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Abstract†

A fixed premium interest-sensitive whole life contract is analyzed in order to estimate its market value. In addition, using various definitions of duration, we determine the duration of the contract for each definition. The results of this analysis have implications for market value accounting of life insurance liabilities and for life company portfolio management.

Key words and phrases: *cash flow, accounting, assets, liabilities, surrender benefits, yield curve*

1 Introduction

American life insurance companies traditionally have not assessed the market value of their liabilities. Life actuaries compute statutory reserves under strictly defined rules. Neither state regulatory bodies

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nor the Financial Accounting Standards Board (FASB) have required or allowed market value liability reporting.

Until recently, neither the nature of the life insurance products sold nor the financial markets have made it necessary for companies to assess the market risk or the market value of their liabilities. Market values are sensitive to exogenous factors such as changing interest rates. For much of this century financial markets enjoyed enough stability that risk assessments were not necessary. Life products were relatively simple and margins relatively wide; insolvencies were generally not due to the lack of liability market risk measurements.

But needs have changed. The U.S. Congress is applying pressure on state regulators to tighten statutory reporting requirements. Financial reporting under Generally Accepted Accounting Principles (GAAP) is moving in a market value direction. Complicated new products funding complex new securities are accentuating the effects of the heightened financial market volatility of the last 15 years. The market value of life company surplus has become volatile enough that effective "insolvencies" can result. These insolvencies can be masked by traditional accounting standards, but are revealed by market value methods.

Life company financial management generally has not implemented the technology necessary to measure the risk of surplus volatility. Although statutorily defined cash flow testing procedures adequately reveal certain cases of financial ruin, they also can provide false indications of problems. Most valuation actuary certifications are designed to protect contract holders without regard to stockholders; as a consequence, these certifications constitute another test of solvency rather than of surplus volatility. Even internal portfolio managers remain too far removed from liabilities to apply established market value assessment and risk management procedures to liabilities.

Securities-type valuation and risk management methodologies can be applied to a particularly complex type of life product: the interest-sensitive whole life contract (ISWL). Over the past decade, American life companies have added these and similar universal life contracts with an aggregate face amount of approximately \$1.5 trillion. As a result, interest-sensitive products represent a significant portion of life company obligations. It is not clear that either the literature or the industry has assessed the valuation or risk of this particular liability in a thorough way.

This article analyzes a fixed premium interest-sensitive whole life contract with the intent to:

- Develop an approach to estimating its market value; and

- Assess various notions of its duration.

The results of these analyses have implications for market value accounting life insurance liabilities and for life company portfolio management.

The FASB, through Statement of Accounting Standard (SFAS) 115, has taken dramatic steps to implement mark-to-market accounting for securities portfolios. The typical insurance company has responded by holding about 80 percent of its bond portfolio in the "available for sale" category, with annual changes in the market values reflected in GAAP surplus adjustments. The problem for insurance companies is that SFAS 115 does not provide for a parallel market value accounting of liabilities. As a consequence, a company can have no economic exposure to changing interest rates (i.e., be perfectly matched) and yet be reporting enormous changes in GAAP surplus from year to year. This hardly serves investor needs, which is one of the objectives of GAAP accounting. Some analysts call for a return to amortized cost accounting for bonds. A better approach may be to encourage the FASB to adopt market value accounting for insurance liabilities. This paper attempts to further the discipline of liability fair valuation.

Many product managers and portfolio managers attempt to duration-match their assets with their liabilities. These managers believe that interest rate risk is not adequately compensated for by markets. In the past life contracts were generally considered to call for a very long bond portfolio. That is not necessarily the case with interest-sensitive business, due to the resetting feature of the crediting rate. Having developed a methodology to estimate the fair value of liabilities, we can then re-estimate their values after shocking market rates. The relationship can then inform portfolio managers on how to invest bonds to match the value sensitivities.

2 The Contract

2.1 Provisions

ISWL contracts are similar to traditional whole life contracts. Whole life products are structured to produce a level premium at any issue age for a large pool of insureds. This premium can be regarded as the periodic contribution to a fund that, when accumulated at an implicit rate of interest, is expected to produce periodic death benefits and is exhausted at the end of the pool of lives. This condition holds after allowing for loadings for coverage of company issuance and maintenance expenses and a competitive return to capital. An ISWL contract initially is structured with a level premium.

ISWL contracts differ from traditional whole life contracts in two ways. First, although the contracted premium must be paid in order to avoid nonforfeiture status, ISWL premiums can have a more complex structure. Much of this complexity can be considered as discrete contract owner options to adjust the mix of premium paid and coverage amount within constraints imposed by the issuer and tax code. The contract owner uses private information in exercising these options. The issuer can respond by adjusting the cost of insurance assessment within contractual and legal constraints. In a yet more general type of permanent life contract, universal life (i.e., flexible premium interest-sensitive life), the contract owner effectively has continuous options to adjust the premium/coverage mix.[1]

Second, in an ISWL contract the accumulation interest rate is explicit and adjustable. Whatever rate structure the issuer originally used to price the premium structure, the contract fund value is credited with a rate that resets periodically at the option of the issuer, subject to a floor.

ISWL contracts also allow the contract owner to remove the accumulated fund value by:

- Taking a loan (which continues to commit the policy owner to the premium schedule and can expose the loaned portion of the fund value to a lower crediting rate);
- Making a partial withdrawal; or
- Canceling the entire contract (which typically exposes the policy owner to a schedule of surrender assessments).

ISWL contracts also provide the standard array of nonforfeiture options, ignored in this analysis. See the appendix for a summary of terms and in force assumptions of the ISWL contracts analyzed.

2.2 Cash Flow Components

Contract cash flows belong to four classes:

- Premium inflow;
- Surrender benefit;

[1] The application of game theoretic modeling to this field could provide rich insights into the optimum use of this option.

- Death benefit; and
- Servicing expense and commission.

Projecting surrender benefit cash flows entails certain complexities. First, the industry comparable product crediting rate must be modeled. Second, the company crediting strategy must be established. Third, a function of contract holder surrender behavior with respect to these differential rates must be integrated into the overall model.

2.2.1 Industry Interest-Sensitive Life Crediting Function

Industry interest-sensitive life crediting rates track fixed income market rates with lags for several institutional reasons:

- Company investment committees often establish crediting rates as a spread to the asset book value earnings rate. Once a particular asset is on the books at a given rate of return for purposes of accruing interest income, its rate of return will be assumed to remain constant until those particular assets are rolled into a new security with a new rate of return. This practice delays the recognition that market rates have changed;
- The institution then needs to declare the new rate;
- Companies are reluctant to change rates too often for marketing purposes; and
- The effective crediting rate can be delayed further due to certain contractual provisions allowing new rates to be credited only at the policy's anniversary; hence, contracts containing this provision have a built-in expected delay of six months.

The crediting rate series comes from monthly data from January 1985 through December 1992 from the TULAS industry survey published by the actuarial consulting firm Tillinghast.[2] The period covers a substantial fall in rates and changes in yield curve shape.

Let ICR_t be the industry crediting rate at month t, $T3M_t$ be the three month risk-free bill rate at month t; and $T5Y_t$ be the five year risk-free note rate at month t. The subscript $t - m$ refers to the rate m months prior to the interest-sensitive life crediting rate. A linear regression with

[2]For more information of the TULAS industry survey, write to: Tillinghast, 245 Park Avenue, 18th Floor, New York NY 10167-0128, USA.

ICR_t as the dependent variable and $T3M_{t-6}$, $T5Y_t$ and $T5Y_{t-18}$ yields the following equation:

$$ICR_t = 3.78 + 0.119 \times T3M_{t-6} + 0.342 \times T5Y_t + 0.146 \times T5Y_{t-18} \quad (1)$$

with $R^2 = 0.72$. Equation (1) is not useful in describing the response of the crediting rate to a change in market rate because the nonconstant term coefficients sum to significantly less than one. For example, the model indicates that the crediting rate would never fully respond to a permanent parallel shift in rates.

The following is a similar model, fit without the constant term, and weighs recent observations more heavily. This regression yields the following equation:

$$ICR_t = 0.424 \times T5Y_t + 0.297 \times T5Y_{t-6} + 0.328 \times T5Y_{t-18} \quad (2)$$

with $R^2 = 0.99$. In this case equation (2) also appears to give a good fit; in addition, because its coefficients sum nearly to one, the equation merely describes the timing by which the crediting rate will reflect changed market rates.

2.2.2 Company Crediting Strategy

The crediting rate structure that a company establishes is the result of myriad marketing, investment, legal, and game theory considerations. Crediting rate rationale is not well understood from a theoretical perspective. Companies attempting to establish market share often credit above-market rates. Companies often engage in "bait and switch tactics" (i.e., crediting above-market rates on new money and below-market rates at subsequent policy years). When a company has closed a block and is running off the reserves, it often credits low rates. Company crediting rates are often not closely related to the company's claims-paying capacity. For this analysis the rate is set at the industry rate less 125 basis points.[3]

2.2.3 Contract Holder Surrender Profile

Contract holder behavior is affected fundamentally by the spread between the rate earned by a particular contract and the rate provided

[3] One basis point is the smallest measure used in quoting yields on mortgages, bonds and notes. It represents 0.01% of yield. Thus a 5.77% rate that drops by 125 basis points becomes 4.52%.

by a similar contract issued by an industry competitor. Surrenders also are relatively higher during the early stages of the contract. Surrender behavior can be volatile at critical points in the life of the contract, such as premium redetermination periods and the end of the surrender charge period. Furthermore, it is affected by critical ages, such as retirement, of the contract holder.

The analysis assumes baseline surrenders of 7.5 percent annually with adjustments for:

- Early policy years due to buyer remorse;
- Later policy years, when surrender charges are lower;
- Higher insured ages, when contract owners typically have a postretirement need for cash and when they find surrender to be tax-efficient; and
- Policy-crediting rates that are different from rates on policies of competitor companies. Over the projection horizon the crediting rate is set at the projected industry rate less 125 basis points. The low crediting rate used in this analysis produces moderately higher surrenders.

To date, little empirical analysis has been done on contract holder surrender profile. There are significant barriers to conducting such research:

- Lack of product homogeneity;
- Differences in sales methods;
- Differences in product management;
- Differences in perceived insurance company risk; and
- Absence of usable historical data maintained by companies.

Contrast this with the extensive analytic work that has been conducted on the prepayment behaviors of single family homeowners. The mortgage is relatively a homogeneous product. Good data are available for at least 15 years, and there are few company-specific issues related to them.

This asymmetric behavior reveals the fundamental interest rate option that the contract holder owns. When market rates rise there is the potential problem of disintermediation, i.e., the insured can surrender to pursue higher new money rates elsewhere (subject to a declining surrender charge). Consequently, the company needs to choose between raising the crediting rate to keep the policy or to maintain the rate and cash out the policy. In the former case the company pays more interest over time, perhaps even a greater rate than it earns on its investment. In the latter case the company fails to recoup a portion of its acquisition cost and tends to liquidate bonds at depressed prices.

If rates fall, however, the results are not symmetrically bad for the contract holder. The insured merely keeps the policy knowing that the company can reduce its rate only to the guaranteed rate.

The value of this interest rate option is affected by the same two fundamental variables that affect all other interest rate options: interest rate volatility and time to expiry. Greater market rate volatility provides a greater range of opportunity for the insured to exercise. New money rates can become higher, providing more incentive to pursue them. They also can be driven lower, increasing the chance that the insured enjoys a guaranteed rate in excess of new money rates. In addition, these contracts often have long lives, which provides further opportunity for rates to move to the insureds' advantage. The effect of this option on insurance company surplus is exacerbated by the likelihood that the company has written similar options in its mortgage-backed security and callable bond portfolios.

This analysis uses a quadratic function to derive excess surrenders (i.e., surrenders in excess of baseline) stimulated by the contract holders' desire to earn higher rates. Thus

$$ES_t = (1 + \rho \times TD_t)^2 - 1$$

where

$$\begin{aligned}
ES_t &= \text{Excess surrenders at time } t;\\
\rho &= \text{Propensity to achieve higher rates through surrender;}\\
TD_t &= \max\left\{\left[CR_t - PR_t - 0.0115 - \frac{SC_t(1+PR_t)^{-SP_t}}{SP_t}\right], 0\right\}\\
CR_t &= \text{Identified competitor's credit rate at time } t;\\
PR_t &= \text{Product credit rate at time } t;\\
SC_t &= \text{Remaining surrender charge at time } t;\\
SP_t &= \text{Number of surrender charge periods remaining at time } t.
\end{aligned}$$

Note that the rate of differential must exceed 115 basis points (0.0115) before any excess surrenders occur, and that the remaining surrender charge is then amortized off. This paper uses the 125 basis points spread between the competitor rate and the product rate. The effect is to moderate excess surrenders in most forward rate and surrender charge environments.

Surrender cash flows include new policy loans. Policy loans typically do not pay cash interest and, in any case, are usually not repaid. Instead, interest is capitalized, increasing the loan amount. New policy loans are modeled as a premature cash outflow from the fund. When the contract is extinguished by surrender, the surrender benefit is reduced by the amount of the loan; when it is extinguished by ultimate death of the insured, the death benefit is reduced by the amount of the loan.

We assume future loan balances to be a constant proportion of contract cash values. Under older whole life policies the policy loan was a further powerful option enjoyed by the contract holder. In high rate environments the insured withdrew the policy's cash value at a rate fixed at policy inception and invested in high market rates elsewhere. Under newer interest-sensitive contracts the loan rate depends on market rates. Consequently, there may be no further benefit to taking the loan in high or low rates. In high interest rate scenarios, however, cash values increase quickly, loan balances capitalize quickly, and cash paid upon death of the insured is reduced.

In high interest rate scenarios our model shifts cash flows to earlier periods and from the death benefit component to the surrender benefit component. This modeling assumption does not distort aggregate contract duration, but may affect the respective durations of the surrender component and death benefits component.

2.2.4 Cash Flow Generation

Various interest rate scenarios are generated in estimating market values and durations. Each scenario is associated with a yield curve. Under each scenario for each projected month premium cash flows into, and benefit and expense cash flows out of, the fund are generated for each contract remaining in force. In addition, the fund accumulates by capitalizing interest at the credited rate and is reduced by the cost of insurance and expense charges deducted for each policy in force. Industry and company crediting rates are functions of medium term forward rates implied by the scenario yield curve. Contract surrenders are triggered, generating surrender cash flows. Finally, mortality cash flows are generated, further reducing the number of policies in force. The process is repeated for each month and each yield curve.

2.2.5 Duration

The market value of financial instruments is sensitive to interest rates at various points on the yield curve. The concept of duration provides a measure of the extent of this sensitivity. The basic duration measure is the Macaulay duration; see, for example, Boyle (1992, Chapter 3). It is defined as follows for an arbitrary series of cash flows C_k paid at time k, $k = 1, 2, \ldots, n$ with yield-to-maturity rate of y_n

$$\text{Macaulay Duration} = \frac{\sum_{k=1}^{n} kC_k(1+y_n)^{-k}}{\sum_{k=1}^{n} C_k(1+y_n)^{-k}}. \tag{3}$$

For example, a one year Treasury bill has only one cash flow; its duration is explained by what happens to the one year rate. A five year investment grade corporate bond's duration is explained largely by the five year rate, but also by the change in rates at the same term as coupon payments. A mortgage-backed security can be affected by medium term rates because its cash flow is centered in that region, and also because refinance rates, which affect the amount and incidence of cash flow itself, are centered there. Interest-sensitive life contracts can have highly complex cash flow magnitude and incidence of cash flow profiles.

In general, the duration of a financial instrument can be calculated as follows: let P_0 be the current price of the financial instrument, P_+ be the price after a very small increase of y_+ in the interest rate, and P_- be the price after a very small decrease of y_- in the interest rate. The duration D is defined as:

$$D = \frac{P_- - P_+}{(y_+ + y_-)P_0}. \tag{4}$$

For example, if a bond currently trading at par (i.e., $P_0 = 100$) is subjected to an instantaneous rate increase of one half of one percent ($y_+ = 0.005$) and its price drops to 98 ($P_+ = 98$), while if it is subjected to an instantaneous rate decrease of one half of one percent ($y_- = 0.005$) and its price increases to 102 ($P_- = 102$), then $D = 4$.

Duration increases as the time to maturity increases, other things being equal. Floating rate instruments usually have quite short durations. Although duration was initially introduced for fixed income instruments, all financial instruments have a duration as defined in equation (2).

3 Market Value Estimate

Consider a comparison to corporate debt whereby the accumulating ISWL fund represents the proceeds from company debt issuance in the same way as an industrial firm can issue debt as part of the financing structure for a capital project. In either case the issuer is accumulating assets with the promise to repay the debt, with interest, at a future date. The issuing firm (in the case of debt issuance) and the life company (in the case of contract issuance) are charged a certain number of basis points as a risk premium by entities providing the funds. Issuers in both cases incur issuance costs and enjoy related tax advantages. Because the ISWL contract provides risk-reducing services to the provider of funds (the contract holders), some or all of such cost of funds is offset. The insurance company is rewarded for providing risk intermediation services. Consider the aggregate of these costs and their offsets to be a cost of funds basis point spread to Treasury.

The spread can be estimated for a line of business or a block of policies at any time in its life. To accomplish this, solve for the spread that when added to the respective risk-free zero coupon rates discounts all future expected contract cash flows to the market value of funds that insureds willingly provide to the insurer. Immediately prior to issuance such assets equal zero; after the initial premium such assets equal the premium reduced by the commission and other acquisition costs incurred in issuance; at any time such assets equal the sum of all net cash flows and their investment earnings at risk-adjusted rates.

Let $CF_{c,t}$ be the cash flow of component c at projection month t where c is a nominal integer parameter taking possible values from 1 to 4 (1 = premium, 2 = surrender benefit, 3 = death benefit, and 4 = expense), and $t = 1, 2, \ldots, 480$ (that is, 40 years). Thus, we are interested in projecting cash flows component by component over 40 years. The cash flows can depend on the level and shape of the yield curve at any projection month, and each month's discount rate also can be different. This component by component approach is consistent with pricing of mortgage-backed securities and derivatives that have been in place for about a decade. See, for example, Roll (1988).

Let z_t be equal to the risk-free zero coupon rate at projection month t, and $\xi_{k,t}$ be an instantaneous shift in the zero coupon rate at time t for key rate k, $k = 1, 2, \ldots, 9$. The shift is applied to z_t at all t near the term of a Treasury on-the-run instrument. Finally, let s be the spread. In this analysis the resultant presumed spread is set to 20 basis points for the entire projection period. Using the semi-annual coupon convention of the investment literature, let d be the market price discounting factor such that, at any month t,

$$d_t = \left(1 + \frac{z_t + \xi_{k,t} + s}{2}\right)^{-t/6}. \tag{5}$$

A simple procedure provides the market value (MV) of ISWL,

$$MV = \sum_{c=1}^{4} \sum_{t=1}^{\infty} CF_{c,t}\, d_t. \tag{6}$$

This analysis uses 480 months as a practical analog for infinity. Applying the methodology produces Table 1. Because this market value measure is that of a liability, cash flows of the premium component have a negative sign.

Table 1
Base Market Value ISWL

Component	Market Value
Premium	(71,687,488)
Surrender Benefit	53,262,613
Death Benefit	34,190,301
Expense	7,852,738
Aggregate	23,618,164

A few points are worth noting about these values relative to the various in force values in the appendix:

- The aggregate market value is only about 50 percent of the statutory reserve, reflecting the conservative nature of statutory accounting principles;
- The surrender benefit is even higher than the fund or cash values due primarily to the inclusion of future premiums in this component; and
- The death benefit is only about 5 percent of face amount, indicating the impact of withdrawals and the time value of money.

4 Notions of Duration

In general duration is a measure of the sensitivity of a stream of cash flows to an interest rate change. Different notions of interest rate shifts are depicted graphically in Figures 1 and 2. Figure 1 shows parallel and nonparallel (i.e., dampened) shifts from a base case yield curve. Figure 2 shows a piecewise dampened shift. Each shift produces a different sensitivity, or duration, of the present value of the product cash flow.

Consider the following four concepts of duration.

Spread Duration: Spread duration is the percent sensitivity of market value to a shift in s. This is similar to Macaulay duration and modified duration which are measures of sensitivities to changes in yield for fixed and certain cash flows. It is useful as an average life index and as a measure of the capitalized value of a unit of higher cost of funds. Mathematically,

$$\textit{Spread Duration} = \frac{1}{MV}\frac{dMV}{ds}. \tag{7}$$

Parallel Effective Duration: Parallel effective duration is the percent sensitivity of market value to a parallel shift in z simultaneously over all k. That is, $\xi_{k,t}$ are equal at all k. This is the type of duration customarily used in asset and liability matching. Mathematically,

$$\textit{Parallel Effective Duration} = \frac{1}{MV}\frac{dMV}{dz}. \tag{8}$$

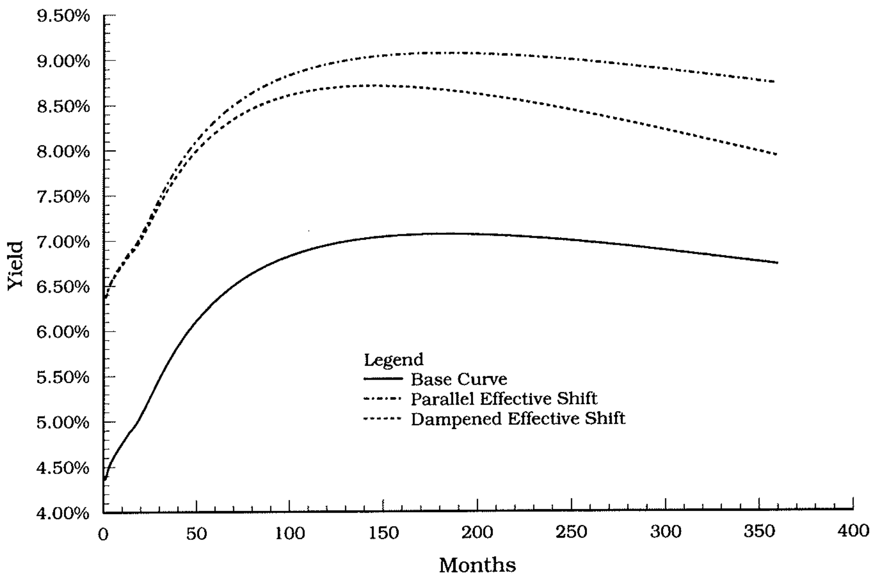
Dampened Effective Duration: Dampened effective duration is the percent sensitivity of market value to a function f that produces nonparallel shift vector in z over all k such that: $\xi_{k,t} > \xi_{k+1,t}$. Rates for months within a region also are shifted according to the nonparallel function f. This is similar to effective duration, but recognizes that short rate volatility typically exceeds long rate volatility. In this analysis the average shift over the region $k = 9$ is about 60 percent of the average shift over the region $k = 1$. Mathematically,

$$\textit{Dampened Effective Duration} = \frac{1}{MV}\frac{dMV}{d\tilde{z}} \tag{9}$$

where the vector $\tilde{z}$ is produced according to f.

Key Region Effective Duration: Key region effective duration is the percent sensitivity of market value to a set of shifts in z sequentially over each k. Measures under either of the following two subsets of this notion can be considered partial durations:

Figure 1
Base and Shifted Yield Curves



Parallel: This duration measures shifts by region of the curve. Mathematically,

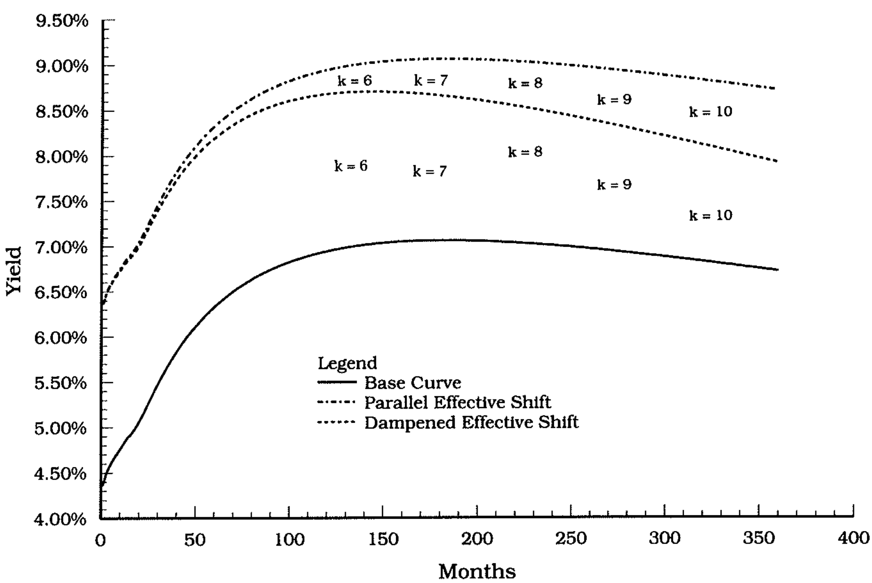
$$\textit{Parallel Duration} = \frac{1}{MV}\frac{dMV}{d\xi_{k,t}}. \tag{10}$$

The methodology developed in this paper provides a duration, termed *piecewise duration*, to each significant point on the yield curve. It effectively decomposes overall duration to assess the impact of changes of pieces of the yield curve. For example, the duration of a five year bond is about 4.2 percent; about 70 percent of this duration can be attributed to the five year rate. For instruments with fixed cash flows, piecewise durations approximately sum to the aggregate durations. For instruments whose cash flows are interest contingent piecewise durations may sum to a different number. The piecewise (region-by-region) shifts in the yield curve are similar to Ho's (1992) key rate duration methodology. This analysis shifts entire segments of the yield curve by cutting and excising these portions. This happens over all t within a given region k. Ho uses a series of triangle functions in shifting Treasury on-the-run rates. Both methodologies create kinked undifferentiable curves. The curves used in this paper are also discontinuous. This key region methodology, however, improves the performance of the aggregation of piecewise duration.

Dampened Duration: Dampened duration is similar to dampened effective duration, but relates to the effect of dampened shifts by region of the curve. Mathematically,

$$Dampened\ Duration = \frac{1}{MV}\frac{dMV}{d\bar{\xi}_{k,t}}. \tag{11}$$

Figure 2
Base and Shifted Yield Curves: Key Rates



5 Duration Calculations

Consider point estimates of these measures shown in Table 2. These figures are scaled to a 100 basis point shift. Under dampened measures figures are scaled to reflect an approximately 100 basis point shift over the region $k = 1$. We show only the dampened category of key region measurements.

Table 2
Point Estimate Durations

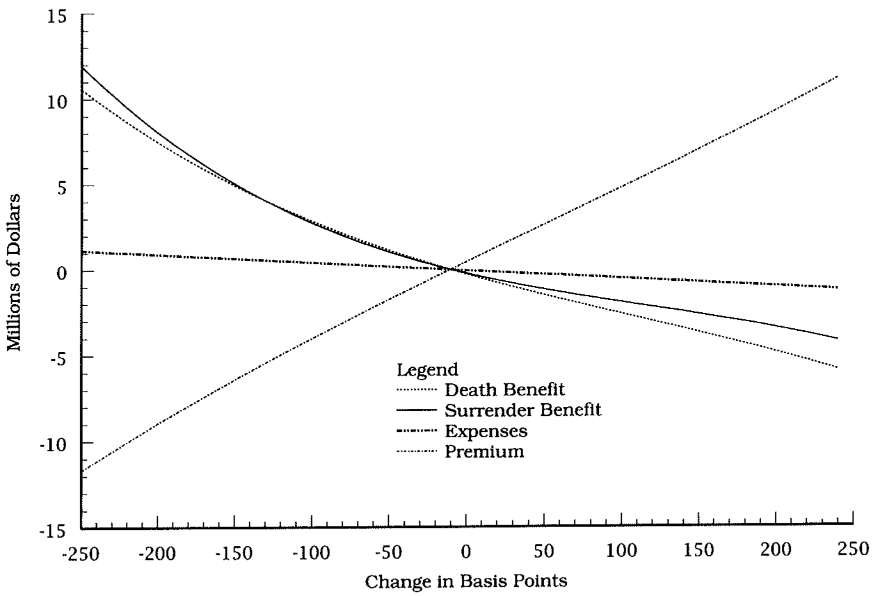
Component	Spread	Parallel Effective	Dampened Effective
Premium	-5.94	-5.94	-4.94
Surrender Benefit	10.86	3.78	3.55
Death Benefit	14.15	7.93	6.51
Expense	5.49	5.49	4.59
Aggregate	28.79	3.82	3.95

Component	1	2	3	4	5	6	7	8	9	Total
Premium	-0.01	-0.03	-0.11	-0.02	-0.44	-1.11	-1.85	-0.99	-0.16	-4.94
Surrender Benefit	0.01	0.05	0.12	0.35	0.5	1.92	1.32	0.02	0.45	4.74
Death Benefit	0	0.02	0.06	0.17	0.29	1.37	3.45	2.13	0.21	7.71
Expense	0.01	0.04	0.12	0.24	0.44	1.08	1.58	0.88	0.19	4.59
Aggregate	0	0.05	0.06	0.42	0.37	3.31	2.89	0.34	1.75	9.18

Premium duration figures are provided with respect to their effect on the value of the liability. For example, at a higher discount spread of 100 basis points the market value of premium will become less negative, producing a higher overall liability. In this sense premium has a negative duration as well as a negative market value.

Consider also the more intuitive market value change by component shown in Figure 3. See also the aggregate market value changes by effective and dampened effective-type shifts shown in Figure 4. Each of the figures shows a market value change from the base case resulting from the indicated shift.

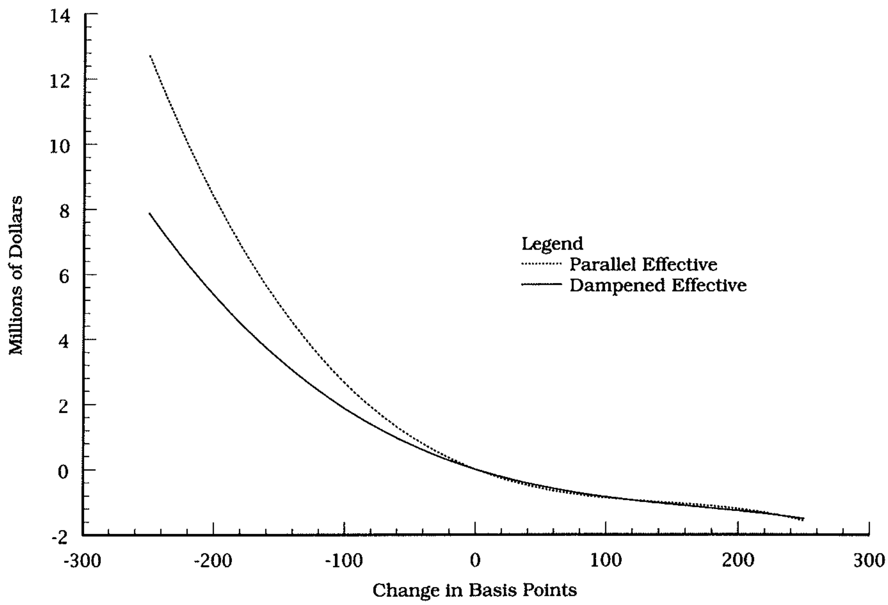
Figure 3
Market Value Changes By Component Parallel Effective Shifts



6 Discussion

Estimating the cost of funds is a complex process. The rate credited to the fund accumulations explains less of such cost than do analogous rates in depository institution or corporate bond environments. A simplistic comparison between a life company's asset earning rate and its crediting rate provides inadequate information with respect to its net spread. Such comparison in a depository context, however, provides much more significant information. Market value approximations for complex insurance liabilities, however, can be made. Actuaries can provide market value estimates using the ideas presented in this paper.

Figure 4
Aggregate Market Value Changes By Shift Type



The death benefit component is highly positively convex[4] due to the mere length of the cash flows and to the effect of policy loans. Our model of policy loans causes the death benefit component to have greater absolute cash flows in lower interest rate environments, a combination sure to produce convexity. Liability convexity is not desirable. The surrender benefit component is also highly positively convex due primarily to the presence of the guaranteed rate in the contracts, which acts as an interest rate floor. The effective duration of the surrender benefit component, primarily a floating rate set of cash flows, is long due to the lagged nature of adjustments to crediting rates.

[4]The concept of convexity refers to the extent that the price function changes over greater yield intervals. A financial instrument is said to be positively (negatively) convex when its duration increases (decreases) as the yield decreases. Positive convexity is desirable for assets but not for liabilities.

Financial managers need to decide whether to use the dampening notion of effective duration or whether to use the strictly parallel approach. The alternative methods can give significantly different results. Long-tailed cash benefits and the customary front-loaded premium structure of whole life combine to produce a net asset at the beginning of contract time and a net liability at the end of such time. The present value of this configuration is subject to yield curve twists such as are emphasized in the difference between effective and dampened effective durations.

Spread duration, indicating average lives of cash flow components, are long with respect to the following components:

- Surrender benefits, as interest is capitalized rather than paid currently on the surviving contracts;
- Death benefits, because as the insureds of surviving contracts grow older, cash flow increases; and
- Aggregate, because it is leveraged, in a sense, by negative premium flows in earlier periods. The leveraged characteristic of this component renders its spread duration a less reliable measure of cash flow life. It remains a measure of the value of a basis point change in discounting spread.

Spread and effective durations are equal for fixed cash flow components such as premium and expense, and are different for interest rate-sensitive cash flows such as surrender and death benefits.

Dampened effective durations and aggregate dampened effective partial durations are equal for fixed cash flow components and different for cash flow components. Premium cash flows are fixed over different scenarios even though surrender cash flows change. We have fixed the incidence of surrender in the analysis by leaving the crediting strategy unchanged—hence the constant premium cash flows. On the other hand, the amount of surrender cash flows can change dramatically under different forward rate scenarios.

Because the partial durations with respect to interest-sensitive cash flows are relatively high compared to an equivalent dampened effective duration (in the context of Reitano (1990)), it appears that the interest-sensitive nature of the cash flows has the effect of adding durational leverage to the component. Other elements capable of causing the effect include negative partial durations, portfolios of securities, and short positions. Option-adjusted spread methodology would provide yet additional texture to the duration results. Additionally, analogous shocks can be made in mortality and surrender scenarios to reveal additional risk measures.

A further consideration is the duration of cash flows distributable to the stockholders. Under life insurance accounting conventions in most states, dividends generally can be paid without restriction to the extent of the prior year statutory earnings. In this case statutorily defined accounting and legal conventions actually drive a cash flow stream. In most cases these conventions have a significant historical cost component that stabilizes earnings. As a result, intracompany durations and the durations of returns to the ownership rights to such company may diverge substantially. Management and stockholders rarely synthesize an optimal duration position.

7 Conclusions

Life insurance companies can adapt market pricing methodologies that customarily are used in the financial area to the cash flow provisions of their particular contracts. Adapting such methodologies can provide rich insight into the market value of liabilities. These modeling methodologies can guide investment strategies and provide management with measures of the market value of life company surplus.

References

Boyle, P.P. *Options and the Management of Financial Risk,* Schaumburg, Ill.: Society of Actuaries, 1992.

Ho, T.S.Y. "Key Rate Durations: Measures of Interest Rate Risks." *The Journal of Fixed Income* 2, no. 2 (September 1992): 29-44.

Reitano, R.R. "Non-parallel Yield Curve Shifts and Durational Leverage." *The Journal of Portfolio Management* 16, no. 4 (Summer 1990): 62-67.

Roll, R. "Stripped Mortgage-Backed Securities." In *The Handbook of Mortgage-Backed Securities,* (edited by F.J. Fabozzi). Chicago, Ill.: Probus, 1988.

Appendix—Hypothetical ISWL Contract Terms and In Force Assumptions

Following is a sample cell of the ISWL model used in the analysis.

Table A1
Representative Cell Information

Field	Value
Gender	Male
Weighted Average Issue Age	32
Smoking Classification	Nonsmoker
Premium Mode	Monthly
Underwriting Class	Nonmedical
Weighted Average Policy Age	10 Years
Number of Policies	70
Basic Annual Premium	$5.60/$1000
Total Face Amount (Policy yrs 1-10)	$7,605,500
Total Face Amount (Policy yrs 11+)	$6,844,950
Total Fund Value	$266,633
Total Cash Value	$226,639
Total Statutory Reserve	$250,740
Total Policy Loan	$22,664

Common to all 102 cells of the analysis are a guaranteed crediting rate of 5.5 percent; a 15 year declining surrender charge schedule as a percentage of existing fund value; a provision whereby interest crediting on loaned balances can be less than on unloaned balances; a specified agent commission schedule; and a standard servicing expense structure. The descriptive statistics for the insured groups, weighted by face amount, are: (i) issue age = 41.5, and (ii) months seasoned = 55.0.

Other beginning characteristics of the model contract are a face amount of $709,585,000; cash value of $36,348,600; fund value of $50,426,020; statutory reserve of $47,150,500; annualized premium in projection month one of $5,886,800; and 6,951 policies. The analysis assumes a simple mortality experience equal to 80 percent of the 1975-80 basic select and ultimate mortality table.