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Physics, Chapter 1: Fundamental Quantities

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Part One

MECHANICS

1

Fundamental Quantities

1-1 The Scope of Physics

Physics is a fundamental science dealing with matter and energy. By convention, the subject matter of physics has been divided into such topics as mechanics, heat, sound, light, and electricity. In addition to these general classifications, present-day physics includes atomic physics, nuclear physics, solid-state physics, chemical physics, biophysics, and many other subdivisions. It is impossible to include all aspects of physics in a single definition or paragraph, and to distinguish physics clearly from its nearest neighbors, the other physical sciences—astronomy, chemistry, and geology.

Like other scientists, the physicist studies nature, and, although the scientist is himself part of the world, he attempts to describe nature as it exists without his interference. Inspired by the conviction that nature is orderly and "rational," the physicist has sought to find that order and to express it as elegantly and as concisely as possible. Physics has attempted to study the least complex aspects of nature, or, if you will, the most fundamental, and has found that quantitative descriptions have been most fruitful. Physics has thus become a quantitative science involving precise measurements of certain quantities and the formulation of mathematical relationships among them.

Like other sciences, physics is an ahistorical subject. This does not mean that there is no history of physics but rather that, at a particular moment in time, the history of the development of physical ideas is not pertinent to the question of the accuracy of the description of nature. Because we can devote but little space to the history of physical ideas, the student will find that the time spent in consulting an encyclopedia on some of the topics studied in this text, and in reading biographies of physicists, will enrich his knowledge of the subject and may even stimulate his own inventiveness.

1-2 The Relationship of Physics to Engineering

Engineering has often been called applied physics, as though the physicist discovered all fundamental truth and the engineer merely applied it; as though the entire structure of engineering rested on the shoulders of physics. This is true only in the very limited sense that fundamental knowledge is called physics, and that the application of that knowledge is called engineering. In actual fact, there is so great an interplay between physics and engineering that activity in one field is stimulated by the needs and the successes of the other. The entire radio industry owes its birth to the researches of physicists such as Maxwell, Hertz, and J. J. Thomson during the latter half of the last century. Much of today's experimental physics would not be possible without the developments which have taken place in the technology of radio through the efforts of such men as Edison, De-Forest, Fleming, Marconi, and Armstrong.

1-3 The Fundamental Concepts of Mechanics

Mechanics deals with the behavior of various types of bodies such as *particles*, *rigid bodies*, *liquids*, and *gases*, when subjected to the action of *forces*. Under special conditions the bodies may be in *equilibrium* under the action of these forces; under other conditions the motions of these bodies will be changed or *accelerated*. The quantitative concepts used in mechanics can be classified into two groups, one known as the *fundamental concepts* consisting of three quantities, *length*, *time*, and *mass*, which form the bases of mechanics, and a second group known as the *derived concepts* consisting of the other quantities used in mechanics.

The difference between these two sets of quantities lies in the fact that each one of the derived concepts can be defined accurately in terms of the fundamental concepts. But the fundamental concepts of length, time, and mass cannot be defined in any such manner; they require special consideration. The method of treating each of these concepts is to set forth a series of rules for its measurement or to outline a series of experiments for determining its magnitude. Two steps are usually involved in each case: one, that of setting up a *standard* and, two, that of setting up rules for producing multiples or subdivisions of the standard. These steps will be discussed for each of the fundamental concepts.

The mechanics of particles and rigid bodies is subdivided under the more formal titles of *statics*, *kinematics*, and *dynamics*. *Statics is the study of bodies at rest and the conditions under which they remain at rest. Kinematics is the study of the motion of bodies without regard to the cause of that motion.* It is the language in whose terms motion is described. *Dynamics is the study of motion in relation to the forces which cause that motion.*

1-4 Length. Standards and Units

Most of us are familiar, in a general way, with the operations involved in measuring the length of an object or of measuring the distance between two points. Such a measurement involves the use of some measuring rod or tape whose length is known in terms of a standard of length. Only in comparatively modern times have standardized measuring rods been readily available. Such units as the length of a man's foot, the distance between his outstretched hands (the fathom), and the distance between his footprints (the pace) have always been available and are still in common use, but these are inadequate for recording the measurements of the physicist or the engineer. Today, *the legal standard of length in the United States is the meter*, defined as the distance between two lines marked on a special platinum-iridium bar known as the *standard meter* when that bar is at 0°C. This standard meter is deposited at the International Bureau of Weights and Measures at Sèvres, France. Accurate copies of this standard meter have been distributed to various national physical laboratories such as the National Bureau of Standards in Washington, D.C. As a result of subsequent developments in physics, it has been possible to measure the meter in terms of the wavelength of a particular kind of light. If this wavelength measurement, or a similar one, is adopted as a standard by international agreement, we would then have an indestructible standard of length.

In English-speaking countries other units of length find conventional usage. In the United States the yard is defined legally as 3,600/3,937 of the standard meter.

TABLE 1-1 PREFIXES USED IN THE METRIC SYSTEM

Latin Base			Greek Base		
deci	1/10	10^{-1}	deka	10	10^{1}
centi (c)	1/100	10^{-2}	hecto	100	10^{2}
milli (m)	1/1,000	10^{-3}	kilo (k)	1,000	10^{3}
micro (μ)	1/1,000,000	10^{-6}	mega (M)	1,000,000	10^{6}

Symbols for prefixes in common use are shown in parentheses.

While multiples and submultiples of the yard are defined in a rather complex way in the English system of units, in the metric system multiples and submultiples of the meter are based upon the decimal system. Latin prefixes are attached to the name of the standard to indicate smaller units, while Greek prefixes indicate larger units. The prefixes in common use are indicated in Table 1-1.

Some of the more common multiples and submultiples of the yard and meter are given in the appropriate columns of Table 1-2.

TABLE 1-2 UNITS OF LENGTH

English	Metric
3 feet = 1 yard	100 centimeters = 1 meter
36 inches = 1 yard	10 millimeters = 1 centimeter
5,280 feet = 1 mile	1,000 microns = 1 millimeter
1,000 mils = 1 inch	1,000 meters = 1 kilometer

It is often necessary to convert the value of a quantity expressed in one set of units to its value expressed in a different set of units. For this purpose the appropriate conversion factor must be used. To obtain this, it is necessary to refer to the original definitions of the standards of the two systems and then compute the conversion factor. Once done, these factors may be listed in a table for convenience and may be used with confidence. Table 1-3 lists some conversion factors for units of length. Others may be calculated when necessary.

TABLE 1-3 CONVERSION FACTORS

1 kilometer	= 0.6214 mile
1 meter	= 39.37 inches
1 foot	= 30.48 centimeters
1 inch	= 2.540 centimeters

Illustrative Example. Convert miles per hour to feet per second. The conversion from one set of units to another is executed by a series of multiplications by unity. Since multiplication by the pure number, 1, cannot affect the truth of an equation, the resulting calculation, correctly performed, must also be a true equality. Note that the equation 5,280 ft = 1 mi can be written as

$$\frac{5{,}280 \text{ ft}}{1 \text{ mi}} = 1;$$

similarly,

$$\frac{1 \text{ hr}}{3{,}600 \text{ sec}} = 1.$$

We can thus write

$$1 \frac{\text{mi}}{\text{hr}} = 1 \frac{\text{mi}}{\text{hr}} \times \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{3{,}600 \text{ sec}}$$

$$= \frac{5{,}280}{3{,}600} \frac{\text{ft}}{\text{sec}},$$

or

$$1 \frac{\text{mi}}{\text{hr}} = 1.467 \frac{\text{ft}}{\text{sec}}.$$

All conversions from one set of units to another may be accomplished in this way.

1-5 Angular Measure

While many systems of linear measure are in general use, only two systems of angular measurement are ordinarily encountered. In the degree-minute-second system, a circle is first divided into 360 equal parts called degrees. Each degree is divided into 60 equal parts called minutes, and each minute is divided into 60 equal parts called seconds. This method of subdivision comes from the Babylonians, who used a number system based on 60; the terminology derives from the Latin for "first small part" and "second small part" (small = minute). The other system of angular measurement is based on the radian as the unit of angle, where a radian is defined as the angle subtended at the center of a circle by an arc whose length is equal to the radius. To determine an angle in radian measure, we draw a circle of arbitrary size centered at the intersection of the sides of the angle, as shown in Figure 1-1. If r is the radius of the circle and s is the length of the inscribed arc, then the angle ϕ (Greek letter phi) is determined by dividing the length of the arc s by the length of the radius r; thus

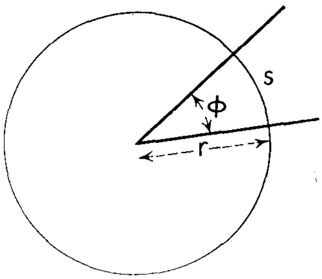

Fig. 1-1

$$\phi = \frac{s}{r} \cdot \tag{1-1}$$

Note that neither the degree nor the radian has any dimensions. The radian is a pure number. It is simply a length divided by length. The degree, minute, or second is likewise a pure number, simply indicating the fraction of a full circle represented. The designation "radian" or "degree" is merely a description of the manner in which the measurement was made.

To convert from degrees to radians, we observe that a complete circle may be represented as 360° or as 2π radians, so that

$$1 \text{ radian} = \frac{360°}{2\pi} = 57.3°.$$

Some interesting examples of the measurement of distances through the measurement of angles are the determination of the radius of the earth, the diameter of the earth's orbit, and the distance of some of the nearer stars. For example, to measure the radius of the earth, simultaneous sightings of a distant star may be made from two points A and B on the earth's surface a known distance s apart (see Figure 1-2). Because of the great distance of the star from the earth, the lines of sight of the star from the two locations may be assumed to be parallel. We assume the plumb bob to point to the center of the earth (although this is not strictly true). The

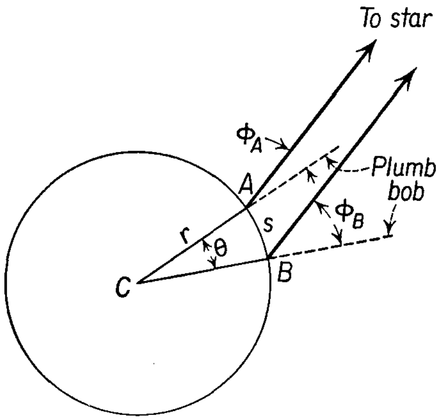

Fig. 1-2 Method of determining the radius of the earth.

angle between the line of sight and the plumb bob is measured at each of these two points; let us call these angles ϕ_A and ϕ_B, respectively. From geometry, the angle θ (theta) subtended at the center of the earth is given by the difference between the angles ϕ_B and ϕ_A. Thus

$$\theta = \phi_B - \phi_A.$$

Now

$$\theta = \frac{s}{r},$$

or

$$r = \frac{s}{\theta},$$

from which

$$r = \frac{s}{\phi_B - \phi_A}.$$

Since the quantities on the right-hand side of the equation are measurable, the radius of the earth can be determined. As the result of a great many such measurements, the earth is known to be an oblate spheroid, a slightly squashed-down sphere, of a radius of approximately 4,000 miles.

1-6 The Concept of Time

Just as man has a built-in concept of length, so he has a built-in concept of time. The natural cycles of the day, the lunar month, and the year have always provided man with measures of long time intervals, while the pulse has provided something of a measure of smaller time intervals. It is said that a musician's sense of tempo is based upon his pulse beat, and that variations in the tempo of performance of a symphony orchestra have been observed to accompany changes in the conductor's pulse.

The measure of time is still based upon the period of the earth's rotation on its axis and on the period of its revolution about the sun. For this reason the fundamental measurement of time has been the domain of the astronomer. Because of the very practical importance of accurate knowledge of time in ship navigation in the determination of longitude, the accurate measurement of time in the United States is in the hands of the U.S. Naval Observatory. *The standard time interval used in scientific work is the mean solar day.* This standard makes use of the apparent motion of the sun with respect to an observer on the earth; this apparent motion is due to the rotation of the earth on its axis. The instant at which the sun passes the meridian (directly overhead) at the observer's place is called *local noon.* The solar day is the time between two successive noons. Careful measurements have shown that solar days differ slightly at different times of the year, owing to the earth's position in its orbit and the speed and direction of its motion. The mean solar day is an average value taken over the year.

The *unit* of time universally used is the *second;* this is $1/(24 \times 60 \times 60) = 1/86{,}400$ of the mean solar day. The instrument for measuring time is the clock. The astronomical clock makes use of the periodic motion of a pendulum or of the periodic motion of a balance wheel connected to a fine spring. The clock is used to subdivide the day into submultiples.

Basic principles of physics such as the principles of the conservation of energy and of angular momentum, which will be studied in detail later in this text, indicate that the period of rotation of a rigid earth should be constant. The earth loses about a second a year owing to mechanical energy losses in the motion of the tides, and so on. The shape of the earth is continually undergoing slight changes as a result of volcanic action, earthquakes, and the action of the tides. Consequently, although the average second is well defined, two successive seconds of time may not correspond to the same change in the earth's rotational position. Physicists have sought to establish a means of measuring time which does not depend on the motion of the earth and have only recently succeeded in building a clock which bases its measurement of time on the internal vibrations of the ammonia molecule. This clock is said to be accurate to 1 part in 24 billion, or to about 1 sec in 300 years and can, perhaps, be used to determine the irregularities in the motion of the earth.

1-7 The Concept of Mass

Most of us have some idea of the meaning of the term *mass of a body,* but unfortunately this is generally confused with the term *weight.* Our muscles provide us with an intuitive conception of weight, and it is often difficult for us to make the required distinction between weight and mass. The concept of mass is related to the "amount" of matter present in a particular object, while weight refers to the force of the earth's attraction on that object. While the weight may vary from place to place on the earth, and certainly will vary if the object is removed to the moon, the mass remains constant, except at speeds comparable to the speed of light, and is independent of position in the universe.

A real understanding of the concept of mass can come only after a study of dynamics; for the present we shall confine our discussion to a statement of the standard of mass, the method of measuring mass, and the units commonly used.

The standard of mass is a *kilogram* (abbreviated kg). This is the mass of a certain piece of platinum kept at the International Bureau of Weights and Measures at Sèvres, France. The instrument used in comparing masses is the equal-arm balance. The two bodies whose masses are to be compared are placed in the pans of this instrument and, if a balance is obtained, are said to have equal masses. Two accurate copies of the standard kilogram are deposited at the National Bureau of Standards.

The kilogram is not only the standard of mass, but it is widely used as the *unit* of mass in science and engineering. The system of units based upon the *meter* as a unit of length, the *kilogram* as the unit of mass, and the *second* as the unit of time is called the *mks system.* A second metric system of units is based upon the *centimeter* (cm) as the unit of length, the *gram* (gm) as the unit of mass, and the *second* as the unit of time. This system is known as the *cgs system.*

Other units of mass are commonly used. Thus the *pound mass* (lb mass) is legally defined as $\frac{1}{2.20462}$ kg. The *British gravitational* system of units utilizes a unit of mass called the *slug,* which cannot be properly defined until we study dynamics. In this system of units the unit of length is the *foot* (ft), and the unit of time is the *second.*

1-8 Force

The concept of force is best derived formally through the definitions of length, mass, and time, and the application of Newton's laws of motion. For most purposes the common spring balance is a convenient device for the measurement of force. Although a precise definition of force again must await the study of dynamics, the units of force in the several systems may be named now. The unit of force in the mks system is called the *newton* (nt). The unit of force in the cgs system is called the *dyne.* The unit of force in the British gravitational system is the weight of a pound mass at a place where the acceleration of gravity has the "standard" value of 32.17398 ft/sec^2; we shall call it a *pound force* (lb f).

1-9 Position

In many physical problems it is necessary to describe the position of an object, and this is most conveniently done by coordinate systems such as those used in analytic geometry. A coordinate system has an origin and a net of fixed reference lines. The location of a point on a two-dimensional plane surface is often specified in terms of a rectangular (Cartesian) coordinate system by giving the x and y coordinates of the point. Where three dimensions are involved, a rectangular coordinate system utilizing three mutually perpendicular axes is normally used; the position of a point is specified by its x, y, and z coordinates.

Equations relating the x and y coordinates are used to describe a plane figure. For example, $y = mx + b$ is the equation of a straight line in the xy plane, where m is the slope of the line, and b is the y intercept. Equations relating x, y, and z coordinates are used to describe a surface in space. Thus $x^2 + y^2 + z^2 = r^2$ is the surface of a sphere of radius r whose center is at the origin.

The position of a point in a plane may also be specified through the use of the polar coordinates r and θ. The length of the line from the origin of the coordinates O to the point P is the coordinate r, while the coordinate θ is the angle between the x axis and the line OP, measured in the counterclockwise direction.

1-10 Displacement. Scalar and Vector Quantities

If an object initially at a position P_1 with coordinates x_1, y_1 is displaced to a point P_2 with coordinates x_2, y_2, its *displacement* may be described in terms of the distance it has been moved parallel to the x axis and the distance it has moved parallel to the y axis. We say that the x displacement is the final x coordinate minus the initial x coordinate. Thus if in Figure 1-3 the point P_1 is at $(x, y) = (10, -5)$ and P_2 is at $(5, 10)$, the x displacement is $5 - 10 = -5$; that is, the object has been displaced 5 units in the direction of the negative x axis. Similarly, the y displacement is given as $10 - (-5) = +15$; that is, the object has been displaced 15 units in the positive y direction. The actual displacement from P_1 to P_2 is specified by the arrow in Figure 1-3 and designated by the boldface letter $\mathbf{r}$; the x and y components of the displacements are shown in dotted lines. The magnitude of the displacement is the length of the arrow, and its direction is from P_1 to P_2. To specify completely the displacement of an object, both the magnitude of the displacement and its direction must be given. One way of specifying a displacement is to give its x and y components. A second way to describe the displacement from P_1, the initial point, to P_2, the final point, is to imagine a new coordinate system with axes parallel to x and y whose origin is at P_1. In terms of the new coordinate system, the displacement is described by the polar coordinates of the point P_2 which give both the magnitude r and the direction θ along which the displacement is to be made.

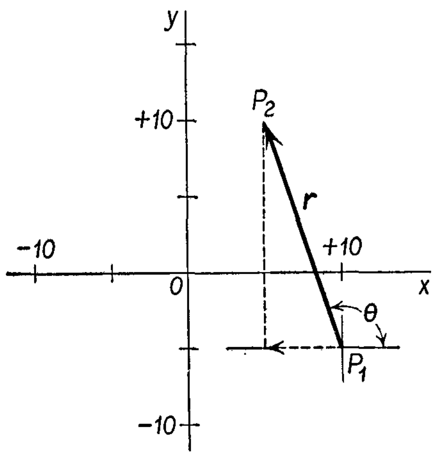

Fig. 1-3 Displacement.

We shall be dealing with two types of quantities in physics: *scalar* quantities, of which length, mass, and time are examples; and *vector* quantities, of which displacement and force are examples.

A scalar quantity has magnitude only and is specified by a number together with the appropriate unit, for example, a length of 2 m or 78.74 in. When describing a vector quantity, we must specify its *direction* as well as its magnitude. The method for doing this is suggested by our study of displacements; a vector quantity is represented by a *vector* which consists of an arrow drawn in the direction of the vector quantity with the head of the arrow showing the *sense* of the vector, and the length of the arrow showing the magnitude of the vector drawn to some arbitrary scale. For example, a displacement of 6 mi 30° north of east can be represented by an arrow 1 in. long, as shown in Figure 1-4, with its line at an angle of 30° to the conventional east direction. The scale of this drawing is 1 in. = 6 mi. Other vector quantities can be represented in a similar manner with a suitable choice of scales.

1-11 Addition of Vectors

The mathematical operations of addition, subtraction, multiplication, and division that are familiar to all of us have been developed for scalar quantities. We must now proceed to develop a set of mathematical operations suitable for vector quantities. In developing these operations we shall be guided by the results of physical processes. For example, the process of addition of vectors can be based upon the process involved in the addition of displacements. We can then extend this to the addition of other vector quantities, checking the results to see whether the method is applicable.

Suppose that we consider the displacement of a particle from point P_0 to point P_1, as shown in Figure 1-5; this to be followed by the displacement from P_1 to P_2. The same result could have been accomplished by the single displacement from P_0 to P_2. Thus the displacement represented by the vector $\mathbf{P_0P_2}$ is equal to the sum of the displacements $\mathbf{P_0P_1}$ and $\mathbf{P_1P_2}$. We thus have a rule for the addition of two vectors **A** and **B**. This rule may be stated as follows: Starting at any arbitrary point, and using any convenient scale, draw vector **A** equal and parallel to itself and pointing in its direction.

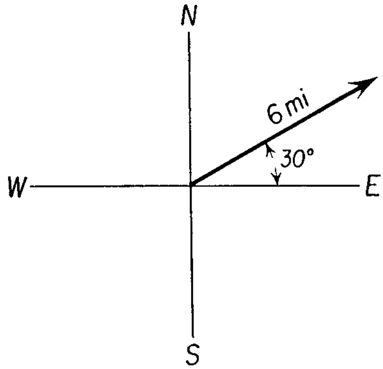

Fig. 1-4

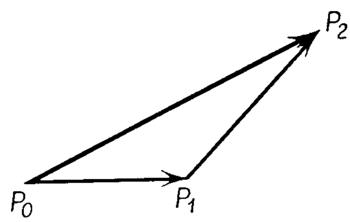

Fig. 1-5 Addition of displacements.

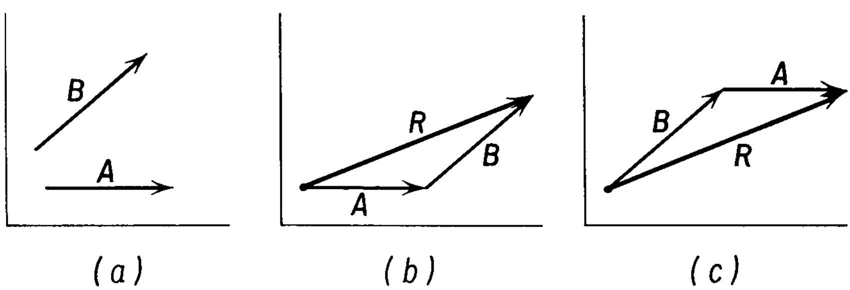

Fig. 1-6 Vector addition.

At the head of vector **A**, start the tail of vector **B** and draw **B** equal and parallel to itself and in its direction. To find the sum of vector **A** and vector **B**, draw a vector from the origin, or tail, of **A** to the end, or head, of **B**. This vector **R** is the sum of **A** and **B**, as shown in Figure 1-6. It is also called the *resultant* of **A** and **B**. Thus

$$\mathbf{R} = \mathbf{A} + \mathbf{B}.$$

The order in which this addition is performed is immaterial, as shown in Figure 1-6(c), in which **B** is drawn first and **A** is added to it. Thus

$$\mathbf{R} = \mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}. \tag{1-2}$$

It may be remarked here that identical results could have been obtained by a geometrical method by starting with **A** and **B** at a common

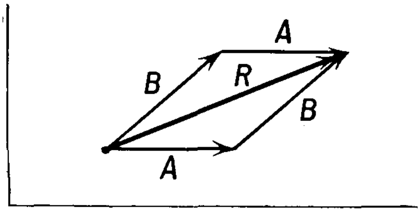

Fig. 1-7 Parallelogram method for the addition of two vectors.

point, as shown in Figure 1-7, and drawing a parallelogram with **A** and **B** as sides; the diagonal of the parallelogram is the resultant **R**. It will be noted that each of the earlier figures is one half of the parallelogram with **R** common to each triangle.

The above method of adding two vectors is called the *parallelogram method.* It can be extended to any number of vectors by first determining the resultant of two of the vectors, then adding a third vector to this resultant, and so forth.

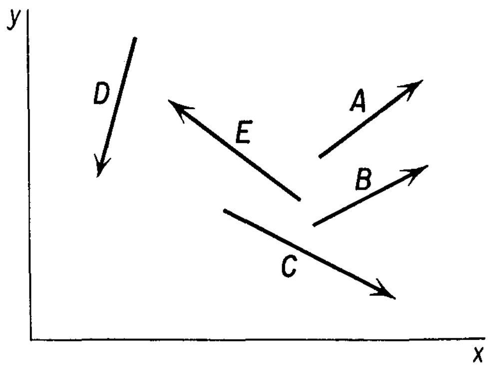

Fig. 1-8

In the preceding discussion we have used **boldface type** to represent *vector* quantities. This procedure will be followed throughout the text.

Suppose that the arrows of Figure 1-8 represent a set of instructions for the displacement of an object from the point P_0. These displacements can be applied to the object in any sequence with the same result. In Figure 1-9 the vector **A** has been transferred to the point P_0. The head of the arrow at the point P_1 represents the first new position of the object. To the point P_1 as a new origin, we apply the vector **B**, and then successively add vectors **C**, **D**, and **E** in head-to-tail fashion to find the final position of the object at the point P_5. A single displacement given by the vector

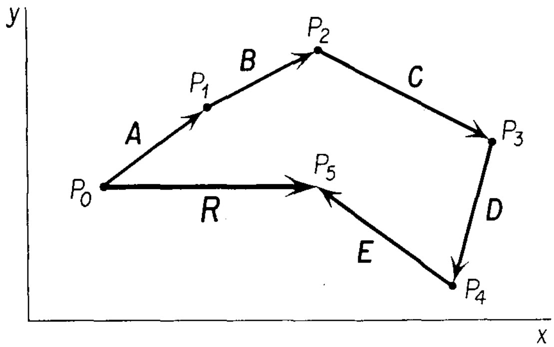

Fig. 1-9 Polygon method for the addition of vectors.

R, Figure 1-9, will also locate the object at the point P_5. The vector **R** is therefore completely equivalent to the set of operations **A**, **B**, **C**, **D**, and **E**. The resultant **R** is said to be the sum of the vectors. This result may be expressed in equation form as

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E}. \tag{1-3}$$

The procedure just described is called the *polygon method for the addition of vectors*, for the resulting figure is a closed polygon. *To add a number of vectors by the polygon method, the vectors are redrawn in sequence from some arbitrary origin, placing the vectors head to tail. The sum of the several vectors is the single vector drawn from the tail of the first vector to the head of the last vector.* Just as in the case of two vectors, the order in which vectors are added makes no difference in the final result.

When summing two vectors by the triangle method, the resulting figure is a triangle, made up of the two vectors and their resultant. It is often convenient to apply the law of sines or the law of cosines to find both the magnitude and the direction of the resultant.

Illustrative Example. Find the sum of vectors **A** and **B** in Figure 1-10. Vectors **A** and **B** are redrawn, connected head to tail, as shown in the figure. Vector **R** represents their sum. Since A, B, and θ are known, the magnitude of **R** may be determined, using the law of cosines. Thus

$$R^2 = A^2 + B^2 - 2AB\cos\theta.$$

Now that the magnitude of R has been found, the angle ϕ which describes the orientation of **R** with respect to the x axis may be found from a second application of the law of cosines, or from the law of sines:

$$\frac{\sin \phi}{B} = \frac{\sin \theta}{R},$$

$$\sin \phi = \frac{B \sin \theta}{R},$$

from which

$$\phi = \arcsin \frac{B \sin \theta}{R}.$$

In this example, as elsewhere in the text, the italic letters A, B, and R have been used to represent the magnitude of the vectors **A**, **B**, and **R**.

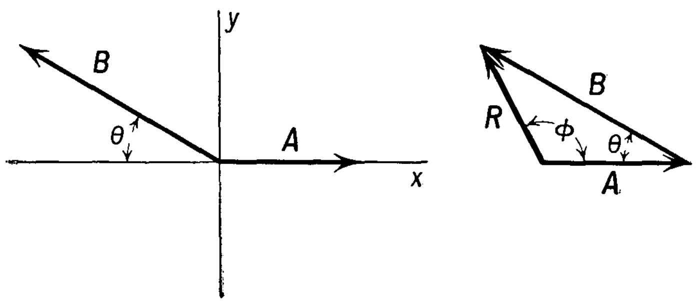

Fig. 1-10

1-12 Subtraction of Vectors

The subtraction of vectors may be accomplished by the rules developed for the addition of vectors once the meaning of a negative vector has been established, for we shall adhere to the notation developed in algebra that

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}). \tag{1-4}$$

The negative of a vector is a vector of the same length but pointing in the opposite direction. The sequence of operations performed in subtracting a vector **B** from a vector **A** is described in Figure 1-11(a). First the vector **B** is reversed in direction to become the vector $-\mathbf{B}$. Then the vector $-\mathbf{B}$ is added to the vector **A** by the triangle method. Finally the resultant of the operation $\mathbf{A} - \mathbf{B}$ is displayed. In Figure 1-11(b) the sum of the same two vectors is displayed. Just as in algebra, the difference of two quantities differs from their sum.

Because the addition of two vectors requires consideration of direction as well as magnitude, the algebra of vectors is distinctly different from the algebra of scalars, leading to some superficially strange results. For example, a vector of length 5 added to a vector of length 3 does not necessarily yield a vector of length 8. The addition may lead to a vector of any length between 2 and 8, depending on the orientations of the two vectors. Similarly, a vector of length 3 subtracted from a vector of length 5 does not

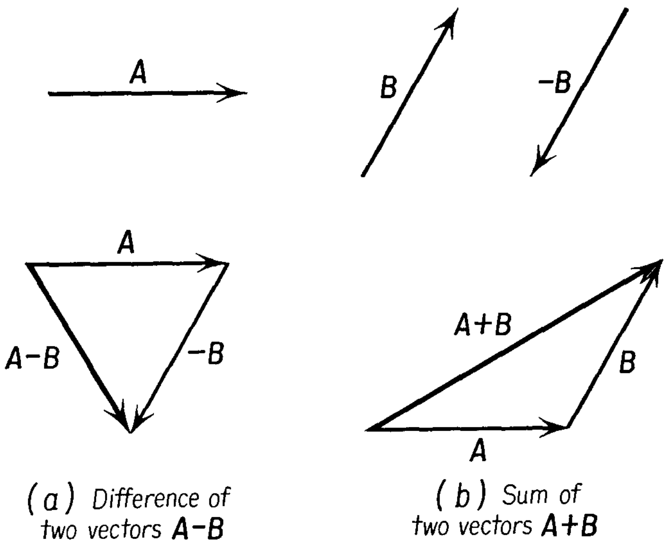

Fig. 1-11 (a) Difference of two vectors, **A** − **B**. (b) Sum of the same two vectors, **A** + **B**.

necessarily yield a vector of length 2. Again, the subtraction may yield a vector of any length between 2 and 8, depending upon the orientations of the two vectors.

1-13 Resolution of Vectors; Components

Any number of vectors which, added together, form a resultant **R** may be considered as *components* of **R**. Thus a vector may be resolved into any desired number of components.

Most frequently, the components which are sought are those which are mutually independent, with one component in some specified direction. Any two vectors which are perpendicular to each other are mutually independent. For example, if we refer the displacement to an x-y Cartesian coordinate system, a displacement of a body in the y direction produces no change in its x coordinate; similarly, a displacement in the x direction produces no change in the y coordinate of the body. Thus, if we are interested in determining two components of a vector **A** one of which is parallel to the x axis, the other component must be parallel to the y axis. Referring to Figure 1-12, if the vector **A** makes an angle θ with the x axis, its x component is of magnitude

$$A_x = A \cos \theta, \tag{1-5a}$$

and its y component is of magnitude

$$A_y = A \sin \theta. \tag{1-5b}$$

The magnitude of A can be found in terms of its components in a very simple way, since

$$A = (A_x^2 + A_y^2)^{1/2}. \tag{1-6}$$

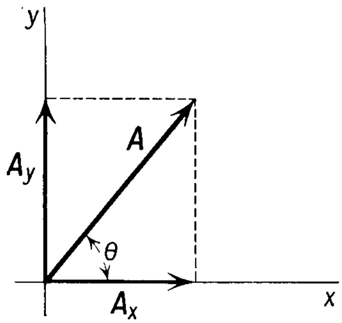

Fig. 1-12 Resolution of a vector into two rectangular components.

The concept of rectangular components of a vector can be utilized for an analytical method of adding vectors. Suppose that we wish to obtain the resultant of a number of coplanar vectors, that is, vectors all in one plane, the x-y plane; these may be written as

$$\mathbf{A} + \mathbf{B} + \mathbf{C} + \cdots = \mathbf{R}. \tag{1-7}$$

Let us call the x components of these vectors $\mathbf{A}_x$, $\mathbf{B}_x$, $\mathbf{C}_x$, . . . respectively, and similarly for the y components $\mathbf{A}_y$, $\mathbf{B}_y$, $\mathbf{C}_y$ . . . . Let θ_1 be the angle between **A** and the x axis, θ_2 the angle between **B** and the x axis, and so forth. Then

$$A_x = A \cos \theta_1, \tag{1-8a}$$

and

$$A_y = A \sin \theta_1. \tag{1-8b}$$

In a similar manner

$$B_x = B \cos \theta_2,$$

$$B_y = B \sin \theta_2, \quad \text{and so on.}$$

The x component of the resultant R_x is

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x + \mathbf{C}_x + \cdots, \tag{1-9}$$

and similarly for the y component of the resultant.

From a knowledge of the components of the resultant **R**, the magnitude of the resultant may be obtained from the Pythagorean theorem.

$$R = (R_x^2 + R_y^2)^{1/2}. \tag{1-10}$$

The angle θ that $\mathbf{R}$ makes with the x axis is given by

$$\cos\theta = \frac{R_x}{R}, \tag{1-11a}$$

or

$$\sin\theta = \frac{R_y}{R}, \tag{1-11b}$$

or

$$\tan\theta = \frac{R_y}{R_x}. \tag{1-11c}$$

These results may be easily extended to three dimensions.

Illustrative Example. Find the resultant of the vectors **A**, **B**, **C**, in the x-y plane, as shown in Figure 1-13. For purposes of illustration, we shall consider

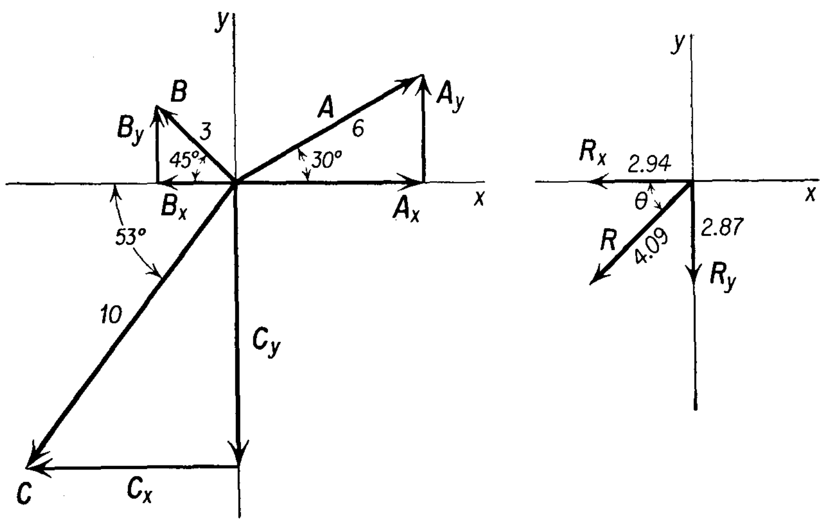

Fig. 1-13

that these vectors represent displacements and that all lengths are given in feet. The x component of the resultant is

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x + \mathbf{C}_x, \tag{1-9}$$

so that

$$R_x = A\cos 30° - B\cos 45° - C\cos 53°$$
$$= 6 \times 0.866 - 3 \times 0.707 - 10 \times 0.602,$$

which yields

$$R_x = -2.94 \text{ ft}.$$

The y component of the resultant is

$$\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y + \mathbf{C}_y,$$

so that

$$R_y = 6 \sin 30° + 3 \sin 45° - 10 \sin 53°$$
$$= 6 \times 0.500 + 3 \times 0.707 - 10 \times 0.799,$$

which yields

$$R_y = -2.87 \text{ ft.}$$

The magnitude of the resultant is given by

$$R = (R_x^2 + R_y^2)^{1/2} \qquad (1\text{-}10)$$
$$= [(-2.94)^2 + (-2.87)^2]^{1/2},$$

so that

$$R = 4.09 \text{ ft.}$$

The angle that **R** makes with the x axis, denoted by (R, x), is given by

$$\cos (R, x) = \cos \theta = \frac{2.94}{4.09} = 0.719,$$

$$\theta = 46° = 0.80 \text{ radian.}$$

Since **R** lies in the third quadrant, we state the result as: The angle made by the resultant with the positive x axis is 226°, or 3.94 radians.

1-14 Other Vector Operations

Methods for obtaining the sum and difference of vectors have already been displayed. One additional operation should be discussed here. A vector may be multiplied by a scalar $+a$, and the result is still a vector. The new vector points in the same direction as the old but has a times the magnitude of the old vector. The result of multiplication of a vector by -1 is a vector of the original length but pointing in the opposite direction. Combining these two definitions, it is now possible to multiply a vector by a negative scalar. The definition further permits the division of a vector by a scalar, for division of a vector by a scalar b is equivalent to multiplication by the scalar $1/b$. The result of this operation is always a vector quantity. Thus velocity is a vector quantity, for it is the result of dividing displacement, a vector, by time, a scalar.

Problems

1-1. Check the conversion factors given in Table 1-3, starting with the legal definition of the yard.

1-2. A ship at sea receives radio signals from two radio transmitters A and B 125 mi apart, one due south of the other. The direction finder shows that transmitter A is 30° south of east, while transmitter B is due east. How far is the ship from each transmitter?

1-3. What are the polar coordinates of a point having the following rectangular coordinates?

(a) $x = 3, y = 4$ (b) $(x, y) = (5, -5)$ (c) $(x, y) = (-12, 6)$

1-4. What are the rectangular coordinates of a point having the following plane polar coordinates?

(a) $(r, \theta) = (6, 30°)$ (b) $(r, \theta) = (10, 53°)$

(c) $(r, \theta) = (12, 0.5 \text{ radians})$ (d) $(r, \theta) = (-9, 3.4 \text{ radians})$

1-5. A displacement vector lying in the x-y plane has magnitude 5 m and makes an angle of 60° with the x axis. Find its x and y components.

1-6. A displacement vector has an x component of +3 ft and a y component of +4 ft. Find the magnitude and direction of the vector.

1-7. A displacement vector has an x component of +3 m, a y component of +5 m, and a z component of +9 m. Find the magnitude of the resultant, and the angles made with the x, y, and z axes.

1-8. Find the resultant of two vectors, the first of length 7 ft making an angle of 25° with the positive x axis, the second of length 15 ft making an angle of 75° with the same axis, all angles being measured counterclockwise: (a) using the method of x and y components; (b) using the law of sines, the law of cosines, or both.

1-9. The coordinates (x, y) of the head and tail of two vectors are:

Vector	Head Coordinates	Tail Coordinates
A	(0, 3)	(3, 7)
B	(5, 2)	(9, −3)

Find the sum $\mathbf{A} + \mathbf{B}$ and the difference $\mathbf{A} - \mathbf{B}$ of these vectors (a) using the method of components and (b) using the law of sines, the law of cosines, or both.

1-10. Find the components of the resultant of the following displacements: 2 ft north, 6 ft southwest, 5 ft south, 7 ft northwest.

1-11. Three coplanar vectors **A**, **B**, **C** are drawn radially outward from the origin of coordinates; the polar coordinates (r, θ) of their heads are **A**: (5, 30°), **B**: (7, 150°), and **C**: (4, 240°). (a) Find their resultant. (b) Find the magnitude and direction of **R** where

$$\mathbf{R} = 3\mathbf{A} - 2\mathbf{B} + \frac{\mathbf{C}}{2}.$$

1-12. Derive the law of cosines by use of the component method for the addition of vectors.

1-13. Derive the law of sines by making use of vector concepts.

1-14. A vector of length A is in the positive x direction, and a vector of length B is in the positive y direction. Their sum is a vector of length 5 which makes an angle of 37° with the positive x axis. Find the magnitudes of **A** and **B**.

1-15. A vector A of length 4 in the positive x direction is added to a vector B to yield a resultant of length 7. The resultant is in the first quadrant and makes an angle of 45° with the x axis. Find the magnitude and direction of the vector **B**.