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Participating GICs: Performance Attribution Analysis

Alec Stais* and John P. Toohey III†

Abstract

The increasing popularity of participating GICs has created a need for an objective understanding of their performance. The fixed income attribution techniques are not adequate for measuring participating GIC performance because they typically restrict performance measurement to concepts such as duration management, sector rotation, and issue selection. We develop an attribution technique based on four components or effects that are helpful in explaining the changes in credited rates. They are the constant duration effect, the reinvestment effect, the cash flow effect, and the investment effect. The underlying mathematical approach to calculating these effects is presented along with examples.

Key words and phrases: investment, duration, yield, spread, cash flow, investment manager

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1 Introduction

Defined contribution plans often offer a stable value fund as an investment alternative. Stable value funds provide a fixed credited rate and a guarantee of principal to participants. Plan sponsors historically have invested in nonparticipating insurance company contracts called GICs (guaranteed investment contracts) to support the principal plus interest guarantees to participants. The Stable Value Association estimates that aggregate stable value funds currently account for approximately $300 billion in assets.¹ Recent years have seen the advent of participating GIC products (often referred to as synthetic or separate account GICs). These products reflect the investment experience of a specific asset or portfolio of assets. Participating GICs now command over half of all stable value sales, according to a recent Stable Value Association survey.²

The increasing use of participating GICs within GIC/stable value portfolios has created a need for an objective understanding of participating GIC performance. Performance measures were not required for traditional GICs because performance essentially was guaranteed (ignoring default risk) in the form of a fixed credited rate. Fixed income attribution techniques³ are not sufficient for measuring participating GIC performance. Such techniques typically restrict performance measurement to concepts such as duration management, sector rotation, and issue selection. These techniques decompose portfolio experience into components in order to isolate the performance effect of various investment strategies.

Other concepts are often more important, however, in explaining changes in participating GIC credited rates. Rennie (1994) deals with some of these concepts but he does not explain the mechanics of an attribution. As a result we have developed an attribution technique for explaining changes in participating GIC credited rates. In particular, we develop four key components or effects that are helpful in explaining the changes in credited rates:

- The constant duration effect;

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³Fixed income attribution techniques decompose income security total returns into components based on the sources or factors that contributed to the return. See, for example, Dietz, Fogler and Hardy (1980) and Fong, Pearson and Vasicek (1983).
The reinvestment effect;
• The cash flow effect; and
• The investment effect.

Likewise, the investment effect consists of five subcomponents: investment market, spread; investment manager, time-weighted and nonconstant duration.

This approach described in this paper is not the only way to attribute changes in credited rates, but we believe our approach offers several advantages. It helps plan sponsors to manage their stable value portfolios by explaining, in an intuitive way, why credited rates on participating GICs change. The framework can be integrated with the investment manager's attribution of investment performance. The framework provides a mechanism to compare the participating GIC products of different providers.

To explain the conceptual framework behind our approach, we discuss the characteristics of participating GICs, the basic resetting formula and introduce our notation in Section 2. In Section 3 we develop our attribution methodology, which is based on the four key components or effects described above that are helpful in explaining the changes in credited rates. Section 4 consists of two detailed hypothetical examples illustrating our attribution methodology.

2 Background

2.1 Participating GIC Characteristics

Participating GIC products typically have these characteristics:

• They combine a market value fixed-income portfolio (bond fund) with a book value guarantee (wrapper) that covers benefit payments to participants. A credited interest rate is established at the inception of the contract and reset periodically (quarterly, semiannually, or annually) to reflect contract experience.

• They are similar to bond funds in that performance is recognized immediately in the market value. In addition, participating GICs possess other characteristics of bond funds: active management, total return, market value, yield, and duration. Unlike bond funds, however, performance is not immediately passed to participants but is reflected prospectively in the form of adjustments to the credited interest rate.
• They possess some of the characteristics of traditional GICs: a credited rate and a guarantee of principal. Unlike traditional GICs that have a fixed credited rate that is guaranteed for the term of the contract, participating GICs have a credited rate that changes to reflect contract experience.

• Most participating GICs are designed as evergreen products with no maturity date and assets managed around a constant duration.

We discuss only this common form of participating GICs, although our framework can be modified to accommodate participating GICs with a target maturity date.

2.2 The Resetting Formula

The formula used for resetting the credited rate for participating GICs at time $t$ ($t = 0, 1, 2, \ldots$) typically is:

$$
(mv_t + wc_{ft}) \times (1 + y_t - e_t)^d = (cv_t + wc_{ft}) \times (1 + cr_t)^d
$$

where the variables below are defined with respect to portfolio values at time $t$:

- $mv_t$ = Market value of assets in fixed income portfolio;
- $cv_t$ = Contract value or book value of liability;
- $wc_{ft}$ = Cash flow expectation for the period between resets;
- $y_t$ = Actual annualized market yield to maturity of the assets;
- $e_t$ = Contract expenses/fees;
- $d_t$ = Duration in years (usually the portfolio duration); and
- $cr_t$ = Credited rate

with $mv_0 = cv_0$ and $cr_0 = y_0 - e_0$ at the inception of the contract.

Equation (1) can be solved directly for $cr_t$ as a function of several variables:

$$
cr_t = cr(mv_t, cv_t, wc_{ft}, y_t - e_t, d_t)
$$

$$
= \left(\frac{mv_t + wc_{ft}}{cv_t + wc_{ft}}\right)^{1/d_t} \times (1 + y_t - e_t) - 1.
$$

Equation (2) illustrates that the credited rate reflects current investment yields plus the amortized effect of any differences between market value and contract value.
2.3 Notation

Throughout the rest of this paper, the following notation is used in subscripts and superscripts to denote quantities subjected to the particular effect:

- **CD** = Constant duration;
- **RI** = Reinvestment;
- **CF** = Cash flow;
- **IV** = Investment;
- **IM** = Investment market;
- **SP** = Spread;
- **IX** = Investment index;
- **MG** = Investment manager; and
- **ND** = Nonconstant duration.

In addition, without loss of generality, we will change our time reference point from the time of the inception of the contract to the time of the prior (previous) reset, i.e.,

- as time $t - 1$ represents the prior (previous) reset, events at time $t - 1$ are denoted with the subscript 0; and
- as time $t$ represents the current reset, events at time $t$ are denoted with the subscript 1.

Also, fees are assumed to be held constant between the prior reset and the current reset, i.e., $e_t = e_0 = e$ for $0 \leq t < 1$.

3 The Attribution Methodology

3.1 The Four Components

Let $\Delta cr_0$ denote the change in the credited rate that occurs from prior reset (time 0) to current reset (time 1), i.e.,

$$\Delta cr_0 = cr_1 - cr_0.$$  \hfill (3)

The values of the variables $mv_0$, $cv_0$, $wc_0$, $y_0$, $e_0$, and $d_0$ serve as the starting point for credited rate reset attribution. All effects are measured relative to these values. Each of the intermediate steps involved
in changing the values of these variables at time 0 to their respective values at time 1 corresponds to a component or effect.

The change in the credited rate can be decomposed into four components: a constant duration effect, a reinvestment effect, a cash flow effect and an investment effect. The fundamental equation for the decomposition of the change in the credited rate is as follows:

$$\Delta cr_0 = \Delta cr_0^{(CD)} + \Delta cr_0^{(RI)} + \Delta cr_0^{(CF)} + \Delta cr_0^{(IV)}$$  \hspace{1cm} (4)$$

where

$$\Delta cr_0^{(CD)} = \text{The constant duration effect;}$$
$$\Delta cr_0^{(RI)} = \text{The reinvestment effect;}$$
$$\Delta cr_0^{(CF)} = \text{The cash flow effect; and}$$
$$\Delta cr_0^{(IV)} = \text{The investment effect.}$$

In addition, the investment effect is decomposed further into five subcomponents as follows:

$$\Delta cr_0^{(IV)} = \Delta cr_0^{(IM)} + \Delta cr_0^{(SP)} + \Delta cr_0^{(MG)} + \Delta cr_0^{(TW)} + \Delta cr_0^{(ND)}$$  \hspace{1cm} (5)$$

where

$$\Delta cr_0^{(IM)} = \text{The investment market effect;}$$
$$\Delta cr_0^{(SP)} = \text{The spread effect;}$$
$$\Delta cr_0^{(MG)} = \text{The investment manager effect}$$
$$\Delta cr_0^{(TW)} = \text{The time-weighted effect; and}$$
$$\Delta cr_0^{(ND)} = \text{The nonconstant duration effect.}$$

This attribution framework is sufficiently flexible to accommodate a wide variety of methods of measuring yields and durations including:

- Portfolio, index, or model yields and durations;
- Duration-weighted or market value-weighted yields;
- Option-adjusted yields and durations; and
- Macaulay or modified durations.

In our examples in Section 4, we use duration-weighted portfolio yields, Macaulay portfolio durations as the amortization period, and annual resets. We do not use time-weighted cash flows.

Next we develop the formulas needed to compute magnitude of each effect.
3.2 The Constant Duration Effect

Participating GICs have a constant duration product design, i.e., they do not have a maturity date. Assets are managed around a constant (or target) duration, \( d \), which is consistent with plan sponsor objectives. As equation (2) shows, any difference between \( mv_1 \) and \( cv_1 \) is amortized over the constant duration, \( d \). The amortization process acts as a smoothing mechanism, reducing the volatility of returns to participants. It also will cause \( mv_t \) and \( cv_t \) to converge to each other (as \( t \to \infty \)) in a stable interest rate environment.

Assume market value is below contract value. As equation (2) indicates, the credited rate will be lower than the market yield, as the loss is amortized. In a constant duration product the difference grows smaller each year, but is still amortized over the initial duration. This creates a declining drag on the credited rate; see Figure 1. With a fixed maturity product design (declining duration) the credited rate would have remained unchanged until the end of the fourth year, at which time market value would have equaled contract value.

With a constant amortization period the credited rate will change even if all other prior reset assumptions are realized. Over time the credited rate will approach the portfolio's net yield. In summary, the constant duration effect captures the effect of a constant duration product design as opposed to a fixed maturity product design.

The constant duration effect is calculated by assuming that the portfolio returns its net yield and the contract value grows at the credited rate. In mathematical notation,

\[
\Delta cr_0^{(CD)} = cr_1^{(CD)} - cr_0
\]

where

\[
\begin{align*}
mv_1^{(CD)} & = (mv_0 + wc_f_0) \times (1 + y_0 - e) \\
cv_1^{(CD)} & = (cv_0 + wc_f_0) \times (1 + cr_0) \\
cr_1^{(CD)} & = cr(mv_1^{(CD)}, cv_1^{(CD)} , 0, y_0 - e, d_0) \quad \text{and} \\
cr_0 & = cr(mv_0, cv_0, wc_f_0, y_0 - e, d_0).
\end{align*}
\]

3.3 The Reinvestment Effect

As interest rates change, the effect on the bond portfolio is twofold. A drop in interest rates tends to increase the portfolio's market value and decrease the portfolio's yield. In an instantaneous interest rate
shift these effects are largely offsetting, and the credited rate will remain stable.

As the portfolio moves through time in a lower interest rate environment, however, it tends to earn less coupon income. Thus, at the end of the year the portfolio's market value will reflect the impact of the lower coupon income. Consequently, the credited rate will be reduced. In other words, the overall impact on the credited rate should be in the same direction as the movement in market interest rates. In summary, the reinvestment effect captures the responsiveness of the credited rate to the movement in market interest rates.

The reinvestment effect is calculated by assuming that the yield on the portfolio changes by the change in interest rates and the portfolio return reflects the change in yield. We use Treasury yields as a proxy for interest rates. Using Treasury yields and temporarily ignoring the effect of spread, credit, and prepayment factors, the portfolio return is
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given by:

\[ R_{RI} = (1 + y_0 - e + \frac{(T_1 - \bar{T}_e)}{2}) \left( \frac{1 + y_0 - e}{1 + y_0 - e + (T_1 - \bar{T}_e)} \right)^{d_0} - 1 \]  \hspace{1cm} (11)

where

\[ T_t = \text{The Treasury rate at time } t \ (0 \leq t \leq 1); \]
\[ \bar{T}_e = \frac{mv_0 \times T_0 + wc_{f0} \times \bar{T}_{av}}{mv_0 + wc_{f0}}; \text{ and } \]
\[ \bar{T}_{av} = \text{A weighted average of Treasury rates over time.} \]

Note that \( \bar{T}_{av} \) is given in the examples that follow. Also, \( \bar{T}_e \) is the weighted average of Treasury rates using the initial market value and the expected cash flows as weights.

In mathematical notation,

\[ \Delta cr_0^{(RI)} = cr_1^{(RI)} - cr_1^{(CD)} \]  \hspace{1cm} (13)

where

\[ mv_1^{(RI)} = (mv_0 + wc_{f0})(1 + R_{RI}) \]  \hspace{1cm} (14)
\[ y_1^{(RI)} = y_0 + T_1 - \bar{T}_e \text{ and } \]
\[ cr_1^{(RI)} = cr(mv_1^{(RI)}, cr_1^{(CD)}, 0, y_1^{(RI)} - e, d_0). \]  \hspace{1cm} (16)

3.4 The Cash Flow Effect

Many participating GICs accept ongoing contributions and make benefit payments at book value. Cash flow expectations are reflected in the reset formula, i.e., in the prior reset assumption \( wc_{f0} \). If actual cash flow experience (amount and timing) matches expectations, we define the cash flow effect to be zero. Therefore, our cash flow effect incorporates deviations from both the expected amount of cash flow and expected interest rates (at which the cash flow was invested). In summary, the cash flow effect captures the effect of unexpected cash flows on the credited rate.

The cash flow effect is calculated by adjusting the market value and contract value to reflect the actual as opposed to the expected cash flows.

\[ R_{CF} = (1 + y_0 - e + \frac{(T_1 - \bar{T}_{ac})}{2}) \left( \frac{1 + y_0 - e}{1 + y_0 - e + (T_1 - \bar{T}_{ac})} \right)^{d_0} - 1 \]  \hspace{1cm} (17)
where
\[ T_{\text{ac}} = \frac{m_{0} \times T_{0} + \sum_{i=0}^{\infty} c_{f_{i}} \times T_{t_{i}}}{m_{0} + \sum_{i=0}^{\infty} c_{f_{i}}} \] (18)
and \( c_{f_{i}} \) is the actual \( i \)-th cash flow at time \( t_{i} \), with \( 0 = t_{0} < t_{1} < t_{2} < \ldots \). Note that \( T_{\text{ac}} \) is the weighted average of Treasury rates using the initial market value and the \textit{actual} cash flows as weights.

In mathematical notation,
\[ \Delta c_{r_{0}}^{(\text{CF})} = c_{r_{1}}^{(\text{CF})} - c_{r_{1}}^{(\text{RI})} \] (19)
where
\[
\begin{align*}
acf_{0} &= \text{The actual cash flow received;}
m_{1}^{(\text{CF})} &= (m_{0} + acf_{0}) \times (1 + R_{\text{CF}}) \quad (20) \\
cc_{1}^{(\text{CF})} &= cv_{1} \quad \text{and} \\
cc_{1}^{(\text{CF})} &= cr(\text{m}_{1}^{(\text{CF})}, cv_{1}^{(\text{CF})}, 0, \gamma_{1}^{(\text{RI})} - e, d_{0}). \quad (22)
\end{align*}
\]

3.5 The Investment Effect

This investment effect component can be further divided into five subcomponents: investment market, spread, investment manager, time-weighted and nonconstant duration. From equation (4), we know that
\[ \Delta c_{r_{0}}^{(\text{IV})} = \left( c_{r_{1}} - c_{r_{0}} \right) - \left( c_{r_{1}}^{(\text{CD})} - c_{r_{0}} \right) \\
- \left( c_{r_{1}}^{(\text{RI})} - c_{r_{1}}^{(\text{CD})} \right) - \left( c_{r_{1}}^{(\text{CF})} - c_{r_{1}}^{(\text{RI})} \right) \\
= c_{r_{1}}^{(\text{CF})}. \quad (23) \]
It follows that the sum of the effects of the five subcomponents must satisfy equation (23). We now review each of these subcomponents individually.

3.5.1 The Investment Market Effect

The investment market effect captures the impact of the chosen investment universe on the credited rate. The investment universe is represented by the index or other benchmark selected to evaluate the portfolio's performance. Participating GIC contracts allow for a variety of portfolio structures and benchmarks, developed by mutual agreement between the plan sponsor and the investment manager.

The investment market effect is designed to provide a basis for evaluating benchmark performance by a manager. (The basis or proxy is
indexed management or a manager whose return exactly matches the chosen benchmark.) The performance of the index is compared to the rate needed to support the initial credited rate. This is expressed in terms of a Treasury-plus-spread benchmark. Treasuries are used because they are the universal underpinning of all fixed income prices and returns.

We calculate the investment market effect by performing a rate reset with a market value calculated using the index return. In mathematical notation,

\[ \Delta cr_0^{(IM)} = cr_1^{(IM)} - cr_1^{(CF)} \]  

(24)

where

\[ R_{IX} = \text{The index return.} \]

\[ mv_1^{(IX)} = (mv_0 + acf_0)(1 + R_{IX}) \] and  

(25)

\[ cr_1^{(IM)} = cr(mv_1^{(IX)}, cv_1, 0, y_1^{(RI)} - e, d_0). \]  

(26)

The comparison of a standard market index, such as the Lehman aggregate or government/corporate index, to a Treasury-plus-spread benchmark implicitly assumes the use of non-Treasury assets. The actual impact on the credited rate is derived from the fact that the total return of the index incorporates initial spread plus changes in spreads, while the Treasury-plus-spread proxy only incorporates the initial spread.

For example, the Lehman MBS index may yield 100 basis points over Treasuries at the start of the year, with a four year index duration. Assume that mortgage spreads widen 50 basis points during the year, with Treasury rates and durations remaining unchanged. The mortgage portfolio will underperform duration-matched Treasuries by roughly 75 basis points during the year (assuming spread change occurs uniformly over the year). The investment market effect will be roughly -44 basis points (-75 basis points actual performance versus +100 basis points assumed performance produces a 175 basis point difference amortized over four years). The spread widening will be reflected in the spread effect (see below) of +50 basis points, more than offsetting the investment market effect.

3.5.2 The Spread Effect

Spread widening/tightening may offset the investment market effect results because widening asset spread to Treasuries negatively impacts portfolio return. The impact of spread changes on portfolio return is
recognized in the investment market effect. The impact on the yield to maturity is recognized in the spread effect. These effects therefore should be viewed together rather than individually. An increase in spread translates directly into a like increase in yield which in turn translates directly into a like increase in the credited rate. The opposite holds for a decrease in spread. Thus

\[ \Delta cr_0^{(SP)} = cr_1^{(SP)} - cr_1^{(IM)} \]  

(27)

where

\[ cr_1^{(SP)} = cr(mv_1^{(IX)}, cv_1, 0, y_1 - e, d_0). \]  

(28)

3.5.3 The Investment Manager Effect

The investment manager effect is designed to show the effect of a manager’s performance (relative to a benchmark) on the credited rate. The manager’s relative performance (also called the excess return) is \( R_{MG} - R_{IX} \) where \( R_{MG} \) is the portfolio return.

We compute the investment manager effect by performing a rate reset with a market value calculated using the actual portfolio return. In mathematical notation,

\[ \Delta cr_0^{(MG)} = cr_1^{(MG)} - cr_1^{(SP)} \]  

(29)

where

\[ mv_1^{(MG)} = (mv_0 + acf_0)(1 + R_{MG}) \quad \text{and} \]  

(30)

\[ cr_1^{(MG)} = cr(mv_1^{(MG)}, cv_1, 0, y_1 - e, d_0). \]  

(31)

This effect can be analyzed in further detail using standard portfolio management attribution techniques. These techniques typically allocate the manager’s excess return to categories such as duration management, yield curve management, sector and security selection, and other factors relative to the performance benchmark. This analysis can be incorporated easily into the credited rate attribution framework by applying a factor of \( (1/d_0) \) to each category in the portfolio attribution analysis. For example, if the manager added 15 basis points of return through duration management for a five year duration portfolio, the credited rate impact would be \( (15/5 =) \) 3 basis points.
3.5.4 The Time-Weighted Effect

So far we have used the time-weighted rates of return\(^4\) adjusted for the impact of actual cash flows for three previous effects (cash flow effect, the investment market effect, and the investment manager effect). As a result, there is a residual effect, which we call the time-weighted effect that is given by:

\[
\Delta cr_0^{(TW)} = cr_1^{(TW)} - cr_1^{(MG)}
\]  

(32)

where

\[
cr_1^{(TW)} = cr(mv_1, cv_1, 0, y_1 - e, d_0).
\]  

(33)

3.5.5 The Nonconstant Duration Effect

The duration of the portfolio at the current reset (time 1) probably will change from the duration at the prior reset (time 0). Regardless of the reason (rate anticipation strategy, revised client objectives, duration drift), duration changes will affect both the portfolio return (investment manager effect) and the amortization period. For example, duration shortening will accelerate the recognition of any gains or losses.\(^5\)

The duration effect is calculated by substituting the actual portfolio duration at time 1. In mathematical notation,

\[
\Delta cr_0^{(ND)} = cr_1^{(ND)} - cr_1^{(TW)}
\]  

(34)

where

\[
cr_1^{(ND)} = cr(mv_1, cv_1, 0, y_1 - e_1, d_1).
\]  

(35)

3.6 Other Considerations

3.6.1 Investment Risks

Plan sponsors are accustomed to thinking in terms of investment risks including reinvestment risk, credit risk, prepayment risk, liquidity risk, and active management risk. These risks are inherent in one or more of the four components or effects introduced in Section 2 above. In particular,

\(\text{4} The dollar-weighted return may have been used instead, but it requires more complicated calculations.\)

\(\text{5} This effect refers to the end-of-year portfolio duration, not the management of the portfolio duration during the year. Duration management has been accounted for in the investment manager effect.\)
• Reinvestment risk is captured by the reinvestment effect;

• Credit risk, i.e., a decline in credit quality, appears as a negative impact in the investment market effect and a positive impact in the spread effect;

• Prepayment (extension) risk appears as a negative impact in the investment market effect and presumably a positive impact in the spread effect. In addition, prepayments (extensions) reduce (increase) duration and thus the amortization period. The direction of the impact on the duration effect depends on the ratio of market value to book value;

• The realization of liquidity risk appears as a negative impact in the investment market effect and a positive impact in the spread effect; and

• Active management risk affects the investment manager effect.

3.6.2 Treatment of Cash Flows

We ignore future cash flow expectations for the current reset, i.e., we assume \( wc_f = 0 \). If \( wc_f \) does not equal zero, the future cash flow assumption will affect the credited rate because it affects the market-to-contract value ratio. Positive assumed cash flow increases the credited rate when market value is less than book value. Negative assumed cash flow decreases the credited rate when market value is less than book value. The effects are reversed in a situation where market value is above contract value.

Though cash flow is not a significant element for most contracts, some contracts have significant and frequent cash flows. For these contracts cash flows can be handled more precisely by using dollar-weighted portfolio returns instead of time-weighted portfolio returns. Using dollar-weighted returns will eliminate the time-weighted effect described in Section 3.5.4. Otherwise, the change in credited rate may be incorrectly allocated between the cash flow effect, the investment market effect, and the investment manager effect. The allocation problem concerns the timing of any market or manager outperformance during the reset period.

3.6.3 Participant Equity

The smoothing mechanism raises the issue of equity among plan participants. For instance, when the market value is below contract
value, i.e., \( m v_t < c v_t \), the credited rate will be lower than current the market yield, i.e., \( c r_t < y_t \) as indicated by equation (2). A participant who withdraws funds at contract value causes the portfolio to realize a market value loss. As the remaining participants participate in the experience of the portfolio, they will be hurt by the withdrawal in the form of lower future credited rates.

The participant equity issue is inherent in any blended rate stable value plan. The extent of the participant equity issue depends on the overall plan design and operation, not just the characteristics of one of the plan's investments. By helping plan sponsors to better understand the behavior of one possible investment (the participating GIC), the attribution methodology can help plan sponsors better address the equity issue.

4 Two Examples

4.1 Example 1: Prior Reset Assumptions Realized

4.1.1 Prior Reset Assumptions

These assumptions were made one year ago (i.e., at time \( t = 0 \)), the last time the credited interest rate was reset:

- The portfolio had a 2.1% market value loss. This implies that \( m v_0 = (1 - 0.021) \times c v_0 = 97.9\% \) of \( c v_0 \);
- \( m v_0 = 93.0 \) and \( c v_0 = 95.0 \);
- A $5 cash flow expected at \( t = 0.5 \), i.e., \( w c f_0 = 5.0 \);
- Cash flows are not time-weighted;
- The portfolio net yield will be \( y_0 - e = 8\% \);
- Interest rates will remain unchanged; and
- On average, funds are to mature in four years, making \( d_0 = 4 \).

From equation (2) we have

\[
c r_0 = \left( \frac{93.0 + 5.0}{95.0 + 5.0} \right)^{1/4} \times (1 + 0.08) - 1
= 0.0746.
\]
4.1.2 Actual Activity Since Prior Reset

As we prepare to reset the interest rate today (i.e., at time $t = 1$), we must review the actual performance of the contract. For this example, let us assume all prior reset assumptions were realized, meaning:

- Actual cash flow of $5 at $t = 0.5$;
- No change in interest rates;
- The portfolio actually returned a net yield of 8%; and
- On average, funds are now expected to mature in three years, making the duration $d_1 = d_0 - 1 = 3$.

4.1.3 The Calculations

Once the assumptions are exactly realized, the credited rate must be equal to the constant duration credited rate, i.e.,

$$cr_1 = cr_1^{(CD)}.$$

So, using equations (7) and (8) we see that:

$$mv_1 = 105.84 \text{ from equation (7)};$$
$$cv_1 = 107.46 \text{ from equation (8)}; \text{ and}$$
$$y_1 - e = y_0 - e = 8\%.$$

Equation (9) gives

$$cr_1 = cr(105.84, 107.46, 0, 8\%, 3) = 7.46\%.$$

As the assumptions were exactly realized, it is not surprising that the credited rate is unchanged from the initial credited rate established at time 0.

4.2 Example 2: Prior Reset Assumptions Not Realized

In developing this example, we use the same set of prior reset assumptions that are described in Section 4.1.1. As a result $cr_0 = 7.46\%$ as before.
4.2.1 Actual Activity Since Last Reset

As we prepare to reset the interest rate today, the actual activities since last reset are listed below:

- Interest rates (using Treasuries as a proxy) declined 1% from time 0 to time 1. Specifically, Treasury rates were $T_0 = 7.50\%$, $T_{0.5} = 7.00\%$, $T_1 = 6.50\%$, and $\bar{T}_{av} = 7.00\%$;

- The only actual cash flow received was $\$15$ ($acf_0 = 15$) immediately following the prior reset at time 0;\(^6\)

- $R_{IX} = 0.11$, i.e., the index returned 11% from time 0 to time 1 (net of fees $e$);\(^7\)

- The manager returned 11.50% from time 0 to time 1 (net of contract fees $e$), i.e., $R_{MG} = 11.5\%$. (The manager outperformed the index by 50 basis points);

- The portfolio duration at time 1 is 4.25 years, i.e., $d_1 = 4.25$. In other words, the portfolio lengthened by 1/4 year;

- The actual portfolio net yield at time 1 is 7.10%, i.e., $\gamma_1 - e = 7.10\%$;\(^8\)

This gives

$$
\begin{align*}
mv_1 &= (93 + 15) \times 1.115 = 120.50 \\
cv_1 &= (95 + 15) \times 1.0746 = 118.21 \quad \text{and} \\
cr_1 &= cr(120.50, 118.21, 0, 7.10\%, 4.25) = 7.58\%.
\end{align*}
$$

4.2.2 The Component Effects

Constant Duration Effect: From equation (6),

$$
\begin{align*}
mv_1^{(CD)} &= (93 + 5) \times 1.08 = 105.84 \\
cv_1^{(CD)} &= (95 + 15) \times 1.0746 = 107.46 \\
cr_1^{(CD)} &= cr(105.84, 107.46, 0, 8\%, 4) = 7.59\%.
\end{align*}
$$

\(^6\)The additional cash flow was received while the contract was in deficit (market value below contract value). This unexpected cash flow reduced the deficit and thus increased the credited rate. The subsequent fall in interest rates during the year has been captured in the reinvestment effect.

\(^7\)The index returned 58 basis points less than the Treasury-plus-spread bogey.

\(^8\)Thus, yields fell 90 basis points (from 8% to 7.10%) from time 0 to time 1 while interest rates fell 100 basis points. In other words, spreads increased 10 basis points.
giving
\[ \Delta cr_0^{(CD)} = 7.59\% - 7.46\% = 0.13\%. \]

**Reinvestment Effect:** From equation (12), \( \hat{T}_e = 7.47 \), so \( R_{RI} \) can be calculated from equation (11) to give \( R_{RI} = 11.47\% \). This rate ignores the effect of spread, credit, and prepayment factors. Thus, from equations (14), (15) and (16), we have

\[
\begin{align*}
mv_1^{(RI)} &= (93 + 5) \times (1 + 0.1147) = 109.24 \\
\gamma_1^{(RI)} - e &= \gamma_0 - e + T_1 - \hat{T}_e = 7.03\% \\
\cr_1^{(RI)} &= \cr(109.24, 107.46, 0, 7.03\%, 4) = 7.47\%
\end{align*}
\]

giving
\[ \Delta cr_0^{(RI)} = 7.47\% - 7.59\% = -0.12\%. \]

**Cash Flow Effect:** To isolate the effect of cash flows we adjust the \( mv \) and \( cv \) terms for the amount and timing of actual cash flow, \( acf_0 \). As there was only one cash flow and it occurred at time 0, then \( \hat{T}_{ac} = T_0 = 7.5\% \). Accounting for unexpected cash flow, the portfolio returned 11.58\%, i.e., \( R_{CF} = 0.1158 \) (after using equation (17)).

\[
\begin{align*}
mv_1^{(CF)} &= (93.0 + 15.0) \times (1 + 0.1158) = 120.51 \\
\gamma_1^{(CF)} &= \gamma_1 = 118.21 \\
\cr_1^{(CF)} &= \cr(120.51, 118.21, 0, 7.03\%, 4) = 7.55\%
\end{align*}
\]

giving
\[ \Delta cr_0^{(CF)} = 7.55\% - 7.47\% = 0.08\%. \]

**Investment Market Effect:** Using equations (2), (25) and (26) to get:

\[
\begin{align*}
mv_1^{(IM)} &= (93.0 + 15.0) \times (1 + 0.11) = 119.88 \\
\cr_1^{(IM)} &= \cr(119.88, 118.21, 0, 7.03\%, 4) = 7.41\%
\end{align*}
\]

giving
\[ \Delta cr_0^{(IM)} = 7.41\% - 7.59\% = -0.14\%. \]
Spread Effect: Using equation (28) we find:

\[ cr_1^{(SP)} = cr(119.88, 118.21, 0, 7.10\%, 4) = 7.48\% \]

giving

\[ \Delta cr_0^{(SP)} = 7.48\% - 7.41\% = 0.07\%. \]

Investment Manager Effect: Using equations (30) and (31), we find:

\[ mv_1^{(MG)} = (93.0 + 15.0) \times (1 + 0.115) = 120.42 \]
\[ cr_1^{(MG)} = cr(120.42, 118.21, 0, 7.10\%, 4) = 7.60\% \]

giving

\[ \Delta cr_0^{(MG)} = 7.60\% - 7.48\% = 0.12\%. \]

Time-Weighted Effect: Using equations (32) and (33), we find:

\[ cr_1^{(TW)} = cr(120.42, 118.21, 0, 7.10\%, 4) = 7.60\% \]

giving

\[ \Delta cr_0^{(TW)} = 7.60\% - 7.60\% = 0.00\%. \]

Note that the time-weighted return equals the dollar weighted return since the only cash flow occurs at time 0.

Duration Effect: Using equations (34) and (35) we find:

\[ cr_1^{(ND)} = cr(120.42, 118.21, 0, 7.10\%, 4.25) = 7.57\% \]

giving

\[ \Delta cr_0^{(ND)} = 7.57\% - 7.60\% = -0.03\%. \]

4.3 Summary of Attribution

The credited interest rate in our example changed from 7.46 percent to 7.57 percent, a net increase of 11 basis points in a declining interest rate environment. The credited rate benefited from the amortization of a previous market value loss, unexpected cash flow when market rates were favorable, manager outperformance, and widening spreads. Adverse factors included a declining interest rate environment, a longer portfolio duration, and underperformance by the portfolio’s investment universe relative to initial assumptions.
Table A1
Summary of Components

<table>
<thead>
<tr>
<th>Prior Credited Rate</th>
<th>7.46%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Constant Duration Effect</td>
<td>+0.13%</td>
</tr>
<tr>
<td>2. Reinvestment Effect</td>
<td>-0.12%</td>
</tr>
<tr>
<td>3. Cash Flow Effect</td>
<td>+0.08%</td>
</tr>
<tr>
<td>4. Investment Effects</td>
<td></td>
</tr>
<tr>
<td>A. Investment Market</td>
<td>-0.14%</td>
</tr>
<tr>
<td>B. Spread</td>
<td>+0.07%</td>
</tr>
<tr>
<td>C. Investment Manager</td>
<td>+0.12%</td>
</tr>
<tr>
<td>D. Time-Weighted</td>
<td>0.00%</td>
</tr>
<tr>
<td>E. Duration</td>
<td>-0.03%</td>
</tr>
<tr>
<td>Total</td>
<td>+0.02% +0.02%</td>
</tr>
</tbody>
</table>

New Credited Rate | 7.57%

5 Conclusion

The increasing use of participating GICs within GIC/stable value portfolios has created a need for objective measures of performance attribution. When a credited rate is reset, providers should expect to provide detailed attribution analyses that explain how the new credited rate was derived.

Current explanations discuss, in general terms, such concepts as issue selection and yield curve positioning. These explanations ignore, however, other important concepts such as product design (constant duration effect), credited rate responsiveness (reinvestment effect), cash flows (cash flow effect), initial return assumption (investment market effect), and amortization period (duration effect). We have presented a framework to explain the performance of participating GICs by isolating the effects of all relevant factors.

Our approach analyzes these effects in static fashion and in a particular order. We recognize that in actuality these components are likely to affect performance in a dynamic way.