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Hybrid of Natural Element Method (NEM) with Genetic Algorithm (GA) to find critical slip surface

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Abstract One of the most important issues in geotechnical engineering is the slope stability analysis for determination of the factor of safety and the probable slip surface. Finite Element Method (FEM) is well suited for numerical study of advanced geotechnical problems. However, mesh requirements of FEM creates some difficulties for solution processing in certain problems. Recently, motivated by these limitations, several new Meshfree methods such as Natural Element Method (NEM) have been used to analyze engineering problems. This paper presents advantages of using NEM in 2D slope stability analysis and Genetic Algorithm (GA) optimization to determine the probable slip surface and the related factor of safety. The stress field is produced under plane strain condition using natural element formulation to simulate material behavior analysis utilized in conjunction with a conventional limit equilibrium method. In order to justify the preciseness and convergence of the proposed method, two kinds of examples, homogenous and non-homogenous, are conducted and results are compared with FEM and conventional limit equilibrium methods. The results show the robustness of the NEM in slope stability analysis.

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1. Introduction

Failure in slopes is a common problem in geotechnical engineering. Collapse in these cases, most times, causes serious damage to both life and property of human beings. Therefore, realistic assessments for the factor of safety and the probable slip surface are highly needed.

Engineering approach to slope stability primarily uses factor of safety values to determine whether slopes are away from failure. The principal traditional limit equilibrium methods have been the most commonly-used techniques in evaluation of the stability of slopes. Although many other excellent methods were proposed over the past few decades, due to simplicity, limit equilibrium methods are still the common methods used for stability analysis. The most important outputs of limit equilibrium analysis methods are the factor of safety and the probable slip surface. In these methods, a potential sliding is
assumed prior to the analysis and a limit equilibrium analysis is then performed with regard to the soil mass and/or other loads above the presumed slip surface. Many limit equilibrium methods are available with different degrees of acceptability.

Since these methods are simple, they would not consider the stress-strain distribution in the soil mass before failure, and stress calculations are performed only at the moment of failure.

It is possible to use FEM and obtain both the factor of safety and adequate information on the collapse mechanism. However, it is not easy to achieve a precise factor of safety within the confidence limits achievable by limit equilibrium methods [1]. In order to obtain an accurate factor of safety by FEM, highly refined mesh is required. Furthermore, computer software capable of giving trustworthy results with the Mohr-Coulomb elasto-plastic model for loading states near to failure is needed. It is also necessary to perform a set of analyses with increment ε and tan φ reduction. These analyses become progressively more costly as factor is increased [2].

In the process of slope stability analysis, finite element stress filed prediction is usually needed in both the factor of safety prediction and the probable slip surface estimation technique, like the limit equilibrium methods. Unfortunately, only an approximate factor of safety can be estimated through finite element analysis and also no rigorous mathematical model for prediction of the probable slip surface has become thus far. Usually, through some technical measures, a group of potential sliding surfaces through empirical means is determined prior to analysis, consisting of a series of arcs. Then, the probable slip surface is defined as the surface along which the minimum ratio of resisting force to driving force is achieved. Some researchers [1,3] suggested algorithms for locating the potential slip surfaces in which factor of safety is defined as the ratio of the resisting force to the driving force along a potential slip surface. The above definition of factor of safety is different from its definition based on strength reduction [4], and it is closer to the conventional limit equilibrium methods. The disadvantages of this technique can be summarized as: (a) identification of the element which contains a nodal point on the slip surface; (b) determination of the local co-ordinates of this point; and (c) determination of the element nodal stress values to compute the stress field at the chosen point by interpolating the nodal stresses. The difficulty in Finite Element Method (FEM) is the generation of meshes with elements that are connected together by nodes in a properly predefined manner. The limitations of the FEM with predetermined mesh have the motivation for using Meshfree technique, in particular NEM which is the main scope of present work.

Usually, Meshfree methods are based on Radial Basis Function (RBF) interpolation. Since in domain formulation, any single RBF cannot satisfy the governing equations, obtaining a viable solution would require a large number of collocation points for both domain and boundary of the problem [5]. NEM is local compact support and possess delta kroneker, which would introduce the simplicity usage of the method. Generally, in Meshfree methods, two conditions must be observed:

1. Definition of shape functions is literally based on nodes’ position.
2. The assessment of the nodal connectivity depends on the number of nodes [6].

The difference among Meshfree methods is based on interpolation scattered data techniques [7]. There are some Meshfree methods: Smooth Particle Hydrodynamic (SPH), Partition of Unity Method (PUM), and Diffuse Element Method (DEM) [8]. Some methods were formed by Moving Least Square technique (MLS) that shape functions do not possess kroneker delta property [9].

Most of Meshfree methods need background cells for the definition of numerical integration on domain problem [10]. These methods need to background cells, causing not to define Meshfree methods completely [11]. Another type of Meshfree methods for interpolation scatter data is Element Free Galerkin (EFG); two points are noticeable in EFG:

- Non-element interpolation of field variable.
- Non-mesh integration of weak form [9].

EFG has no kronecker delta property, hence, in the implementation of essential boundary conditions faces problems [12]. One way to overcome this shortfall is Point Interpolation Method (PIM). Although PIM is more accurate than MLS, it may cause to singularity matrix for momentum matrix [9]. Matrix Triangularization Algorithm (MTA) is introduced to solve this problem which is an automatic process to make sure whether the effect of selected node in interpolation is applied [9]. According to what was mentioned, some of the shortfalls of most Meshfree methods are as follow:

1. In some methods, imposition of essential boundary conditions is complicated.
2. Many Gouse points are needed to assess weak form of the problem.
3. Some methods have no performance for scattered data [6].

In this research, Natural Element Method (NEM) is used. This method is based on Voronoi diagram and Delaunay tessellation that have been used as weak form for some mechanical problems NEM possesses kronecker delta, a positive point which is rarely found in other Meshfree methods and covers the mentioned shortfalls. NEM shape functions are \( C^0 \) at node interpolation and \( C^\infty \) elsewhere [13].

The current study uses natural element based method for estimating the probable slip surface and factor of safety described in the following steps.

(1) The natural element method is explained and it is formulated for linear elasto-plastic stress analysis under plane strain assumption. (2) The procedure through which factors of safety are calculated on the potential slip surfaces is described. (3) Genetic Algorithm (GA) is briefly described and it is used to generate and optimize potential slip surfaces (individuals). (4) Examples are provided to justify convergence and robustness of the proposed method, and the results are compared with FEM and conventional limit equilibrium methods.

2. Natural element method

Natural element method is a mesh-less approach which has been developed to solve the partial differential equations (PDEs). Discrete model of a domain \( \Omega \) consists of a set of distinct nodes \( \mathcal{N} \), and a polygonal description of the boundary \( \partial \Omega \). The interpolation scheme used in NEM is known as
Natural Neighbor ($n$-$n$) interpolation [14]. Natural neighbor interpolation is a multivariate data interpolation scheme [15]. To construct the interpolant, natural neighbor interpolation relies on concepts such as Voronoi diagrams and Delaunay tessellations from computational geometry [16]. Despite its simple and alluring structure, $n$-$n$ interpolation has not become popular in the area of multivariate data interpolation, when compared to other schemes such as Shepard’s interpolant [17], moving least-squares approximants [18] and radial basis functions [19]. The $n$-$n$ interpolants are smooth ($C^\infty$) everywhere, except at the nodes where they are ($C^0$) [13]. In one-dimension, NEM is identical to linear FEM. The NEM interpolant is strictly linear between adjacent nodes on the boundary of a convex hull, which facilitates imposition of essential boundary conditions.

NEM interpolants are formulated on basis of the underlying Voronoi tessellation, which is unique for a given set of distinct nodes in the plane. In utilizing the NEM in contrast to the FEM [20], there are no constraints on shape, size and angles of the triangles that make up Voronoi tessellation. In the FEM, interpolation angle restrictions are imposed on triangles to ensure the convergence of the process [13]. A very important feature of NEM is the ability for random configuration of nodes in space without refers to whether the associated Delaunay triangles are acceptable from finite element’s point of view.

A differential boundary value problem solution starts by casting the differential form into an equivalent integral form based on the methods of weighted residuals; then in the natural element context, a set of distinct nodes $N = \{n_1, n_2, n_3, \ldots, n_m\}$ should first be set up at the arbitrarily shaped geometry describing domain $\Omega$ (see Fig. 1). The Voronoi diagram (or first order Voronoi diagram) of set $N$ is a subdivision of the plane into region $T_I$, where each region $T_I$ is associated with a node $n_I$ such that any point in $T_I$ is closer to $n_I$ than to any other node $n_J \in N(J \neq I)$. In mathematical terms, the Voronoi polygon $T_I$ is defined as [21]:

$$T_I = \{x \in \mathbb{R}^2 : d(x, n_I) < d(x, n_J), \forall J \neq I\}$$

where $d(x, n_I)$ is the distance between $x$ and $n_I$ (see Fig. 2(a)).

Delaunay triangulation is constructed by connecting the nodes whose Voronoi cells have common boundaries (see Fig. 2(b)). The important property of Delaunay triangles is the empty circumcircle criterion [22] – if $DT(n_I, n_J, n_K)$ is any Delaunay triangle of the nodal set $N$ then the circumcircle of $DT$ contains no other nodes of $N$ (see Fig. 2(c)).

![Fig. 1](image1.png) Discrete model of region.

**Fig. 2** Construction of natural neighbor co-ordinates: (a) 1st Voronoi diagram, (b) Delaunay triangulation, (c) circumcircle criteria and node $x$ and (d) 1st Voronoidiagram.

In order to quantify the neighbor relation for sample point $x$ introduced into the tessellation, the second-order Voronoi cell of point $x$ is constructed in Fig. 2(d). Therefore, the natural neighbor shape function of $x$ with respect to a natural neighbor $I$ is defined as the ratio of the area of overlap of their Voronoi cells to the total area of the Voronoi cell of $x$:

$$\theta_I(x) = \frac{A_I(x)}{A(x)}$$

where $A_I(x)$ is the overlapping area of Voronoi cell of point $x$ and node $x_I$; $A(x)$ is the total area of the 2nd Voronoi cell of $x$; The four regions shown in Fig. 2(d) are second-order cells, while their union (closed polygon $abcd$) is a first-order Voronoi cell. Referring to Fig. 2(d), the shape function $\theta_I(x)$ is given by

$$\theta_I(x) = \frac{A_{abcd}}{A_{abcd}}$$

Displacement approximations $u^h(x)$ of point $x$ in $\Omega$ can be written as

$$u^h(x) = \sum_{I=1}^{n} \theta_I(x)u_I$$

where $u_I (I = 1, 2, \ldots , n)$ are vectors of nodal displacements at the $n$ natural neighbors of point $x$; and $\theta_I(x)$ are the shape functions associated with each node.

Shape functions, $\theta_I(x)$, given in Eq. (2), satisfy the partition of unity requirement, i.e.

$$\sum_{I=1}^{n} \theta_I(x) = 1 \text{ in } \Omega$$

for $0 \leq \theta_I(x) \leq 1$ and $\theta_I(x) = \delta_{IJ}$

Relations in (6) show that NEM interpolation passes through the nodal values, which is in contrast to most mesh-less
approximations where the nodal parameters $u_I$ are not nodal displacements. Furthermore, the natural neighbor shape functions have $C^\infty$ continuity everywhere except at the nodes where they are $C^0$. The more detailed discussion of the NEM interpolation can be found in Sukumar et al. [13].

3. Natural element stress analysis

3.1. NEM formulation

In order to simplify, consider the following standard two-dimensional linear elastic problem defined over the domain $\Omega$ bounded by $\Gamma$ (see Fig. 3). The partial differential equation and boundary conditions for a two-dimensional problem can be written in the form of

Equilibrium equation : $L^T \sigma + b = 0 \quad \text{in} \; \Omega \quad (7)$

Natural boundary condition : $\sigma n = i \quad \text{on} \; \Gamma$ \quad (8)

Essential boundary condition : $u = \bar{u} \quad \text{on} \; \Gamma_u \quad (9)$

where $L$ is the differential operator defined by Eq. (10); $\sigma^T = \{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\}$ is the stress vector; $u^T = \{u,v\}$ the displacement vector; $b^T = \{b_x,b_y\}$ the body force vector; $i$ the prescribed traction on the traction (natural) boundaries; $\bar{u}$ the prescribed displacement on the displacement (essential) boundaries; $n$ is the vector of unit outward normal at a point on the natural boundary.

$$L = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} & 0 \\
0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial n}
\end{bmatrix} \quad (10)$$

In NEM, the global weak form is used to solve numerically the boundary value problem [23]. The standard global variational (weak) form of Eq. (1) is posed as follows [24]:

$$\int_{\Omega} (L^T \delta u) (D L u) d\Omega - \int_{\Omega} \delta u^T h d\Omega - \int_{\Gamma_b} \delta u^T \bar{i} d\Gamma = 0 \quad (11)$$

where $D$ is the matrix of material property constants regarding the plane strain context.

In order to evaluate the integrals in Eq. (11), the global domain, $\Omega$, is discretized into a set of so-called background-cells that are not overlapping. In NEM, Delaunay triangles are used for this purpose. Evaluation of integrals along the natural boundary involves using a set of non-overlapping curved (for 2D problem) background cells. In NEM, Delaunay triangle edges are used for this purpose.

The problem domain is now represented by a set of field nodes for the purpose of field variable (displacement) approximation. These nodes are numbered sequentially from 1 to $N$ covering the entire problem domain. NEM shape functions to be presented in Section 2 are used to approximate displacements at any point of interest using nodal values of the local support domain of that point.

$$u_I^{(2:x)} = \sum_{I} \left[ \begin{array}{c} \Phi_I \\ 0 \end{array} \right] \left[ \begin{array}{c} u_I \\ v_I \end{array} \right] = \sum_{I} \Phi_I u_I \quad (12)$$

where $\Phi_I$ is the shape matrix of shape functions for node $I$, $n$ is the number of nodes in the local support domain, and $u_I$ are nodal displacement values. In Eq. (12), numbers inside parentheses in the subscript denote matrix dimensions.

In Eq. (12), $u_I$ is the approximated displacement vector at a given point of interest that usually is a sampling point or a quadrature point.

The strain field can be obtained using the approximated displacement values, i.e.

$$e_{I}^{(3:x)} = Lu^{b} = L_{(3:x)} \Phi_{(2:x)} u_{(2:x)} = \sum_{I} B_I u_I \quad (13)$$

where $B_I$ is the strain matrix for node $I$.

$$B_I = \begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} \\
\frac{\partial v}{\partial y}
\end{bmatrix} \quad (14)$$

The stress field can then be obtained by using appropriate constitutive model of the material and the approximate strain field through:

$$\sigma = D e = D_{(3:x)} B_{(3:x)} u_{(2:x)}$$

$$= \sum_{I} D_{(3:x)} (B_I)_{(3:x)} u_{(2:x)} \quad (15)$$

Substituting Eqs. (12) and (13) into the first term of Eq. (11) forms:

$$\int_{\Omega} (L \delta u)^T (D L u) d\Omega = \int_{\Omega} \sum_{I} \sum_{J} \delta u_I^T [B_I^T D B_J] u_J d\Omega \quad (16)$$

Note that the $I$ and $J$ indices are based on local numbering system established for a node to identify its local support domain. The numbering system can now be changed to include all the field nodes over the entire domain in a unique manner numbered from 1 to $N$, the total number of nodes in the problem domain. Hence, both $I$ and $J$ in Eq. (16) can now vary from 1 to $N$. Integrand vanishes when the node $I$ and $J$ are not in the same local support domain. Consequently, their respective integral will be eliminated, as well. Based on this strategy, Eq. (16) can be expressed as:

$$\int_{\Omega} (L \delta u)^T (D L u) d\Omega = \int_{\Omega} \sum_{I} \sum_{J} \delta u_I^T [B_I^T D B_J] u_J d\Omega \quad (17)$$

Moving the integral operator inside the summation ones yields:

$$\int_{\Omega} (L \delta u)^T (D L u) d\Omega = \sum_{I} \sum_{J} \delta u_I^T \left( \int_{\Omega} B_I^T D B_J d\Omega \right) u_J = \delta U^T K U \quad (18)$$

where $K_{I,I}$, which is a 2x2 matrix, is called the nodal stiffness matrix and is defined as:
\[ K_U = \int_\Omega (B_I^{T})_{(2 \times 3)}D_{(3 \times 3)}(B_I)_{(3 \times 2)}d\Omega \]

(19)

\( K \) is the global stiffness matrix.

Since nodal stiffness matrices are \( 2 \times 2 \) and the total number of nodes in the problem domain is \( N \), the dimension of \( K \) is \( 2N \times 2N \).

In a similar way, nodal force vector is constructed as follows, starting with the virtual work statement for body forces:

\[
\int_{\Omega} \delta u^T b d\Omega = \sum_{T} \frac{N}{T} \int_{\Omega} \Phi_{i}^T b d\Omega = \delta U^T F^{(b)}
\]

(20)

\[ F_I^b = \int_{\Omega} \Phi_{i}^T b d\Omega \]

(21)

where \( F_{I}^{b} \) is the nodal body force vector and \( F^{(b)} \) is the global body force vector assembled using nodal body force vectors from all nodes of the domain. Length of vector \( F^{(b)} \) is \( 2N \) since nodal body force vectors are \( 2 \times 1 \) and the total number of nodes in the problem domain is \( N \).

Virtual work due tractions can be written as:

\[
\int_{V_t} \delta u^T d\Gamma = \sum_{T} \frac{N}{T} \int_{\Omega} \Phi_{i}^T d\Gamma = \delta U^T F^{(t)}
\]

(22)

\[ F_{I}^t = \int_{V_t} \Phi_{i}^T d\Gamma \]

(23)

where \( F^{(t)} \) is the global traction force vector assembled using the nodal traction force vector \( F_{I}^t \). Length of vector \( F^{(t)} \) is also \( 2N \).

Substituting Eqs. (18), (20), and (22) in Eq. (11) yields:

\[
\delta U^T K U - \delta U^T F^{(b)} - \delta U^T F^{(t)} = 0
\]

(24)

Or

\[
\delta U^T [K U - F^{(b)} - F^{(t)}] = 0
\]

(25)

Since \( \delta U \) is arbitrary, the above equation can be satisfied only if

\[ K U = F^{(b)} + F^{(t)} \]

(26)

Or

\[ K U = F \]

(27)

where \( F \) is the global force vector given by

\[ F = F^{(b)} + F^{(t)} \]

(28)

Eq. (27) is the final system of equations for the NEM. Nodal displacements can be obtained by solving Eq. (27) after enforcing the displacement boundary conditions. After obtaining nodal displacements, the strain and stress components can be retrieved using Eqs. (13) and (15), respectively. Since in this paper displacement and stress fields in the soil are due to static load (self weight), the term \( F^{(t)} \) vanishes from Eq. (26).

3.2. Numerical implementation

The problem domain is discretized into a set of Delaunay triangles. Hence, a global integration can be expressed as a summation of integrals over these cells:

\[
\int_{\Omega} G d\Omega = \sum_{k}^{n_d} \int_{\Omega_{k}} G d\Omega
\]

(29)

where \( n_d \) is the number of Delaunay triangles, \( G \) represents the integrand, and \( \Omega_{k} \) is the domain of \( k \)-th Delaunay triangle.

Gaussian integration scheme, commonly used in FEM, is employed to perform integrations numerically over these cells (triangles). Where Gaussian points are used in each Delaunay triangle, Eq. (29) changes as follow:

\[
\int_{\Omega} G d\Omega = \sum_{k}^{n_d} \int_{\Omega_{k}} G d\Omega = \frac{1}{2} \sum_{k}^{n_d} \sum_{i=1}^{n} \tilde{w}_i G(x_{Qi}) |J^b_{ik}| \]

(30)

where \( \tilde{w}_i \) is the Gaussian weight at \( i \)-th Gauss point, \( x_{Qi} \), and \( J^b_{ik} \) is the Jacobian matrix for the area integration of the Delaunay triangle \( k \)th, in which the Gauss point \( x_{Qi} \) is located.

Using Eqs. (19) and (30), nodal stiffness matrix \( K_{U} \) can be written as:

\[ K_{U} = \frac{1}{2} \sum_{k}^{n_d} \tilde{w}_i B_I^{T}(x_{Qi}) DB_I(x_{Qi}) |J^b_{ik}| \]

(31)

where \( K_{U}^{b} \) is defined as:

\[ K_{U}^{b} = \frac{1}{2} \tilde{w}_i B_I^{T}(x_{Qi}) DB_I(x_{Qi}) |J^b_{ik}| \]

(32)

where \( K_{U}^{b} \) is a \( 2 \times 2 \) matrix.

Note Eq. (31) means that the nodal stiffness matrix \( K_{U} \) is obtained numerically from the summation of stiffness contributions made from all the quadrature points whose local support domains include both \( I \) and \( J \) nodes. In NEM framework, these local support domains are circumcircles of Delaunay triangles. If node \( I \) and node \( J \) are not natural neighbors of the quadrature point at \( x_{Qi} \), \( K_{U}^{b} \) vanishes.

Similarily, nodal body force vector \( F_{I}^{b} \) given in Eq. (20) can be written as:

\[ F_{I}^{b} = \frac{1}{2} \sum_{k}^{n_d} \sum_{i=1}^{n} \tilde{w}_i \Phi_{i}^{T}(x_{Qi}) b(x_{Qi}) |J^b_{ik}| \]

(33)

where \( F_{I}^{b} \) is defined as:

\[ F_{I}^{b} = \frac{1}{2} \tilde{w}_i \Phi_{i}^{T}(x_{Qi}) b(x_{Qi}) |J^b_{ik}| \]

(34)

where \( F_{I}^{b} \) is a \( 2 \times 1 \) matrix.

4. Genetic algorithm

Genetic Algorithm (GA) is a free-derivative method based on natural selection and evaluation procedure [25]. Genetic algorithm has been widely used since 1970 in many fields of engineering and science [26]. High-speed computers with greater memory capacity have increased the use GA as an optimizer tool in many fields of engineering [27–29]. Generally, GA includes eight main steps: 1-genetic representation, 2-initial population, 3-evaluation function, 4-reproduction selection tool in many fields of engineering [27–29]. Generally, GA includes eight main steps: 1-genetic representation, 2-initial population, 3-evaluation function, 4-reproduction selection, 5-genetic operators, 6-generational selection scheme, 7-stopping criteria, 8-GA parameters [30].

Local Optimization Algorithm (LOA) often is used to overcome the disadvantages as the inability of fine local tuning [30]. Local Search Algorithm is based on slight changes in randomly or selected individuals and the best one will be kept in the population [31]. This type of strategy in a GA is called a Hybrid Genetic Algorithm (HGA). Generally, each generation
includes of a number of individuals and the fitness value of each individual is evaluated which influence the next-generation production. The optimization procedure starts with a population of M individuals (parent generation) and next generations are created by crossover function and mutation. The flowchart for the solution procedure is shown in Fig. 4.

The crossover operator is defined as:

\[
\begin{align*}
    v_{0j+1} &= v_{0j} \times r_c + (1.0 - r_c) \times v_{ni,j} \\
    v_{0j+2} &= v_{ni,j} \times r_c + (1.0 - r_c) \times v_{0j}
\end{align*}
\]

where \( v_{0j+1} \) and \( v_{0j+2} \) are the \( l \)th element of the mother parent vector \( v_{mi} \) and father parent vector \( v_{fi} \). \( v_{0j+1} \) and \( v_{0j+2} \) are defined as follow:

\[
\begin{align*}
    V_{0j+1} &= v_{0j+1} \times r_m + (v_{0j+1} - v_{lim}) \times \left( 1.0 - \frac{r_m}{r_{max}} \right) \times r_{nd} \quad \text{if } r_{nd} \leq 0.5 \\
    V_{0j+1} &= v_{0j+1} + (v_{max} - v_{0j+1}) \times \left( 1.0 - \frac{r_m}{r_{max}} \right) \times r_{nd} \quad \text{if } r_{nd} > 0.5
\end{align*}
\]

\( r_m \) and \( r_{nd} \) are random values in the interval \([0,1]\). \( v_{lim} \) and \( v_{max} \) are the lower and upper bounds. Table 1 shows the parameters used in this research.

5. Nodal layout

Numerical methods, in particular FEM, are techniques that can be used to solve systems of partial differential equations defined over a bounded or an unbounded domain. In FEM, the problem domain is replaced by a mesh of finite elements, and an equivalent integral form of the differential equations is evaluated over the elements. FEM is a very general and powerful computational method. However, if the evaluation of field variables in a particular layout of nodes is required, the mesh generation will face complications in FEM. These complexities include choice of element type, introduction of element connection in the required mesh, control of the field variants’ continuity at inter-element nodes, and the approximation which is employed to access the shape function. To circumvent these complications of FEM, it has become necessary to refine the mesh and interpolate the field variables at specific nodes. However, mesh refinement increases the volume of computations and hence the calculation time, which eventually can lead to numerical instability [13].

In NEM framework, all computations depend on nodal coordinates and there are no restrictions on shape, size, and angle of the Delaunay triangles. In this study, circular slip surface is considered initially. In order to evaluate the factor of safety on this assumed surface, one needs to compute the stresses directly at each node of the slip surface. Choosing circular slip surfaces is needed so to compare the results with those obtained from conventional limit equilibrium methods. Whereas...
NEM is capable of working with irregular nodal configurations, thereby making it possible to obtain irregular shape slips surfaces.

In order to find the critical slip surface, including the minimum factor of safety, Genetic Algorithm (GA) is hybridized with the outcomes of NEM analysis. Every probable critical slip surface (individual) will be generated by GA. The position and geometry of each individual varies in every iteration. Since the coordinates and radius of each probable circular slip surface differs from other ones, the position of generated nodes on every individual varies. As a result, it can be conceived that a problem with floating node position is formed.

All slip surfaces start and finish on the boundary within the slope limits. If the end points of a slip surface are not within the slope limits the slip surface is not analyzed and includes penalty. Sections of the external boundary between slope limits define slope surfaces that can be analyzed. Centers of randomly generated circles are located outside of the slope domain as slip centers. Based on distances from slip center to the slope surface, for each slip center, suitable arcs are determined (Fig. 5).

The main purpose of this nodal configuration is to obtain nodal stresses at nodes on slip surface directly, and eliminate one stage of interpolation which leads to more accurate results. Nodal configuration on slip surface is made based on equal distances on the circular slip surface. Since nodes can be added easily to the problem domain in NEM framework, the number of slip surfaces as well as associated nodes can be increased routinely. It is important to mention that nodes can be placed on circular slip surface’s base with similar angle division. This procedure can provide suitable distribution of nodes on slip surfaces as well as the problem domain. In addition, computation of factor of safety which is discussed in the next section is greatly simplified. Fig. 6 illustrates a typical similar angle division.

As noted earlier, each slip center is located outside of the slope domain. In order to alter the position of slip surfaces, a rectangular domain is determined in which the slip surface coordinates can be altered in every iteration. Hence, a square region of slip centers is considered outside of the slope to sweep for potential slip surfaces. This square region is the variable’s boundary in GA. It is worthwhile to note that the proposed approach of considering each slip center and its unique final nodal layout during each calculation phase can create serious remeshing issues in finite element framework. In order to analyze slope stability problem by FEM in addition of difficulties associated with nodal layout changes at each stage, it is necessary to create a new mesh at the beginning of each stage of calculation. However, NEM handles this problem by omitting the concept of elements all together and providing a meshless computational framework. Fig. 7 depicts the randomly generated nodal layouts and Delaunay triangles in each computational phase.

### 6. The factor of safety calculation

In slope assessment, engineers use the factor of safety to determine if a slope is probable to fail. Abramson et al. [32] have listed several definitions commonly used in slope stability analysis. Here, the overloading definition is used to calculate the factor of safety (FOS) and is given as [33]:

\[
\text{FOS} = \text{Min} \left( \int_{s} \tau ds / \int_{s} \tau ds \right) \quad s \in S \tag{37}
\]

In this equation, \( S \) is a set of potential slip surfaces and \( s \) is a slip surface of the set.

If the slip surface \( s \) is divided into \( n \) number of sections with dissimilar lengths \( \Delta L_{i} \), Eq. (37) can be written in the following form:

\[
\text{FOS} = \text{Min} \left( \sum_{i=1}^{n} \tau_{i} \Delta L_{i} / \sum_{i=1}^{n} \tau_{i} \Delta L_{i} \right) \quad s \in S \tag{38}
\]

The first nodal configuration where circular slip surfaces are generated with nodes placed on these slip surface takes into consideration similar angle division so that \( \Delta L_{i} \) will be identical for all values of \( i \) and hence Eq. (38) can be replaced with an easier one:

\[
\text{FOS} = \text{Min} \left( \sum_{i=1}^{n} \tau_{i} \Delta L_{i} / \sum_{i=1}^{n} \tau_{i} \Delta L_{i} \right) \quad s \in S \tag{39}
\]
where \(\tau_i\) and \(\tau_f\) are the tangential shear stress values of the circular slip surface and shear strength at node \(i\), respectively. Physical meaning of FOS \(o\) is the ratio of total resisting moment to total driving moment.

Shear strength of soil can be obtained by several criteria. In this article, \(\tau_f\) is computed at nodes using Mohr-coulomb’s criterion

\[
\tau_f = c + |\sigma_n|\tan\phi \\
\]

where \(c\) and \(\phi\) are cohesion and angle of internal friction for the soil, respectively. \(\sigma_n\) is the normal stress acting at node \(i\) on \(s\).

Based on Eqs. (39) and (38), it is clear that stresses have a crucial role in both factor of safety and shear strength. Therefore, nodal displacements are obtained by solving Eq. (27)
after enforcing the displacement boundary conditions. Then the strain and stress components are retrieved using Eqs. (13) and (15), respectively.

Factors of safety for the potential slip surfaces are obtained from Eq. (37). Then the slip surface with least factor of safety is selected as the most probable slip surface from slip surface set $S$. To convey the concept of minimum FOS in every iteration, Fig. 8 shows contours for factor of safety in a square grid placed above the slope.

Square grid of the slip centers is used to sweep slip surfaces. Each slip surface includes a set of radius and slip center in calculation phase. Moreover, for each slip surface there is a factor of safety. Now, let

$$f(x_{ci}, y_{ci}, r_{ci}) = \text{FOS}_i^0$$

(41)

where $x_{ci}, y_{ci}$ are center coordinates, $r_{ci}$ is the radius, and FOS$_i^0$ is the minimum factor of safety of the $i^{th}$ slip surface. Eq. (41) is used as objective function in GA optimization procedure.

### Table 2: Design variables, variable bounds, and goal function in this research.

<table>
<thead>
<tr>
<th>Case</th>
<th>Design variables</th>
<th>Variable bounds (m)</th>
<th>Goal function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>$(x_c, y_c, r_c)$</td>
<td>(25, 15, 10) to (40, 33, 20)</td>
<td>Eq. (41)</td>
</tr>
<tr>
<td>Example 2</td>
<td>$(x_c, y_c, r_c)$</td>
<td>(50, 70, 40) to (82, 95, 60)</td>
<td>Eq. (41)</td>
</tr>
</tbody>
</table>

### Table 3: Geometric dimensions for homogenous slope and information about the slip surfaces.

<table>
<thead>
<tr>
<th>$(x_c, y_c)$ (m, m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x_1, y_1)$</td>
</tr>
<tr>
<td>$(x_2, y_2)$</td>
</tr>
<tr>
<td>$(x_3, y_3)$</td>
</tr>
<tr>
<td>$(x_4, y_4)$</td>
</tr>
</tbody>
</table>

Numbers of nodes on each slip surface: 25

A slope of height $H = 10$ m, sloping at angle $45^\circ$ with the following shear strength parameters: friction angle $\phi = 20^\circ$ and cohesion $c = 12.38$ kPa, the unit weight $\gamma = 20$ kN/m$^3$, and the elastic parameters $E = 20,000$ kN/m$^3$ and $v = 0.35$. Table 2 lists the geometric dimensions of slope as well as details of slip surfaces for each stage of computation (see Table 3).

7. **Illustrative example**

In the following examples, three quadrature points are used for each Delaunay triangle during numerical integration. For simplicity, the material is assumed to be linear elastic. Failure criterion is Mohr-coulomb and no pore pressure is considered.

The proposed four stages in slope stability analysis, i.e. generation of random probable slip surface by GA, computing the stresses of a homogeneous slope by NEM, calculating the factor of safety of slip surfaces, and detecting the probable slip surface in an iterative procedure are demonstrated. Results are compared with conventional limit equilibrium methods assuming circular surfaces and FEM analysis which is performed by [34].

7.1. **Homogeneous media**

A slope of height $H = 10$ m, sloping at angle $45^\circ$ with the following shear strength parameters: friction angle $\phi = 20^\circ$ and cohesion $c = 12.38$ kPa, the unit weight $\gamma = 20$ kN/m$^3$, and the elastic parameters $E = 20,000$ kN/m$^3$ and $v = 0.35$. Table 3 lists the geometric dimensions of slope as well as details of slip surfaces for each stage of computation (see Table 3).

It should be noted that material in the FEM analysis is assumed to be elastic- plastic, with a Mohr-Coulomb’s yield criteria. Zheng et al. [34] implemented 1340 four-node quadrilateral isoperimetric finite element meshes for a homogeneous slope. The number of nodes used by NEM to analyze this slope is about 480 in each stage, and totally 2535 slip surfaces are checked out. The final results from NEM + GA as well as other methods are given in Table 4.

Fig. 9 depicts the probable slip surface which is introduced after running the program and an approximate factor of safety.
of 1.09 is computed. According to Fig. 9, the probable slip surfaces predicted by NEM + GA and FEM are larger and deeper than those obtained by conventional equilibrium methods. Moreover, the probable slip surface obtained by NEM + GA is in good agreement with the FEM.

### 7.2. Non-homogeneous media

The second example includes two layers. The slope height $H = 20$ m, sloping at angle $40^\circ$ with the following parameters: a layer of 5 m height (Soil 2) is on a layer of 15 m (Soil 1). Soil 1 includes friction angle $\phi = 45^\circ$ and cohesion $c = 7$ kPa, the unit weight $\gamma = 20$ kN/m$^3$, and the elastic parameters $E = 20,000$ kN/m$^3$ and $\nu = 0.30$. Soil 2 includes friction angle $\phi = 35^\circ$ and cohesion $c = 5$ kPa, the unit weight $\gamma = 18$ kN/m$^3$, and the elastic parameters $E = 18,000$ kN/m$^3$ and $\nu = 0.25$ (see Table 5).

Table 6 lists the geometric dimensions of slope and details of slip surfaces for each stage of computation. The method of simplified Bishop is used and the results of proposed method are verified with Bishop. Table 6 shows the results of NEM + GA method in comparison with simplified Bishop.

![Fig. 9](image1.png) Comparison among current study analysis, FEM, and limit equilibrium methods.

![Fig. 10](image2.png) Comparison between proposed method and simplified Bishop.

### Table 4 Factors of safety and details of homogenous slip surfaces.

<table>
<thead>
<tr>
<th>Methods</th>
<th>$x_c$ (m)</th>
<th>$y_c$ (m)</th>
<th>$r_c$ (m)</th>
<th>Factor of safety</th>
<th>Slip color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bishop simplified</td>
<td>33.36</td>
<td>27.58</td>
<td>17.82</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>Janbu simplified</td>
<td>33.36</td>
<td>27.58</td>
<td>17.82</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>Spencer</td>
<td>33.36</td>
<td>27.58</td>
<td>17.82</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>FEM (Zheng)</td>
<td>31.41</td>
<td>25.26</td>
<td>15.26</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>NEM + GA</td>
<td>32</td>
<td>27</td>
<td>17.016</td>
<td>1.09</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5 Geometric dimensions for non-homogenous slope and information about the slip surfaces.

<table>
<thead>
<tr>
<th>$x_i$, $y_i$</th>
<th>(m, m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$, $y_1$</td>
<td>(0, 20)</td>
</tr>
<tr>
<td>$x_2$, $y_2$</td>
<td>(20, 20)</td>
</tr>
<tr>
<td>$x_3$, $y_3$</td>
<td>(30, 12)</td>
</tr>
<tr>
<td>$x_4$, $y_4$</td>
<td>(50, 12)</td>
</tr>
</tbody>
</table>

Numbers of nodes on each slip surface: 35
Numbers of slip surfaces in each computational stage for each slip center: 20

### Table 6 Factors of safety and details of non-homogenous slip surfaces.

<table>
<thead>
<tr>
<th>Methods</th>
<th>$x_c$ (m)</th>
<th>$y_c$ (m)</th>
<th>$r_c$ (m)</th>
<th>Factor of safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bishop simplified</td>
<td>77</td>
<td>93</td>
<td>51.26</td>
<td>1.37</td>
</tr>
<tr>
<td>NEM + GA</td>
<td>74</td>
<td>90</td>
<td>52.23</td>
<td>1.42</td>
</tr>
</tbody>
</table>
8. Conclusions

This research uses Natural Element Method (NEM) simultaneously with Genetic Algorithm (GA) in order to find the probable and critical slip surface in slope stability problems. Regarding the limit equilibrium criteria, it is necessary to propose a randomly selected slip surface and then analyze the problem domain. Consequently, the factor of safety for the proposed slip surface would be available. Obviously, if the geometry of the slip surface changes, the nodal position of the problem will be altered. Therefore, conventional numerical methods encounter difficulties in these kinds of problems. NEM, as a mesh-free method, is assessed in this research and special features are extracted from the current study as follow:

- The stresses obtained by the NEM are found to yield more accurate results when computing the factor of safety.
- Probable slip surface is obtained using much fewer nodes than regular FEM studies.
- Due to the difficulties in FEM to determine both critical slip surface and its factor of safety, NEM allows particular nodal layout that is suitable for obtaining probable slip surface choice to get rid of these problems.
- Ability of NEM to handle irregular nodal layouts makes it very beneficial for slope stability analysis, where other geometries of slip surfaces such as spiral, parabolic or multi lines are naturally implementable in FEM framework.
- By omitting the concept of element and therefore remeshing phase in the NEM, this method can be easily used in order to sweep all the slip surfaces which are produced by changing the position of nodes at each stage.
- The test case demonstrates the validity of the proposed NEM + GA framework.
- Genetic algorithm is in a good convergence with illustrated results.
- The proposed method is useful in preliminary stages of slope stability analysis.

References