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Learning Opportunities of Multiplication Fluency in Open Source K-5 Mathematics Curriculum

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LEARNING OPPORTUNITIES OF MULTIPLICATION FLUENCY IN OPEN SOURCE K-5 MATHEMATICS CURRICULUM

by

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LEARNING OPPORTUNITIES OF MULTIPLICATION FLUENCY IN OPEN SOURCE K-5 MATHEMATICS CURRICULUM

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Three open source K-5 mathematics curricula, *Bridges in Mathematics, Eureka Mathematics,* and *Texas Go Math!,* were analyzed and coded to determine what learning opportunities they provided for students to develop multiplication fluency. Multiplication fluency is achieved when conceptual and procedural knowledge are layered to allow the learner to process the information. Across the three sets of materials, 429 items were coded, with 69.93% being coded as items that afforded the development of procedural knowledge. One of the goals of this study was to determine how curricular materials support rote learning versus more layered [scaffolded] learning. It was determined that *Texas Go Math!* was the curricular material that provided the most layered learning opportunities for multiplication fluency.

Keywords: multiplication fluency, rote learning

DEDICATION

To Schyler Fujimoto, and Yvonne Johnson, whom I would not have survived this process without. Thank you for your countless hours of reading and providing feedback as well as encouragement!

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CHAPTER 1: INTRODUCTION

Problem Statement

Understanding how mathematics curriculum is intended to be used to build multiplication fluency allows teachers to create opportunities for students to learn multiplication fluency in the most productive way. When teachers do not understand the intentions of the written curricular materials, they may choose to emphasize a concept that the curriculum author did not. Therefore, it is seldom shown in the curriculum, which can lead to an incomplete understanding of the concept. With a better understanding of the curriculum, teachers could make choices to support students' understanding of the concepts and procedures to improve multiplication fluency for their students.

Curriculum materials are often given to teachers, and they are expected to know how to use them. In my experience, teachers are given several curricular materials and told to produce lessons. It is important that teachers first understand the curricular materials they are given so the lessons they produce align with the written curriculum as it is written, before enactment. Curriculum materials come in many different formats, such as curriculum maps, scope and sequences, textbooks, lesson plans, and district mandated assessments. Teachers sometimes make decisions about curriculum without knowing the full implications of those decisions. When elementary teachers are given mathematics curricula, they often do not know the reasoning behind the authors' layout of the materials, problem selection, and specific language used. Remillard (1996, 1999, 2000) conducted multiple studies about how teachers use mathematics curricular materials and found "Not only did the two teachers read entirely separate parts of the textbooks (exercises for students as opposed to supplementary activities), but they also read for different purposes (potential activities and assignments as opposed to big ideas to guide planning)" (Remillard, 2005, p. 222).

Framework

Framework Lens

When studying mathematics curricula, a researcher can focus on different aspect of the curriculum or curriculum in use, such as: how the teachers interpret these materials, how the author or authors intend the materials to be used, or how teachers' own self-efficacy determines their reliance on the materials (Remillard, 2005). This thesis focuses on textbooks or curricula materials without taking into account teachers' perspectives. Remillard (2005) goes on to say, "Studying the relationships between written curriculum material and the enacted curriculum necessarily involves understanding teachers' processes of constructing the enacted curriculum. Including the role that resources, such as curriculum materials, play in the process" (p. 213). Therefore, the lens that will be used as a framework is what possible opportunities to learn are presented in the written curricular materials.

Sociocultural Theory

Researchers ground their work in theory. It was important for me to work within constructivism and relate my work to foundational theorists such as Bruner, Dewey, Piaget, and Vygotsky. Curricular materials can be viewed as Vygotskian "cultural tools" from his idea of sociocultural theory because they communicate language, stories, works of art, signs and models to students in classrooms (Aubrey & Riley, 2017). How language is communicated through the "cultural tool" is of utmost importance. Mathematics has its own unique language and curricula materials need to communicate that language effectively in order to improve mathematics literacy and fluency.

Vygotsky is considered by many to be an early constructivist, but he is also credited with developing sociocultural theory. Researchers use sociocultural theory to interpret how teachers interact with their curricular materials, but it should not be used to interpret the materials themselves because “within a sociocultural perspective learning mathematics involves participation in certain established mathematics cultural practices” (Steele, 2001, p. 404). Since there is no participation of students or teachers, sociocultural theory is not the lens that I feel is appropriate for this research. Therefore, constructivist theory, as developed by Vygotsky, Piaget, and Bruner, will be used to analyze what opportunities to learn multiplication fluency are presented in the written curricular materials.

Constructivist Theory

Constructivism is the theory that students create meaning based on exposure to new learning and relating that exposure to previous experiences. Clements & Battista (1990) say "…in constructivist instruction, students are encouraged to use their own methods for solving problems. They are not asked to adopt someone else's thinking but encouraged to refine their own” (p. 35), and teachers act as facilitators to help students make those connections to previous knowledge. For a curricula material to support constructivist learning, it needs to allow students to explore different ways to build their understanding. Multiplication can be thought of as repeated addition, skip counting, arrays, area models, number lines, combinations, and grouping (Smith & Smith, 2006).

Since multiplication is taught and understanding is constructed in so many formats, curricular materials' view of multiplication can be analyzed from a constructivist view.

Scaffolding

Bruner presented the idea of scaffolding student understanding. "Scaffolding, in practice, involves the learning being helped by an adult or another child (who possesses a greater level of knowledge) by starting tasks, simplifying problems and highlighting errors to a point where the child can do tasks by themselves" (Aubrey & Riley, 2017, p. 109). One example of scaffolding in curricular materials is when an idea is presented in a simple manner in the opening section, presented more in depth later in the chapter, and then summarized at the end of the chapter by asking the students to create their own meaning and understanding of the concept. For scaffolding to extend to a student's understanding of multiplication, the text might present the idea of skip counting first and then later advance to models such as arrays and repeated addition. At the end of the chapter or section, there might be a problem or task that asks students to create a real-life problem and model it using a multiplication technique they learned in the process. Scaffolding is only successful if students complete the entire process and reflect on their learning. Steele (2001) brings this point to light when she says "The teacher created a context in which students explored, reflected, and communicated their ideas, while they made connections from their ordinary personal language to formal mathematical language" (p. 411). The students were able to make these connections because the lesson was scaffolded in a way to create meaningful learning and then students reflected before sharing their ideas.

Piaget is known for his stages of development. For mathematical curricular materials, specifically for multiplication, manipulatives are a useful tool (Steffe & Kieren, 1994) and often require concrete modeling to build students' conceptual understanding. Piaget's third stage of development - concrete operations stage - encourages students to build their understanding of the world with hands-on materials (Aubrey & Riley, 2017). Ojose (2008) goes on to say:

> As students use the materials, they acquire experiences that help lay the foundation for more advanced mathematical thinking. Furthermore, students' use of materials helps to build their mathematical confidence by giving them a way to test and confirm their reasoning. (p. 28)

This can be evident in curricular materials by the way a chapter or section is scaffolded.

Purpose and Research Question

The purpose of this thesis is to determine how three different open source K-5 mathematics curricula provide multiplication fluency learning opportunities. The research question to be answered is: *What opportunities to learn multiplication fluency are present in three open source K-5 mathematics curricular series?*

By opportunity to learn, I base my definition on Smith, Males, Dietiker, Lee, and Mosier (2013) and Floden's (2002) explanations. Smith et al. (2013) took students' "opportunity to learn" as what does the written curriculum provide for students to develop deep understanding of measurement when teachers allocate significant instructional time to measurement? Floden (2002) took "opportunity to learn" to mean how long will it take all students to learn the content, when given sufficient time? The definition of "opportunity to learn" I used was what exactly does the curricular material provide in examples, problems, and texts for all students to have a deep understanding of the content assuming that adequate time is given.

Methods Overview

I selected three open source K-5 mathematics curricular materials to code and analyze to determine what opportunities to learn multiplication fluency were present. Materials were selected due to availability and alignment with the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). The first chapter regarding multiplication was selected to be coded, because opportunities to learn fluently were presented.

Definition of Key Terms

Multiplication fluency: Fluency, in reference to multiplication, is the ability to recall basic facts accurately and quickly (Brendefur, Strother, Thiede, & Appleton, 2015).

Rote learning: Rote learning is the method of practice, practice, practice, until the concept becomes memorized through the process (Geary, 2011).

CHAPTER 2: LITERATURE REVIEW

Overview

The purpose of this literature review is to understand the processes involved with achieving arithmetic fluency, specifically pertaining to multiplication. While reading about mathematical fluency and the different ways that can be achieved, research literature revealed that there is a high demand on brain function as it processes the new skills and stores the information (Brown, 2018; Rinne, Ye, & Jordan, 2020). Mathematical fluency is complicated to achieve because it requires practice in various forms (Geary, 2011; Lemaire & Siegler, 1995; Reed, Gemmink, Broens-Paffen, Krirchner, & Jolles, 2014), and multiple levels of procedural and conceptual understanding. It is no longer sufficient to say that a student is fluent in multiplication when they have memorized the times tables.

Different themes arose from the readings. The themes are cognitive demand when achieving fluency, different ways mathematical fluency is presented in texts, rote learning and its' implications for multiplication fluency, and procedural and conceptual ways to achieve multiplication fluency.

Cognitive Demand When Achieving Fluency

As a new skill is learned, the information is stored in various parts of the brain. Brown (2018), used functional magnetic resonance imaging (fMRI) to determine how the brain, specifically the "Default Mode Network," activates and deactivates as it processes tasks that start as unfamiliar, and become familiar over time. An unfamiliar task is when a new skill is presented for the first time. After practicing that skill, it becomes familiar. Brown (2018) found:

> Our interpretation of these results is that individuals who are less proficient with their math facts may need to rely more on counting and other strategies, so they deactivate the DMN [Default Mode Network] to a greater extent to make resources available for deliberate slower processes (i.e. attention, working memory, strategy use). (p. 64)

This means that students who are not fluent with multiplication are having to calculate the answer rather than retrieve it from a stored place within their brain. Therefore, they are placing a higher cognitive demand on their brain.

Rinne at al. (2020) suggest that students may not have an issue with multiplication, but rather their reading ability and fluency, which affects their ability to process written mathematical problems. During their study, students' multiplication fluency was tested by administering a paper-and-pencil test with a time limit. Students were required to read each problem and write an answer. If a student struggles with their reading fluency, they most likely do not have multiplication fluency; their ability to perform well on a timed assessment will likely be hindered. Rinne et al. (2020) explain "…poor reading fluency may have a greater effect on response times for calculations of products" (p. 112). As students achieve multiplication fluency, they are able to retrieve the stored multiplication fact and produce the answer within a timely manner, assuming the student had reading fluency.

Different Methods for Developing Mathematical Fluency

Throughout the process of reading articles for this literature review, it became apparent that there are many different ways textbooks, researchers, and classroom teachers support students in developing mathematical fluency by presenting facts, such as addition, subtraction and multiplication. During Geary's (2011) study, students were interviewed and asked to solve an arithmetic question which they had to respond to orally. Student explanations to the questions were coded as counting on fingers, verbal counting, retrieval, or decomposition. Counting on fingers and verbal counting were then categorized in more specific terms such as "min, max, sum, and other." "Min" implied that the student started at the larger addend, and then counted the number of times equivalent to the smaller addend. "Sum" implied that the student started at one and counted both addends. "Max" implied that the student started at the smaller addend, and then counted the number of times equivalent to the larger addend. "Retrieval" was when the student knows the fact well and produces the answer automatically. "Decomposition" was when a student broke the problem up into facts that they knew which were equivalent to the original problem.

When allowing students to use paper-and-pencil to problem solve and answer a question that required students to use multiplication fluency, Lemaire & Siegler (1995) found that students' answers could be coded as retrieval, repeated addition, counting sets of objects, writing down the problem, or responding with "I don't know." Reed et al. (2014) created an intervention where one group had to produce the answers to the multiplication problems by whatever method they were most comfortable with, while the other group had to select the correct answer. Unfortunately, Reed et al. (2014) did not specifically study the methods that students used to produce answers. Seeing as the study took place in the Netherlands, it cannot be assumed that the students used the same strategies advocated for in the Common Core States Standards for Mathematics for solving multiplication problems. The strategies listed in the Common Core States

Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) are:

> 1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 3. Construct viable arguments and critique the reasoning of others, 4. Model with mathematics, 5. Use appropriate tools strategically, 6. Attend to precision, 7. Look for and make use of structure, and 8. Look for and express regularity in repeated reasoning. (p. 6-8)

Regardless of how mathematical fluency is presented, it is important that students have basic arithmetic skills because, according to research conducted by Geary (2011), "early arithmetic skills are important for later mathematics achievement, above and beyond the influence of domain-general abilities and several other quantitative competencies" (p. 1549). Students can learn these skills through practice. However, without fluency of these basic skills, learning multiplication becomes challenging. There are many more multiplication facts than addition and subtraction (Rinne et al., 2020) because subtraction and addition facts are patterned, and learning the basic patterns is more easily achievable than with multiplication.

Rote Learning and Implications for Multiplication Fluency

Rote learning is the process of memorization for before achieving procedural fluency. Knowing that three times five is fifteen by practicing writing 3 x 5 = 15 over and over again is a form of rote learning; using flashcards with 3 x 5 = _____ on them is another form of rote learning. In the Common Core Standards for Mathematics (Governors Association Center for Best Practices and the Council of Chief State School

Officers, 2010) third-grade mathematics standards, there is one multiplication standard, that says:

> 7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. (p. 23)

Oftentimes, teachers run out of time during the school year to teach all the standards (Tye & O'brien, 2002). They have to make decisions about which standards to cut or shorten. Unfortunately, in my experience as a former mathematics teacher, this particular standard is shortened, because it does not address how fluency should be taught. In fact, the use of the word "memory" might lead teachers to believe that rote learning is the only appropriate way to obtain fluency. Memorization is important, and acceptable, once students understand the conceptual knowledge connected to the rote procedural knowledge. Berrett and Carter (2017) suggest that students should be modeling the problems or process for conceptual understanding before memorization. They state "… students learn to conceptualize and memorize more complex math facts through an interactive modeling and practice process" (p. 235).

Foster (2018) studied the effects of having students learn concepts embedded in rich tasks. An "etude" is defined as a rich mathematical task, where the focus is on students performing a mathematical procedure that is embedded within the task. He goes on to say, "…an etude cannot simply be a problem which provides an *opportunity* for students to use the desired procedure; it must place that procedure at the centre of the students' activity and force its repeated use" (p. 123), meaning that teachers need to be thoughtful when implementing tasks to ensure that the students are practicing the procedure, and therefore building fluency. Harvey-Swanston (2017) understood the development of fluency to be “a flexible approach to derive new ones [facts] (Russell, 2000) demands a deep conceptual understanding of multiplication" (p. 20) rather than just the memorization of facts.

Procedural and Conceptual Ways to Teach Multiplication for Fluency

Brown (2018), Izsák and Beckmann (2019), and Rittle-Johnson, Siegler, and Alibali (2001) write about the importance of procedural and conceptual fluency when teaching mathematics. Izsák & Beckmann (2019) say:

> We think that mathematics education as a field should seek more completely worked out, coherent approaches to the MCF [multiplicative conceptual field] based on consistency and logical interconnections. The absence of such articulation may be constraining our capacity to help students and teachers use prior knowledge and experience to effectively relate topics and construct interconnected bodies of knowledge. (p. 100)

“Consistency and logical interconnections” relates directly to Brown’s (2018) work relating cognitive demand to understanding. When a student lacks understanding of a concept, there is a high cognitive demand on the brain to piece previous knowledge together to gain a new understanding of the concept. Interconnections are scaffolded conceptual and procedural knowledge that “…span topics that would likely be taught at different grade levels” (Izsák & Beckmann, 2019, p. 100).

Rittle-Johnson et al. (2001) understand procedural and conceptual fluency to develop together rather than in a specific order. When applying this to multiplication, the problem 14 x 2 might be solved procedurally by rewriting the problem vertically with fourteen above two. The problem is then solved by multiplying 4 x 2 first and then 1 x 2 last. Conceptually, the problem is broken up into two separate problems and their sum gives the final answer. The first problem is 4 x 2, and the second problem is 10 x 2. Both processes produce the same answer of twenty-eight, however the decomposition allows for a much deeper conceptual understanding of multiplication. Ideally, multiplication would be taught with iterative conceptual understanding and procedural fluency (Rittle-Johnson et al., 2001). The National Council of Teachers of Mathematics (NCTM) produced *Principles to Action* (2014) with eight teaching practices to follow. The sixth practice says:

> **Build procedural fluency from conceptual understanding**. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. (p. 10)

This sixth practice aligns perfectly with the idea of scaffolding conceptual and procedural knowledge to produce the most in-depth understanding because it uses the iterative process of building knowledge that Rittle-Johnson et al. (2001) mention.

Summary

In order to develop fluency, the brain hardwires pathways of related facts; recalling these facts strengthens the cognitive demand allowing for quick retrieval (Brown, 2018). However, this is not the case for all, because multiplication fluency is related to reading fluency (Rinne et al., 2020). If a student is not reading fluently, it is possible they will not demonstrate multiplication fluency, regardless of their age. In order to create opportunities for students to build fluency, teachers might want students to gain conceptual knowledge through the use of enriched tasks (Foster, 2018), which in turn can improve their procedural knowledge (Rittle-Johnson et al., 2001), and therefore improve overall fluency.

CHAPTER 3: METHODS

Overview

In this study, I analyzed three open-source third grade mathematics curricular materials to determine what opportunities to learn multiplication fluency they provided. The materials were selected based on availability. They are free and downloadable. Due to the need for an expedited analysis, I chose to analyze the chapter or section where multiplication was first introduced. Analyzing the first chapter is appropriate because this provides insight into how students may first be introduced to multiplication, which has implications for the development of multiplication fluency. The materials were also selected for their alignment with the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). The coding schema was adapted from Smith, Males, Dietiker, Lee, and Mosier's (2013) study that investigated students' opportunities to learn to measure length presented in three elementary curricular series and whether the opportunities addressed known students learning challenged with length measurement.

Context of the Study

Due to COVID-19, I had to modify my study. My original study involved an intervention with third grade students in an after-school program to determine how students develop multiplication fluency. Since meeting with students was not possible, I chose to investigate the problem from a different perspective. I chose to analyze published curricula materials to determine what learning opportunities are available for students to develop multiplication fluency. This study is still valuable as mathematics curricular materials have a strong influence on what teachers plan and enact and therefore what students have opportunities to learn (Brown & Edelson, 2003; Tarr, Chávez, Reys, & Reys, 2006). Multiplication has been taught differently as of 2010, when the Common Core State Standards emphasized focus on conceptual understanding rather than procedural fluency [rote memorization] (Bidwell, 2014). Since this change has been implemented, many students no longer have the procedural fluency and when they get to later grades it becomes a frustration/problem for the students as well as the teachers who feel like they have to teach content below their grade-level (Sezer, Güner, & Ispír, 2012). The hope was that these curricula materials would present numerous opportunities for students to learn multiplication conceptually and procedurally so they would have fluency.

Materials Used

I selected three mathematics curricula based on availability and alignment with Common Core State Standards for Mathematics. The materials are available online for download. All materials were published in 2015 and were specific to third-grade. Third-grade is the first time in standards that students are formally introduced to multiplication. The first section or chapter where multiplication is introduced was the focus since the goal was to understand what opportunities to learn multiplication fluency were present and examining these first chapters provides insight into how students are introduced to multiplication. The materials were coded in three parts. First, to determine the audience, either the student or teacher. Then, the knowledge type was determined as either conceptual or procedural. Lastly, the materials were coded as demonstrations, worked examples, statements, questions, or problems. Coding was selected as the methodology because it was an objective way to determine how the curricular materials presented opportunities to learn multiplication fluency. I was a mathematics teacher and have a Bachelor of Science in mathematics with an emphasis in education. The three curricula materials I selected were: *Eureka Math* (Great Minds, 2015) - henceforth *EM; Bridges in Mathematics* (The Math Learning Center, 2015) - *BIM*; and *Texas Go Math!* (Houghton Mifflin Harcourt Publishing Company, 2015) - *TGM.*

BIM's "Home Connections" book was supplemental material that went along with the lessons and provided students the opportunity to build multiplication fluency (Math Learning Center, n.d.).

Data Collection & Analysis

As Rittle-Johnson et al. (2001) suggested, to build conceptual and procedural knowledge is an iterative process and "Increases in one type of knowledge lead to gains in the other type of knowledge, which in tun lead to further increases in the first" (p. 347). When selecting a coding schematic, it was important that the schematic incorporated the different types of knowledge. Therefore, I adapted the coding schematic from Smith et al. (2013).

The adapted schema (see Figure 1) differs from the schematic Smith et al. (2013) created since multiplication fluency does not fit within conventional knowledge. They defined conventional knowledge to be knowledge that is arbitrary to the mathematical concept but necessary for defining measurement (e.g., the unit of inches). Smith et al. (2013) produced a coding technique that broke the curricular materials into Teacher Version and Student Version. If a particular problem was only in the teacher version, then it could influence the opportunity to learn for the students (Smith et al., 2013). Within each version, the texts were coded based on procedural, or conceptual knowledge. Rittle-

Johnson and Star (2007) define procedural knowledge as "...the ability to execute action sequences to solve problems, including the ability to adapt known procedures to novel problems…" (p. 562). They go on to define conceptual knowledge as a generalizable and flexible grasp of mathematical ideas. After the knowledge type was determined, Smith et al. (2013) coded the curricula materials into demonstrations, worked examples, statements, questions, and problems. They used those specific coded texts because they are "textual forms that expressed length measurement content" (p. 404). I kept with Smith et al. (2013) coded texts because those codes also pertain to multiplication fluency: however, the coded texts do not align perfectly with Smith et al. (2013), so they are defined further. Demonstrations were games where students had rules to follow in order to demonstrate their understanding. Worked examples were examples from the text that had been partially completed or had some type of modeling to show what was expected. Statements were actions such as draw, write, describe. Questions were queries, such as, the following question presented in *TGM* on page 171: "Rereading the examples at the top of the page. How does this example show the Commutative Property of Multiplication?" Problems were skills that were completed for practice and without context. For example, skip counting by fives without context was coded as a problem.

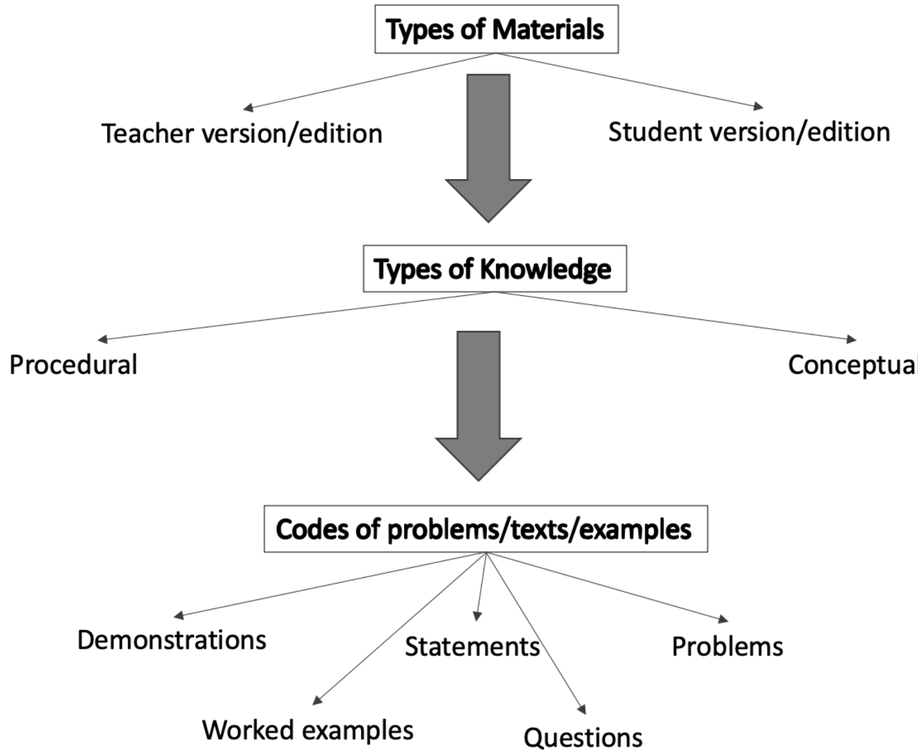


Figure 1. The schematic used for coding, adapted from Smith et al. (2013)

Coding Procedures

The first step in my analysis was to determine whether the curricular material was related to multiplication. If it was, I assigned a materials type code (i.e., teacher version, student version). I then assigned a knowledge type code (i.e., conceptual, procedural). Lastly, I assigned a coded text (i.e., demonstration, worked example, statement, question or problem).

Each curricular material had its own way of showing conceptual knowledge. *BIM* used “Count-Arounds”, which were a whole class way to skip count. This was considered conceptual knowledge because students were stopped and asked to make observations and notice patterns. The materials indicated the teacher should keep a list on the white board when they counted by three’s and then underneath it in a different color, they could make a new list when counting by sixes. *EM* provided an opportunity for students to develop conceptual knowledge through correction. For example, in lesson 1, in the problem set on page 26 says “2. The picture below shows 2 groups of apples. Does the picture show 2 x 3? Explain why or why not.” The picture has one circle with three apples in it and another circle with two apples in it. Students have to understand what 2 x 3 means and then create a mental image and determine if their mental image aligns with the picture or not. Then they must explain their thinking through writing. *TGM* built conceptual understanding by asking students to write their own word problems, and then write the multiplication sentence to solve the problem they wrote.

Procedural knowledge was shown in a number of different ways throughout each curricular material. *BIM* appeared to have components of a scripted curriculum. “Scripted [curricular] materials reflect a focus on explicit, direct, systematic skills instruction…” (Ede, 2006, p. 29), and *BIM* had sections within the text that said:

> Tell students that they will do one more count-around with 8’s. Ask them the following questions:
>
> - Will there be more multiples or fewer multiples of 8? Why?
> - Will everyone get to call out a number? Why or why not?
> - Can you estimate how many people will get to call out a number? Tell us more about your estimate.
> - What happens as the multiples by which we are counting become larger? (p. 9)

Asking specific questions and telling students to do things explicitly aligns with scripted curriculum, and through these scripted sections opportunities to learn procedural knowledge were being presented. *EM* and *TGM* were not scripted curricular materials. They provided opportunities to learn procedural knowledge by group counting and sprint fluency practice, as well as games, and sequences of skip counting without any reasons or directions.

Table 1.

Coded texts from curricular materials in percent

		Procedural Knowledge					Conceptual Knowledge				
		D	**WE**	**S**	**Q**	**P**	**D**	**WE**	**S**	**Q**	**P**
BIM	**Student version**	11.54	0.96	0	11.54	4.81	2.88	1.92	0	2.88	0.96
	Teacher version	1.92	27.88	1.92	4.81	5.77	1.92	9.62	1.92	1.92	4.81
EM	**Student version**	0	3.26	31.52	9.78	13.04	0	0	6.52	2.72	0
	Teacher version	5.98	0	11.41	5.43	2.17	1.09	1.63	2.72	1.09	1.63
TGM	**Student version**	0.71	0	7.09	26.95	0.71	0.71	0	7.80	8.51	0
	Teacher version	0	3.55	2.84	4.96	5.67	0.71	4.26	12.06	9.22	4.26

D = demonstration; WE = worked examples; S = statements; Q = questions; P = problems

The teacher version of statements was the biggest area of conceptual knowledge questions. Table 1 shows the percentages of the coded texts. It is interesting to note that, within each curricula material, not all codes were used. *BIM* did not provide opportunities for students to build conceptual or procedural statements, while *EM* did not provide the opportunity for students to build conceptual or procedural knowledge through demonstrations. *TGM* provided no opportunities for students to build conceptual or procedural knowledge through worked examples. The distribution of knowledge was not equal among any of the resources, but *TGM* has the most equal distribution with 52.48% as procedural and 47.52% as conceptual knowledge.

Summary

Overall, *BIM, EM,* and *TGM* provide numerous learning opportunities for students to build their procedural knowledge of multiplication. *BIM* and *EM* provided more opportunities for building student's procedural knowledge than conceptual knowledge. The materials were first coded based on who the audience was, or where the material was found - either the student or teacher edition. Then, the materials were coded into what knowledge was being presented, either procedural or conceptual. Lastly, the materials were coded based on what was being asked (i.e., demonstration, worked example, statement, question or problem).

CHAPTER 4: FINDINGS

Overview

BIM had a total of 104 items coded within the Grade 3 - Module 2 - Unit 2 curricula material that I analyzed. Seventy-four of the items were considered procedural knowledge with the majority being included in the teacher version as statements. Most conceptual knowledge came from the teacher version as statements, as well. *EM* had 184 items coded within the Grade 3 - Module 1 - twenty-one lesson series. Note, not all twenty-one lessons were coded because they did not all pertain directly to multiplication; several lessons were about division and were omitted. One hundred fifty-two items were coded as procedural knowledge, with the majority being in the student version as statements. The largest conceptual understanding came from the student version as statements. *TGM* had 141 items coded within Grade 3 - Unit 2 - Module 6. Seventy-four items were coded as procedural knowledge with the majority being embedded in the student version as questions.

There were several similarities in the opportunities for developing multiplication fluency in *BIM, EM,* and *TGM*. There was an emphasis on layering knowledge, and the majority of the coded items were addressed to students in the student version. There were also a few common themes for building students' multiplication fluency. I describe these similarities below.

Layering Knowledge

Layering can be viewed as a form of scaffolding, since it provides conceptual and procedural knowledge to build a concept. *Figure 2* shows the percentage of knowledge types within each curricular material. It does not distinguish between the student and teacher versions. The figure illustrates that *BIM* and *EM* had an uneven distribution of procedural knowledge. That does not mean that they did not provide opportunities to build students' multiplication fluency. *TGM* had an almost even distribution and provided more conceptual opportunities for students to build their understanding of multiplication than the other materials. All materials provided layering as an opportunity to build fluency. However, the focus was on procedural knowledge.

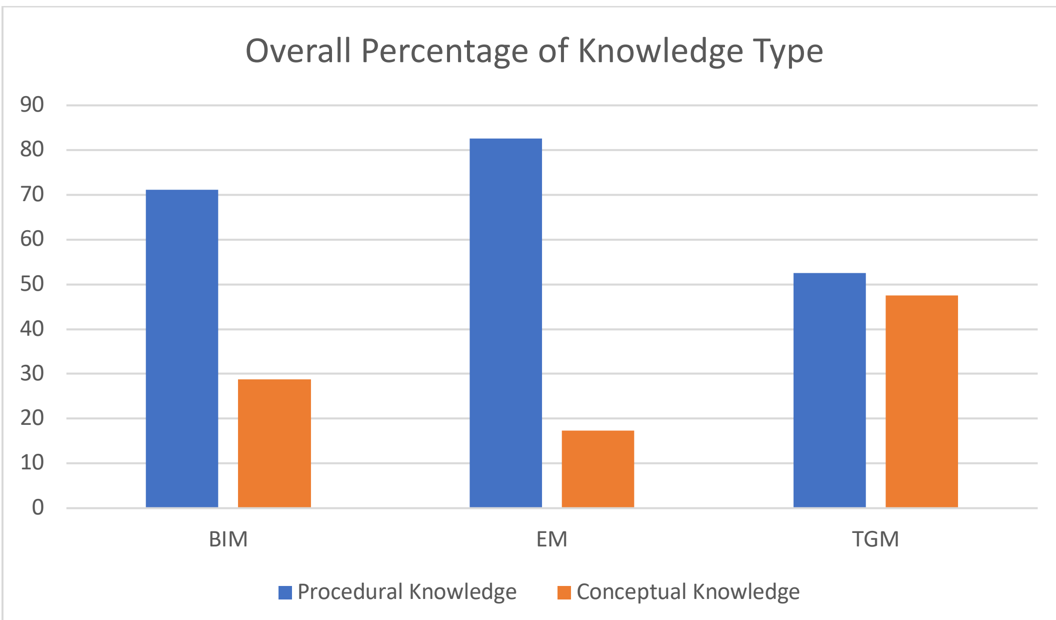
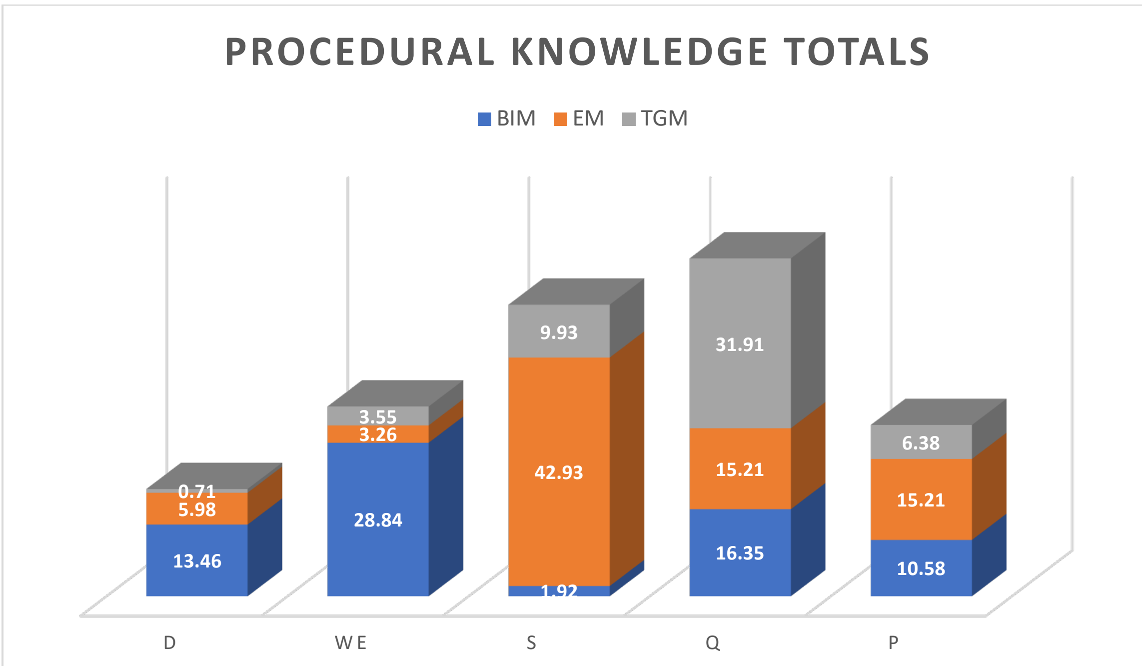


Figure 2. Overall percentage of knowledge types within each curricular material.

BIM was broken up into sections and distinguished between teacher's guide and student book. On average, within each lesson, there was one opportunity to build conceptual understanding to four opportunities to build procedural knowledge. *EM* had "Problem Sets" that students were expected to complete during each lesson before the exit ticket. These problem sets provided opportunities for students to gain procedural and conceptual knowledge by implementing numerous statements. For example, in Problem Set 14, on page 191, number 3 says "Trina makes 4 bracelets. Each bracelet has 6 beads.

Draw and label a tape diagram to show the total number of beads Trina uses." This was coded as a student version of procedural knowledge written as a statement. *TGM* had a "Daily Assessment Task" in each lesson. In these tasks, students were asked procedural and conceptual questions. Through these tasks, students were provided an opportunity to become fluent in multiplication because the knowledge was layered.

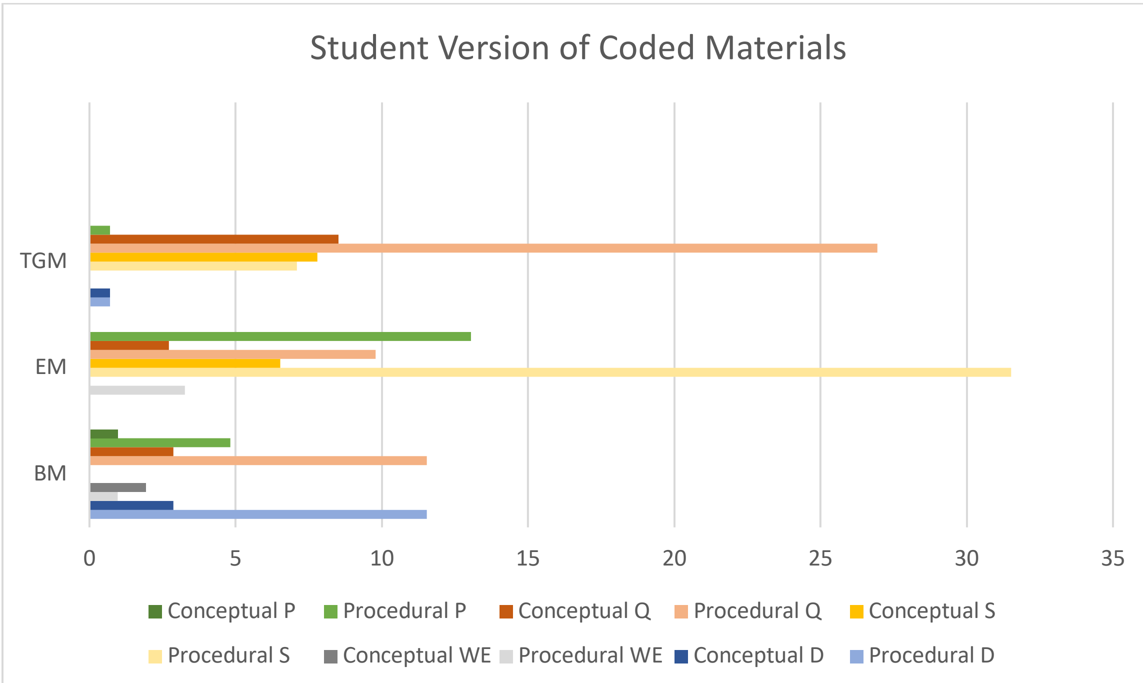


D = demonstration; WE = worked examples; S = statements; Q = questions; P = problems

Figure 3. Total percentage of each coded text type that was considered procedural knowledge.

Focus on Student Version

Figure 4 shows that the student version of each curricular material emphasized different texts/examples/problems. *BIM* emphasized procedural questions and demonstrations, *EM* emphasized procedural statements, and *TGM* emphasized procedural questions. Procedural questions tend to ask "how many times longer/bigger/greater…?" or "how many ______ are there?". Whereas, procedural statements are "match the answer to the correct mathematical expression...”, “draw the array...”, “fill in the equations…”, or “solve the equation...”.



D = demonstration; WE = worked examples; S = statements; Q = questions; P = problems

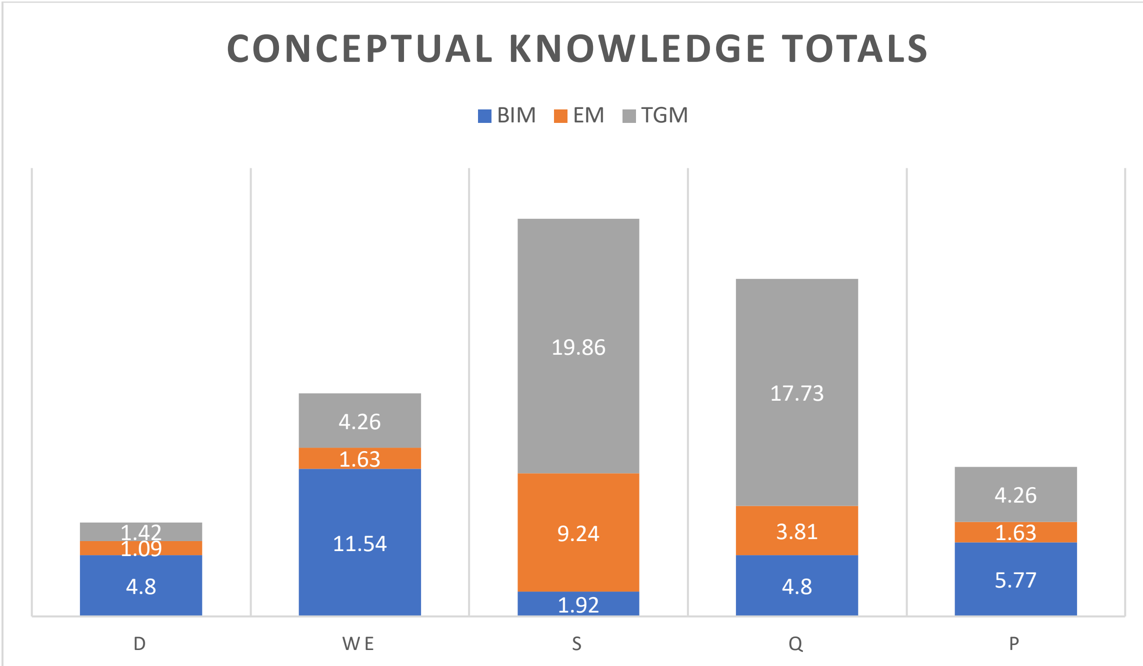
Figure 4. Student version of coded texts of all conceptual and procedural knowledge types.

Both procedural questions and statements provide opportunities to learn multiplication fluency. Questions are designed to have students understand the specific mathematical language involved. “How many times greater …” requires one to understand what “times greater” means mathematically, and how to apply that understanding to answer the question. Statements are designed to show understanding of the procedure by drawing arrays, tape diagrams, or modeling with pictures and provide opportunities for students to become familiar modeling problems. These techniques are considered procedural because they are a form of repeated practice. Matching expressions with the correct statement requires an understanding of the mathematical expression and the picture that is to be matched. When fluent in multiplication, looking at an expression, such as 2 x 3, should automatically produce the image of two groups of three items, or two rows of three columns. From automatically envisioning that, one can match the picture to the expression without lag time.

Common Themes for Building Fluency

After looking strictly at procedural knowledge (see *Figure 3)*, *BIM* focused on worked examples, *EM* focused on statements and *TGM* focused on questions. *BIM* had teacher modeling in their lessons, which was coded in the teacher version, as conceptual knowledge written as statements.

When building conceptual knowledge, the curricula materials focused on statements, and questions (see *Figure 5*). As previously mentioned, statements and questions contribute to the opportunity to learn multiplication fluency because they require a deeper understanding of the mathematical language and modeling involved. Conceptually, these types of items asked for a comparison or explanation of thinking. An example of this is in *EM* on page 137, lesson 10 homework problem 3b. It says "Adriana calculates the total number of books as shown below [6 x 3 = 15 + 3 = 18]. Use the array you drew to help explain Adriana's calculation." Asking the student to build upon their own picture solidifies the previous understanding, which relates Piaget's part of the framework (Aubrey & Riley, 2017).



D = demonstration; WE = worked examples; S = statements; Q = questions; P = problems

Figure 5. Total percentage of each coded text type that was considered conceptual knowledge.

Summary

My analysis indicates that *BIM, EM,* and *TGM* provide different opportunities to learn multiplication fluency. Each curricular material has its own strengths when providing these opportunities. The content that appeared in the student versions provided more opportunities in the form of procedural questions and statements.

CHAPTER 5: DISCUSSION AND CONCLUSIONS

Overview

In my study I analyzed three sets of curricular materials to determine what opportunities each provided for students to develop fluency. Conceptual and procedural knowledge needed to be layered so learners had the opportunity to transition skills and concepts from their short-term memory to their long-term memory (Brown, 2018), which allows for more automatic recall and better retrieval. *TGM* was the curricular material that did this best. However, this study could be expanded upon and other materials should be considered.

Discussion

What does layering knowledge types, as a technique, mean for opportunities to learn multiplication fluency? Since each curricula material layered conceptual and procedural knowledge with a heavier emphasis on procedural knowledge, understanding the goals of the procedural knowledge is key. *Figure 3* breaks down the procedural knowledge by specific curricular material and code. Statements and Questions were techniques that were used most often to create opportunities to learn multiplication fluency. *EM* and *TGM* supported these techniques the most because they were built into the layering models within each curriculum. Overall, providing conceptual and procedural knowledge creates opportunities to learn multiplication as fluently as possible.

What does focusing on the student version mean in the context of multiplication fluency? Opportunities to become fluent in multiplication are provided in all three curricular materials. These opportunities are embedded in the student version of the materials when questions or statements are made that use procedural knowledge. It is important to remember that procedural knowledge does not always correlate to rote learning and timed multiplication assessments but is embedded in everyday questions and statements.

From the literature review, it is most ideal when students have opportunities to develop a conceptual understanding of multiplication and then procedural understanding is layered in order to create a multilayered understanding for the student. Since all curricular materials provided layering of knowledge, they provided the most ideal way to learn multiplication fluency (Rittle-Johnson et al., 2001). This may allow students to process the information in systematics ways and build upon that knowledge in the next lessons. Students potentially have the opportunity to develop procedural knowledge because the teacher is modeling step-by-step and students can relate that to what they already know. This aligns with Brown's (2018) chapter on calculation. She learned that as students become fluent in mathematical operations, their cognitive demand lowered because the facts were available to retrieve rather than having to calculate by use of a strategy. Creating opportunities for students to build upon their previous knowledge is part of the constructivist view and these materials are in that framework, regardless of it was the author's intention or not.

TGM appeared to provide the best learning opportunity for multiplication fluency due to the way the concepts were layered with conceptual and procedural knowledge. *TGM* had the teacher model through either a demonstration or worked example, and then the students would work through questions to build their understanding of the concept or procedure before attempting the homework assignment. This process was repeated throughout each lesson. This aligns with Harvey-Swanston (2017) when he said, "I would argue that both fluency and conceptual understanding in multiplication are needed and that we can develop this by moving beyond a focus on rote-learning strategies and speed of recall" (p. 22). *BIM* provided the next best opportunity to learn multiplication fluency. It had a similar approach, but also implemented games that students were expected to play with peers after the teacher modeled it to the whole class. Where *BIM* fell short was in their conceptual and procedural statements. *BIM* did not provide as many opportunities for students to draw, write, or describe their understanding of multiplication. Steele (2001) says, "The act of representing encouraged them to focus on the essential characteristics of a situation, made the mathematical ideas more concrete, and provided the foundation for the teacher to help students build meaningful mathematical language" (p. 412). By not providing these opportunities, *BIM* is limiting the student's opportunities to build meaningful mathematical language.

Some curricular materials provide opportunities to build multiplication fluency better than others. It is important for educators to realize this because it can help guide decisions on what materials should be adopted, and how teachers should be trained to use the adopted materials. Teachers often do not have time to read through the entire teacher edition of the textbook and understand why the author's focus is on a particular set or type of problems when they are responsible to teach science, writing, spelling, reading, health, social studies, and much more. Providing training to teachers and giving them insight into the goals and intent of the curricular materials would be beneficial. From a teacher's perspective, when viewing curricular materials, it is important to consider what the authors' intent of the materials is and how that impacts the types of questions and problems they use. Knowing how many procedural and conceptual questions are being asked in the lessons could be important because it allows teachers to potentially supplement with questions or problems that align with their teaching style and the goals of their particular school and district.

Conclusions

In conclusion, providing opportunities to learn multiplication fluently occurs when procedural and conceptual knowledge are layered, which allows the learner to build upon their conceptual understanding with the procedures. From analyzing and coding *BIM, EM,* and *TGM* it became evident that not all curricular materials are created equal. *BIM* and *EM* had a larger emphasis on procedural than conceptual knowledge but that does not diminish their abilities to provide opportunities to fluently learn multiplication. Overall, *TGM* provided the most layered knowledge; therefore, of the three analyzed in this study, it may provide more opportunities for students to develop multiplication fluently.

Limitations

The coding was subjective since there was only one person coding all the curricular materials. In *BIM* there were several problems where a number line was presented with various multiplication facts listed (e.g., 2 x 6, 5 x 6, and 6 x 6) and students were expected to place the resulting answers on the correct place on the number line below the corresponding fact. These problems could have been coded in multiple different ways depending on the specific instructions/directions that were from the text, and if it was the first time students were introduced to this particular type of problem. For example, this type of problem could have been a conceptual knowledge problem if the facts were not already listed on the number line, and the directions said place the mathematical expressions in order from least to greatest on the number line. Check your answers by simplifying each expression.

Not all of the materials selected were the same length. *EM* had twenty-one lessons, and had an entire unit dedicated to teaching multiplication and division. *BIM* and *TGM* had five and six lessons/sessions, respectively. Since *EM* was significantly larger and not all lessons pertained to multiplication, only lessons 1 through 4, 7 through 10, and 14 through 16 were coded. This could be considered a limitation because when teaching division, there might be multiplication fluency built into the lessons.

Selection of the curricular materials may have been too narrow. As a graduate student without association to a school district, it is not easy to get curricular materials that are available online. This limitation influenced the materials that were selected.

Future Research

A broader study analyzing more than three curricular material could be beneficial, especially for school districts that have more than one textbook or curricular material adopted by the mathematics department. It might also be worthwhile to see how intervention curricular materials differ from the standard materials. Many students are placed into mathematics inversion classes due to their lack of multiplication fluency. Understanding the curricular materials could give insight to concepts that need more time or explanation for all students, which might lead to less students being placed in intervention classes. Another potential study is how the curricular materials are enacted in classrooms and how those opportunities to learn differ from what was presented in this study.

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