile irrigation equipment. In this study, a combination of analytical and simulation modeling approach was used.

2.1 Crop Water Production Function (CWPF)

To model the functional relationship between crop yield and irrigation water, I use a crop water production function (CWPF). The Crop Water Production Function (CWPF) is a mathematical relationship that describes the response of crop yield to water use, typically focusing on the effects of irrigation (Figure 1(a)). It is an essential tool in agricultural research, as it helps in understanding the interactions between crop yield and water inputs, and can be used to optimize irrigation management, water use efficiency, and water allocation in agricultural systems.

The complexity of crop responses to water deficits has historically led to the use of empirical production functions to assess crop yields in relation to water (Hsiao, 1973; Hsiao et al., 1976; Bradford and Hsiao, 1982). One widely adopted method, which has been used extensively by irrigation planners, economists, and engineers, is described in FAO Irrigation & Drainage Paper no. 33, "Yield Response to Water" (Doorenbos and Kassam, 1979).

Empirical and mathematical approaches to modeling crop water production functions (CWPF) aim to estimate the relationship between crop yield and water use. These approaches can be broadly categorized into two groups: empirical methods and process-based mathematical models. Empirical methods estimate crop water production functions using statistical techniques to fit functional relationships to observational data on crop yield responses to varying irrigation levels, typically obtained from field experiments or farm surveys.

Numerous studies have used empirical methods to develop crop water production functions, such as Hexem and Heady's (1978) widely cited research and other similar studies (for example, Barrett & Skogerboe, 1980; Ayer & Hoyt, 1981; Dinar et al., 1986; Zhang & Oweis, 1999; Kipkorir et al., 2002; Igbadun et al., 2012). These studies often report a curvilinear relationship between seasonal irrigation and crop yield but also highlight variability due to differences in management practices and climatic conditions.

Crop coefficient models are a simple and widely used approach to simulating crop yield responses to variable irrigation water inputs (Scheierling et al., 1997; Shani et al., 2004; Smilovic et al., 2016), and have been extensively applied in hydro-economic (Sunantara and Ramírez, 1997; Cai, 2008; Esteve et al., 2015; Giuliani et al., 2016; Noël and Cai, 2017) and water resource planning models (e.g., WEAP) (Yates et al., 2005) to support agricultural water management and climate change adaptation. Crop coefficient models typically involve multiplying a reference evapotranspiration value by a crop-specific coefficient that represents the crop's water use relative to the reference crop.

Driven by the limitations of crop coefficient models and recent advancements in computing power, there has been a growing interest in using more complex process-based crop growth models for simulating crop yield response to water in hydro-economic research and economic water policy analysis. Crop growth models such as AquaCrop, DSSAT, and APSIM are mathematical models that simulate the growth, development, and yield of a crop under specific environmental conditions and management practices. These models offer detailed representations of crop physiological responses to water deficits, allowing for a more accurate estimation of crop-water production relationships.

Often, crop models are linked with optimization algorithms to determine the optimal scheduling of limited seasonal irrigation; however, this approach assumes that farmers have perfect foresight of daily weather conditions and face no constraints on irrigation timing. In reality, farmers face high levels of uncertainty about future weather conditions and use heuristics to make irrigation decisions, which are often affected by various constraints (Foster and Brozović, 2018).

There is a need for a comprehensive approach that simulates farmers' expectations, while accounting for intraseasonal irrigation constraints and stochastic climate conditions in irrigated agricultural production systems (Foster and Brozović, 2018). Stochastic methods have been used in several studies to account for uncertain climatic conditions when modeling Crop Water Production Functions (CWPF) (García-Vila et al., 2012; Foster et al., 2014). In the presence of uncertain climatic conditions, stochastic methods can be used to model CWPF. These methods incorporate probability distributions for key variables, such as precipitation or temperature, and can help capture the variability in crop yield responses to water across a range of potential environmental and management conditions.

In most economic studies, it is assumed that the crop water production function has a concave shape and can be divided into roughly four sections (Foster et al., 2014). In Zone 1, most of the applied water is used by the crop, resulting in a nearly linear slope. As the applied water increases (Zone 2), marginal returns decrease because of non-beneficial processes to the plant, such as soil evaporation, deep percolation, and surface runoff. The yield reaches its maximum in Zone 3, beyond which further irrigation will not increase yield and may even reduce it in Zone 4 owing to waterlogging and anaerobic conditions that limit transpiration. This shape has significant implications for irrigation decision-making and agricultural water management (English, 1990; English et al., 2002).

However, the idealized concave shape may not always apply, such as in drought years requiring a minimum level of irrigation to avoid crop failure (Shani et al., 2004; Foster et al., 2015) or when limited seasonal irrigation is scheduled sub-optimally owing to water availability or information constraints (Foster et al., 2014; Smilovic et al., 2016). The concave shape also fails to consider intra-seasonal irrigation constraints and stochastic climatic conditions within a given season. A non-concave crop water production function is more realistic because it can capture the variability due to weather, crop type, and soil type. For example, in the case of rainfed agricultural practices, the shape of the crop water production function may not necessarily be strictly concave, and the non-concave portion becomes a major portion of the production function. Similarly, the crop water production function in the case of water-intensive crops constitutes a major portion of the production function in the non-concave part because it requires a large amount of water before we start obtaining any yield. Thus, it becomes particularly important in studies that use a generic crop water production function in their methodology to analyze both strictly concave and non-concave crop water production functions.

Rad et al. (2020) developed a methodology for using a cumulative distribution function (CDF) to model the relationship between crop yield and irrigation water. The main objective of the research was to understand how groundwater availability affects agricultural production and irrigation decisions and to evaluate the effects of different aquifer management policies. Research showed that CWPFs could be represented using CDF, enabling a transition from an empirical simulation problem to a more economic modeling approach.

CDFs are inherently monotonically increasing functions, and when the underlying data possess a central tendency, they tend to display a similar pattern to a CWPF (Figure 1(a) and 1(b)). The CDF is advantageous as it can be normalized to be bounded between 0 (minimum crop yield) and 1 (maximum crop yield) and tends to be non-concave or concavo-convex. Its lower tail captures the fact that there is a minimum level of irrigation needed before harvestable crop yield can develop and its upper tail captures the plateau of crop yield beyond which the marginal revenue of additional water reaches zero (i.e., when the crop yield reaches its biophysical maximum). This implies that the marginal value of water is highest at some intermediate amounts of water applied and not for the first or last units of water applied. The model also considers the stochastic nature of seasonal and intra-seasonal irrigation decisions.

The shape of the CDFs can be changed by varying the values of the mean and standard deviation of the distribution. This property allows for the study of concave and non-concave CWPF. When we increase the value of mean, non-concave portion of the production function increases (Figure 3), and when we increase the standard deviation, the curve undergoes a transformation from a sigmoid shape to a more linear one. I considered solely the variation in the mean in this study because the goal is to see the difference in our objective functions for concave and non-concave CWPF, which is dictated by the mean of the distribution.

The basic construct that I build my model on is the CWPF, which can be represented by the following equation:

$$Y = f(X) \tag{1}$$

where Y is the crop yield and X is the input variable, which in our case is the irrigation water applied.

Following the work of Rad et al. (2020), I adopt a normal cumulative distribution function to represent the CWPF mathematically.

$$Y = \Phi(w; \mu, \sigma) \tag{2}$$

where Φ represents a normal cumulative distribution function (CDF), w is the irrigation water applied, μ is the mean, and σ is the standard deviation of the CDF.

Upon defining the CWPF, maximization problems are examined using the CWPF and constraints on water availability. These problems can be classified into two primary categories: yield maximization and profit maximization. These objective functions are selected because of their relevance in the fields of agronomy and economics. Maximizing crop yield is widely used by agronomists when assessing agricultural systems, and it is also essential for policymakers in the agricultural development sector who aim to enhance food security in a given region. In the realm of water resource economics, one of the standard and well established approach involves analyzing agricultural water use decisions under the assumption of risk neutrality and solving for profit maximization.

2.2 Yield Maximization:

The primary research question addressed a scenario with multiple homogeneous agricultural fields, where the ability to irrigate is subject to varying extents (none, some, or all fields), constrained by the total time allocated for irrigation. Each individual field has its own CWPF.

Here, the yield maximization problem is solved to maximize the yield across all irrigated fields. This maximization solution may not necessarily lead to yield maximization for all the individual fields. In this context, the maximizer represents an authoritative figure or policymaker with the goal of enhancing food security within a region in which the introduction of irrigation-as-a-service is aimed at assisting farmers who previously lacked access to irrigation resources.

In the yield maximization problem, I solve for a water allocation across all fields so the aggregate yield is maximized. The objective was to determine the optimal water allocation (w_i) for each field (i) to maximize the aggregate yield (Y). The total available water ($\overline{W}$) is used as a constraint, so the sum of water allocated to all fields cannot exceed $\overline{W}$. Function $\Phi(\mu, \sigma)$ follows a normal cumulative distribution function (CDF), which captures the characteristics of the crop-water production function.

For yield maximization, I formulate the problem as follows:

Objective: Maximize the total yield Y across all fields

$$\max_{\{w_1, w_2, \ldots\ldots, w_I\}} Y = \sum_{i=1}^{I} \Phi_i(w_i; \mu, \sigma) \tag{3}$$

Subject to:

$$\sum_{i=1}^{I} w_i \leq \overline{W} \tag{4}$$

$$w_i \geq 0\ \forall i \tag{5}$$

where $\overline{W}$ is the total available water and $w_i \geq 0$ for all i.

Here, Y is the total yield, $\Phi_i(w_i; \mu, \sigma)$ is the yield of field i as a function of water allocated to that field (w_i), and Φ_i is the normal CDF, which captures the characteristics of the crop-water production function and its shape is a function of the parameter mean (μ) and standard deviation (σ). The objective is to maximize the total yield across all fields, considering the constraint that the total allocated water ($\sum_{i=1}^{n} w_i$) should not exceed the total available water ($\overline{W}$).

Since, $\Phi_i(w_i; \mu, \sigma) = \Phi(w; \mu, \sigma)\ \forall_i$, I can simplify the objective function in Equation (3) as:

$$\max_{\{w\}} Y = I\Phi(w; \mu, \sigma) \tag{6}$$

Subject to:

$$Iw \leq \bar{W} \tag{7}$$

$$w \geq 0 \tag{8}$$

Using analytical and simulation approaches, I solve the yield maximization problem to determine the optimal water allocation across all fields, which maximizes the aggregate yield under limited water supply and mobile irrigation equipment constraints.

2.3 Profit maximization:

In the profit maximization problem, I solve for water allocation across all fields so that aggregate profit is maximized. In this case, the goal is to allocate the available water across multiple fields to maximize the total profit (Π) instead of the yield. The profit function (π_i) for each field depends on the amount of irrigation water (w_i) allocated. Similar to the yield maximization problem, the total available water ($\bar{W}$) is a constraint. However, in this case, the sum of the water allocated to all fields can be less than or equal to $\bar{W}$.

Objective: Maximize the total profit Π across all fields

$$\max_{\{w_1, w_2, \ldots\ldots, w_I\}} \Pi = \sum_{i=1}^{I} [\Phi_i(w_i; \mu, \sigma) - cw_i] \tag{9}$$

Subject to:

$$\sum_{i=1}^{I} w_i \leq \overline{W} \tag{10}$$

$$w_i \geq 0 \; \forall i \tag{11}$$

where $\overline{W}$ is the total available water and $w_i \; \geq \; 0$ for all i. Here, Π is the total profit, $\Phi(w_i; \mu, \sigma)$ is the output of field i as a function of the water allocated to that field (w_i), and the parameters μ and σ. The objective is to maximize the total profit across all fields, considering the constraint that the total allocated water ($\sum_{i=1}^{n} w_i$) should not exceed the total available water ($\overline{W}$). I have chosen crop yield to be a numéraire good, where c is the cost of water. Here, crop yield is considered the benchmark commodity against which the cost of water is measured. It is assumed that the fuel cost is linear and the lift is similar across all fields.

Since, $\Phi_i(w_i; \mu, \sigma) = \Phi(w; \mu, \sigma) \; \forall_i$, I can simplify the objective function in Equation (9) as:

$$\max_{\{w\}} \Pi = \sum_{i=1}^{I} I[\Phi(w; \mu, \sigma) - cw] \tag{12}$$

Subject to:

$$Iw \leq \overline{W} \tag{13}$$

$$w \geq 0 \;\; \forall_i \tag{14}$$

In the profit maximization problem, the optimal water allocation depends on the cost of water and output price**.** If $MC_w \; > \; VMP_w$, then an additional water application will not increase the profit.

$$\phi(w_i; \mu, \sigma) = VMP_{w_i} \tag{15}$$

The simulation modeling approach can be used to explore various irrigation decision-making scenarios for both yield and profit maximization, including the conditions under which both objectives are equal, one is greater than the other, or the conditions under which the preference switches between the two. By simulating different conditions and constraints, decision makers can gain insights into optimal water allocation strategies that balance yield and profit objectives.

3.0 Results and discussion

In this section, I discuss the key results. The complexity of the results depends on the curvature of CWPF. Here, I define two specific cases: Concave CWPF and Non-concave CWPF. For each case, I investigate both yield and profit maximization. Table 1 highlights the main results of irrigation decision making under IaaS.

3.1 Concave production function

These results refer to the case in which the CWPF is concave. Concave production functions can be used to represent geographical areas that are mostly rainfed but could benefit from supplemental irrigation during the critical period of crop growth. The concave nature of the CWPF enables me to solve the objective functions analytically and discuss the allocation of water across multiple fields in IaaS without the need to run simulations.

A concave CWPF indicates that the yield of a crop strictly increases at a decreasing rate as more water is applied. This means that each additional unit of water contributes less to the overall yield. For a concave function, the first derivative is always positive, whereas the second derivative is always negative. In a maximization problem where the objective function is strictly concave and constrained, first-order conditions (FOCs) are necessary and sufficient, and the maximum that is found is the global maximum.

3.1.1 Yield maximization

To solve the yield maximization problem, the objective function is set up subject to the constraints outlined in Equation 3. As the constraints are in the form of inequalities (Equations 4 and 5), the optimality conditions must be analyzed using the Kuhn-Tucker conditions.

By solving the Kuhn-Tucker conditions (as outlined in Appendix 1), it can be determined if all the first-order conditions (FOCs) hold. If a potential solution satisfies both the Kuhn-Tucker conditions and FOCs, it can be considered a valid solution.

By solving these conditions, I found that only one solution is possible in this case. The solution to the problem is unique and can be expressed as:

$$w^* = \frac{\overline{W}}{I} \tag{16}$$

where w^* is the optimal water allocation to each individual field in order to maximize the total yield across all fields. The solution indicates that all available water will be used. The shadow value of the constraint equals the Marginal Physical Product (MPP), and it can be observed that:

$$\phi\left(\frac{\overline{W}}{I}\right) = \lambda_1 > 0 \tag{17}$$

In this particular case, the constraint is binding and the value of ϕ represents the Marginal Physical Product (MPP) at the constraint, which is positive. This suggests that if more water was available, it would be beneficial to use it because the objective function would increase by an amount equal to the shadow value, λ_1, which represents the per field marginal yield gained by loosening the constraint by 1. Further details of the analysis are provided in Appendix 1.

Result 1: For a strictly concave CWPF, the yield maximization problem will always use all available water ($\bar{W}$) to maximize aggregate yield. This corresponds to the shadow value of water being always positive.

Result 2: All fields are irrigated and will receive the same amount of irrigation water. Result 2 follows because all the fields are identical. If they received different amounts of water, the MPPs would also differ, and a higher yield could be achieved by reallocation. Therefore, the optimal water allocation for each field can be represented by the following equation:

$$\frac{\bar{W}}{I} = w_i^* \forall_i \tag{18}$$

Maximizing the total yield across all fields results in equalizing marginal physical product (Figure 4, top left panel).

3.1.2 Comparison to a fixed irrigation system

In the case of a fixed irrigation system, the total water available can be applied to only one field. This is equivalent to the problem above, where $I = 1$. In a concave CWPF, MPP is positive which implies that this field will receive all the water available ($\bar{W}$) to maximize yield. Thus, in both the fixed and mobile irrigation system scenarios, all the available water will be used. This means that the amount of water used is the same in both cases. Therefore, the asset utilization rate is the same because the irrigation system will be operational to its maximum capacity.

Result 3: The total yield will be higher in IaaS as compared to a fixed system. This is an application of Jensen's inequality.

$$\Phi\left(\frac{\bar{W}}{I}\right) > \frac{\Phi(\bar{W})}{I} \tag{19}$$

Intuitively, this implies that for concave production functions, the total crop yield obtained by applying $1/I$ of water to each of the I fields is greater than the crop yield obtained by applying all the water to one field.

3.1.3 Profit maximization

Next, I analyzed the profit maximization solution. The profit maximization problem is solved analytically by setting up an objective function subject to the constraints (Equation 6).

I consider the yield to be a numéraire good to simplify the economic analysis by providing a reference point for the measurement of input and output prices. In this case, the objective function is subject to inequality constraints (Equations 7 and 8). In order to solve the optimality conditions the Kuhn-Tucker conditions need to be analyzed.

On solving the Kuhn-Tucker conditions (Appendix 2), this problem has three possible solutions that depend on the value of c or the cost of water. These three conditions hold and suffice for the FOCs.

- Case 1: In the first case, $c \leq \phi\left(\frac{\bar{W}}{I}\right)$: if water price is low enough we are going to use all the water, and we get a corner solution, which means that $VMP\left(\frac{\bar{W}}{I}\right) > MC$. All the fields will receive equal amounts of water because they are identical, and equalization of MPP will apply.
- Case 2: $\phi\left(\frac{\bar{W}}{I}\right) < c < \phi(0)$: if c is large enough then this case applies. $VMP(0) \ < \ MC$, this case binds, but is not an interesting and profitable case. Thus, it is optimal not to apply any water.
- Case 3: In the last case, $c \geq \phi(0)$ if the cost of water is high, we have an interior solution and both inequalities will hold. $\phi(w^*) = c$, which means $VMP = MC$. Because of strict concavity, this point is a global maximum.

Result 4 (Profit maximization): For a concave CWPF, all available water may or may not be used.

Result 5: When the cost of water is high enough, there is an interior solution where both inequality constraints will hold. If any field is irrigated, the irrigation amount is at a point where $VMP_i = MC \ \forall_i$.

Corollary: The decision is to irrigate all fields or none of them. This is because of the concave nature of CWPF and the assumption that all fields are identical.

3.1.4 Comparison to a fixed irrigation system

Result 6: The asset utilization rate may remain the same or increase in the case of a mobile irrigation system. However, profits always increase in the case of mobile irrigation systems relative to fixed irrigation systems. This is because in case of mobile irrigation systems as soon as we reach a point where $VMP_i = MC \; \forall_i$, we will choose to move to the next field and keep applying water until we reach a point where $VMP_i = MC \; \forall_i$. This is not the case for fixed irrigation systems.

The asset utilization rate remains the same in the case of a fixed irrigation system and mobile irrigation system when we are in a corner solution. This is because if the irrigation systems would be used all the time so that all the water is used. However, if we are in an interior solution, the asset utilization rate will increase for mobile irrigation systems. This is because as we reach a point where $VMP_i = MC \; \forall_i$, we will choose to irrigate the next field in case of mobile irrigation equipment. Thus, the average yield and profits are also higher in the mobile irrigation system than in the fixed system.

3.1.5 Comparison between yield and profit maximization

For non-zero water prices, the profit-maximizing and yield-maximizing irrigation rates differ owing to the concave nature of the crop water production function. As water prices rise (c>0), it becomes economically optimal for farmers to use deficit irrigation by applying water below full crop requirements to maximize economic productivity (Foster et al., 2015).

In this section, I identify the key differences between yield maximizing and profit maximizing solutions. For yield maximization, we apply at least as much water as for profit maximization. When water is costless, both yield and profit maximizing results converge, and the profit maximizing water allocation for individual fields becomes the same as the yield maximizing result.

Yield maximization should always have weakly more output than profit maximization because we will always use all available water in the case of yield maximization, but this is not always true for profit maximization.

In terms of asset utilization rate, yield maximization leads to higher asset utilization than profit maximization. However, this may result in lower average returns per acre on irrigation technology in terms of yield maximization compared to profit maximization. This is because yield maximization does not account for the irrigation costs.

3.2 Non-concave production function

The previous section considered a strictly concave CWPF in which optimal solutions are analytically defined, and global maxima are clearly defined. However, an idealized concave shape may not always apply, such as in drought years requiring a minimum level of irrigation to avoid crop failure (Shani et al., 2004; Foster et al., 2015), or when limited seasonal irrigation is scheduled suboptimally due to water availability or information constraints (Foster et al., 2014; Smilovic et al., 2016).

A non-concave Crop Water Production Function (CWPF) represents a more complex relationship between the crop yield and the amount of irrigation water applied (refer to Figure 1). In the case of non-concave CWPF, FOCs are not sufficient, and if we find a maximum, it is not guaranteed that it is the global maxima and SOCs only guarantee local maxima, and not global maxima.

Optimizing water allocation with a non-concave CWPF generally requires simulation modeling. Simulations allowed me to explore various water allocation strategies and determine the best solution for maximizing yield or profit.

3.2.1 Yield maximization

I used simulation to find the optimal solution for yield maximizing problem for non-concave CWPFs. I use MATLAB Version: 9.13.0 (R2022b) for running the simulations. The simulation results are presented as follows.

Result 1: All water available will be used because the MPP is always positive (Figure 4).

Corollary: All irrigated fields will have the same water applied, that is, equalizing the marginal physical product. However, not all fields must be irrigated at an optimum.

Result 2: Under non-concave CWPFs, one or more fields may be irrigated (Figure 4). The non-concavity of the function implies that applying a large amount of initial water to the field may not result in a significant increase in yield. Thus, when deciding how to allocate additional water, it is important to compare the yield increase from applying more water to the current field with the potential yield increase from applying water to the next field.

Corollary 1: As you loosen the constraint and increase the available water, it may or may not increase the number of irrigated fields. However, this always leads to an increase in total yield.

Corollary 2: The decision to apply marginal water to previously irrigated fields or to introduce a new field hinges on comparing the marginal physical product of water for the existing number of fields n^{th} and the scenario with an additional $(n+1)^{th}$ field (Figure 5).

Corollary 3: For an individual field, an additional unit of available water may either increase or decrease the total water applied to previously irrigated fields (Figure 6). The optimal choice is to apply this additional unit of water to the field with the largest increase in yield due to the additional water (i.e., the field that yields the largest marginal physical product of water).

3.2.2 Comparison to a fixed irrigation system

My results indicate that in both fixed and mobile systems, we use all available water. In the case of a fixed system, we use all the water in that field, but in the case of a mobile system, we may irrigate only one field or more fields.

The decision to irrigate one field or more fields depends on the shape of the CWPF. If the initial crop response to water is low, such that we have to apply a large amount of water to obtain any yield response, we are better off irrigating only one field. If the initial response to water is high, it is preferable to irrigate more than one field. However, the decision to add a new field is determined by comparing the MPP of adding marginal water to the existing field and the MPP of adding that water to the new field. Figure 4 provides a good representation of irrigation decision-making in the case of three fields. Here, μ determines the shape of the CWPF and low μ means that initial crop response to irrigation water applied is low. As the value of μ increases, the decision is to allocate water to all three fields instead of applying all the water to one field. This is because even for small amounts of irrigation water, the crop response is high so the decision is to irrigate maximum number of fields for high values of μ.

3.2.3 Profit maximization

In this section, I discuss the profit-maximizing results.

Result 1: Considering the cost of irrigation, it may be optimal to not utilize all available water, even if it means that some or all fields receive no irrigation (Figure 7). The simulation results indicate that in non-concave solutions, there are situations where some of the fields receive no water. The fields that are irrigated are either at an interior solution or a corner solution for the subset of fields that are irrigated, but there may be other fields that receive zero irrigation.

Corollary 1: The choice to use marginal water on previously irrigated fields or to add a new field depends on comparing the marginal value product of water for the n^{th} and the scenario with an additional $(n+1)^{th}$ field because of non-concavity.

Corollary 2: In any field that is currently irrigated, and we increase available water, the amount of water might remain the same, go up, or go down. The asset utilization rate may remain the same or go up. Profits will stay the same or go up.

3.2.4 Comparison to a fixed irrigation system

My results show that we may or may not use all available water in the case of both a fixed system and under IaaS.

The profit under IaaS may remain the same or increase compared to a fixed irrigation system. This depends on the shape of the CWPF and compares the MVP of irrigating one field and adding a new field.

The asset utilization rate may remain the same or increase in the case of IaaS compared to a fixed system.

3.2.5 Comparison between yield and profit maximization

All water is used for yield maximization, but all water may or may not be used for profit maximization.

The asset utilization rate is higher in yield maximization, but the returns per irrigation water applied are higher in profit maximization.

The number of fields irrigated under yield and profit maximization may or may not be the same.

3.3 Discussion

If we are in a situation with supplemental irrigation where water is cheap and we have multiple fields that could benefit from getting small amounts of irrigation water, it is better to adopt IaaS and apply a small amount of water to all fields. This can be the case for solar irrigation, where pumping water is inexpensive.

If we are in a situation with supplemental irrigation where we have to pay the fuel cost, as in the case of electric pumps or diesel pumps, then we are in an interior solution. In this case, IaaS will work, and we will get higher average yield and higher average profits as compared to using that pump only in one field. The asset utilization rate also will increase in the case of IaaS compared to owning that pump.

If we are in a very non-concave setting, if we make more water available, you will not irrigate any more fields. In this situation, use of the technology is required almost all the time. Even in one field, we are capacity constrained, and there may not be many benefits from IaaS. In this case, IaaS does not assure higher yield or profits, and the asset utilization rate also will be the same as that of a fixed system. In this case, the IaaS does not seem to work, and it is better to buy a fixed irrigation system and use it only in that field. Geographical areas that are dry (sandy soil or receive very little rainfall) are typical examples of non-concave CWPFs.

4.0 Policy implications

In this section, I present the policy implications of my analysis.

4.1 Cost-effectiveness of donor funds

Development banks and multilateral agencies seek to improve food security and increase incomes in rural areas, with small-scale irrigation (SSI) emerging as a promising approach. The issue is that optimal strategies that maximize yield, maximize profit, and maximize asset utilization rate may be quite different but it is often assumed that these objectives are interchangeable. My findings indicate that, for the cases considered, we need to carefully define objectives as they lead to different irrigation strategies across the landscape, and presumably different policies to implement. Therefore, it is essential for multilateral agencies and donors to prioritize one of these objectives when establishing programs for smallholder farmers.

My results show that there is a clear distinction between yield maximization and profit maximization results in the case of both concave and non-concave production functions. The asset utilization rate also differs with the choice of objective function. Thus, we need to be careful when we talk about increasing yields, increasing profits, increasing social welfare and increasing asset utilization rate for smallholders especially in the case of irrigation-as-a-service.

For example, the government or multilateral agencies may establish a pilot project in a sub-humid region with a strictly concave production function. This region receives erratic rainfall during the growing season. In such areas, they deploy mobile solar irrigation systems due to their low cost of pumping water. The objective of these projects is twofold: to increase irrigation access for smallholder farmers and to enhance the asset utilization rate of the deployed pumps. The optimal and cost-effective strategy in this case is to fully use the pumps such that we achieve aggregate profit maximization across the landscape.

Another example could be a sub-humid region with mobile diesel pumps or electric pumps where pumping water is costly as compared to solar pumps. In this case, if the government aims to increase irrigation access to smallholders by promoting irrigation service providers and to use the funds cost-effectively, the optimal strategy would be to irrigate the maximum number of fields until we reach a point where marginal revenue equals marginal cost. Each individual field will be irrigated such that marginal physical product is equalized.

However, if the multilateral agencies donate funds to increase smallholder irrigation in arid regions (non-concave production function), our strategies differ from the above cases. The optimal strategy in this case would depend on the level of aridity or non-concavity of the production functions and water availability. In the case of arid regions, our results show that it may be reasonable to use the available water to irrigate one or a few fields intensively rather than irrigating many fields equally. There are potential cases where the production function is highly non-concave and IaaS may not be the solution for those type of landscapes. It might be a better policy strategy to subsidize pumps for individual pump ownership for these geographies. In geographies where we have arid climatic conditions, we need to be careful in assessing the level of non-concavity in production functions.

Small Scale Irrigation (SSI) also typically involves subsidizing individual farmers' purchase of pumps to irrigate their crops with the goal of increasing irrigation access; however, concerns exist about the cost-effectiveness of these donor funds allocated to SSI programs.

In light of these concerns, it is important to explore alternative approaches that could lead to better outcomes. Since smallholders do not own the assets in IaaS, and these assets tend to be used more frequently than under individually-owned systems, policymakers need to assess whether IaaS can offer a more cost-effective solution in certain contexts. Addressing this question, which has not yet been thoroughly explored in the literature, may lead to more efficient allocation of donor funds and improved outcomes in terms of food security and income generation in rural areas.

My research suggests that IaaS might be a more cost-effective way to support SSI than traditional subsidies. Investing funds in IaaS can potentially encourage better accessibility to irrigation technologies and increase the efficiency of water use.

For instance, my research indicates that in geographical areas that are rainfed, such as Sub-Saharan Africa, providing temporary supplemental irrigation to all farmers in the landscape might help achieve higher returns on funds than providing subsidies to a small number of farmers to install or purchase their own pumps.

For geographical areas where irrigation is well developed, such as South Asia where most farmers own their pumps, my findings suggest that providing subsidies to increase pump ownership may lead to underutilization of irrigation equipment. Instead of incentivizing farmers to purchase subsidized irrigation equipment, sharing irrigation equipment in this type of landscape could be a more cost-effective approach.

We can set performance metrics and standards for IaaS providers based on near real-time data to monitor and evaluate their performance regularly to ensure that funding is allocated to providers who deliver results to farmers. Furthermore, IaaS providers can invest in more advanced and efficient irrigation systems, leading to higher returns for farmers and reducing the environmental impacts of irrigation. Overall, IaaS has the potential to achieve a greater impact on donor funds in terms of food security and poverty alleviation, while also promoting sustainable water use.

4.2 Support entrepreneurs providing irrigation-as-a-service

Currently, policies do not encourage IaaS entrepreneurship. For example, renting or lending irrigation pumps is explicitly prohibited in Rwanda. This may be due to the perception that informal trading networks are undesirable, and because IaaS is often associated with such networks, it has been overlooked and even actively discouraged by governments. In the literature, informal trading is frequently portrayed negatively, leading to smallholder irrigation policies that disallow and discourage pump rental markets. Subsidies are typically not granted to pumps intended for rentals or lending.

My analysis indicates that a well-managed IaaS system can generate higher profits per pump than a system that does not allow mobility or rentals. In this case, both entrepreneurs and smallholders could be better off in an IaaS system than if smallholders owned their own pumps. Governments should consider encouraging water entrepreneurship and providing oversight or governance to ensure that smallholders benefit.

Although lending and renting are disallowed, field evidence suggests that these practices occur regardless, indicating that enforcement in smallholder settings is difficult. This implies that government subsidy schemes may be generating more benefits than anticipated due to the existence of untracked and unregulated informal markets. From an analytical perspective, third-party evaluators should be open to understanding how these rental markets function because some government schemes may be generating significantly more benefits than expected. By overlooking informal markets, governments may underestimate the value created in a landscape through existing schemes.

4.3 Scaling beyond smallholders

To date, the focus has been on IaaS as a policy tool for smallholder farmers. However, it is also important to explore their applicability to medium and large farms.

My model was developed with the intent of addressing smallholder irrigation, but its framework is general and can be adapted to various contexts. The primary features of the model include mobile irrigation technology capable of irrigating more than one field per day without limiting the technology or field size. Hose reel systems, often tractor-towed, are well suited for larger fields, highlighting the model's adaptability to broader settings.

The analysis remains applicable to larger scenarios, as long as irrigation technology can handle multiple fields within a day. For example, a system that irrigates 50-60 hectares daily, covering multiple 10–20-hectare plots, aligns with our model and framework. Companies catering to these markets employ mobile irrigation technologies such as hose reel systems, which can serve larger fields effectively. Entrepreneurs such as AgriRain are already operating on this scale. This further validates the model and approach. This demonstrates that IaaS, along with its associated irrigation technology, can meet the needs of various farm sizes, provided that it can efficiently irrigate multiple areas in a single day.

5.0 Limitations and Future work

The current analysis employed several simplifications to facilitate modeling and enhance the manageability of the model. It presumes all fields to be identical with the same Crop Water Production Functions (CWPFs), which may not hold in real-world scenarios. This research also does not take into account any optimization related to stocks or flows of water resources, nor does it take any bio-physical constraints on water resources into the optimization. This would entail more advanced computational techniques and expertise in modeling these complex scenarios. Nevertheless, the findings from this analysis can be extended to cases where fields differ, and various crop types are cultivated.

Furthermore, the study assumes that the time required to move irrigation equipment between farms is negligible, which is a strong assumption. The time spent on equipment relocation incurs a significant opportunity cost and can substantially influence the results. Future research could delve into the impact of introducing time constraints on the existing model, elucidating how these would affect the combinations that maximize yield and profit.

Another possible policy implication is that Irrigation-as-a-Service (IaaS) has the potential to mitigate groundwater abstraction, a critical issue in informal groundwater markets (IGMs). In the current analysis, there was a constraint on the total available water. By replacing this constraint with permissible water use limits, we can explore trade-offs and scenarios that simultaneously generate profits and mitigate water abstraction in a given landscape.

The present study developed a theoretical model to solve the objective functions of yield and profit maximization in an IaaS setting, where the irrigation equipment is mobile and can be shared between multiple fields with different owners. To validate and refine these findings, data and empirical analysis are necessary.

6.0 Conclusions

In summary, the potential of IaaS has been overlooked by researchers and policymakers in the context of small-scale and farmer-led irrigation. However, my research demonstrates that when irrigation equipment is mobile, IaaS may serve as a profitable, cost-effective, and scalable approach to enhance food security. The modality of IaaS varies significantly based on factors such as the responsiveness of the Crop Water Production Function (CWPF) to applied irrigation water and the proximity of fields requiring irrigation.

My results suggest that, to consider providing irrigation to multiple nearby fields, analyzing the CWPF and understanding the value of marginal irrigation are crucial for determining the optimal irrigation strategy for each field to maximize both yield and profit. Interestingly, the asset utilization rate of irrigation equipment often increases in the case of IaaS compared with installing a permanent irrigation system. Counterintuitively, the results show that the asset utilization rate in IaaS can be lower relative to a fixed system, even when the overall profitability of the landscape is increasing.

In situations where the incremental application of water results in increased crop yield, the marginal increase in profit is inadequate to offset the associated pumping costs.

In this case, we are better off not using the irrigation equipment, thereby decreasing the asset utilization rate.

My results have policy implications for the cost-effectiveness of donor funds. In particular, the results show that IaaS can increase water-use efficiency, and provide higher returns on invested funds and higher asset utilization rates in some cases.

Furthermore, my findings offer insights into how governments may approach subsidy policies for smallholder farmers and entrepreneurs. My results indicate that if the government incentivizes businesses providing IaaS, it might potentially generate more return per pump compared to a fixed system.

The economic model I developed is generic and considers the crop water production function and the constrained available water. The results can be applied directly to different situations in which irrigation water needs to be allocated across multiple fields. The analysis from this model can also be applied to medium and large farms, suggesting its scalability across different irrigation landscapes.

This research considers the time required to move between fields to be negligible relative to the operating time. In future research, a compelling question to explore involves considering the travel time necessary for relocating equipment between fields, as this would offer a more accurate representation of real-world scenarios.

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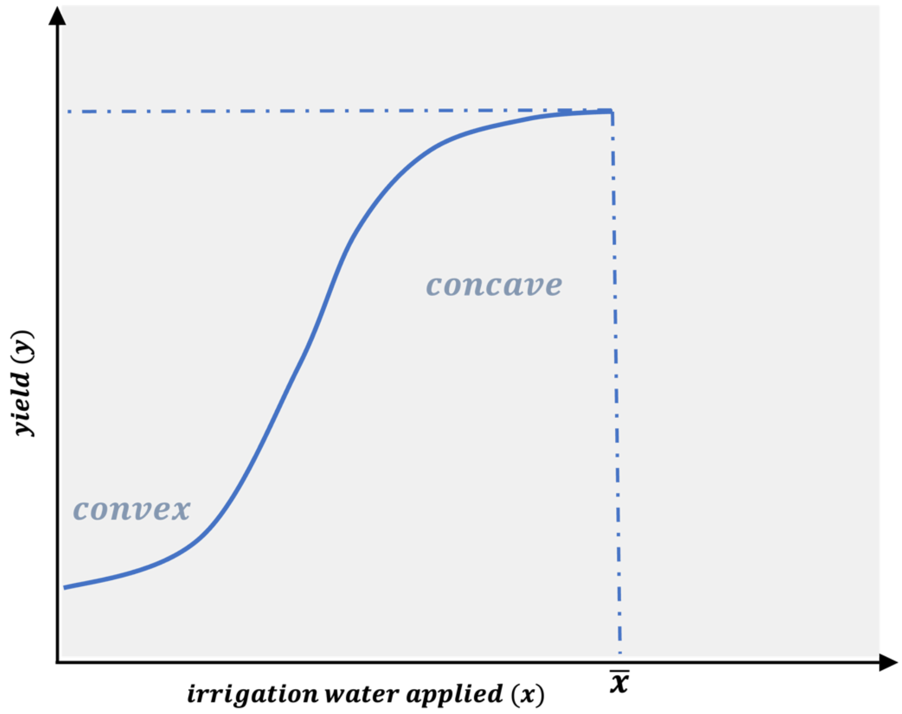
Table

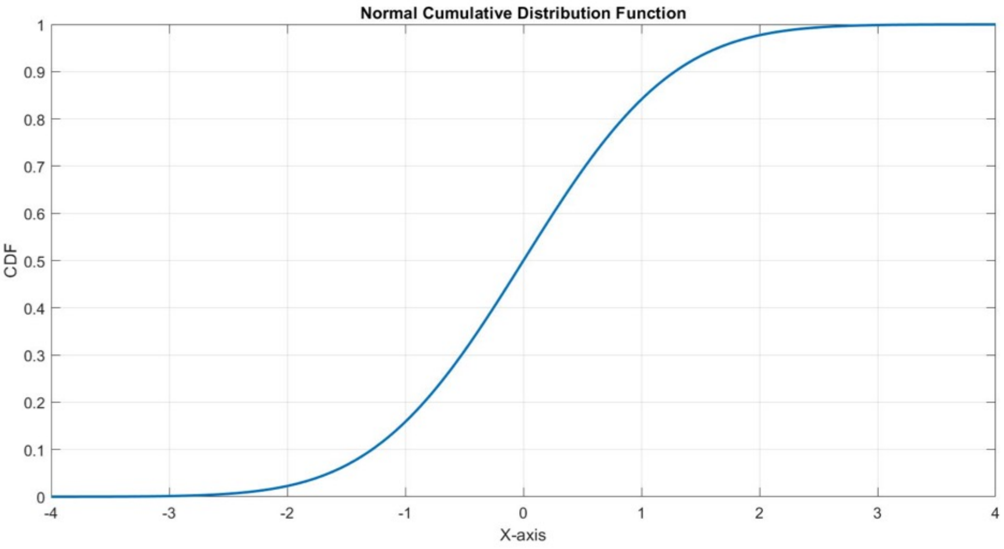
	Yield Maximization		**Profit Maximization**	
	Concave	**Non-Concave**	**Concave**	**Non-Concave**
Irrigate all fields	YES	MAYBE	MAYBE (None or all)	MAYBE
Use all water	YES	YES	MAYBE	MAYBE

Table 1. Table showing irrigation decision making under IaaS for the case of concave and non-concave CWPFs.

Figures



(a)



(b)

Figure 1: (a) Generic crop water production function (CWPF) showing concave and convex portion of a typical CWPF (b) Normal cumulative distribution function (CDF). These two diagrams show the similarity in curvature of a typical CWPF and a normal CDF.

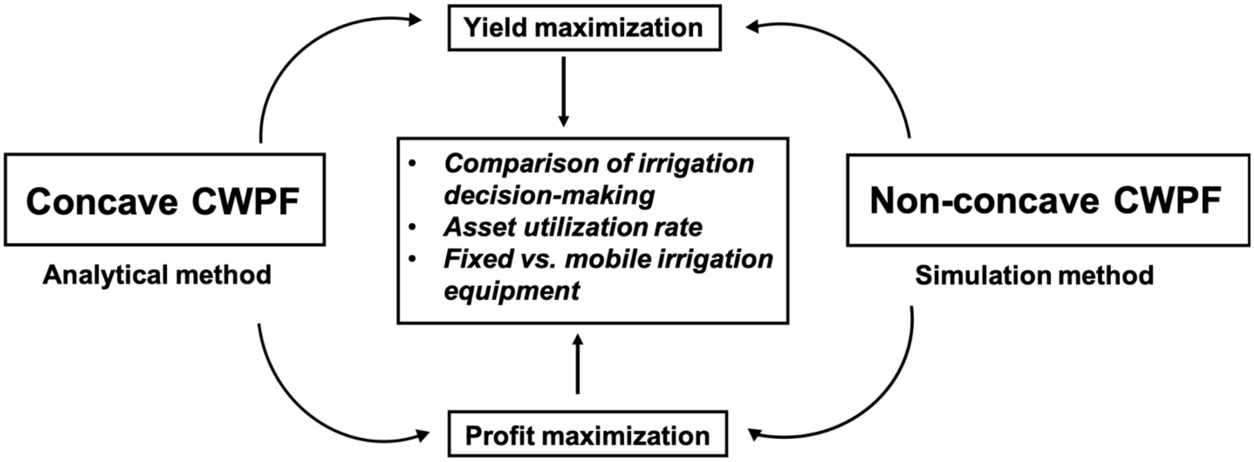


Figure 2: Flowchart showing the different components described in the methodology section and analyses performed for concave and non-concave CWPFs.

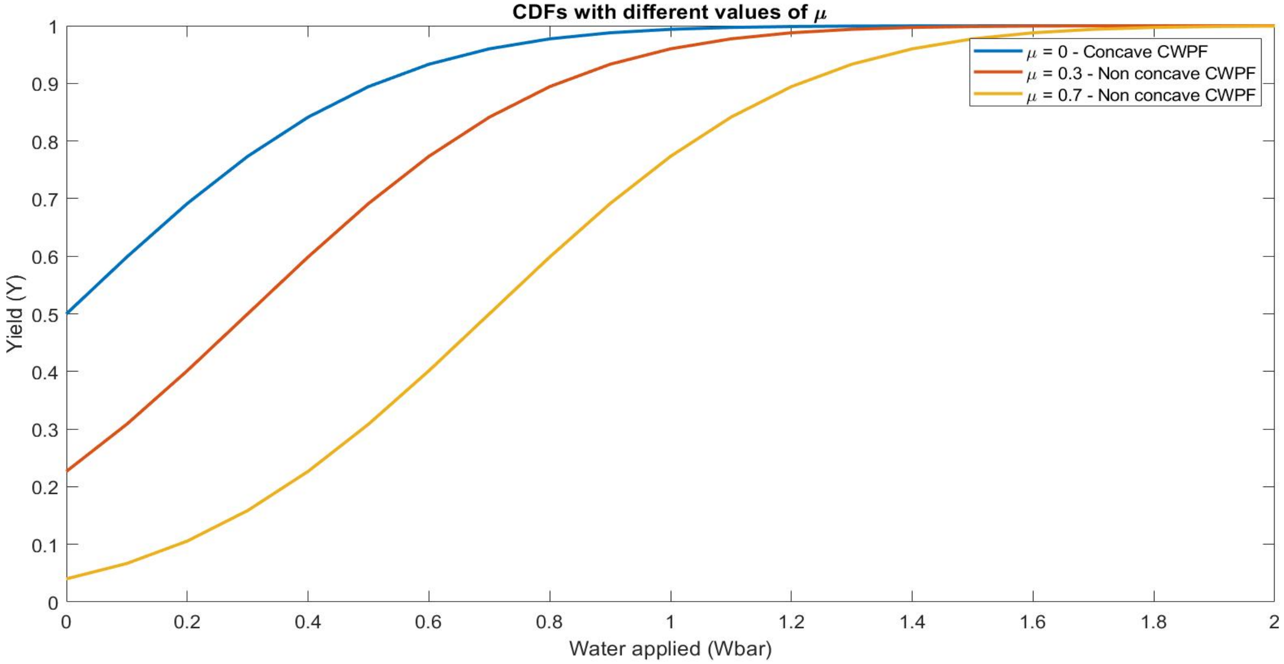


Figure 3: Plot showing the CWPFs for three different values of the mean of normal cumulative distribution functions. This figure illustrates the difference between a strictly concave and non-concave CWPFs. This difference in the initial curvature of CWPF has a major role in driving the irrigation decision-making under irrigation-as-a-service.

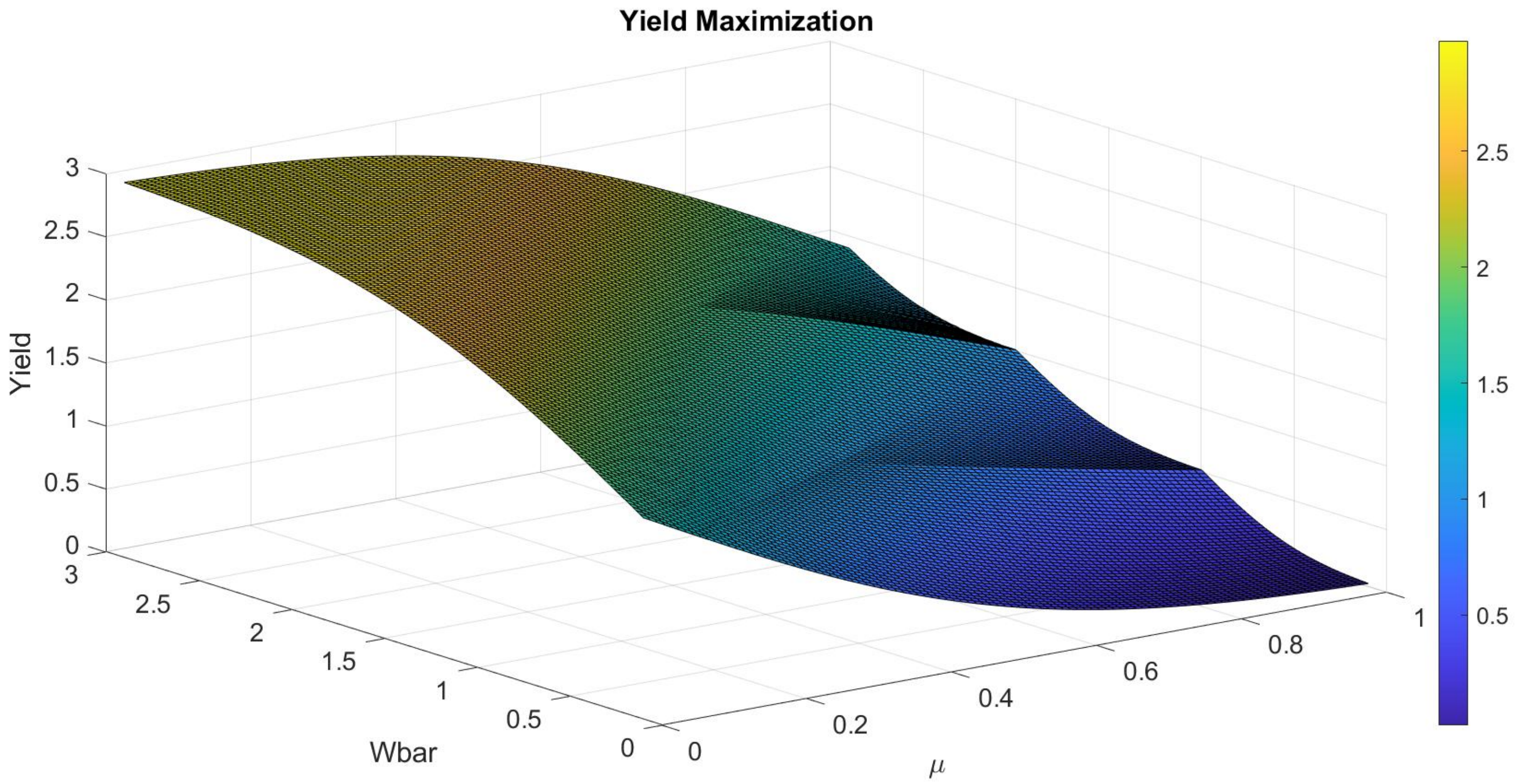


Figure 4: Surface showing yield maximizing combinations in the case of a three field scenario. It shows the interaction between $\overline{W}$ (total water available), μ (mean of the CWPF that depicts the concavity of the CWPF) and Y (crop yield). The surface of the plot is not smooth and the contours on the plot show the points where we are indifferent in our decision to keep irrigating the current field as the water availability increases or to add a new field.

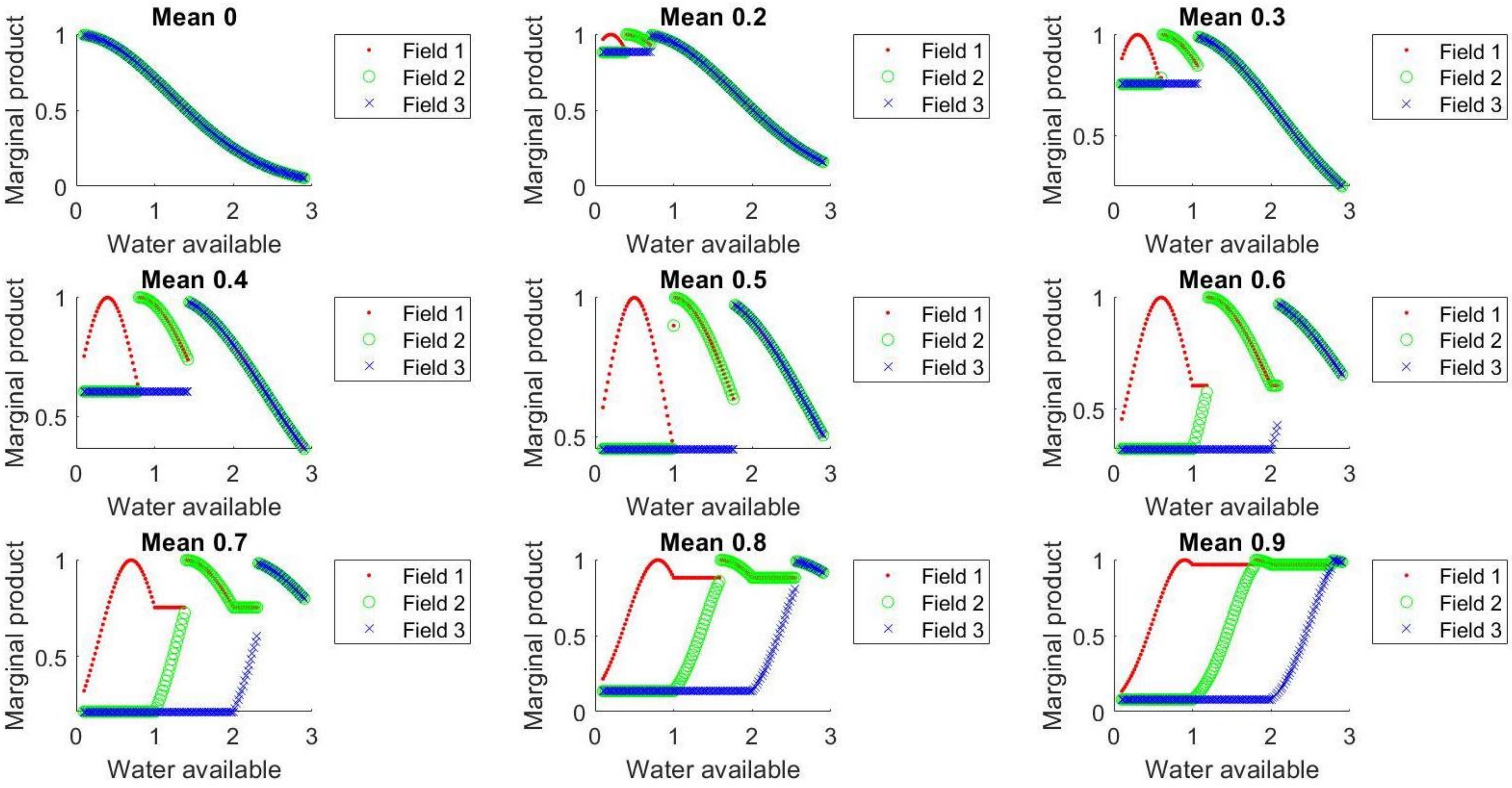


Figure 5: Plots showing marginal product of crop yield for each additional unit of water on individual fields due to the curvature of the CWPF (via different values of mean of the CDF) for the case of three identical fields. The marginal product plot at $\mu = 0$ is the derivative of a strictly concave CWPF and as the value of μ increases the non-concavity of the CWPF increases. Each subplot depicts the variation in the marginal product of Field 1, Field 2, and Field 3 with the availability of an additional unit of water.

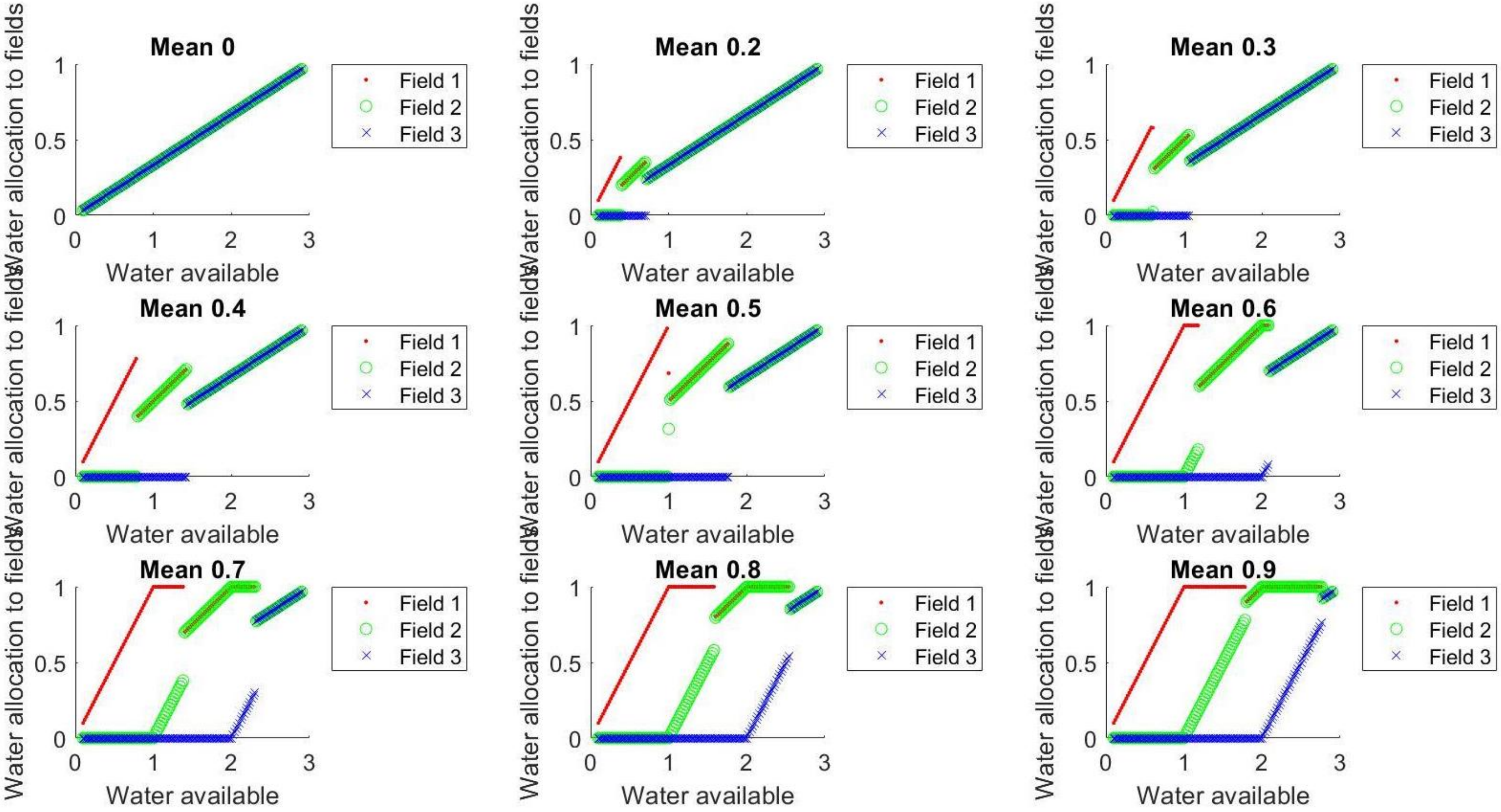


Figure 6: Plots showing how water allocations to individual fields change due to the shape of the CWPF (different values of mean) for the case of three identical fields. The plot at μ = 0 shows that the available water will be allocated equally between the three fields in case of a strictly concave CWPF. Each subplot depicts the variation in water allocations to Field 1, Field 2, and Field 3 with the availability of an additional unit of water.

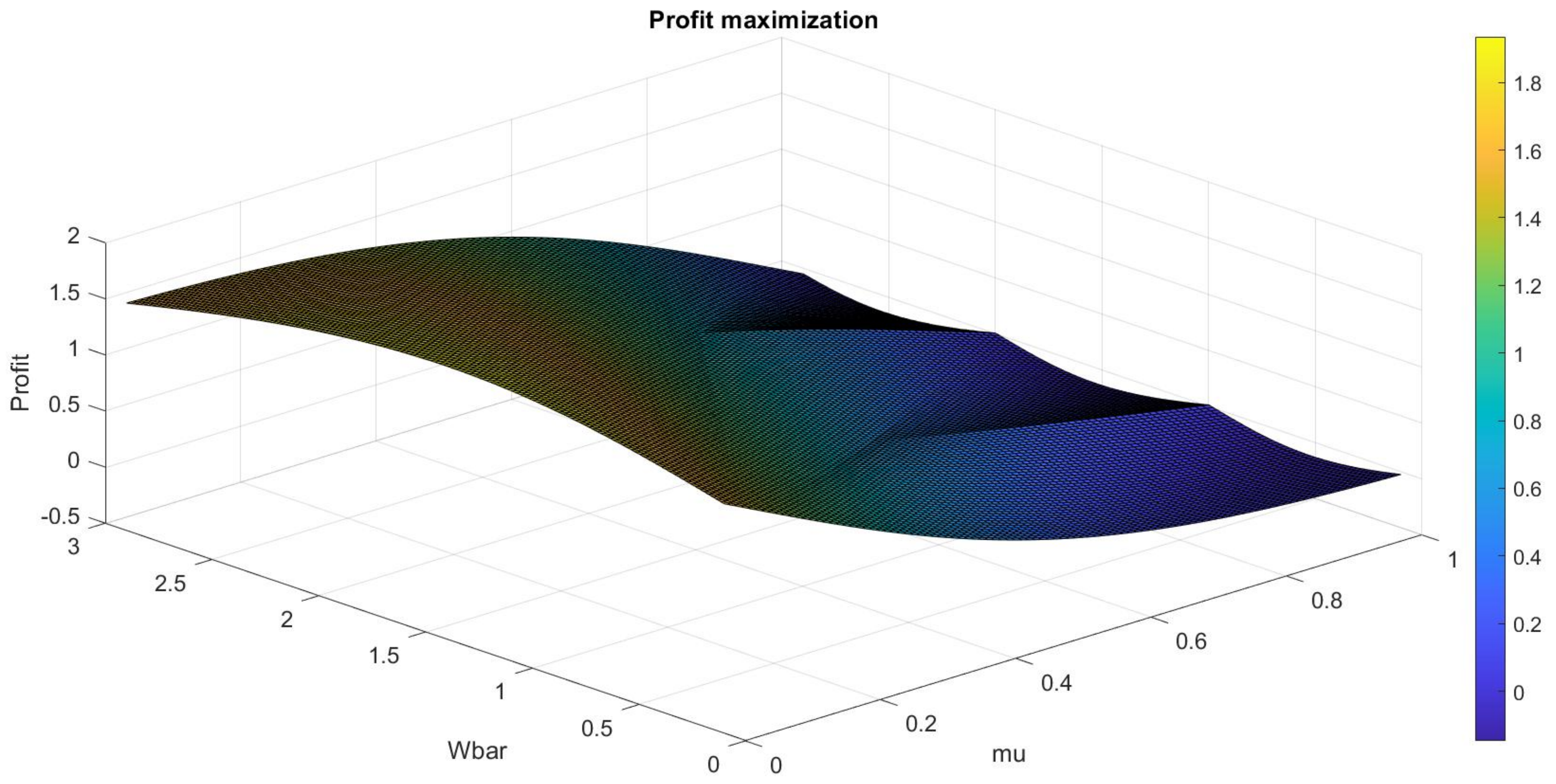


Figure 7: Surface showing profits from optimal irrigation-as-a-service solutions when yield is considered to be a numéraire good and the cost of water is set to 0.5 for the case of a three field scenario. It shows the interaction between $\overline{W}$ (total water available), μ (mean of the CWPF that depicts the concavity of the CWPF) and Π (Profit).

Appendix:

A.1 Yield maximization solution including Kuhn-Tucker conditions

Objective function:

$$\max_{\{w_1, w_2, \ldots\ldots, w_I\}} Y = \sum_{i=1}^{I} \Phi_i(w_i; \mu, \sigma) \tag{3}$$

Subject to:

$$\sum_{i=1}^{I} w_i \leq \bar{W} \tag{4}$$

$$w_i \geq 0 \ \forall_i \tag{5}$$

Since $\Phi_i(w_i; \mu, \sigma) = \Phi_i(w_i; \mu, \sigma) \ \forall_i$, we can simplify the above equations.

$$\max_{\{w\}} Y = I\Phi(w; \mu, \sigma) \tag{i}$$

Subject to:

$$Iw \leq \bar{W} \tag{ii}$$

$$w \geq 0 \tag{iii}$$

Introducing slack variables to convert the inequality constraints in Equations (ii) and (iii) into equalities,

$$\bar{W} - Iw - a^2 = 0 \tag{iv}$$

$$w - b^2 = 0 \tag{v}$$

Any solution that obeys these two equality constraints will also obey the inequality constraints.

I set up the Lagrangian and introduce the Langrange multiplier to solve the First-order conditions (FOCs),

$$\mathcal{L} = I\Phi(w;\mu,\sigma) + \lambda_1[\bar{W} - Iw - a^2] + \lambda_2\, w - b^2 \tag{vi}$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial w} = I\phi - I\lambda_1 + \lambda_2 = 0 \tag{vii}$$

$$\frac{\partial \mathcal{L}}{\partial a} = -2a\lambda_1 = 0 \tag{$viii$}$$

$$\frac{\partial \mathcal{L}}{\partial b} = -2b\lambda_2 = 0 \tag{ix}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \bar{W} - Iw - a^2 = 0 \tag{x}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = w - b^2 = 0 \tag{xi}$$

Kuhn-Tucker conditions:

1. $\lambda_1 = 0,\ \lambda_2 = 0$

   It means that $0 < w^* < \frac{\bar{W}}{I}$ and plugging the value of w^* in Equation (vii) will give us $\frac{\partial \mathcal{L}}{\partial w} = I\phi = 0$ which is mathematically not possible so we reject this solution.

2. $\lambda_1 > 0,\ \lambda_2 = 0$

   It means that $w^* = \frac{\bar{W}}{I}$ (from Equation (x) a will be 0 when $\lambda_1 > 0$) and plugging the value of w^* to Equation (vii) will give us $\frac{\partial \mathcal{L}}{\partial w} = I\phi\left(\frac{\bar{W}}{I}\right) - I\lambda_1 = 0 \Rightarrow \phi\left(\frac{\bar{W}}{I}\right) = \lambda_1 > 0$. **This is a valid solution.**

3. $\lambda_1 = 0,\ \lambda_2 > 0$

It means that $w^* = 0$ (from Equation (xi)) and plugging the value of w^* into Equation (vii) gives us $I\phi(0) + \lambda_2 = 0$. This is not a mathematically possible result so we reject this solution.

4. $\lambda_1 > 0,\ \lambda_2 > 0$

   It means that $w^* = \frac{\bar{W}}{I}$ from Equation (x) and $w^* = 0$ from Equation (xi). Two values of w^* is not possible so we reject this solution.

A.2 Profit maximization solution including Kuhn-Tucker conditions

Objective function:

$$\max_{\{w_1,w_2,\ldots\ldots,w_I\}} \Pi = \sum_{i=1}^{I} [\Phi_i(w_i;\mu,\sigma) - cw_i] \tag{9}$$

Subject to:

$$\sum_{i=1}^{I} w_i \leq \bar{W} \tag{10}$$

$$w_i \geq 0\ \forall_i \tag{11}$$

Since $[\Phi_i(w_i;\mu,\sigma) - cw_i] = [\Phi_i(w_i;\mu,\sigma)\ - cw_i]\ \forall_i$, we can simplify the above equations.

$$\max_{\{w\}} \Pi = I[\Phi(w;\mu,\sigma) - cw] \tag{i}$$

Subject to:

$$Iw \leq \bar{W} \tag{ii}$$

$$w \geq 0 \tag{iii}$$

Introducing slack variables to convert the inequality constraints in Equations (ii) and (iii) into equalities,

$$\bar{W} - Iw - a^2 = 0 \tag{iv}$$

$$w - b^2 = 0 \tag{v}$$

Any solution that obeys these two equality constraints will also obey the inequality constraints.

I set up the Lagrangian and introduce the Langrange multiplier to solve the First-order conditions (FOCs),

$$\mathcal{L} = I[\Phi(w;\mu,\sigma) - cw] + \lambda_1[\bar{W} - Iw - a^2] + \lambda_2\, w - b^2 \qquad (vi)$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial w} = I\phi - Ic - I\lambda_1 + \lambda_2 = 0 \qquad (vii)$$

$$\frac{\partial \mathcal{L}}{\partial a} = -2a\lambda_1 = 0 \qquad (viii)$$

$$\frac{\partial \mathcal{L}}{\partial b} = -2b\lambda_2 = 0 \qquad (ix)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \bar{W} - Iw - a^2 = 0 \qquad (x)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = w - b^2 = 0 \qquad (xi)$$

Kuhn-Tucker conditions:

1. $\lambda_1 = 0,\ \lambda_2 = 0$

   It means that $0 < w^* < \frac{\bar{W}}{I}$ and plugging the value of w^* in Equation (vii) will give us $\frac{\partial \mathcal{L}}{\partial w} = I\phi(w^*) - Ic = 0 \Longrightarrow \phi(w^*) = c$. This further means that $VMP = MC$. **This is the interior solution to the problem and is a valid solution.**

2. $\lambda_1 > 0,\ \lambda_2 = 0$

   It means that $w^* = \frac{\bar{W}}{I}$ (from Equation (x) a will be 0 when $\lambda_1 > 0$) and plugging the value of w^* to Equation (vii) will give us $\frac{\partial \mathcal{L}}{\partial w} = I\phi\left(\frac{\bar{W}}{I}\right) - Ic = I\lambda_1 \Longrightarrow \phi\left(\frac{\bar{W}}{I}\right) - c = \lambda_1 > 0 \Longrightarrow \phi\left(\frac{\bar{W}}{I}\right) > c$. This further means that $VMP\left(\frac{\bar{W}}{I}\right) > MC$**. This is a corner solution to the problem and is a valid solution.**

3. $\lambda_1 = 0,\ \lambda_2 > 0$

   It means that $w^* = 0$ (from Equation (xi)) and plugging the value of w^* into Equation (vii) gives us $I\phi(0) - Ic + \lambda_2 = 0 \Rightarrow I\phi(0) + \lambda_2 = Ic$. This further means that $VMP(0) < MC$. **This is a corner solution and is valid but it is not an interesting solution in our case. This is because the cost of water is very high even at the constraint. Hence, it is not a profitable solution.**

4. $\lambda_1 > 0,\ \lambda_2 > 0$

   It means that $w^* = \frac{\bar{W}}{I}$ from Equation (x) and $w^* = 0$ from Equation (xi). Two values of w^* is not possible so we reject this solution.

A.3 MATLAB code used for running simulations

```
function f = choices(numfields, granularity)
% CHOICES provides all combinations of allocations between 3
% fields where the total sums to 1. Higher GRANULARITY values
% mean more data i.e. (1/GRANULARITY) is the step size. Both
NUMFIELDS and
% GRANULARITY must be positive integers.
% The function uses nsumk which provides the number and
listing of
% non-negative integer n-tuples summing to k.

% The code could be generalized to deal with any number of
fields but
% keeping it like this for clarity of how the filter is
constructed to
% impose the weakly decreasing condition

% Generate all possible values, without paying attention to
ordering
[N,K] = nsumk(numfields, granularity);

% Filter the values so that each row is weakly decreasing
% Example code for NUMFIELDS = 3
% f = (K( (K(:,1)>=K(:,2)) & (K(:,2)>=K(:,3)),:)) ./
granularity;
% This generalizes to applying the condition iteratively
column by column

for i=1:(numfields-1)

    K = K( (K(:,i) >= K(:,(i+1))) ,: );

end

f = K./granularity;

% Remove rows with zero values
%f = f(find( f(:,1)>0 ),:);

% PICKZ is a script that will calculate yield-maximizing
allocations amount
% fields, then find how many fields are irrigated, and finally
plot the
% relevant surface

% First, run choices e.g. f = choices(3,100);
```

```
% Then run pickz
% mu = mean, z = water allocted, sigma = std. dev
% W = water applied to each field, Yield = cdf of water
applied
% Ishani Lal, March 16, 2022

sigma=0.4; % ignore variation in sigma for now

z.min = 0.1;
z.max = size(f,2)-z.min;
z.step = 0.02;

mu.min = 0;
mu.max = 1;
mu.step = 0.01;

[MU, Z] = meshgrid(mu.min:mu.step:mu.max, z.min:z.step:z.max);

% Make the output arrays for speed; Y is max yield; FIELDS is
the number of
% fields irrigated

Yield = zeros(size(MU));
FIELDS = zeros(size(MU));

W = cell(size(f,2),1);

for n = 1:size(f,2)

    W{n} = zeros(size(MU));

end

for i = 1:numel(MU)

    g = (f*Z(i)) .* ((f(:,1)*Z(i))<=1);
    g = g(find(g(:,1)>0),:);

    % calculate max yield at each point

    [Yield(i), INDEX] = max( sum(normcdf(g,MU(i),sigma),2) );

    % calculate the slope of production function for different
value of MU
    %slope = normpdf(Yield(i));

    % calculate how many fields are irrigated at max yield
```

```
    FIELDS(i) = sum(g(INDEX,:)>0);

    % pull out yields for each field
    for n = 1:size(f,2)

        W{n}(i) = g(INDEX,n);

    end

end

% Plot results
% Plot yields, numbers of fields
% Plot pdf
clf
figure(1)

% surf(MU,Z,Yield); hold on
% contour3(MU,Z,FIELDS,[1:size(f,2)],'r-'); hold off
% xlabel('mu'); ylabel('Z'); zlabel('Yield/Number of fields');

surf(MU,Z,Yield);
colorbar;
hold on
set(gca, 'FontSize', 12);
xlabel('mu'); ylabel('Z'); zlabel('Yield/Number of fields');

% Plot water allocated per field
figure(2)
tiledlayout(1,size(f,2))

for n=1:size(f,2)

    nexttile

    surf(MU,Z,W{n})
    xlabel('mu');ylabel('Z');zlabel('Water
applied');title("Field "+n);
    set(gca, 'FontSize', 12);

end

% Plot pdf
%%
figure(3)

subplot(3,3,1)
```

```
musample = 0.0;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
hold off
xlabel('Water available');ylabel('Marginal
product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,2)
musample = 0.2;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
hold off
xlabel('Water available');ylabel('Marginal
product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,3)
musample = 0.3;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
```

```
hold off
xlabel('Water available');ylabel('Marginal product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')


subplot(3,3,4)
musample = 0.4;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayName','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayName','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Marginal product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,5)
musample = 0.5;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayName','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayName','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Marginal product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,6)
musample = 0.6;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
```

```
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
hold off
xlabel('Water available');ylabel('Marginal
product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend({'Field 1','Field 2','Field
3'},'Location','bestoutside')

subplot(3,3,7)
musample = 0.7;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
hold off
xlabel('Water available');ylabel('Marginal
product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend({'Field 1','Field 2','Field
3'},'Location','bestoutside')

subplot(3,3,8)
musample = 0.8;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
hold off
```

```
xlabel('Water available');ylabel('Marginal
product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend({'Field 1','Field 2','Field
3'},'Location','bestoutside')

subplot(3,3,9)
musample = 0.9;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
hold off
xlabel('Water available');ylabel('Marginal
product');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend({'Field 1','Field 2','Field
3'},'Location','bestoutside')


%figure 4
%%

figure(4)

subplot(3,3,1)
musample = 0.0;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,2)
```

```
musample = 0.2;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,3)
musample = 0.3;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')


subplot(3,3,4)
musample = 0.4;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')
subplot(3,3,5)
musample = 0.5;
t=W{1}(MU==musample);
```

```
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,6)
musample = 0.6;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,7)
musample = 0.7;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,8)
musample = 0.8;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
```

```
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

subplot(3,3,9)
musample = 0.9;
t=W{1}(MU==musample);
t2=W{2}(MU==musample);
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.','DisplayName','Field 1')
plot(Z(MU==musample),t2,'go','DisplayName','Field 2')
plot(Z(MU==musample),t3,'bx','DisplayName','Field 3')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');title("Mean "+musample);
set(gca, 'FontSize', 12);
legend('Field 1','Field 2','Field 3','Location','bestoutside')

%figure 5
%%
figure(5)

musample = 0.5;
subplot(2,3,1)
t=W{1}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t,musample,sigma),'r.','DisplayNa
me','Field 1')
hold off
xlabel('Water available');ylabel('Marginal product');
set(gca, 'FontSize', 12);

subplot(2,3,2)
t2=W{2}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t2,musample,sigma),'go','DisplayN
ame','Field 2')
hold off
xlabel('Water available');ylabel('Marginal product');
set(gca, 'FontSize', 12);
```

```
subplot(2,3,3)
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),normpdf(t3,musample,sigma),'bx','DisplayN
ame','Field 3')
hold off
xlabel('Water available');ylabel('Marginal product');
set(gca, 'FontSize', 12);

subplot(2,3,4)
t=W{1}(MU==musample);
hold on
plot(Z(MU==musample),t,'r.')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');
set(gca, 'FontSize', 12);

subplot(2,3,5)
t2=W{2}(MU==musample);
hold on
plot(Z(MU==musample),t2,'go')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');
set(gca, 'FontSize', 12);

subplot(2,3,6)
t3=W{3}(MU==musample);
hold on
plot(Z(MU==musample),t3,'bx')
hold off
xlabel('Water available');ylabel('Water allocation to
fields');
set(gca, 'FontSize', 12);

sgtitle('Marginal product and Water allocation plots for Mean
0.5')

figure(6)
py = 1;
profit = (py * Yield) - Z;
surf(Z,profit,MU)
colorbar;
xlabel('MU');ylabel('Z');zlabel('Profit');title("Profit
maximization at output price = "+py);
```

```
set(gca, 'FontSize', 12);
```