2005

Algebraic Isomorphisms and Spectra of Triangular Limit Algebras: Erratum

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We have found an error in the proof of Theorem 4.1 in [2]. This error affects only Theorems 4.1 and 4.3. As we have been unable to find an alternative proof of Theorem 4.1, we do not know if these theorems are true in full generality. Restricted to limit algebras generated by their order-preserving normalizers, the proof is correct. The theorems are known to be true in this case by somewhat different methods [1].

The precise error in the proof is the claim, in the middle of the last paragraph on page 1145, that weak-$*$-convergence implies there is $N \in \mathbb{N}$ so that for $n \geq N$, $\phi_{J_n}(e_k) = 1$. Below we give an counterexample. Precisely, there is a limit algebra $A = \varinjlim(A_m, \alpha_m)$, a sequence of completely meet irreducible ideals J_n in A so that ϕ_{J_n} converges weak-$*$ to $\rho \in \mathrm{Spec}(A)$, associated to a chain of matrix units (e_k) but there is no N as claimed.

Let A_m be 2^m by 2^m upper-triangular matrices with matrix unit system $e_{i,j}^{(m)}$, $1 \leq i, j \leq 2^m$; we omit the superscript in the matrix unit system when the algebra is clear. Let $\alpha_m : A_m \to A_{m+1}$ be the $*$-extendible embedding given by sending

$$\begin{aligned} e_{i,i} &\text{ to } e_{2i-1,2i-1} + e_{2i,2i}, \\ e_{i,i+1} &\text{ to } e_{2i,2i+1} + e_{2i-1,2i+2}. \end{aligned}$$

The images of all other matrix units in A_m are then determined. That is, α_m is the nest embedding which sends each superdiagonal matrix unit to a block of the form $\left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$.

To construct the counterexample, we associate a spectral functional ρ to the following matrix unit chain: $e_{1,4} \in A_2$, $e_{2,7} \in A_3$, $e_{4,13} \in A_4$ and so on. In

Indiana University Mathematics Journal ©, Vol. 54, No. 4 (2005)

general, the matrix unit in A_{m+2} is e_{2^m,k_m} where $k_m = 2k_{m-1} - 1$ and $k_0 = 4$. Notice that ρ is the functional associated to the unique CMI ideal J so that $e^{(2)}_{1,4} \in J^+ \backslash J$.

We define J_n to be the unique CMI ideal so that the matrix unit e_{2^n-1,k_n+1} of A_{n+2} is in $(J_n)^+ \backslash J_n$.

Notice that the ϕ_{J_n} do converge weak-* to ρ and that, for example, $e_{1,4} \in A_2$ is the unique matrix unit of A_2 where each ϕ_{J_n} takes the value one. However, $e_{1,4}$ is *not* an element of the CMI chain for J_n, which does not begin until we reach the matrix unit e_{2^n-1,k_n+1} of A_{n+2}. Simply put, $e_{1,4} \notin J_n^+$ for all n. Likewise, for any matrix unit in the matrix unit chain for J, say $e_{2^m,k_m} \in A_{m+2}$, this matrix unit is not in J_k^+ for all $k > m$. Thus, there is no matrix unit e_k we can choose so that the statement in the proof ("$\ldots e_k \in J_n^+$ for all $n \geq N$.") holds.

REFERENCES

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KEY WORDS AND PHRASES: limit algebras, spectra, algebraic isomorphisms.
2000 MATHEMATICS SUBJECT CLASSIFICATION: Primary: 47L40; Secondary: 47L50.
Received: *May 21st, 2004.*