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Simulation of Investment Returns for a Money Purchase Fund

M. Zaki Khorasanee*

Abstract†

This paper examines the problem of investment risk in money purchase pension plans. The disadvantages of modeling equity returns as independent, identically distributed random variables are considered, and a modified stochastic model of equity returns is proposed. This modified stochastic model is used to estimate the variability in a plan member's retirement fund and to compare various alternatives to investing 100 percent of the assets in ordinary shares. Varying conclusions are drawn about the likely success of these alternative investment strategies in reducing investment risk.

Key words and phrases: pension plan, defined contributions, dividend yield, investment risk

1 Introduction

1.1 Defined Benefit and Money Purchase Pension Plans

In defined benefit pension plans the pension benefit is calculated from a set mathematical formula. The most common approach is for the pension to equal a fixed fraction of the member's salary close to retirement multiplied by the number of years of service with the employer. Such arrangements usually are described as final salary plans.

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From the employees' perspective, final salary plans have the advantage of providing pensions linked to their retirement income needs. New entrants to the plan can predict what fraction of their earnings will be replaced by the plan should they stay in service until retirement. Moreover, provided that an employee's salary increases at a rate not lower than the rate of price inflation, the real value of the pension (in terms of its future purchasing power) has a lower bound.

A money purchase pension plan is fundamentally different; it is a defined contribution plan where the objective is to set aside a fraction of the member's salary for contributions to the pension plan. The fraction is determined by an agreed upon mathematical formula. The pension at retirement is an annuity purchased by the member's accumulated fund, the value of which depends on investment returns over the same member's period of service.

A comparison by Bodie (1989) based on historic United Kingdom investment and earnings data for a money purchase plan in which contributions of 10 percent of earnings are invested in ordinary shares shows that the pension of a United Kingdom employee with 20 years of service retiring in one of the years from 1970 to 1987 would have varied between 13 percent and 41 percent of final salary.

Nevertheless, money purchase plans have become increasingly prevalent in both the United States and the United Kingdom for a variety of reasons. This paper examines the problem of investment risk in such plans and assesses the validity of various strategies that may be employed to limit this risk.

1.2 Outline of Paper

Our approach is first to develop a stochastic investment model for equity returns net of wage inflation. The reasons for focusing on returns net of wage inflation are twofold. First, contributions to money purchase plans are usually a fixed percentage of the employee's salary. Second, it is desirable for an employee's retirement fund to be measured relative to the projected salary at retirement. Thus, it is natural to use currency units adjusted for future wage inflation, in which case returns also must be measured relative to wage inflation.

In particular, in Section 2 we derive formulae for the mean and variance of a money purchase fund, assuming net annual returns are independent and identically distributed lognormal random variables. It is shown that this model, however, overstates the variability in the retirement fund. In Section 3 we develop a modified stochastic investment model that employs certain aspects of Wilkie's (1986) model. This mod-
ified model is used throughout the rest of the paper. Sections 4, 5, and 6 examine the impact of switching the fund to low risk assets near to retirement, the impact of balanced investment strategies, and the use of derivative-based investment products, respectively. Section 7 contains a summary and implications for pension plan design.

1.3 Notation

We assume that a contribution of one unit is paid annually at the start of each year into a pension plan member's fund. All amounts and returns are expressed in terms of constant earnings.

Let:

\( x_0 \) = Youngest entry age to a money purchase plan;
\( y \) = Normal retirement age, e.g., 62 or 65;
\( n = y - x_0 \) = Maximum number of years to normal retirement;
\( x = x_0 + t_0 \) = Current age of a plan member in mid-career;
\( t \) = Time since member was age \( x_0 \), with \( t = 0, 1, \ldots \);
\( F(t) \) = Actual fund at time \( t \);
\( F(0) = 0 \); and
\( \delta_t \) = Average force of interest between \( t \) and \( t + 1 \).

It follows that:

\[
F(t + 1) = (F(t) + 1)e^{\delta_t}.
\] (1)

2 The Independent Lognormal Returns Model

We now derive expressions for the expected value and variance of the fund at retirement for a member at any age. The following assumptions are needed:

Assumption 1: The annual investment returns (net of wage inflation) form a sequence of independent, identically distributed, lognormal random variables; and

Assumption 2: A member age \( x \), where \( x_0 \leq x < y \), already has accumulated a fund equal to its expected value on entering the plan at age \( x_0 \).

Assumption 1 implies that

\[
\delta_t \sim N(\mu, \sigma^2)
\] (2)
and that the annual investment return (net of wage inflation) in any year is independent of the fund value at the start of that year. This independence makes it easy to derive expressions for the mean and variance of $F(t)$. The second assumption is intended to cover the case of members who enter the plan in mid-career, bringing with them transfer values.

2.1 Mean and Variance of the Fund

Because $e^{\delta t}$ is lognormally distributed, then from Assumption 1 and equation (1) we can deduce that:

$$E[F(t + 1)] = (E[F(t)] + 1)e^{\mu + \frac{1}{2}\sigma^2}. \quad (3)$$

As $F(0) = 0$, equation (3) yields

$$E[F(t)] = \frac{(1 + r)}{r} ((1 + r)^t - 1) \quad (4)$$

where

$$r = e^{\mu + \frac{1}{2}\sigma^2} - 1$$

is the expected annual return net of wage inflation.

We now derive the variance of the projected retirement fund of a member age $x$ who has accumulated a fund equal to its expected value. It is well known [see, for example, Bowers et al. 1986, Chapter 2, equation (2.2.11), p. 29] that for any two random variables $W$ and $V$,

$$\text{Var}[W] = \text{Var}[E[W|V]] + E[\text{Var}[W|V]]. \quad (5)$$

So, from equation (1), let

$$W = F(t + 1) = (F(t) + 1)V \quad \text{and} \quad V = e^{\delta t}.$$

Note that $V$ is a lognormal random variable with

$$E[V] = e^{\mu + \frac{1}{2}\sigma^2} \quad \text{and} \quad \text{Var}[V] = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1).$$

It follows from Assumption 1 above that

$$\text{Var}[F(t + 1)] = \text{Var}[(E[F(t)] + 1)e^{\delta t}] + E[\text{Var}[F(t)]]$$

$$= (E[F(t)] + 1)^2 \text{Var}[e^{\delta t}] + e^{2\mu + 2\sigma^2} \text{Var}[F(t)]$$

$$= e^{2\mu + \sigma^2}(E[F(t)] + 1)^2(e^{\sigma^2} - 1)$$

$$+ e^{2\mu + 2\sigma^2} \text{Var}[F(t)]. \quad (6)$$
Now equation (6) is the first order linear difference equation in $\text{Var}[F(t)]$. From Mickens (1987, Chapter 2.2), the solution to a first order linear difference equation of the form

$$y_{t+1} = p_t y_t + q_t, \quad t = 0, 1, 2, \ldots$$

for $t = 1, 2, \ldots$, is given as

$$y_t = \left( \prod_{i=0}^{t-1} p_i \right) \left[ y_0 + \sum_{i=0}^{t-1} \left( \frac{q_i}{\prod_{j=0}^{i} p_j} \right) \right]. \tag{7}$$

To solve equation (6), let

$$p_t = e^{2\mu + 2\sigma^2}$$

$$q_t = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1) (E[F(t)] + 1)^2.$$

Hence, as $\text{Var}[F(0)] = 0$, we have

$$\text{Var}[F(t)] = e^{2t\mu + (2t-1)\sigma^2} (e^{\sigma^2} - 1) \sum_{i=0}^{t-1} e^{-2(i(\mu + \sigma^2))} (1 + E[F(i)])^2. \tag{8}$$

This equation can be simplified further because of the simple form that $E[F(t)]$ takes in equation (4).

### 2.2 Parameters Estimated From Past Equity Returns

Estimators for the mean and standard deviation of the force of interest are obtained from United Kingdom equity index returns and average earnings data from 1950 to 1993. The equity returns are taken from the BZW\(^1\) equity index and the earnings data from government statistics. The following estimators are obtained for $\mu$ and $\sigma$:

$$\hat{\mu} = 0.052 \quad \text{and} \quad \hat{\sigma} = 0.2556$$

The estimate $\hat{\sigma}$ is larger than expected, particularly if one believes that equity returns are correlated, to some extent, with wage and price inflation. The data suggest, however, that there is little correlation

\(^1\)The BZW equity index is a representative stock price index for ordinary shares traded in the United Kingdom. This index is compiled by the investment bank Barclays de Zoete Wedd (hence BZW). We have used annual returns on the index as calculated by BZW, which allow for the reinvestment of gross dividends.
when returns are measured over annual intervals.\textsuperscript{2} In addition, the period covered includes the crash/recovery scenario of 1974 and 1975, which has a significant effect on the measured standard deviation.

Using equations (4) and (8) and the above estimates $\hat{\mu}$ and $\hat{\sigma}$ gives, for example:

$$E[F_{20}] = 54.5 \quad \text{and} \quad \sigma(F_{20}) = 55.8.$$  

Although it is possible (given the skewed nature of the distribution) for the standard deviation of the fund to exceed its expected value, the figure obtained is nevertheless implausibly high and not consistent with the empirical studies to which Bodie refers.

It may be incorrect to assume that annual equity returns net of wage inflation are independent when estimating the variability in funds accumulated over long periods. In making such an assumption, we ignore the fact that the average dividend yield on ordinary shares tends to fluctuate around a central value that may be comparatively stable. This effect will tend to reduce the variability in returns over long periods, without necessarily affecting the measured variability in annual returns.

A central feature of Wilkie's stochastic model\textsuperscript{3} for the simulation of equity returns is the explicit treatment of dividend yield. This aspect of Wilkie's approach is adapted next to simulate equity returns net of wage inflation.

\section{Modification of the Simple Lognormal Model}

The end-of-year dividend yield on the BZW equity index ranges from 4 percent to 6 percent in 35 of the 44 years from 1950 to 1993. This has had a profound effect on long-term stability in equity returns, as there has been a tendency for the market to correct itself when overvalued or undervalued by historical standards.

\textsuperscript{2}Economic theory implies that equity returns and wage inflation should be correlated over long periods, as both are driven by growth in the national income. Over relatively short intervals, however, there is little evidence of this correlation.

\textsuperscript{3}In the mid-1980s, A.D. Wilkie developed a stochastic investment model that simulates United Kingdom investment returns that since has become a standard tool for many United Kingdom actuaries. Wilkie devised four connected models: (1) for United Kingdom price inflation; (2) for ordinary share dividend yields; (3) for growth in ordinary share dividends; and (4) for yields on fixed interest government bonds. The price inflation time series from model (1) is used as an input variable for each of the subsequent models. Model (2) assumes that the natural logarithms of share dividend yields are correlated over adjacent periods. This is probably the most robust feature of Wilkie's models as far as adherence to the data is concerned.
United Kingdom actuaries implicitly have recognized this phenomenon by using a discounted cash flow (or actuarial) value for equities in valuations of defined benefit plans. Actuarial values differ from market values in that price changes arising from fluctuations in dividend yields (as opposed to a rise or fall in dividend income) are not recognized.

Following Thornton and Wilson (1992), the actuarial force of net interest, $\gamma_t$, is defined by the relationship

$$
\gamma_t = \delta_t + d_t - d_{t-1}
$$

where $d_t = \ln(D_t)$ and $D_t$ is the average equity index dividend yield at the end of year $t$. Equation (1) now can be rewritten as:

$$
F(t + 1) = (F(t) + 1)e^{\gamma_t + d_{t-1} - d_t}.
$$

We now model the $\gamma_t$'s (the actuarial force of net interest) as a sequence of independent, identically distributed, normal random variables. The historical data over 1950-1993 give the following estimates of the mean and standard deviation of $\gamma_t$:

$$
\hat{\mu}(\gamma_t) = 0.0428 \quad \text{and} \quad \hat{\sigma}(\gamma_t) = 0.0646.
$$

The variability is reduced compared with returns on market values, $\delta_t$. What matters for a plan member, however, is the market value of the fund at retirement. Given $F(t_0)$, let $F(t, t_0)$ denote the value of the fund at time $t$ ($t_0 \leq t \leq n$). Then,

$$
F(t, t_0) = F(t_0) \exp[d_{t_0} - d_t + \sum_{k=t_0+1}^{t} \gamma_k]
$$

$$
+ \sum_{j=t_0}^{t-1} \exp[d_j - d_t + \sum_{k=j+1}^{t} \gamma_k].
$$

We therefore require a model for the way that dividend yields change over time.

### 3.1 Dividend Yield Model

Dividend yields ($D_t$) must lie in the range zero to infinity, so it may appear reasonable to assume that $d_t$ can be modeled as a normally distributed random variable with mean $\mu_d$ and standard deviation $\sigma_d$.

Wilkie (1986) observes that the average dividend yield on United Kingdom equities tends to vary about a long-term average and that
yields in adjacent periods exhibit significant positive correlation. We estimate the autocorrelation\(^4\) of \(d_t\) from the year-end dividend yield on the BZW equity index from 1919 to 1993.

\[
\begin{array}{c|c}
 k & \rho(d_t, d_{t+k}) \\
--&--
\end{array}
\]
\[
\begin{array}{c|c}
 1 & 0.512 \\
 2 & 0.204 \\
 3 & 0.030 \\
 4 & -0.008 \\
\end{array}
\]

where \(\rho(d_t, d_{t+k})\) is the correlation coefficient between \(d_t\) and \(d_{t+k}\).

It seems that dividend yields in adjacent years exhibit significant positive correlation, which is consistent with the idea that changes in market valuations occur in response to a continuous stream of price sensitive information. The data confirm that an autoregressive\(^5\) model, as used by Wilkie, is appropriate. Wilkie uses an autocorrelation parameter of 0.6 for \(d_t\) and \(d_{t-1}\) and also assumes that the rate of price inflation has a direct effect on \(d_t\). Because we require a model that operates in real values, we ignore the latter feature of Wilkie’s model and use an autocorrelation parameter of 0.5 for \(d_t\) and \(d_{t-1}\) in accordance with our own data.

This leads to the following first order autoregressive formula for \(d_t\):

\[
d_t = 0.5d_{t-1} + 0.5\mu_d + \frac{\sqrt{3}}{2}\sigma_d N_t
\]

(12)

where the \(N_t\)'s form a sequence of independent normal random variables with mean zero and unit variance. The coefficients in equation (12) have been selected so that \(E[d_t] = \mu_d\) and \(\text{Var}[d_t] = \sigma_d^2\). The historic data give the following estimates of the mean and standard deviation:

\[
\hat{\mu}_d = -3.008 \quad \text{and} \quad \hat{\sigma}_d = 0.240.
\]

### 3.2 Expected Value of Fund

Assuming that the change in the equity dividend yield over any period is independent of the actuarial return over the same period, we can deduce expected value of the fund at age \(x_0 + t\) for a new entrant age \(x_0\) as follows: let \(t_0 = 0\) and \(F(0) = 0\) in equations (9) and (11). Assuming

---

\(^4\)The term \textit{autocorrelation} refers to the correlation of a sequence of random variables with itself.

\(^5\)For a more detailed description and analysis of autoregressive processes, see Box and Jenkins (1976, Chapter 3).
that the change in dividends $d_j - d_t$ and the $\gamma_k$'s are independent, then we see that

$$E[F(t)] = E \left[ \sum_{j=0}^{t-1} \exp[d_j - d_t + \sum_{k=j+1}^{t} \gamma_k] \right]$$

$$= \sum_{j=0}^{t-1} E[e^{(d_j - d_t)}]E[\exp[ \sum_{k=j+1}^{t} \gamma_k]]$$

$$= \sum_{j=0}^{t-1} E[e^{(d_j - d_t)}]e^{(t-j)(\nu + \frac{1}{2} \sigma^2_j)}.$$  (13)

But from equation (12), we have

$$E[e^{(d_j - d_t)}] = \exp[\frac{\sigma^2_d}{2} (1 + (1 - \beta)^2)]$$  (14)

where $\beta = 2^{-(t-j)}$. Although the value of this expression does not depend on $t$ and $j$, we find that for our estimate of $\sigma_d$, it is fairly close to unity for all $t$ and $j$. Thus for the purpose of estimating the fund in mid-career, we shall use the approximation:

$$F_{t_0} \approx \sum_{j=0}^{t-1} e^{(t-j)(\nu + \frac{1}{2} \sigma^2_j)}.$$  (15)

### 3.3 Use of Simulation to Obtain Percentiles

As Bodie notes, the standard deviation is not a particularly useful parameter for the skewed distribution of the fund at retirement. What is required are values of the fund at various percentiles so that we can estimate the probability of a plan member's benefits lying within a particular range. The relevant probability density function is difficult to obtain, so these values have to be estimated through simulation.

For $n = 40$ and $t_0 = 0, 20, 30, \text{ and } 35$, there are 1000 simulations performed for each combination of $n$ and $t_0$ using the modified stochastic model described in Section 3.1 above. The values of the retirement fund at various percentiles, as a multiple of its mean value over each run of 1000, are shown in Table 1.
Table 1

<table>
<thead>
<tr>
<th>Percentile Points of $F_{40}$</th>
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</thead>
<tbody>
<tr>
<td>$t_0$</td>
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<tr>
<td>------</td>
</tr>
<tr>
<td>0</td>
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<tr>
<td>20</td>
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<tr>
<td>30</td>
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<tr>
<td>35</td>
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</tbody>
</table>

There are two main conclusions to be drawn from Table 1.

- Even though the stochastic model allows for long-term stability in dividend yields, the variability in the projected fund of a new entrant 40 years from retirement is still high; the ratio of the 75th percentile to the 25th percentile is 1.66. In other words, an employee whose working career coincides with a period of moderately favorable equity returns would end with a fund 66 percent greater than that of an similar employee whose working career coincides with a period of moderately unfavorable equity returns.

- The variability in the projected retirement fund reduces slowly as the employee gets closer to retirement. At only five years from retirement, the ratio of the 75th percentile to the 25th percentile is as high as 1.45. There is still a 1 in 4 chance that the fund will be less than 80 percent of its expected value and a 1 in 20 chance that it will be less than 60 percent of its expected value.

The results obtained over a 40 year period of service are broadly consistent with those of Knox (1993), based on the experience of an Australian managed fund. This appears to be a coincidence, however, as the stochastic model used by Knox assumes independent, identically distributed returns combined with a low standard deviation. Hence for periods of service less than 40 years, Knox's model would imply significantly less variability in the fund.

### 3.4 Practical Problems Created by Investment Risk

Some practical implications of the results shown in Table 1 are discussed below.

- **Uncertainty in future benefit levels**: An employee in a money purchase plan may have little idea of what the real value of his or her future pension will be, which makes planning for retirement
difficult. The projected future pension arising from a given rate of contribution can be estimated, but these estimates need to be updated frequently and may be wide of the mark. Even if the contribution rate is varied, following regular benefit projections the plan member may find that either:

1. The retirement fund is too small to purchase the required pension; or
2. The retirement fund is larger than required, and the surplus savings it contains must be used to purchase an annuity.

The second problem is a consequence of United Kingdom legislation that limits the amount of a member's fund that can be taken as a lump sum.

- Inequity between employees: It can be argued that a money purchase plan is the most equitable form of pension provision, as the same contribution rate can be paid for each employee, who always would receive his or her asset share by definition.

We believe this definition of equity is valid only for individual pension contracts, where the member effectively hires an insurance company to manage his or her personal savings and retains control over the choice of insurer and type of fund.

In an employer-sponsored plan, the member usually has less control over the money invested on his or her behalf. Furthermore, the option to receive salary in lieu of pension contributions is not normally available. It follows that the benefit being provided by the employer is not the contribution, but the pension derived from the contribution. In a money purchase plan, this pension will depend on whether the employee’s period of service happens to coincide with a period of favorable or unfavorable investment experience. Thus, different generations of employees with identical salary and service histories may end with different pensions.

If a government requires its citizens to invest social security contributions in money purchase arrangements, the economic consequences of inequity between the generations could be severe, as an entire generation of newly retired pensioners could end with inadequate pensions and could require additional financial support from the working population.
4 Switching to Low Risk Assets

The results obtained in Section 3 are for a money purchase fund invested fully in ordinary shares. The first variant from this investment strategy to be examined is one frequently employed: switching the existing fund and future contributions to low risk assets at some time close to retirement.

Before investigating the optimal time to switch, we should consider what low risk assets are appropriate for such a switch. Most insurance companies in the United Kingdom writing unit-linked business have funds invested in cash and/or government bonds, specifically to meet the needs of risk-averse policyholders. Individuals with unit-linked pension policies can switch their assets into these funds at any time, sometimes subject to a small administration fee. Cash and fixed interest bonds give no guaranteed protection against inflation, however, so switching into a fund investing in index-linked government bonds may be more appropriate.

The real yield (net of price inflation) on United Kingdom index-linked bonds usually has been around 3 percent to 4 percent, which is approximately 1 percent above the annual growth in United Kingdom average earnings over the post-war period. We assume for modeling purposes that a pension plan member always can switch into assets that guarantee a fixed return of 1 percent above the increase in United Kingdom average earnings. Let $F^{(sw)}(n, t_0)$ be the fund at retirement after switching at time $t_0$. Then:

$$F^{(sw)}(n, t_0) = F(t_0)(1.01)^{(n-t_0)} + \tilde{s}_{n-t_0}$$

(16)

evaluated at 1 percent.

Switching to index-linked assets partly solves the problem of having an unpredictable pension at retirement—at least the real value of the fund is now fairly predictable, although one still must contend with uncertain future annuity rates. The earlier the switch is made, the easier it is to plan for retirement and to afford any extra contributions that may be required to obtain the desired pension. If the switch is made too early, however, the projected fund at retirement will be far below the fund expected from continued investment in equities.

Under the stochastic model used in this paper, the equity dividend yield at the time of switching has an important bearing on the decision.

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6A *unit-linked* product offers its policyholders a number of investment funds in which their assets can be invested. As with mutual funds, no investment guarantees are provided, and the policyholder's maturity value is linked directly to the market value of the underlying assets.
The argument for switching would be strengthened if the dividend yield were below its long-term average, because of the greater risk of a fall in the equity market. The reverse would apply if the dividend yield were above its long-term average.

Simulations are performed to compare the fund obtained after switching into index-linked bonds at time \( t_0 \) with that obtained by remaining in equities, assuming that the equity dividend yield at \( t_0 \) were either equal to, 1 percent below, or 1 percent above its long-term average.

Table 2 shows the value of the fund obtained after switching into index-linked bonds, as a fraction of the mean fund from continued investment in equities, for switches made at different durations from retirement and at different equity dividend yields. For comparison, the 25th and 50th percentiles of the fund obtained from continued equity investment (from Table 1) also are shown.

<table>
<thead>
<tr>
<th>Fund Obtained by Switching to Low Risk Assets</th>
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<tbody>
<tr>
<td>Switch to Low Risk Assets</td>
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<tr>
<td>Value of ( D_{t_0} )</td>
</tr>
<tr>
<td>( t_0 )</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>20</td>
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<tr>
<td>30</td>
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<td>35</td>
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</tbody>
</table>

As one might expect, the ratio of the switched retirement fund to the mean fund from continued investment in equities is always less than one. The amount by which this ratio falls below unity is the insurance premium paid in order to obtain a guaranteed fund at retirement.

By comparing these ratios with the percentiles from continued investment in equities, we can assess the degree of risk protection obtained by switching. If the fund remains in equities, the probability of ending with a retirement fund below the 25th percentile is 0.25, a significant risk. If by switching to low risk assets we can guarantee a fund equal to or higher than this, the case for switching is reasonably strong.

According to Young (1994), the most commonly recommended time for a switch to low risk assets is approximately five years before retirement, which corresponds to the case \( t_0 = 35 \). Table 2 confirms that at this duration, the risk of a lower retirement fund by remaining in equities is significant but the magnitude of this risk depends greatly on the prevailing equity dividend yield. (At \( t_0 = 0 \), however, the initial dividend yield is irrelevant, as there is no fund to switch.)
Ideally, the following conditions would hold before switching into low risk assets:

- The projected fund after switching will meet the member's requirements;
- The equity market is overvalued by historic standards;
- There are fewer than ten years before retirement.

If the first condition is true, one would expect the member to be risk-averse, as he or she virtually can guarantee the required fund without having to pay extra contributions. Thus, even a small probability of not achieving the necessary fund might be unacceptable.

If the second condition is true, the first condition is more likely to be true (as the market value of the accumulated fund will be greater), and the risk of ending with a lower retirement fund by remaining in equities would be greater.

If the third condition is true, the risk of ending with a lower retirement fund by remaining in equities would be significant under most conditions. But if the third condition is true and the first condition is not true, there is less time to obtain the required fund by paying extra contributions. A member therefore might prefer to risk continued equity investment in the hope of obtaining the target fund through superior investment performance, i.e., by taking a calculated gamble.

In summary, we can conclude that switching to low risk assets at some point within ten years of retirement is likely to be a suitable strategy for most members of money purchase plans. The precise timing of this switch should flexible, however, depending on the member's projected fund after switching and the level of the equity market at the time of the switch.

5 Balanced Investment Strategies

This section examines the results of following a balanced investment strategy\(^7\) throughout an employee's period of service and compares them with the results obtained for 100 percent investment in equities.

The following balanced investment strategies are considered:

\(^7\)A balanced investment strategy is one involving a combination of different asset types, with a view to achieving a suitable compromise between risk and return.
Khorasanee: Simulation of a Money Purchase Fund

- 75 percent equities, 25 percent index-linked bonds, realigned annually by market values; and

- 50 percent equities, 50 percent index-linked bonds, realigned annually by market values.

Again, 1000 simulations are performed simultaneously for each investment strategy, so that each set of simulations is based on the same sequence of equity returns. This enables the number of times that a particular investment strategy leads to a higher retirement fund than does an alternative strategy to be calculated.

The simulations are carried out for the case \( n = 40, t_0 = 0 \), i.e., for a new entrant at the youngest permitted age of entry, with no accumulated fund. The values of the retirement fund at various percentiles, as a multiple of the mean fund from investing fully in equities, are shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Comparisons of Different Investment Strategies</th>
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<tbody>
<tr>
<td></td>
<td>Investment Strategy (% in Equities)</td>
</tr>
<tr>
<td>Percentile</td>
<td>A = 100%</td>
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<tr>
<td>5th</td>
<td>0.49</td>
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<tr>
<td>25th</td>
<td>0.72</td>
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<tr>
<td>50th</td>
<td>0.93</td>
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<td>75th</td>
<td>1.20</td>
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<tr>
<td>95th</td>
<td>1.75</td>
</tr>
<tr>
<td>Mean</td>
<td>1.00</td>
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</tbody>
</table>

As one would expect, a lower allocation to equities reduces the mean value of the retirement fund, but also reduces its variability. In order to determine whether a balanced investment strategy has anything to offer the individual plan member, the following probabilities are estimated from the simulations:

**Strategy A (100 percent in equities):** Probability of obtaining a fund of less than one-half the mean is equal to 0.056;

**Strategy B (75 percent in equities):** Probability of obtaining a fund of less than one-half A's mean is equal to 0.031; and

**Strategy C (50 percent in equities):** Probability of obtaining a fund of less than one-half A's mean is equal to 0.024.
We can summarize the results by saying that a more balanced investment strategy would result in a lower retirement fund for the majority of members, but also would reduce the already small proportion of members who obtain a severely substandard fund.

So, we could sell investment strategy B to a member by explaining that although his or her expected fund would be 10 percent lower, the risk of ending with only half the expected fund is reduced from 5.6 percent to 3.1 percent. Most members might not feel this is a good deal, and the case for strategy C would be even weaker.

The main advantage of investing in low risk assets is that inequity between different members is reduced significantly. The ratio of the retirement fund at the 75th percentile to that at the 25th percentile is 1.67 for strategy A, 1.48 for strategy B, and 1.31 for strategy C. These ratios have been achieved by leveling down; the actual fund value at the 25th percentile is highest for strategy A.

The results obtained therefore suggest that the case for investing a significant proportion of the fund in low risk assets as a long-term strategy is weak. This does not necessarily argue against short-term tactical switches from the equity market based on the judgment of the fund manager.

6 Guaranteed Equity Products

The final investment strategy to be considered as an alternative to 100 percent investment in equities is one involving the use of guaranteed equity products (GEPs).

GEPs have been marketed by United Kingdom insurance companies as a means of allowing policyholders to participate in the underlying growth of an equity portfolio while also benefiting from a guaranteed minimum fund, either at termination of the contract or at intermediate durations. These guarantees are designed to protect against adverse movements in the equity market.

A typical contract may provide a return on the investor's capital equal to the increase in an ordinary share price index, while guaranteeing that the investor will be repaid the initial capital should the index fall over the term of the contract. In such a contract, the absence of reinvested dividends would pay for the guarantee. Dodhia and Sheldon (1994) describe how the creative use of financial options has enabled leveling down means a reduction in the inequality between two groups, achieved by making the better off group poorer, rather than making the worse off group richer.
the design of a wide variety of contracts, each offering a different type of guarantee.

Consider a contract that provides a rolling guarantee at one year intervals coinciding with the annual investment of contributions to the pension fund. We assume that the contract guarantees a fraction of the capital invested at the start of the year plus the actual equity return (if positive) applied to the minimum guaranteed capital. For modeling purposes we further assume that:

- The guaranteed capital increases in line with United Kingdom average earnings over the year;

- The equity return is based on the equity price index with dividends reinvested, as opposed to the more usual practice of using the price index alone.

Dodhia and Sheldon, on commenting on the feasibility and propriety of the first assumption for pension fund contracts, state that such a guarantee would be possible to provide and would be suitable for pension contracts.

The GEP investment return net of wage inflation in year between times $t$ and $t + 1$ is given by:

$$R_t = f \max\{e^{\delta t}, 1\} - 1$$  \hspace{1cm} (17)

where $f$ is a constant with $0 < f < 1$. The expected value of the retirement fund will be sensitive to the value of $f$ chosen, as this factor will compound over the years to retirement. We choose values for $f$ that produce approximately the same expected fund as from investing in the equity portfolio alone, which trial simulations show to be in the range $0.92 \leq f \leq 0.93$.

Using the modified stochastic model, we perform simultaneously 1000 simulations for contracts with $f$ equal to 0.92, 0.925, and 0.93, respectively, and for investment in the underlying equities alone. As before, these are done for a new entrant at the youngest age with 40 years until retirement. The values of the retirement fund at various percentiles, expressed as a multiple of the mean fund from investing in equities alone, are shown in Table 4.
Table 4
Investing in Guaranteed Equity Products (GEPs)

<table>
<thead>
<tr>
<th>Percentile</th>
<th>$f = 0.92$</th>
<th>$f = 0.925$</th>
<th>$f = 0.93$</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th</td>
<td>0.44</td>
<td>0.50</td>
<td>0.57</td>
<td>0.50</td>
</tr>
<tr>
<td>25th</td>
<td>0.62</td>
<td>0.71</td>
<td>0.82</td>
<td>0.70</td>
</tr>
<tr>
<td>50th</td>
<td>0.80</td>
<td>0.91</td>
<td>1.05</td>
<td>0.93</td>
</tr>
<tr>
<td>75th</td>
<td>1.05</td>
<td>1.21</td>
<td>1.40</td>
<td>1.20</td>
</tr>
<tr>
<td>95th</td>
<td>1.58</td>
<td>1.82</td>
<td>2.11</td>
<td>1.78</td>
</tr>
<tr>
<td>Mean</td>
<td>0.89</td>
<td>1.02</td>
<td>1.18</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 4 indicates that the expected fund from investing in a rolling one year GEP contract is sensitive to the level of guarantee offered. More important, there appears to be no reduction in the variability of the fund at retirement compared with a strategy of investing in the underlying shares alone.

Guaranteed equity products reduce variability in investment returns over short periods, so it is perhaps surprising that a rolling one year contract fails to reduce the same variability over longer periods. An intuitive explanation follows from the fact that the return from a rolling GEP contract depends on how variable the underlying equity returns are. The greater the variability in equity returns, the greater the return from the GEP, as the investor benefits from large positive equity returns while being protected against large negative ones.

Over long periods, however, the variability in equity returns also may be variable—perhaps there will be several crash/recovery scenarios as in 1974 and 1975; perhaps there won't be any. It follows that the long-term return from a GEP may be as variable as the long-term return from the underlying shares.

7 Summary and Implications

7.1 Summary

The main findings of this paper are summarized below.

- Modeling equity returns as an independent, identically distributed, lognormal random variable appears to overestimate the variability in funds accumulated from the investment of annual contributions over relatively long periods.
• In the United Kingdom stochastic models that allow for tendency of the equity dividend yield to move toward a central value produce results that are more consistent with empirical studies. Even when such models are used, however, the variability in the retirement fund of a new entrant to a money purchase plan is large, and this variability reduces only slowly as the member approaches retirement.

• A strong case exists for the individual plan member to switch his or her fund to low risk assets in the period close to retirement. Although the case for switching becomes stronger as the member approaches retirement, the optimal time to do so depends also on the member's target fund and the prevailing equity dividend yield.

• A balanced investment strategy in which a significant proportion of the member's fund is invested in low risk assets throughout his or her period of service reduces both the expected value of the fund at retirement and its variability. Most of the reduction in variability occurs from leveling down—the reduction in the member's downside risk is not significant.

• Over a 40 year period a rolling one year guaranteed equity contract of simple design results in no significant reduction in the variability of the retirement fund, compared with investing purely in equities.

7.2 Implications for Pension Scheme Design

The arguments for investing long-term savings in ordinary shares are strong, both from the viewpoint of maximizing returns and hedging against wage and price inflation. Equities are a highly appropriate asset class for pension plans other than those that consist mainly of retired employees.

In money purchase pension plans, however, investment in equities results in pension benefits that depend excessively on whether the employee's period of service happens to coincide with a period of favorable or unfavorable investment experience. This makes it difficult for individual members to plan for retirement and results in inequity between different generations of employees.

Three strategies for reducing the investment risk associated with equities are examined in this paper:

• Switching to low risk assets close to retirement;
• Balanced investment strategies; and
• The use of derivative-based investment products.

Of these three, only the first is found to offer significant advantages to the individual member. Moreover, a switching strategy does not deal with the fundamental problem—by the time a member gets close to retirement, the damage already may have been done.

A great advantage of defined benefit plans is the implicit smoothing of variable investment returns for different generations of employees, brought by the use of a fixed benefit formula. A good example of such a formula is found in the United Kingdom State Scheme, where a pension equal to a fixed fraction of career-average revalued earnings is granted. The rate of revaluation applied to each year's earnings figure is the increase in an index of average earnings between the year concerned and the year prior to retirement. This example is similar to a money purchase plan in which a fixed percentage of salary is invested for each employee. The only difference is that a guaranteed rate of interest, equal to the increase in the average earnings index, is applied to each member's contributions.

Defined benefit plans have become less popular in both the United Kingdom and the United States. Aside from the costs of complying with increasingly complex legislation, employers have been less willing to accept the open-ended liability of such plans, which may require them to increase their contribution rate to cover a shortfall created by unfavorable experience.

A way must be found to apply the defined benefit principle to defined contribution plans. In some ways, this would be similar to a with-profits insurance fund, and a few United Kingdom pension plans are run on this basis. Unlike a with-profits fund, however, there should be explicit formulae for calculating the benefits paid, ideally based on career-average revalued earnings as used in the United Kingdom State Scheme. In addition, there would have to be rules for varying the rate of benefit accrual, should the experience of the plan deviate too far from the assumptions made by the actuary.

A defined contribution plan with a defined benefit scale that could be adjusted from time to time would represent a more equitable and secure form of pension provision than arrangements based purely on the money purchase principle.
References


Appendix: United Kingdom Equity Dividend Yields and Index Returns

Table A1
U.K. Equity Dividend Yields and Index Returns

<table>
<thead>
<tr>
<th>Year</th>
<th>EQIDY</th>
<th>REQIN</th>
<th>Year</th>
<th>EQIDY</th>
<th>REQIN</th>
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</thead>
<tbody>
<tr>
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<td>-</td>
<td>1938</td>
<td>5.5%</td>
<td>-</td>
</tr>
<tr>
<td>1920</td>
<td>9.5%</td>
<td>-</td>
<td>1939</td>
<td>5.4%</td>
<td>-</td>
</tr>
<tr>
<td>1921</td>
<td>8.9%</td>
<td>-</td>
<td>1940</td>
<td>6.3%</td>
<td>-</td>
</tr>
<tr>
<td>1922</td>
<td>6.0%</td>
<td>-</td>
<td>1941</td>
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</tr>
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<td>1923</td>
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<tr>
<td>1937</td>
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<td>-</td>
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</tbody>
</table>

EQIDY = Equity index dividend yield at year-end.
REQIN = Return on equity index net of increase in average earnings.

Sources: BZW Equity/Gilt Study. The abstract of statistics for social security benefits and contributions and the indices of retail prices and average earnings—Government Statistical Service.
Table A1 (continued)

<table>
<thead>
<tr>
<th>Year</th>
<th>EQIDY</th>
<th>REQIN</th>
<th>Year</th>
<th>EQIDY</th>
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<tbody>
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</tr>
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</tr>
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</tr>
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<td>3.4%</td>
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<td>1975</td>
<td>5.7%</td>
<td>109.1%</td>
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</table>

EQIDY = Equity index dividend yield at year-end.
REQIN = Return on equity index net of increase in average earnings.

Sources: BZW Equity/Gilt Study. The abstract of statistics for social security benefits and contributions and the indices of retail prices and average earnings—Government Statistical Service.